Fluid Mechanics FE Review

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FERC

Fluid Mechanics FE Review

These slides contain some notes, thoughts about what to study, and some practice problems. The answers to the problems are given in the last slide.

In the review session, we will be working some of these problems. Feel free to come to the session, or work the problems on your own. I am happy to answer your email questions or those that you bring to the session.

Good luck!

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Fluid Mechanics FE Review

MAJOR TOPICS

Fluid Properties

Fluid Statics

Fluid Dynamics

Fluid Measurements

Dimensional Anaysis

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Fluid Mechanics FE Review

It will be very helpful to memorize the following concepts and equations:

- Specific weight, density, and specific gravity
- Hydrostatics pressure equation / manometry
- Force magnitude and location due to hydrostatic pressure for horizontal and vertical plane walls
- · Conservation of mass / continuity
- Conservation of energy / Bernoulli and Energy Eqn
- · Darcy Eqn
- Relative roughness equation
- Drag equation
- · How to use the Moody Diagram

Most equations and problems taken from Professional Publications, Inc. FERC Review Course Book

Fluid Mechanics

Properties

Fluids

- Substances in either the liquid or gas phase
- Cannot support shear

DENSITY, P SPECIFIC (UNIT) WT, 8= Pg SPECIFIC GRANITY, SG = P P SPECIFIC GRANITY, SG = P P SURFACE TENSION, O = FORCE Length VAPOR PRESSURE, Pa ; VISCOSITY, W RASSOLUTE OR DYNAMIC MAKE SURE YOU USE THE PROPER VALUES ACCORDING TO H20 TEMPERATURE! TDEAL GAS LAW! P=PRESSURE; T= TEMP.; Po DENSITY; R= GAS CONSTANT P T = P R R R Molecular Wt. R Molecular Wt.

Make sure you use the ideal gas law when calculating properties for gasses!!

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Fluid Mechanics

Properties

Property	Symbol & Equation	Definition	Etc.
Density	$ ho = \frac{m}{\forall}$	<u>mass</u> volume	
Specific Weight	$\gamma = \rho g$	density x gravity	
Specific Gravity	$SG = \frac{\rho_x}{\rho_{water}} = \frac{\gamma_x}{\gamma_{water}}$		
Viscosity	$\mu = \frac{\tau}{du/dy}$	shear stress velocity gradient	
Kinematic viscosity	$v = \frac{\mu}{\rho}$	<u>viscosity</u> density	

Make sure you know the relationship between density, specific weight, and specific gravity!

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Mass & Weight

- 10.0 L of an incompressible liquid exert a force of 20 N at the earth's surface. What force would 2.3 L of this liquid exert on the surface of the moon? The gravitational acceleration on the surface of the moon is 1.67 m/s²
 - (A) 0.39 N
 - (B) 0.78 N
 - (C) 3.4 N
 - (D) 4.6 N

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Fluid Mechanics - Fluid Properties

Traid Meditarios Traid Troportios				
Property	Symbol & Equation	Definition	Etc.	
Vapor Pressure	p_v	Pressure at which liquid and vapor are in equilibrium	Used to predict cavitation (local pressure < vapor pressure	
Bulk Modulus	$ \rho = \frac{m}{\forall} $	mass volume		
Speed of Sound	$c, liquid = \sqrt{k/\rho}$ $c, gas = \sqrt{krt}$	Velocity of propegation of a small wave	k = Bulk modulus $k_{air} = 1.4$	

SPEED OF SOUND

Furation of Back Modulus (which correlates to compressibility)

FOR IDEAL GAS! C= VKRT (T must be in K or OR) k= ratio of specific heats

MACH Nomber $M = \frac{V}{c} = \frac{object}{speed} \frac{velocity}{speed}$

Mach Number

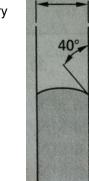
- 2. A jet aircraft is flying at a speed of 1700 km/h. The air temperature is 20 ℃. The molecular weight of air is 29 g/mol. What is the Mach number of the aircraft?
 - (A) 0.979
 - (B) 1.38
 - (C) 1.92
 - (D) 5.28

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Fluid Mechanics – Fluid Properties					
Property	Symbol & Equation	Definition	Etc.		
Surface Tension	$\sigma = \frac{F}{L}$	<u>force</u> length			
Capillary Rise	$h = \frac{4\sigma \cos\beta}{\rho d_{tube}g}$	Distance a liquid will rise (or fall) in a "tube"			
	Ψ ≡ β	d			
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Surface Tension

3. A 2 mm (inside diameter) glass tube is placed in a container of mercury. An angle of 40° is measured as illustrated. The density and surface tension of mercury are 13550 kg/m³ and 37.5 x 10-2 N/m, respectively. How high will the mercury rise or be depressed in the tube as a result of capillary action?



2 mm

- (A) -4.3 mm (depression)
- (B) -1.6 mm (depression)
- (C) 4.2 mm (rise)
- (D) 6.4 mm (rise)

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Fluid Mechanics

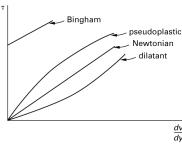
9-1b

Stresses and Viscosity

Shear Stress

- Normal Component: $au_n = p = \frac{Force}{Area}$
- · Tangential Component
 - For a Newtonian fluid: $\tau_t = \mu \frac{d\mathbf{v}}{dy}$ ($\mathbf{v} = \text{velocity}$)
 - For a pseudoplastic or dilatant fluid: $au_t = K \left(\frac{d\mathbf{v}}{dy} \right)^{\alpha}$

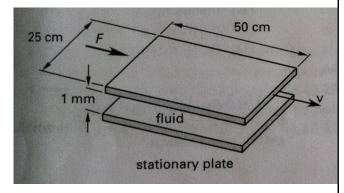
Absolute Viscosity = μ = Ratio of shear stress to rate of shear deformation



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Viscosity

- 4. A sliding-plate viscometer is used to measure the viscosity of a Newtonian fluid. A force of 25 N is required to keep the top plate moving at a constant velocity of 5 m/s. What is the viscosity of the fluid?
 - (A) 0.005 N-s/m^2
 - (B) 0.04 N-s/m²
 - (C) 0.2 N-s/m²
 - (D) 5.0 N-s/m²



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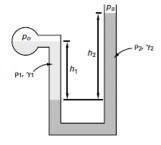
Fluid Statics

PRESSURE: Pabsolute = Parce + Patnospheric = Patnospheric - PVACUUM

HYDEOSTATIC PRESSURE = P=8h # Pressure is ALWAYS NORMAL to a surface!

MANOMETRY: Start w/P on one end i add or Subtract changes in hydrostatic

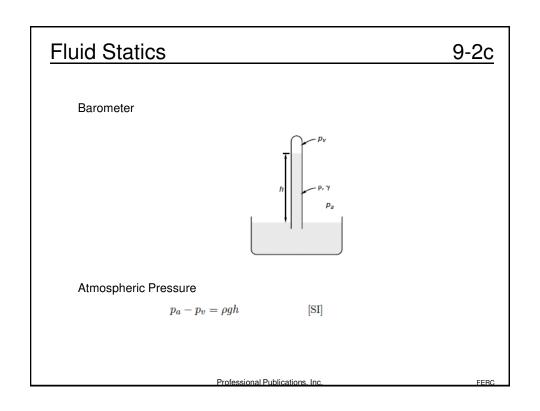
Pressure to fin P at other end C"TAKE A WERNEY "through)



 $p_o-p_a=\gamma_2h_2-\gamma_1h_1$

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Manometry 5. An open water manometer is used to measure the pressure in a tank. The tank ifs half-filled with 50,000 kg of a liquid chemical that is not miscible in water. The manometer tube is filled with liquid chemical. What is the pressure in -chemical the tank relative to the atmospheric 400 mm tank pressure? 225 mm (A) 1.4 kPa (B) 1.9 kPa 10 m (C) 2.4 kPa (D) 3.4 kPa Professional Publications, Inc



Fluid Statics

FORCES ON SUBMERGED SURFACES : FORCES ACT @ CENTER OF PRESSURE

FROM OR F- 8A 2c SIND

LOCATION = CENTROLD OF PRESSURE PRISM OR y = Iys / 2c A ; 2° = Zc A

Volume

BYOYANCY: Forces = 8 fluid Hisplaced

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Hydrostatics

- 6. A 6m x 6m x 6m vented cubical tank is half-filled with water; the remaining space is filled with oil (SG=0.8). What is the total force on one side of the tank?
 - (A) 690 kN
 - (B) 900 kN
 - (C) 950 kN
 - (D) 1.0 MN

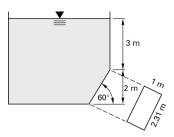
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Fluid Statics Examples

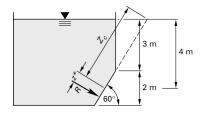
9-2f1

Example 1 (FEIM):

The tank shown is filled with water. At what depth does the resultant force act?



The surface under pressure is a rectangle 1 m at the base and 2.31 m tall.



$$A = bh$$

$$I_{y_c} = \frac{b^3 h}{12}$$

$$Z_c = \frac{4 \text{ m}}{\sin 60^\circ} = 4.618 \text{ m}$$

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Fluid Statics Examples

9-2f2

Using the moment of inertia for a rectangle given in the NCEES Handbook,

$$z^* = \frac{I_{y_c}}{AZ_c} = \frac{b^3h}{12bhZ_c} = \frac{b^2}{12Z_c}$$
$$= \frac{(2.31 \text{ m})^2}{(12)(4.618 \text{ m})} = 0.0963 \text{ m}$$

 $R_{\text{depth}} = (Z_c + Z^*) \sin 60^\circ = (4.618 \text{ m} + 0.0963 \text{ m}) \sin 60^\circ = 4.08 \text{ m}$

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Fluid Statics Examples

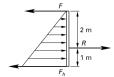
9-2g

Example 2 (FEIM):

The rectangular gate shown is 3 m high and has a frictionless hinge at the bottom. The fluid has a density of 1600 kg/m³. The magnitude of the 1600 kg/m³. The magnitude of the force *F* per meter of width to keep the $p_{ave} = \rho g z_{ave} (1600 \frac{kg}{m^3}) (9.81 \frac{m}{s^2}) (\frac{1}{2}) (3 \text{ m})$ gate closed is most nearly



- (A) 0 kN/m
- (B) 24 kN/m
- (C) 71 kN/m
- (D) 370 kN/m



$$p_{\text{ave}} = \rho g z_{\text{ave}} (1600 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2}) (\frac{1}{2}) (3 \text{ m})$$

= 23544 Pa

$$\frac{R}{w} = p_{ave}h = (23544 \text{ Pa})(3 \text{ m}) = 70662 \text{ N/m}$$

 $F + F_h = R$

R is one-third from the bottom (centroid of a triangle from the NCEES Handbook). Taking the moments about R,

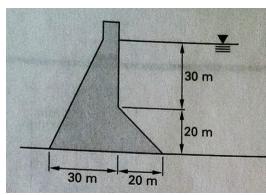
$$2F = F_h \frac{F}{w} = \left(\frac{1}{3}\right)\left(\frac{R}{w}\right) = \frac{70,667 \frac{N}{m}}{3} = 23.6 \text{ kN/m}$$

Therefore, (B) is correct.

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Hydrostatics

- 6. A gravity dam has the cross section shown. What is the magnitude of the resultant water force (per meter of width) acting on the face of the dam?
 - (A) 7.85 MN/m
 - (B) 12.3 MN/m
 - (C) 14.6 MN/m
 - (D) 20.2 MN/m



Buoyancy

Archimedes' Principle and Buoyancy

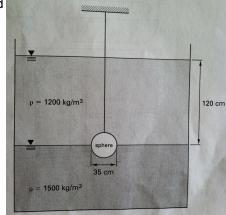
- The buoyant force on a submerged or floating object is equal to the weight of the displaced fluid.
- A body floating at the interface between two fluids will have buoyant force equal to the weights of both fluids displaced.

$$\emph{F}_{ ext{buoyant}} = \gamma_{ ext{water}} \emph{V}_{ ext{displaced}}$$

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Buoyancy

- 8. A 35 cm diameter solid sphere (ρ = 4500 kg/m³) is suspended by a cable as shown. Half of the sphere is in one fluid (ρ = 1200 kg/m³) and the other half of the sphere is in another (ρ = 1500 kg/m³). What is the tension in the cable?
 - (A) 297 N
 - (B) 593 N
 - (C) 694 N
 - (D) 826 N



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Fluid Dynamics

CONTINUITY: Mass is Conserved
$$\dot{m}_1 = \dot{m}_2$$

$$pA_1 v_1 = P_2 A_2 v_2$$

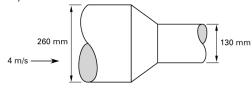
$$pA_1 v_1 = P_2 Q_2$$
For incompressible fluids $Q = A_1 v_1 = A_2 v_2 = E+c$.
(p is constant)

(Might be called the field eqn.)
$$\frac{P_1}{Y_1} + \frac{Y_2^2}{2g} + Z_1 = \frac{P_2}{Y_2} + \frac{V_2^2}{2g} + Z_2 = \frac{P_1}{P_1} + \frac{V_2^2}{2g} + g Z_1 = \frac{P_2}{P_2} + \frac{V_2^2}{2g} + g Z_2$$
(might be called the field eqn.)

Continuity Equation Example

9-3c

Example (FEIM):



The speed of an incompressible fluid is 4 m/s entering the 260 mm pipe.

The speed in the 130 mm pipe is most nearly

- (A) 1 m/s
- (B) 2 m/s
- (C) 4 m/s
- (D) 16 m/s

$$A_1V_1=A_2V_2$$

$$A_1 = 4A_2$$

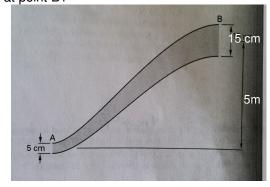
so
$$v_2 = 4v_1 = (4)(4\frac{m}{s}) = 16 \text{ m/s}$$

Therefore, (D) is correct.

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Bernoulli Equation

- 9. The diameter of a water pipe gradually changes from 5 cm at point A to 15 cm at point B. Point A is 5 m lower than point B. The pressure is 700 kPa at point A and 664 kPa at point B. Friction between the water and the pipe walls is negligible. What is the rate of discharge at point B?
 - (A) $0.0035 \text{ m}^3/\text{s}$
 - (B) 0.0064 m³/s
 - $(C) 0.010 \text{ m}^3/\text{s}$
 - (D) $0.018 \text{ m}^3/\text{s}$

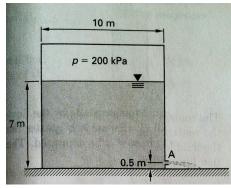


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Bernoulli – Flow from a Jet

- 10. A liquid with a specific gravity of 0.9 is stored in a pressurized, closed storage tank. The tank is cylindrical with a 10 m diameter. The absolute pressure in the tank above the liquid is 200 kPa. What is the initial velocity of a fluid jet when a 5 cm diameter orifice is opened at point A?
 - (A) 11.3 m/s
 - (B) 18.0 m/s
 - (C) 18.6 m/s
 - (D) 23.9 m/s



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PPI

Flow in Pipes – Fluid Dynamics and Friction Steady, Incompressible Flow

EXECUTE GOVERNOM:
$$\frac{P}{y} + \frac{v^2}{2g} + 2$$
, $= \frac{P_2}{y} + \frac{v^2}{2g} + 2z + h_1 + h_1 \leftarrow \text{fittings}$

Frether)

The hose is a function of the and $\frac{v}{h}$; $E = \text{Surface roughness}$

LAMINALE FLOW: $\frac{v}{h} = \frac{32}{2} \frac{M_{\text{LV}}}{VD^2}$

TURBULENT FLOW: USE MOODY DIAGRAM to FIND of

Smooth & Hydraulially smooth pipes:
$$f = \frac{0.316}{Re^{NA}}$$
 (for Re < 100,000)

Can Also use HARLAND ERN FOR ALL TURBULENT FLOW

 $\frac{1}{\sqrt{f}} = -1.8 \log \left[\frac{6.9}{Rc} + \left(\frac{6/5}{3.7} \right)^{1.11} \right]$

Reynolds Number

9-3q

For a Newtonian fluid:

$$Re = \frac{vD\rho}{\mu}$$
 [SI]

$$Re = \frac{vD}{\mu}$$

 $D = \text{hydraulic diameter} = 4R_{H}$

v = kinematic viscosity

 $\mu = dynamic viscosity$

For a pseudoplastic or dilatant fluid:

$$Re' = \frac{v^{2-n}D^{n}\rho}{K\left(\frac{3n+1}{4n}\right)^{n}8^{n-1}}$$

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Fluid Dynamics and Friction

- 11. A steel pipe with an inside diameter of 25 mm is 20 m long and carries water at a rate of 4.5 m³/h. Assuming the specific roughness of the pipe is 0.00005 m, the water has an absolute viscosity of 1.00 x 10⁻³ Pa-s and a density of 1000 kg/m³, what is the friction factor?
 - (A) 0.023
 - (B) 0.030
 - (C) 0.026
 - (D) 0.028

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FERO

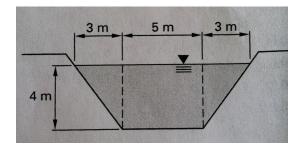
Non-Circular Conduits, Open Channel Flow, and Partially Full Pipes

OPEN CHANNEL:
$$V = \left(\frac{1}{N}\right) R_{ij}^{2/3} S^{1/2}$$
 (SI whits) $N = Manning's Roughness$ coefficient $V = \left(\frac{1.480}{N}\right) R_{ij}^{3/3} S^{3/2}$ (U.S. UNITS) $S = Slope of Channel$

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Hydraulic Radius

- 12. What is the hydraulic radius of the trapezoidal irrigation canal shown?
 - (A) 1.63 m
 - (B) 2.00 m
 - (C) 2.13 m
 - (D) 4.00 m

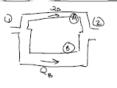


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Hydraulic Grade Line and Energy Grade Line Pump Power HYDRAULIC GRADE LINE & ENERGY GRADE LINE (Included) (Inc

Multi-path Pipelines



3 MAIN PRINCIPLES

- 1) Head Loss in each branch is equal: hf. A = hf. B

 for ha 2g = for Dos Zg

 Zg
- 2) Head loss between the 2 junctions is same as head loss in each branch
- 3) Mass Must be conserved. Therefore the total flow rate equals the sum of the flow rate in each branch.

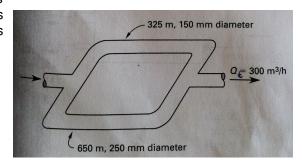
 Q1 = Q2 = QA + QB OR \(\frac{4}{4}\right)_2^2\righty_1^2 = \frac{11}{4}\right)_A^2\righty_A^2 = \frac{11}{4}\right)_A^2\righty_

This simplifies to: $D_1^2 V_1 = D_2^4 V_2 = D_A^2 V_A + D_6^2 V_8$

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Multi-path pipelines

- 13. The Darcy friction factor for both of the pipes shown is 0.024. The total flow rate is 300 m³/h. What is the flow rate through the 250 mm pipe?
 - (A) $0.04 \text{ m}^3/\text{s}$
 - (B) $0.05 \text{ m}^3/\text{s}$
 - (C) $0.06 \text{ m}^3/\text{s}$
 - (D) $0.07 \text{ m}^3/\text{s}$



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Implulse-Momentum

IMPULSE-MOMENTUM
$$\Sigma \Gamma = \rho Q \left(\nabla_2 - \nabla_1 \right) = \left(\frac{Qy}{y} V \right)_{out} - \left(\frac{Qy}{y} V \right)_{in} \quad \text{WATCH YOUR SIGNS!}$$

Be sure to familiarize yourself with how this equation works for bends, enlargements, contractions, jet propulsion, fixed blades, moving blades, and impulse turbines.

Impulse-Momentum

9-6a

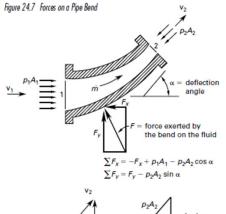
 $\sum \mathbf{F} = Q_2 \rho_2 \mathbf{v}_2 - Q_1 \rho_1 \mathbf{v}_1 \quad [SI]$

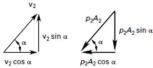
Pipe Bends, Enlargements, and Contractions

$$-F_x=p_2A_2\coslpha-p_1A_1$$

$$+Q
ho({
m v}_2\coslpha-{
m v}_1) \eqno(SI) \qquad \emph{24.39a}$$

 $F_y = (p_2 A_2 + Q \rho v_2) \sin \alpha + m_{\text{fluid}} g$ [SI]



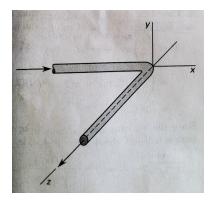


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24.38a

Impulse-Momentum

- 14. Water is flowing at 50 m/s through a 15 cm diameter pipe. The pipe makes a 90 degree bend, as shown. What is the reaction on the water in the z-direction at the bend?
 - (A) -44 kN
 - (B) -33 kN
 - (C) 14 kN
 - (D) 44 kN



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Flow and Pressure Measurement

PHOT TUBE - USED TO MEASURE FLOW VELOCITY V= \(\frac{2(P_0-P_5)}{P} = \sqrt{\frac{2g(P_0-P_5)}{8}} \)

Po= Static pressure i Ps = Stagnation pressure

VENTURI © ORIFICE MOTERS

(See sketches in Ref. Book
for variable descriptions)

Venturi

$$Q = \left(\frac{C_V A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}}\right) - \sqrt{2\left(\frac{P_1}{P_1} + g^2, -\frac{P_2}{P_2} - g^2\lambda\right)}$$
(SE)

$$Q = \left\langle \frac{C_v \Delta_z}{\sqrt{\int_{-\left(\frac{A_z}{A_i}\right)^2}}} \right) \sqrt{2} \frac{P_i}{y} + z_1 - \frac{P_z}{y} - z_2 \quad (us)$$

Orifices

$$G = CA\sqrt{2g\left(\frac{P_1}{6} + z_1 - \frac{P_2}{8} - z_2\right)}$$
 (U.S.)
 $G = CA\sqrt{2\left(\frac{P_1}{6} + gz_1 - \frac{P_2}{8} - gz_2\right)}$ (S.J.)

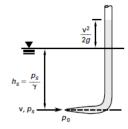
Fluid Mechanics

9-10a

Fluid Measurements

Pitot Tube - measures flow velocity

Figure 25.1 Ptot Tube



 The static pressure of the fluid at the depth of the pitot tube (p₀) must be known. For incompressible fluids and compressible fluids with M ≤ 0.3,

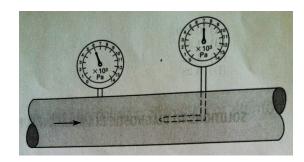
$$\mathbf{v} = \sqrt{\frac{2(p_0 - p_s)}{\rho}} \qquad [SI] \qquad 25.11a$$

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EEDC

Flow and Pressure Measurement

- 15. The density of air flowing in a duct is 1.15 kg/m³. A pitot tube is placed in the duct as shown. The static pressure in the duct is measured with a wall tap and pressure gage. Use the gage readings to determine the velocity of the air.
 - (A) 42 m/s
 - (B) 102 m/s
 - (C) 110 m/s
 - (D) 150 m/s



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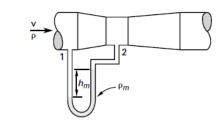
Fluid Mechanics Fluid Measurements

9-10c

Venturi Meters - measures the flow rate in a pipe system

• The changes in pressure and elevation determine the flow rate. In this diagram, $z_1 = z_2$, so there is no change in height.

Figure 25.2 Venturi Meter with Differential Manameter



$$Q = \left(\frac{C_{\mathbf{v}}A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}}\right)\sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)}$$

[U.S.]

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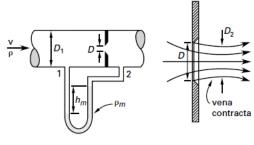
Fluid Mechanics

9-10e

Fluid Measurements

Orifices

Figure 25.3 Orifice Meter with Differential Manometer



(a) orifice meter

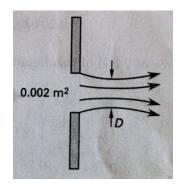
(b) orifice plate

$$Q = CA\sqrt{2g\left(rac{p_{1}}{\gamma} + z_{1} - rac{p_{2}}{\gamma} - z_{2}
ight)}$$
 [U.S.] 25.17b

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Orifice Flow

- 16. Water flows out of a tank at 12.5 m/s from an orifice located 9m below the surface. The cross-sectional area of the orifice is 0.002 m², and the coefficient of discharge is 0.85. What is the diameter D, at the vena contracta?
 - (A) 4.2 cm
 - (B) 4.5 cm
 - (C) 4.7 cm
 - (D) 4.8 cm



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Similitude

Geometric Similarity: Model is true to length, area &volume
Kinematic Similarity: Flow Regimes of model of prototype are the same
Byrnanic Similarity: Ratios of all types of forces are equal for
model + prototype

Met if following simultaneous equations are satisfied for model ? Prototype

In other words... Dimensionless parameters must be equal between the model and prototype (Each dimensionless parameter is a ratio of different types of forces being exerted on the fluid)

For example:

For completely submerged models/prototypes and pipe flow, the Reynolds numbers must be equal For Weirs, dams, ships, and open channels, the Froude numbers must be equal

<u>Similitude – Dimensionless Parameters</u>

inertial force Reynolds number = viscous force inertial force Froude number = gravity force inertial force Mach number = compressibility force inertial force Weber number = surface tension force centrifugal force Strouhal number = inertial force pressure force Pressure coefficient = inertial force $C_D = \frac{\text{drag}}{\frac{1}{2}\rho V^2 A}$ drag force Drag coefficient = inertial force

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Similitude

- 17. A nuclear submarine is capable of a top underwater speed of 65 km/h. How fast would a 1/20 scale model of the submarine have to be moved through a testing pool filled with seawater for the forces on the submarine and model to be dimensionally similar?
 - (A) 0.90 m/s
 - (B) 18 m/s
 - (C) 180 m/s
 - (D) 360 m/s

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ANSWERS!! 1. B 2. B 3. A 4. B 5. A 6. B 7. C 8. C 9. C 10.C 11.C 12.C 13.C 14. D 15.A 16.D 17. D Professional Publications. Inc