Collaborative Action Research: Teaching of Multiplication and Division in the Second Grade

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Abstract

This paper discusses the impact of action research methodology used in the teaching and learning process and professional teacher development. In this study are including 58 students of three second grade classes, 3 teachers of those classes and a university professor. Aiming at using a different approach in their teaching of multiplication and division in the second grade, all three teachers agreed to cooperate and jointly plan the learning activities, to observe systematically their students and to reflect on the outcomes. This way of research doing in their classes enabled them to 'act' effectively in designing an action plan appropriate to students' achievement level. This research was carried out in the period of February 18 to May 31 incorporating several different methods, such as classroom observation, interviewing and worksheets.

Keywords: *Action research; multiplication; division; sharing/partitive; grouping/quotative*

Introduction

The four fundamental operations – addition, subtraction, multiplication and division, and their relations are basic mathematical concepts to be taught at primary education level. Acquisition of those four concepts and their relations enables students to develop their understanding for 'numbers and calculating strategies' as well as associating them with daily life problems. In the curriculum of Kosova's primary education (MASHT, 2004), multiplication and division are presented for the first time in the second grade. According to this curriculum, second graders learn the meaning of multiplication as *repeated addition*, and division as an inverse operation of multiplication (*finding a factor, when the product and the other factor are known)*. As in most traditionally programs, these concepts taught separately with multiplication preceding division. The teaching is very similar in most classes. Each teacher is quite rigorously based on school math textbooks. They use them for preparing the lesson, class organization and as resource for students work. Traditionally, for the first 10 weeks of the second term, in all schools, students learn the 'multiplication table' and after that they start with division (as inverse of multiplication).

Lirika, is a primary school teacher at "Mileniumi i Tretë", which was listed by an external evaluation as achieving the best results in mathematics, compared to other schools within the same municipality. This evaluation was carried out in all fifth grade[10] classes. The evaluation also concluded that there are still some obstacles related to the application of multiplication and division operations by students. Lirika was concerned with these results and had her dilemmas: Should multiplication and division be taught separately and does memorizing the table of multiplication help children understand division concepts? Are the examples in the textbook related with different division situations? Is it possible for students to understand the division concepts only as the inverse of multiplication? How can I better teach these concepts? Thus, Lirika carefully analyzed the existing curriculum and relevant practices in other countries, including the literature related to math teaching at primary education levels. She

[10] MASHT (NjVS-Testi i kl.V - 2009)

found that, there are many arguments that multiplication and division are closely connected to the lesson plan, and they should be taught jointly (Greer, 1992; Carpenter et.al., 1999; Van de Wale, 2004). Mulligan and Michelmore (1997), in a longitudinal study of Grade 2 and 3 students, found that students possessed several *intuitive models* for division when faced with word problems. They defined these models as "internal mental structures corresponding to a class of calculation strategies" (p. 325). So, students should solve problems using their strategies and should be able to explain what they did with numbers, words or drawings.

Firstly, Lirika decided to consult two of her teacher colleagues (Miranda and Shqiponja), who work at the same school as she does, then the school principal, and afterwards she invited the instructor (author) from the Faculty of Education to discuss her dilemmas. After some meetings, an action plan was designed, and a decision was made to carry out an action research related to the teaching of multiplication and division concepts.

The aim of this study is the assessing of the student's ways of experiencing word problems in different situations. Also, this study assesses how students make a conceptual connection between multiplication and division and develop the reasoning skills. The study was carried out within the action research methodology.

Literature review

What is Action Research?

"Action research is any systematic inquiry conducted by teacher researchers to gather information about the ways that their particular school operates how they teach, and how well their students learn. The information is gathered with the goals of gaining insight, developing reflective practice, effecting positive changes in the school environment and on educational practices in general, and improving student outcomes" (Mills, 2003, p.4). Often an action research is considered as a collaborative activity and focuses on the co-creation of knowledge about practices. It is an appropriate methodology since it enables teachers to get involved in joint practical activities, to make changes to their practice and to examine their own teaching and students' learning through descriptive reporting, purposeful conversation, colleagual sharing, and critical reflection for the purpose of improving classroom practice (Miller and Pine, 1990; Wilson, 2009; Mcniff and Whitehead (2010); Koshy, 2010). According to Kemmis and Taggart (2000), action research is represented through spiral cycles, which are repeated. Every cycle is constituted of four stages as following: Planning- planning a change; Acting and observing the process and consequences of the change, reflecting on those processes and consequences and then re-planning the change. Action research is considered as a form of "applied" research, which not only serves for the professional teacher development, but also for increasing the performance of the school and education in general.

The collaborative action research is the joint research between two or more teachers or between universities and teachers. They collaborate and influence in changing the curricular approach, and their main focus is on practical problems of individual teachers or schools. This collaboration between universities and schools may foster communication and mutual respect (Raymond, 2004).

At the very beginning of this research, we introduced the issue of using different approaches related to teaching of multiplication and division in the second grade of primary school. Collaborative action research has directly influenced the application of these new approaches in classroom. This methodology enabled us to find out more appropriate ways of teaching aimed at acquisition of basic mathematical concepts through the spiral cycles of collaborative planning, acting and reflecting.

Research Related to Early Teaching and Learning of Multiplication and Division

Several researchers have studied how young students multiply and divide. Nunes and Bryant (1996) indicated that a general point of view about multiplication and division is that they simply "are inverse arithmetical operations ... that are taught after addition and subtraction" (p. 144). However, they stress that such a viewpoint is incomplete knowing the fact that "multiplication and division represent a significant qualitative change in children's thinking" (p. 144).

The first confrontation of students with multiplication is usually accompanied with situations that include sets with equal number of objects Greer (1992). Although there are other models available that represent multiplication, the model of equal sets (repeated addition) is known as a basic intuitive model for multiplication. A challenge in this situation is the child's reflection on the 'set' as a unit and the addition of those 'units'. In such a case, different expressions are used, such as '3 times 5', 3 multiplied with 5' or '3 with 5 each'. In their study, Gray and Tall (1994) noted that some children are not able to apply repeated addition to find out the product of two numbers. Thus, for instance, they can add 5+5=10, but then they continue to count 11, 12,..15 in order to get to know how much is 3x5. Consequently, a precondition to teach children how to multiply is to teach them first to do repeated addition. Since multiplication is the addition of 'many times' of equal sets, the initial thinking of children related to division is connected to the division of a set of objects in equal portions. Fischbein, et al (1985) discussed two models of division used when either number of portions or the number of items in each portion is known. These are generally known as … division through partitioning (sharing out), partitive division and division by 'chunking' (grouping), quotitive division. According to the model through 'partitioning', the general number of objects represents the dividend, while the divisor represents the total of partitioned parts. For instance, three children should share 6 apples; how many apples each of them will receive? (6:3). Apples are related to the dividend, while the divisor is related to the children. According the model through grouping, the problem is formulated as following: How many children will receive 3 apples if there are 6 apples in total? (6:3) (in this case both the dividend and the divisor are the apples). According to the research, the initial intuitive model used to develop the concept of division is that of 'partitioning', while as a result of teaching the other mode is developed, i.e. through 'grouping' (Fischbein, et al.(1985); Mulligan (1992); Murray, et al. (1992); Kouba (1989)). However, there is often misunderstanding when these two models are discussed. In the first model, the dividend (3) represents the number of 'children'; while in the second model the number (3) represents the 'apples'. From the child's perspective, division situations are often related to the division expression (6:3) rather than the situation itself. Therefore, it is important to pay particular attention if the child is experiencing such differences, i.e. if they understand that number 3 has a different meaning in the division through grouping and another one in the division by partitioning. From research related to these two concepts, we come to the idea that considering multiplication as (always) increasing numbers, while division as inverse operations that (always) decrease numbers and that a smaller number cannot be divided with a big number are wrong ideas (Kouba (1989); Arighileri (1989)). Therefore, understanding multiplication and division as a repeated addition and subtraction represents a future challenge. On the other hand, word problems not only serve as a basis for understanding children's strategies for solving addition, subtraction, multiplication, and division problems, they also can provide a unifying framework for thinking about problem solving in their daily life (Carpenter et.al., 1999). Children's thinking and their reasoning are important parts of the problem solving process (Barmby, (2009). Using practical experiences of children themselves and linking those with informal calculation strategies helps children count easier and clearly see the connections between the concepts and their application in problems solving.

Method

Aim and Research Questions

The aim of this study was to investigate the ways of teaching and learning activities which enable students to use their experiences, consider different ways of calculation and justify word problem solving related to multiplication and division.

The main research question was formulated:

> What is the effect of using the word problem solving in the understanding of division, through sharing/partitive situations and grouping/quotitive situations and their relations to multiplication?

So this research contributes to the understanding of how action research may serve as a 'tool' for teaching activity and assessing the impact of word problem solving to ensure a better understanding of basic mathematical concepts and their application in problem solving.

School Context and Participants

This research is carried out in a non-public funded school called "Third Millennium" which has a student population of 527 and 55 teachers. There are three second grade classrooms with 58 students were the teacher are, Lirika, Miranda and Shqiponja. Lirika graduated as a primary teacher in the Faculty of Education three years ago. Miranda graduated in the same faculty, five years ago and she is working in her Master Theses on school management. Shqiponja graduated in the Higher Pedagogical School and she has a six year experience in teaching. She also finished some in-service teacher courses. This school closely cooperates with the staff of Faculty of Education - University of Prishtina. Thus, Lirika invited me (author) as a staff member of the Faculty of Education to discuss her dilemmas about teaching of multiplication and division in her class. Together, I and Lirika, engaged in this joint effort as co-researchers. The data collection and all activities were carried out in Lirika's classroom during the second term with twenty students (7-8 years old). In that school, the teaching and learning process , from first to fifth grade develops mostly according to the philosophy of the 'Step by step' program[11]. According to this philosophy, interactive teaching and the integration of different subjects have a primary role. At the beginning of the day, known as *the morning meeting*, usually teachers work with the entire classroom where the daily plan is presented. Then, the work is carried out in different learning centers. I took part three times per week, usually when children were learning in the mathematics center. Teacher Miranda and Shqiponja also took part in this research. They collaborated with us and carried out the same activities in their classrooms. Also, the school vice-principal and parents were informed about this study.

Research Design

Action research was used in this study. At the beginning, we carried out a plan for action research in order to explore the word problem as part of 'curriculum' during the teaching and learning of multiplication and division. First, it was compared with the learning outcomes for multiplication and division in the Mathematics Curriculum[12] with the math textbook's content for second grade. Then we designed the action stages:
First, *planning* and selecting appropriate teaching/learning materials, examples and methods for representing mathematical ideas related to multiplication and division were developed. The mathematics learning center was designed to be an activity-based center providing the students with many opportunities to solve different problem situations. Secondly*, interpreting and evaluating* the students' mathematical solutions, their arguments or representations (verbal or written, drawing or modeling), including misconceptions. Also, in this stage, we diagnosed the students achievements, strengths and weaknesses. Because it was a *practical research,* after reflecting we *reassessed* the activities and a*dapted* the tasks for different student needs. Different assessment instruments were used to collect data, including: *classroom observation, interviewing, and worksheets.* The research took place during the second term, three times per week.

In the beginning, I was a passive observer during Lirika's teaching. I observed how she interacted with students, discussed with them and how students discussed among themselves. But when students were working in groups or individually, we both interacted with them. In these cases we used the interviewing which was videotaped or registered as notes in our notebooks. Transcribed

[11] The 'Step by step' program, http://www.kec-ks.org.

[12] MASHT (2004)

materials were then analyzed by us. W*orksheets* were used as data in order to analyze and assess the students reasoning in their problem solutions.

The triangulation technique was used for the validation of this study (Mcniff, at al, 2010). There were different gathering data methods, and the analyses were done from both of us, sometimes together and sometimes separately. Two other teachers and the vice-principal helped us validate our work through the whole process. They were our 'critical friends' and we established trusting relationships which became the grounds for giving and receiving critique (Mcniff and Whitehead, 2010).

Findings and Interpretations

The presentation of the results is divided into three sections. First, we were interested to observe and analyze how students experienced the computation with multiplication and formal division. Formal division here means 'division as the inverse of multiplication' as it is in the existing mathematics curriculum[13]. We analyzed the teacher's instruction and students work in their student's textbook. The second section is related with different strategies that students use to explain their reasoning on word problem solving related with multiplication, and the third section concerns the *division through sharing/partitive situations and grouping/quotitive situations.* The findings of the above sections are included as cases. They are based on classroom observations and student work during the different periods.

Case 1

This is a whole class situation in the 'Morning meeting' where teacher Lirika, expands the daily objectives. She starts with a problem that she takes from math textbook for secondary grade (p.109). Afterwards, she picks out 12 counters from a box and asks three children to come to the board. The teacher than shares out the counters in a 'one for each of them' order and when the counters are shared out, the three children count their counters and then saw that they have four each. She writes in the table, 12:3=4 and explains how it relates with multiplication 4x3=12. She presented another example from the textbook: *Four friends equally share 24 candies. How many candies each of them have?* The students discussed that the answer is related with multiplication and in that case, answer is 6 because 4x6=24.
Thus, it was supposed that students understand the division as 'sharing equally' and as the inverse operation of multiplication. After this situation, the teacher invited children to work in their learning centers, where they have to solve problems in their student's textbook (p.80).
We observed students how they 'filled' their worksheet. Most of them just memorized the multiplication table …and used the calculation (in their mind or using the counters or other things that they had in their learning centers).

Case 2

Here the teacher prepared the supplement worksheet, with three word problems. The reason was: did the students know to relate the 'situations' with multiplication? In this context, students were required to solve the three problems related to daily life and afterwards we analyzed their solutions and reasoning. Below we present one of the analyzed problems.

It is shown in the Figure 1, that a student has used a drawing to solve the problem: *On the table there are 5 plates with 7 biscuits each. How many biscuits are altogether?* A student explaining his correct answer based in his 'drawing'.

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[13] MASHT 2004

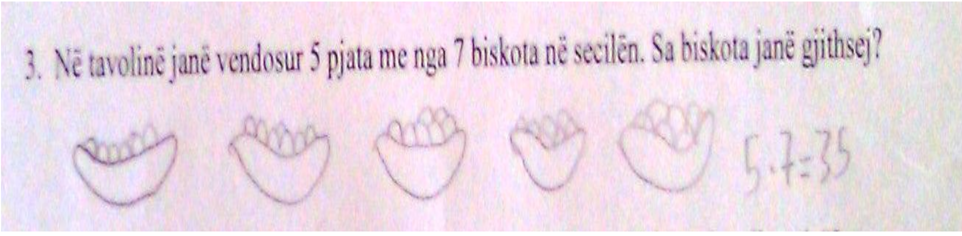

Figure 1. Using Drawing to Solve Problem

Not always students relate their 'modeling' with the context in the correct way. A student, used the same presentation, but he didn't show correctly the relation between the context and the drawing (Figure 2). For this student it is unclear what does the number 5 means. He just draws some circles (biscuits) and plates without numbering them.

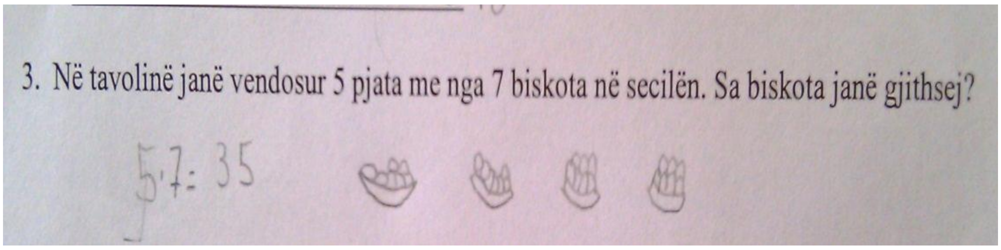

Figure 2. Uncorrected Relation between 'Drawing' and 'Context'

In this example, we found that all students wrote the correct answer, except one. Nine students had correct results without reasoning, 5 of them used drawings, 2 of them had correct answers but they presented their drawing incorrect, 2 students used arraying and 2 students used repeated addition (by 7).

Case 3

As in the above example, we found that most of the second grade students relate their drawing with the context. To find the solution of the problem: *Four girls eat 8 apples equally. How many apples each of them eat?* Most students draw the girls and apples (Figure 3).

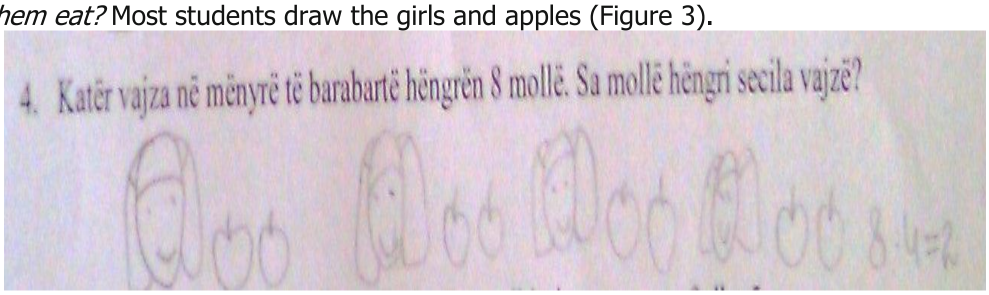

Figure 3. A Student's Solution Using Drawing Related with Context.

In this problem, the dividend concerns apples and the divisor girls. So it is related with partitive division, so 'sharing equally' and drawing was used from most of students. There were 7 of them, who wrote only the correct answer.

It seemed that it was difficult for some of second graders to write the correct reasoning of problem solving. Not always students relate their solution with correct representations. Below, in the Figure 4 is shown a student's solution of this problem: *In the second grade there are 48 students. If they have to divide in 6 clubs, how many students are in each club?* Even though the result is correct, the student misunderstands what 'sharing equally'means. The student considered the procedure '*finding a factor, when the product and the other factor are known'* (she memorized) and fund the correct solution, but it wasn't important for her if there are 8 or more circles in each set (which in this case represent the students and clubs).

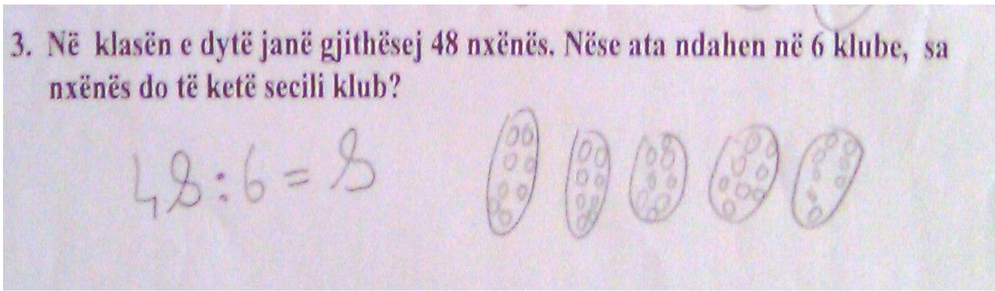

Figure 4. A Student's Solution with Not Correct Reasoning

From the analysis of the student's solutions, we see that the reasoning of most of them were the same (48:6=8 because 6x8=48). Some of them just memorized the multiplication table, and the others used the 'drawing' model in the correct way.

During the textbooks analysis, we didn't find any problem related with measurement or quotative problems. Thus we prepared some additional problems to understand how children think and use their strategy to solve division problems (by grouping). Here is an example:

> Era has 28 balls and some boxes. She places four balls in each box. How many boxes did Era fill?

This problem seems to be harder. It was not a 'routine-problem', so there was some uncertainty. We understood from analyses that some students didn't understand yet how to connect the situations with the dividend, divisor and multiplier (Figure 5). They do the computations, whenever they find numbers and don't worry about the 'context. However, from our observing, them who relate the counting and adding strategy with 'drawing' seem to have no problem to connect 'situations' with division (Figure 6).

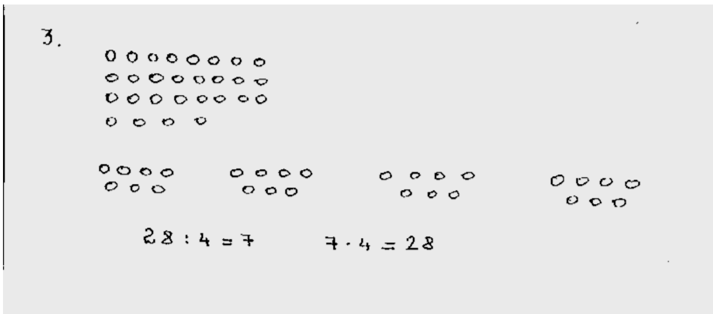

Figure 5. A student's Wrong Representation

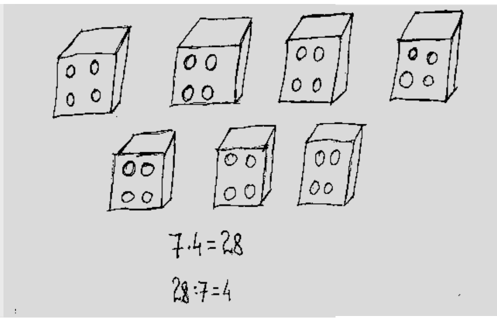

Figure 6. A student who uses the counting and adding strategy and then presents it with a drawing

Adapted Plan

After analyzing the students work, we decided to prepare the 'treatment plan' for students who had difficulties understanding the relation between the 'concepts' and the problem situations. This plan was discussed with the two other teachers too. It was decided to use student interviewing during the problem solving process. So, the supplement worksheets with more illustrations and figures were prepared. They were considered as necessary material for students. For two weeks, teachers worked after regular classes with the identified students in need using individual interviews. All interviews started with similar initial questions, but the follow-up questions depended on the answers that were given. During this process, cubes, counters, and other objects available in the classroom were used, including paper and pencil to take notes. Students were encouraged to freely talk, write and draw. It was required to explain their way of thinking in their solutions. In the same way, teacher Miranda and teacher Shqiponja interviewed their students. Everything that students said and did was registered and then discussed with me in order to analyses and evaluate two aspects of the use of multiplication and division – as operations for calculation and, as operations to solve problems in different situations.

Discussion

The first steps toward engaging in collaborative action research in the mathematics classroom are vital in establishing quality research projects, designed and implemented jointly by classroom teachers and universities (Raymond (2004). This collaborative research helps us not only to engage in the classroom inquiry, but as practical research it contributes to improve teaching and student achievements. The variation of ways in which young students experience word problems has been the focus of this research. The findings illustrate that even students of the same age, have different experiences and capabilities in solving mathematical problems. The drawings and notations made by children in this study illustrate the process of gradual generalisation, from concrete details to abstraction. Van de Wale, suggested that multiplication and division activities should begin with models before word problems (Van de Wale, 2004). So, in Case 1,'sharing equally' shown by the teacher's demonstration was the first confrontation of students with division and the basis for the development of initial concepts related to multiplications and divisions (Greer , 1992; Carpenter at.al, 1999). Also, other intuitive strategies were used, as repeated addition of equal sets, or 'modeling'. 'Modeling' here means, using concrete materials to help the problem solving. Thus, during this case, we concluded that the demonstration of repeated addition with two, with five, with six,… and so on, does not present difficulties if addition operations are excellently acquired. Because, textbooks[14] have most of the examples with 'calculation' it was a routine for students to solve most of them in the same way, using only memorization. However, using only calculation skills and 'routine models' isn't sufficient to understand what the factor and product mean. Even though there were no perceiveable mistakes in the textbook pages 'filled' by students, it doesn't mean that they understand what each of the 'numbers' represents in the problems that were presented in the Case 2.

"Today, mathematics is not about computation, especially pencil-and-paper computation. Mathematics is about reasoning and patterns and making sense of things. Mathematics is problem solving" (Van de Walle, 2004, p.176). Using practical examples and word problems enables children not only to improve their calculation skills, but also to understand the meaning of 'size' presented through those problems, which is very important for the development of the division concept in children (Fischbein at al., 1985; Mulligan, 1992; Gray and Tall, 1994). However, Vergnaud (1983) stated that multiplication, multipliers and product present different links of the 'factors' to the problems of division. According to this research, initial intuitive models were used to develop the concept of division as 'sharing equally', while as a result of teaching, other models were developed, i.e. through 'grouping' (Fischbein, et al.,1985; Mulligan, 1992; Murray, et al., 1992; Kouba, 1989).

"Teaching activities for multiplication and division need to give young learners the opportunity to explore different representations of multiplications and division and to reason about connections between these" (Barmby, 2009, p.60). In Case 3, additional problems were presented, regarding quotative division problems. In general, connecting the situations with the dividend, divisor and multiplier may cause problems in most cases (Neuman, 1999). But, providing children the opportunity to solve not only routine problems is the best way to help them construct the procedures for calculations.

Undoubtedly, individual interviews with students significantly contributed to the analysis of their knowledge and identification of their obstacles in the learning process. Children develop their understanding by constructing relationships, and in order to understand they must speak something and be able to comprehend the relationships (Carpenter et.al., 1999, p.53). So, the 'treatment process' as part of action research methodology impacted directly the improvement of the student's ability to understand multiplication and division as inverse concepts and to solve different problems.

[14] Matematika 2 (2006)

Conclusion

The process of collaboratively working toward the problems solving not only provides a wide range of expertise, but also generates positive working relationships. So, using collaborative research in this study is considered as a very useful educational resource. The planning, interpretation, evaluation, and afterwards the adapted plan can provide useful resources for the improvement of student's abilities and skills. This collaborative research suggest that using different teaching and learning resources, appropriate activities and managing individual interventions in math learning centers /classes helps students construct and develop the basic concepts. Also, this study suggests teachers to teach multiplication and division not as separate concepts but jointly. Also, it suggests teachers to use word problems as tools for concept understanding. They should engage their students in solving and explaining their problem solving strategies, and not to get them textbook 'to do pages'. Teachers should look on the textbook as a teaching resource and not as object of instruction.

Limitations of the Study

Because the research was carried out in a private school where in each classrom there is an avaregae of twenty students, and students stay at school during the whole day, the major limitation of the study is the generalization of its conclusions for other schools, where the student number in classrooms is larger than 30 and math classes run for 40 -45 minutes.

References

Altrichter, H., Posch, P., & Somekh, B. (1993). *Teachers investigate their work: An introduction to the methods of action research*. New York. Routledge.

Anghileri, J. (2000). *Teaching numbers sense*. London: Continuum.

Carpenter,T. P., Fennema, E., Franke, M. L., Levi, L.,& Empson, B. S. (1999). *Children's Mathematics. Cognitively Guided Instruction.* Portsmouth, NH: Heinemann

Clift, R., Veal, M. L., Johnson, M., & Holland, P. (1990). Restructuring teacher education through collaborative action research. *Journal of Teacher Education, 41*(2), 52–62.

Fischbein, E., Deri, M., Nello, M. S., & Marino, M. S. (1985). The role of implicit models in solving verbal problems in multiplication and division*. Journal for Research in Mathematics Education, 16*(1), 3-17.

Gray, E., & Tall, D. (1994). Duality, ambiguity and flexibility: A perceptual view of simple arithmetic. *Journal of Research in Mathematics Education, 25*(2), 115-141.

Greer, B. (1992). Multiplication and division as models of situations. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. New York: MacMillan.

Kemmis, S. ,& McTaggart, R. (Eds.) (1988). *The action research planner* (3rd ed.). Victoria: Deakin University.

Koshy, V. (2010). *Action research for improving educational practice*. *A step-by-step guide* (2nd ed.). London: Sage.

MASHT (2004). *Plani dhe Programi Mësimor, për klasën e dytë*. Retrieved January 9, 2010, from www.masht-gov.net

Matematika 2 (2006). Botimi i tretë, Dukagjini-Pejë.

Mcniff. J, & Whitehead, J. (2010). *You and your action research project.* London: Routledge.

Mills, G. E. (2003). *Action research: A guide for the teacher researcher*. Upper Saddle River, NJ: Merrill/Prentice Hall.

Miller, D. M., & Pine, G. J. (1990). Advancing professional inquiry for educational improvement through action research. *Journal of Staff Development, 2*(3), 56–61.

Mulligan, J. T. (1992). Children's solutions to multiplication and division word problems: A longitudinal study. In G. William, & K. Graham (Eds.). *Proceedings of the Sixteenth PME Conference*, (pp. 144–151), University of New Hampshire, Durham, NII (USA).

Mulligan, J. T., & Mitchelmore, M. C. (1997). Young children's intuitive models of multiplication and division. *Journal for Research in Mathematics Education*, *28*, 309-330.

Neuman, D. (1999). Early learning and awareness of division: A phenomenographic approach, *Journal of Educational Studies in Mathematics*, *40*, 101-128.

Nunes, T., & Bryant, P. (1996). *Children doing mathematics*. Oxford: Blackwell.

Ponte, P. (2002). How teachers become action researchers and how teacher educators become their facilitators. *Educational Action Research, 10*(3), 399–423.

Rafferty, C. D. (1995). *Impact and challenges of multi-site collaborative inquiry initiatives*. Paper presented at the Annual Meeting of the American Association of Colleges for Teacher Education, Washington, DC.

Raymond, A. (2004). *Collaborative action research in mathematics education: A tale of two teacher-researchers.* Retrieved January 3, 2011, from http://www.eric.ed.gov/ERICWebPortal

Sagor, R. (2000). *Guiding school improvement with action research*. Alexandria, VA: ASCD.

Steffe, L. P. (1994). Children's multiplying schemes. In G. Harel, & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 3-39). Albany, NY: State University of New York Press.

Vatanabe, T. (2003). Teaching multiplication: An analysis of elementary school mathematics teachers' manuals from Japan and the United States. *The Elementary School Journal, 104*(2), University of Chicago.

Vergnaud, G. (1983). Multiplicative structures. In R. Lesh, & M. Landau (Eds.), *Acquisition of Mathematics Concepts and Processes*, (pp.127-174), New York: Academic Press.

Van de Wale, & John, A. (2004). *Elementary and middle school mathematics: Teaching developmentally*. Boston, MA: Pearson Education, Inc.

Wilson, E. (Ed) (2009). *School-based research; A guide for education students*. London: Sage.