Title: Introduction to Analytic Number Theory

Julio Andrade*

Institut des Hautes Études Scientifiques (IHÉS)
Le Bois-Marie 35, route de Chartres - 91440, Bures-sur-Yvette - France
E-mail: j.c.andrade@ihes.fr,   j.c.andrade.math@gmail.com,
`http://www.ihes.fr/~jcandrade/`

SUMMARY

9 de janeiro de 2014

1   Level

This is an *introductory* level minicourse.

2   Prerequisites

The only prerequisite assumed is a good course in Calculus and Complex Analysis. It is desirable that the students have some familiarity with elementary number theory, but this requirement is not mandatory for the proper understanding of this minicourse.

3   Syllabus

- **Aims.**

  One of the goals of this minicourse is to nurture the intrinsic interest that many young mathematics students seem to have in number theory. And it is the main aim of this minicourse to introduce to the interested student the techniques, results and terminology of analytic number theory.

  This minicourse is an introduction to classical results in analytic number theory, presenting fundamental theorems with detailed proofs and highlighting the tight connections between them.

  Topics covered might include: the prime number theorem, Dirichlet L-functions, zero-free regions, sieve methods, representation by quadratic forms, and Gauss sums. Including the use of zeta functions and L-functions to prove distribution results concerning prime numbers (e.g., the prime number theorem in arithmetic progressions).

- **Audience.** The minicourse has as target audience Masters and Ph.D. students in Mathematics and Physics. But it can be also attended by advanced undergraduate students.

- **Contents.**

  1. **The Fundamental Theorem of Arithmetic.**
  2. **Arithmetic Functions and Dirichlet Multiplication.**
  3. **Mean Values of Arithmetic Functions.**
  4. **Characters and Dirichlet Characters.**
  5. **Gauss Sums, Primitive Roots and Quadratic Residues.**
  6. **Primes in Arithmetic Progression.**
  7. **Dirichlet Series and Euler Products.**
  8. **Prime Number Theorem.**
  9. **The Riemann Zeta Function $\zeta(s)$ and L–functions.**
  10. **Revisiting the Prime Number Theorem.**

Keywords: *Analytic Number Theory, Prime Numbers, Prime Number Theorem, Riemann Zeta Function, Arithmetic Functions, Dirichlet Series and Dirichlet Characters.*

Minicourse Subjects: Number Theory, Analysis, Complex Analysis and Algebra.

Referências

[1] T.M. Apostol, *Introduction to Analytic Number Theory*, Undergraduate texts in Mathematics, Springer–Verlag, 1976.

[2] R. Ayoub, *An Introduction to the Analytic Theory of Numbers*, American Mathematical Society (AMS), 1963.

[3] H. Davenport, *Multiplicative Number Theory.* Graduate Texts in Mathematics (GTM), Springer–Verlag GTM, 2000.

[4] G.H. Hardy and E.M. Wright, *An Introduction to the Theory of Numbers (with an appendix by Andrew Wiles and Roger Heath–Brown)*, Oxford University Press, 2008.

[5] H. Iwaniec and E. Kowalski, *Analytic Number Theory*, Colloquium Publications, Vol. 53, American Mathematical Society (AMS), 2004.

[6] G.J.O. Jameson, *The Prime Number Theorem.* Cambridge University Press, LMS Student texts vol 53, 2003.

[7] F.B. Martinez, C.G. Moreira, N. Saldanha e E. Tengan, *Teoria dos Números: um passeio com primos e outros números familiares pelo mundo inteiro*, Coleção Projeto Euclides (IMPA), 2013.

[8] H.L. Montgomery and R.C. Vaughan, *Multiplicative number theory: I. Classical theory.* Cambridge University Press, Cambridge Studies in Advanced Math. vol. 97, 2007.

[9] J.P.O. Santos, *Introdução à Teoria dos Números*, Coleção Matemática Universitária (IMPA), 2012.

[10] J. Stopple, *A Primer of Analytic Number Theory*, Cambridge University Press, 2003.