APPENDIX C

Strategies for Learning Multiplication Facts
(Source: Teaching Student-Centered Mathematics Grade 3 – 5, Van deWalle & Lovin)

The multiplication facts can be mastered by relating new facts to existing knowledge. "Mastery" of a basic fact means that a child can give a quick response (in about 3 seconds). Teachers can help students develop an efficient strategy - one that can be done mentally and quickly.

It is also important that students understand the commutative property (turn-arounds) since knowledge of this property `will reduce the number of facts they have to learn.

There are two approaches to introducing the fact strategies:
- Simple story problem designed in such a manner that students are likely to develop a strategy as they solve it. **It is recommended that the discussion of these strategies can be done for 5 – 10 minutes at the beginning of every day.**

- A lesson may revolve around a set of facts for which particular type of strategy is appropriate. You can discuss how these facts might all be alike in some way, or you might suggest an approach and see if students are able to use it on similar facts.

Since arrays are powerful thinking tools for teaching the strategies, provide students with copies of ten-by-ten dot arrays.

There are 100 multiplication facts, from 0 x 0 to 9 x 9. The first 4 of the 5 strategies listed below are generally easier and cover 75 out of the 100 facts. These strategies are suggestions and not rules. Listen to students as they discover other ways to help them think of the facts easily.

1. Zeros and Ones (facts with a 0 or 1)
   - Thirty six facts have at least one factor that is either 0 or 1. Sometimes these facts get confused with the rules children learn about addition facts with 0 or 1. Avoid rules that are without reason such as "Any number multiplies by zero is zero." Rather, these concepts can be best developed through story problems.

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

2. Doubles (facts with a 2)
   - Facts that have 2 as a factor are the same as the addition doubles and are probably already known by students who know their addition facts. Help them to realize that not only is 2 x 7 double 7, but 7 x 2 is also double 7.

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

3. Clock facts (facts with a 5)
   - Focus on the minute hand of the clock. When it points to a number, how many minutes past the hour is it? Connect this idea to the multiplication facts with 5 as a factor.

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

4. Nifty Nines (facts with a 9)
   - Facts with a factor of 9 include the largest products but can be among the easiest to learn. The 9 row and column of a multiplication table includes some nice patterns and are fun to discover. The following two patterns combined are useful to mastering the nines facts. (1) The tens digit of the product is always 1 less than the "other" factor (the one other than 9), and (2) the sum of the two digits in the product is always 9. These two ideas can be used together to get any nine fact quickly. *For 7 x 9, 1 less than 7 is 6, 6 and 3 make 9, so the answer is 63.*

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

   - An alternative strategy for learning the nine facts is also easy. Students may discover that they can relate the 9 fact to the already known 10 fact. For example, notice that 7 x 9 is the same as 7 x 10 less one set of 7 or 70 – 7.

5. Helping Facts – These 25 facts can be learned by relating each to already know fact or *helping fact*.

➤ Double and double again (facts with a 4) When 4 is one of the factors, students can double and double again. Example, find 4 x 6: double 6 is 12 and double 12 is 24.

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	(12)	15	18	21	24	27
4	0	4	8	(12)	(16)	20	(24)	(28)	(32)	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	(24)	30	36	42	48	54
7	0	7	14	21	(28)	35	42	49	56	63
8	0	8	16	24	(32)	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

➤ Double and one more set (facts with a 3) Example: find 3 x 7: double 7 is 14 and add one more 7 to make 21.

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	(9)	12	15	(18)	(21)	(24)	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	(18)	24	30	36	42	48	54
7	0	7	14	(21)	28	35	42	49	56	63
8	0	8	16	(24)	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

➢ Half then double (facts with an even number) Select the even factor and cut it in half. If the smaller factor is known, that product is doubled to get the new product.
Example: find 6 x 7: half the 6 to get 3 x 7. 3 x 7 is 21 and double 21 is 42.

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	(36)	(42)	(48)	54
7	0	7	14	21	28	35	(42)	(49)	(56)	63
8	0	8	16	24	32	40	(48)	(56)	(64)	72
9	0	9	18	27	36	45	54	63	72	81

➢ Add one more set (any fact). Many children prefer to go to a fact that is "close" and then add one more set to this known fact. Example: Think of 6 x 7 as 6 sevens. Five sevens is close. That's 35. Six sevens is only one more 7, so that makes 42.

The relationship between easy and hard facts is useful. Rather than telling students which strategy is best to use, select a fact from one of the strategies and say, "If you didn't know … (for example 6 x 8) how could you figure it out by using something else you know?"

It would be useful for you to go through each of the 20 "hard facts" and see which strategies from the "Helping Facts" section can be used for each one.

"Drill" refers to repetitive non-problem-based activities and it is appropriate ONLY after students understand a strategy but it has not yet become automatic. There is a place for drill of the basic facts but it is critical that it not be used too early.

After students have worked on two or three strategies, they should be given opportunities to look at multiplication facts and select a strategy that is most helpful in finding the answer.