Fluid Mechanics

Definitions

Fluids

- Substances in either the liquid or gas phase
- Cannot support shear

Density

- Mass per unit volume

Specific Volume

$$v = \frac{1}{\rho} \qquad 22.2$$

Specific Weight

$$\gamma = \lim_{\Delta V \to 0} \left(\frac{g \Delta m}{\Delta V} \right) = \rho g$$

Specific Gravity

$$\mathrm{SG} = \frac{\rho}{\rho_{\mathrm{water}}} = \frac{\gamma}{\gamma_{\mathrm{water}}} \qquad 22.5$$

Fluid Mechanics

Definitions

Example (FEIM):

Determine the specific gravity of carbon dioxide gas (molecular weight = 44) at 66°C and 138 kPa compared to STP air.

$$R_{\text{carbon dioxide}} = \frac{8314\ \dfrac{\text{J}}{\text{kmol}\cdot\text{K}}}{44\ \dfrac{\text{kg}}{\text{kmol}}} = 189\ \text{J/kg}\cdot\text{K}$$

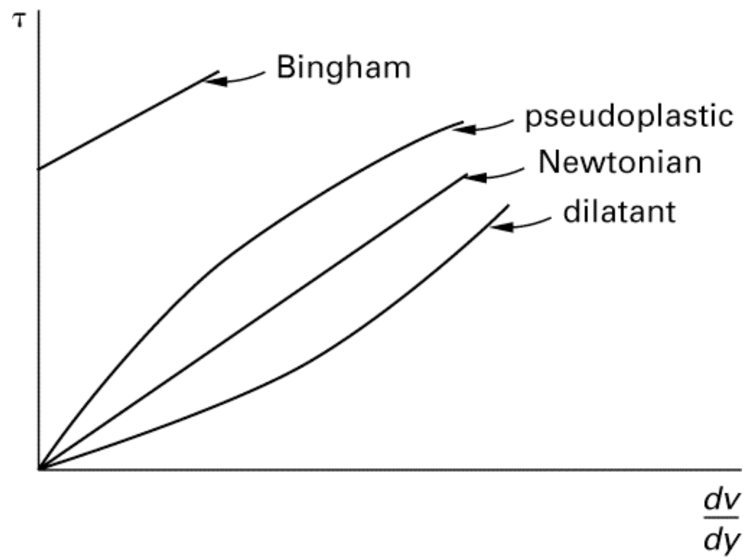
$$R_{\text{air}} = \frac{8314\ \dfrac{\text{J}}{\text{kmol}\cdot\text{K}}}{29\ \dfrac{\text{kg}}{\text{kmol}}} = 287\ \text{J/kg}\cdot\text{K}$$

$$\text{SG} = \frac{\rho}{\rho_{\text{STP}}} = \frac{PR_{\text{air}}T_{\text{STP}}}{R_{\text{CO}_2}Tp_{\text{STP}}} = \left(\frac{1.38\times10^5\ \text{Pa}}{\left(189\ \dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)(66^\circ\text{C}+273.16)}\right)\left(\frac{\left(287\ \dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)(273.16)}{1.013\times10^5\ \text{Pa}}\right) = 1.67$$

Definitions

Shear Stress

- Normal Component: $\tau_n = p$ 22.9
- Tangential Component
  - For a Newtonian fluid: $\tau_t = \mu \frac{dv}{dy}$ 22.11
  - For a pseudoplastic or dilatant fluid: $\tau_t = K\left(\frac{dv}{dy}\right)^n$ 22.12



Definitions

Absolute Viscosity

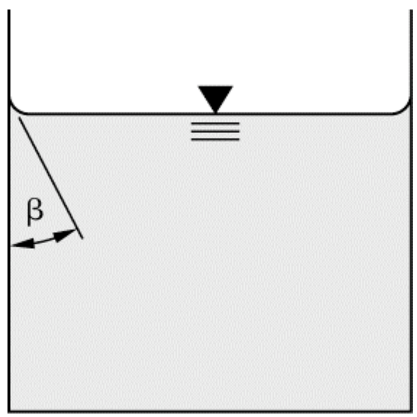
- Ratio of shear stress to rate of shear deformation

Surface Tension

$$\sigma = \frac{F}{L} \qquad 22.14$$

Capillary Rise

$$h = \frac{4\sigma \cos \beta}{\rho d_{\text{tube}} g} \qquad \text{[SI]} \qquad 22.17a$$



Definitions

Example (FEIM):
Find the height to which ethyl alcohol will rise in a glass capillary tube 0.127 mm in diameter.

Density is 790 kg/m^3, $\sigma = 0.0227$ N/m, and $\beta = 0°$.

$$h = \frac{4\sigma\cos\beta}{\gamma d} = \frac{(4)\left(0.0227\ \frac{\text{kg}}{\text{s}^2}\right)(1.0)}{\left(790\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(0.127\times10^{-3}\text{m})} = 0.00923\ \text{m}$$

Fluid Mechanics 9-2a1

Fluid Statics

Gage and Absolute Pressure

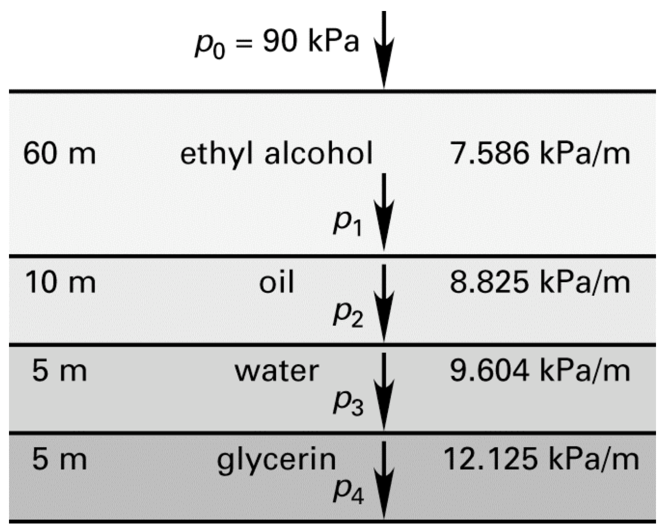
$$p_{absolute} = p_{gage} + p_{atmospheric}$$

Hydrostatic Pressure

$$p = \gamma h + \rho g h$$

$$p_2 - p_1 = -\gamma(z_2 - z_1)$$

Example (FEIM):
In which fluid is 700 kPa first achieved?



(A) ethyl alcohol
(B) oil
(C) water
(D) glyceri

Fluid Statics

$$p_0 = 90 \text{ kPa}$$

$$p_1 = p_0 + \gamma_1 h_1 = 90 \text{ kPa} + \left(7.586 \frac{\text{kPa}}{\text{m}}\right)(60 \text{ m}) = 545.16 \text{ kPa}$$

$$p_2 = p_1 + \gamma_2 h_2 = 545.16 \text{ kPa} + \left(8.825 \frac{\text{kPa}}{\text{m}}\right)(10 \text{ m}) = 633.41 \text{ kPa}$$

$$p_3 = p_2 + \gamma_3 h_3 = 633.41 \text{ kPa} + \left(9.604 \frac{\text{kPa}}{\text{m}}\right)(5 \text{ m}) = 681.43 \text{ kPa}$$

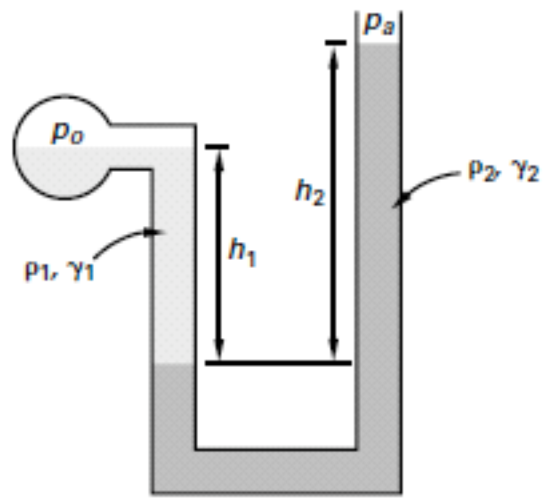
$$p_4 = p_3 + \gamma_4 h_4 = 681.43 \text{ kPa} + \left(12.125 \ \frac{\text{kPa}}{\text{m}}\right)(5 \text{ m}) = 742 \text{ kPa}$$

Therefore, (D) is correct.

Fluid Statics

Manometers

Figure 23.3 Open Manometer

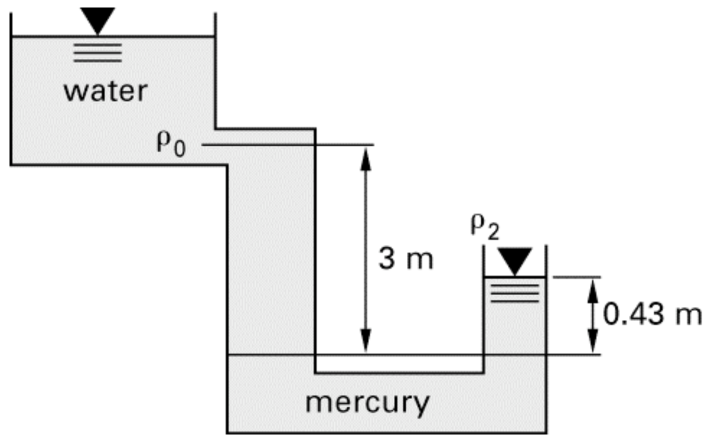


$$p_o - p_a = \gamma_2 h_2 - \gamma_1 h_1 \qquad \text{[U.S.]} \qquad 23.4b$$

Fluid Statics

Example (FEIM):

The pressure at the bottom of a tank of water is measured with a mercury manometer. The height of the water is 3.0 m and the height of the mercury is 0.43 m. What is the gage pressure at the bottom of the tank?



From the table in the NCEES Handbook,

$$\rho_{mercury} = 13560 \ \frac{kg}{m^3} \quad \rho_{water} = 997 \ kg/m^3$$

$$\Delta p = g\left(\rho_2 h_2 - \rho_1 h_1\right)$$

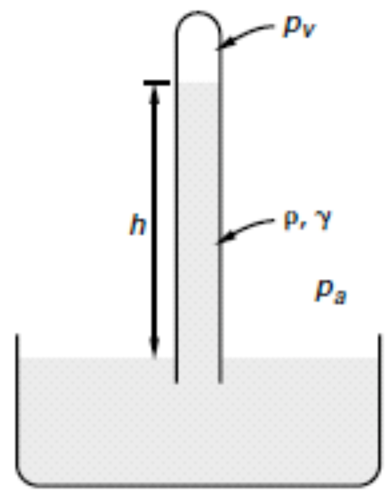
$$= \left(9.81 \frac{m}{s^2}\right)\left(\left(13560 \frac{kg}{m^3}\right)(0.43 \ m) - \left(997 \frac{kg}{m^3}\right)(3.0 \ m)\right)$$

$$= 27858 \ Pa$$

Fluid Statics

Barometer

Figure 23.4 Barometer



Atmospheric Pressure

$$p_a - p_v = \rho g h \qquad \text{[SI]} \qquad 23.7a$$

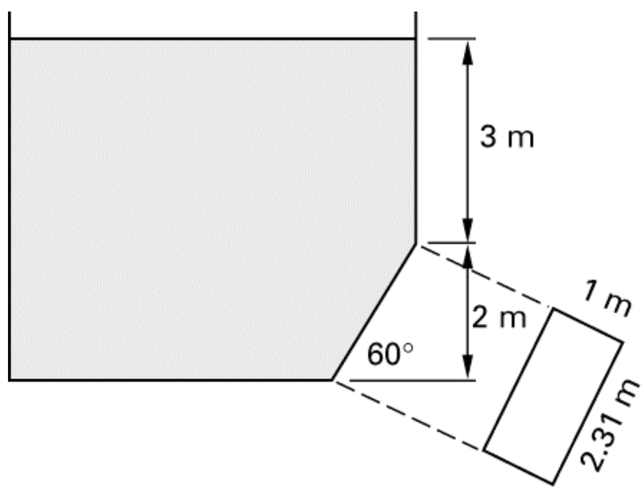
Fluid Statics

Forces on Submerged Surfaces

$$R = pA \qquad 23.8$$

$$\overline{p} = \tfrac{1}{2}\rho g(h_1 + h_2) \qquad \text{[SI]} \qquad 23.10a$$

Example (FEIM):
The tank shown is filled with water. Find the force on 1 m width of the inclined portion.



The average pressure on the inclined section is:

$$p_{\text{ave}} = \left(\tfrac{1}{2}\right)\left(997\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)\left(3\ \text{m} + 5\ \text{m}\right)$$

$$= 39122\ \text{Pa}$$

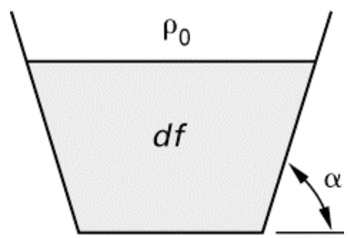
The resultant force is

$$R = p_{\text{ave}}A = \left(39122\ \text{Pa}\right)\left(2.31\ \text{m}\right)\left(1\ \text{m}\right)$$

$$= 90372\ \text{N}$$

Fluid Statics

Center of Pressure



$$y^* = \frac{\rho g I_{yz} \sin\alpha}{p_c A} \quad \text{[SI]} \qquad 23.17a$$

$$z^* = \frac{\rho g I_{yy} \sin\alpha}{p_c A} \quad \text{[SI]} \qquad 23.18a$$

If the surface is open to the atmosphere, then $p_0 = 0$ and

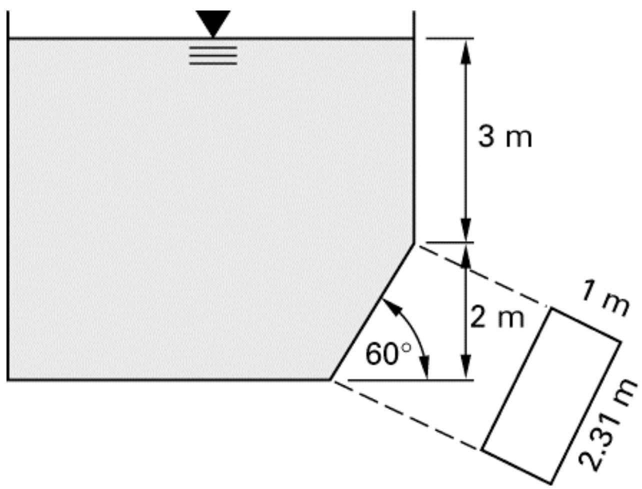
$$p_c = \overline{p} = \rho g z_c \sin\alpha \quad \text{[SI]} \qquad 23.19a$$

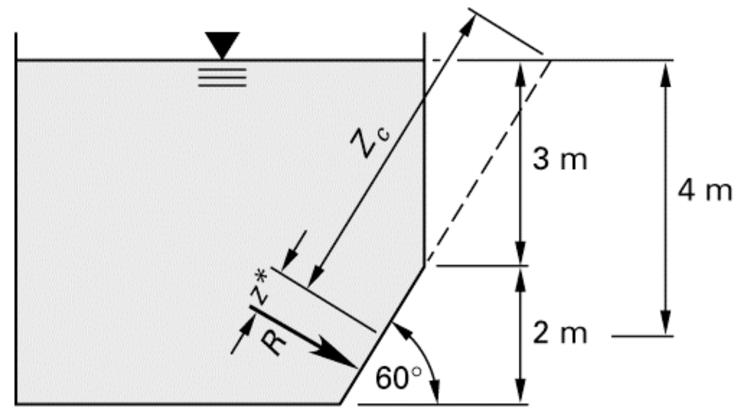
$$y_{cp} - y_c = y^* = \frac{I_{yz}}{z_c A} \qquad 23.20$$

$$z_{cp} - z_c = z^* = \frac{I_{yy}}{z_c A} \qquad 23.21$$

Fluid Statics

Example 1 (FEIM):
The tank shown is filled with water. At what depth does the resultant force act?





The surface under pressure is a rectangle 1 m at the base and 2.31 m tall.

$$A = bh$$

$$I_{y_c} = \frac{b^3 h}{12}$$

$$Z_c = \frac{4\text{ m}}{\sin 60°} = 4.618\text{ m}$$

Fluid Statics

Using the moment of inertia for a rectangle given in the NCEES Handbook,

$$z^* = \frac{I_{y_c}}{AZ_c} = \frac{b^3h}{12bhZ_c} = \frac{b^2}{12Z_c}$$

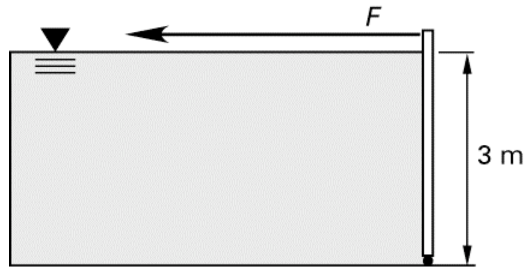
$$= \frac{(2.31\ \text{m})^2}{(12)(4.618\ \text{m})} = 0.0963\ \text{m}$$

$R_{\text{depth}} = (Z_c + z^*)\sin 60° = (4.618\ \text{m} + 0.0963\ \text{m})\sin 60° = 4.08\ \text{m}$

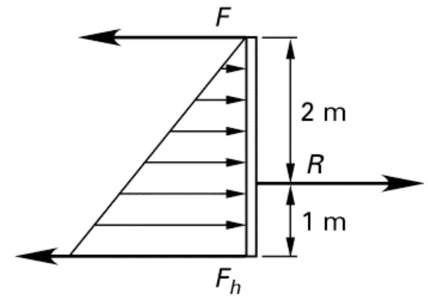
Fluid Statics

Example 2 (FEIM):

The rectangular gate shown is 3 m high and has a frictionless hinge at the bottom. The fluid has a density of 1600 kg/m^3. The magnitude of the force F per meter of width to keep the gate closed is most nearly



(A) 0 kN/m
(B) 24 kN/m
(C) 71 kN/m
(D) 370 kN/m



$$p_{ave} = \rho g z_{ave}(1600\frac{\text{kg}}{\text{m}^3})(9.81\frac{\text{m}}{\text{s}^2})(\tfrac{1}{2})(3\text{ m})$$

$$= 23544 \text{ Pa}$$

$$\frac{R}{w} = p_{ave}h = (23544 \text{ Pa})(3 \text{ m}) = 70662 \text{ N/m}$$

$$F + F_h = R$$

R is one-third from the bottom (centroid of a triangle from the NCEES Handbook). Taking the moments about R,

$2F = F_h$

$$\frac{F}{w} = \left(\frac{1}{3}\right)\left(\frac{R}{w}\right) = \frac{70{,}667\frac{\text{N}}{\text{m}}}{3} = 23.6 \text{ kN/m}$$

Therefore, (B) is correct.

Fluid Statics

Archimedes' Principle and Buoyancy

- The buoyant force on a submerged or floating object is equal to the weight of the displaced fluid.
- A body floating at the interface between two fluids will have buoyant force equal to the weights of both fluids displaced.

$$F_{\text{buoyant}} = \gamma_{\text{water}} V_{\text{displaced}}$$

Fluid Dynamics

Hydraulic Radius for Pipes

$$R_H = \frac{\text{area in flow}}{\text{wetted perimeter}} \qquad 24.26$$

Example (FEIM):

A pipe has diameter of 6 m and carries water to a depth of 2 m. What is the hydraulic radius?

$r = 3 \text{ m}$

$d = 2 \text{ m}$

$\phi = (2 \text{ m})(\arccos((r - d)/r)) = (2 \text{ m})(\arccos \tfrac{1}{3}) = 2.46 \text{ radians}$

(Careful! Degrees are very wrong here.)

$s = r\phi = (3 \text{ m})(2.46 \text{ radians}) = 7.38 \text{ m}$

$A = \tfrac{1}{2}(r^2(\phi - \sin\phi)) = (\tfrac{1}{2})((3 \text{ m})^2(2.46 \text{ radians} - \sin 2.46)) = 8.235 \text{ m}^2$

$$R_H = \frac{A}{s} = \frac{8.235 \text{ m}^2}{7.38 \text{ m}} = 1.12 \text{ m}$$

Fluid Dynamics

Continuity Equation

$$\dot{m} = \rho A \mathrm{v} = \rho Q \qquad 24.2$$

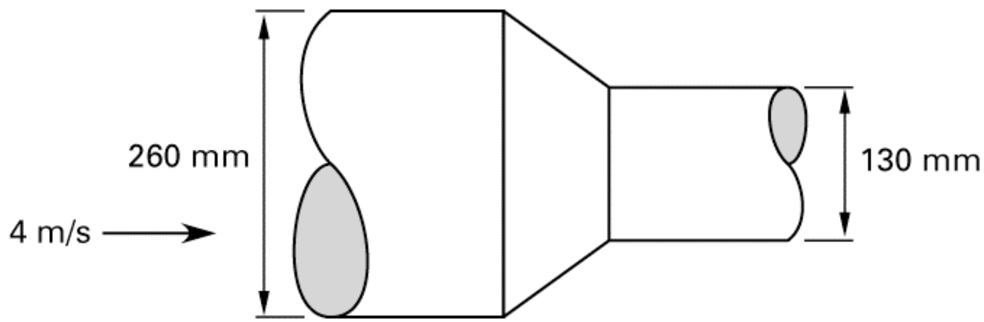
$$\rho_1 A_1 \mathrm{v}_1 = \rho_2 A_2 \mathrm{v}_2 \qquad 24.3$$

If the fluid is incompressible, then $\rho_1 = \rho_2$.

$$Q = A_1 \mathrm{v}_1 = A_2 \mathrm{v}_2 \qquad 24.4$$

Fluid Dynamics

Example (FEIM):



The speed of an incompressible fluid is 4 m/s entering the 260 mm pipe. The speed in the 130 mm pipe is most nearly

(A) 1 m/s
(B) 2 m/s
(C) 4 m/s
(D) 16 m/s

$$A_1 v_1 = A_2 v_2$$

$$A_1 = 4A_2$$

$$\text{so } v_2 = 4v_1 = (4)\left(4\,\frac{\text{m}}{\text{s}}\right) = 16 \text{ m/s}$$

Therefore, (D) is correct.

Fluid Dynamics

Bernoulli Equation

$$\frac{p_1}{\gamma_1} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma_2} + \frac{v_2^2}{2g} + z_2 \qquad \text{[U.S.]} \quad \textit{24.11b}$$

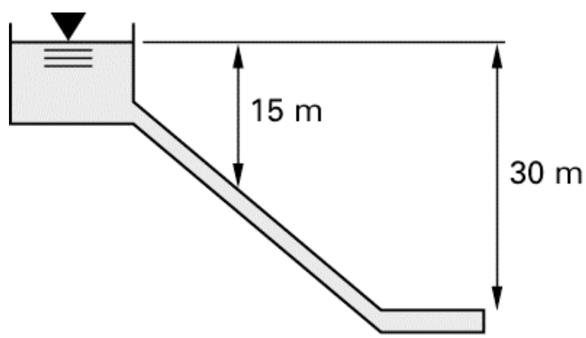
- In the form of energy per unit mass:

$$\frac{p_1}{\rho_1} + \frac{v_1^2}{2} + gz_1 = \frac{p_2}{\rho_2} + \frac{v_2^2}{2} + gz_2$$

Fluid Dynamics

Example (FEIM):

A pipe draws water from a reservoir and discharges it freely 30 m below the surface. The flow is frictionless. What is the total specific energy at an elevation of 15 m below the surface? What is the velocity at the discharge?



Fluid Dynamics

Let the discharge level be defined as z = 0, so the energy is entirely potential energy at the surface.

$$E_{\text{surface}} = z_{\text{surface}}\, g = (30 \text{ m})\left(9.81 \frac{\text{m}}{\text{s}^2}\right) = 294.3 \text{ J/kg}$$

(Note that m^2/s^2 is equivalent to J/kg.)

The specific energy must be the same 15 m below the surface as at the surface.

$$E_{15 \text{ m}} = E_{\text{surface}} = 294.3 \text{ J/kg}$$

The energy at discharge is entirely kinetic, so

$$E_{\text{discharge}} = 0 + 0 + \tfrac{1}{2}\text{v}^2$$

$$\text{v} = \sqrt{(2)\left(294.3 \frac{\text{J}}{\text{kg}}\right)} = 24.3 \text{ m/s}$$

Fluid Dynamics

Flow of a Real Fluid

• Bernoulli equation + head loss due to friction

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f \quad \text{[U.S.]} \quad \textit{24.12b}$$

$$h_f = \frac{p_1 - p_2}{\gamma} \quad \text{[U.S.]} \quad \textit{24.13b}$$

(h_f is the head loss due to friction)

Fluid Dynamics

Fluid Flow Distribution

If the flow is laminar (no turbulence) and the pipe is circular, then the velocity distribution is:

$$v_r = v_{max}\left(1 - \left(\frac{r}{R}\right)^2\right) \qquad 24.20$$

r = the distance from the center of the pipe
v = the velocity at r
R = the radius of the pipe
v_{max} = the velocity at the center of the pipe

Fluid Dynamics

Reynolds Number

For a Newtonian fluid:

$$\mathrm{Re} = \frac{\mathrm{v}D\rho}{\mu} \qquad \text{[SI]} \qquad 24.14a$$

$$\mathrm{Re} = \frac{\mathrm{v}D}{\nu} \qquad 24.15$$

D = hydraulic diameter = $4R_H$

ν = kinematic viscosity

μ = dynamic viscosity

For a pseudoplastic or dilatant fluid:

$$\mathrm{Re}' = \frac{\mathrm{v}^{2-n}D^n\rho}{K\left(\frac{3n+1}{4n}\right)^n 8^{n-1}} \qquad 24.16$$

Fluid Dynamics

Example (FEIM):
What is the Reynolds number for water flowing through an open channel 2 m wide when the flow is 1 m deep? The flow rate is 800 L/s. The kinematic viscosity is 1.23×10^{-6} m²/s.

$$D = 4R_H = 4\frac{A}{p} = \frac{(4)(1\text{ m})(2\text{ m})}{2\text{ m} + 1\text{ m} + 1\text{ m}} = 2\text{ m}$$

$$\text{v} = \frac{Q}{A} = \frac{800\,\frac{\text{L}}{\text{s}}}{2\text{ m}^2} = 0.4\text{ m/s}$$

$$\text{Re} = \frac{\text{v}D}{\nu} = \frac{\left(0.4\,\frac{\text{m}}{\text{s}}\right)(2\text{ m})}{1.23 \times 10^{-6}\,\frac{\text{m}^2}{\text{s}}} = 6.5 \times 10^5$$

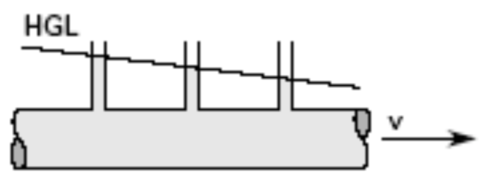
Fluid Dynamics

Hydraulic Gradient

- The decrease in pressure head per unit length of pipe

$$\dot{m} = \rho A v = \rho Q \qquad 24.2$$

Figure 24.2 Hydraulic Grade Line in a Horizontal Pipe



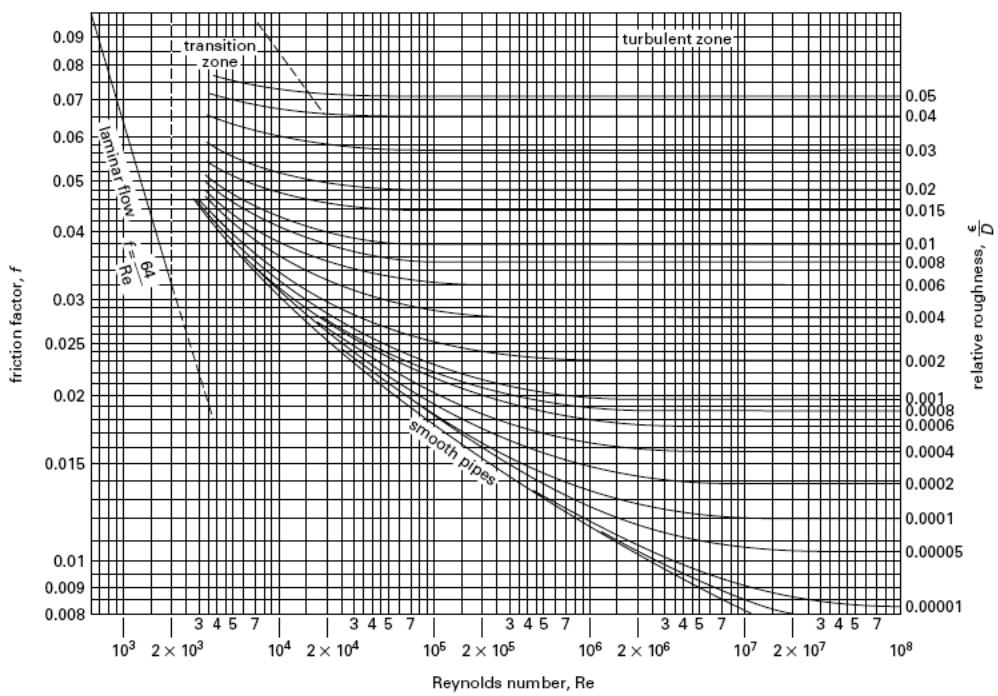
Head Loss in Conduits and Pipes

Darcy Equation

- calculates friction head loss $$h_f = \frac{fLv^2}{2Dg}$$ 24.24

Moody (Stanton) Diagram:

Figure 24.6 Moody Friction Factor Chart



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Fluid Mechanics

Head Loss in Conduits and Pipes

Minor Losses in Fittings, Contractions, and Expansions

- Bernoulli equation + loss due to fittings in the line and contractions or expansions in the flow area

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f + h_{L,\text{fitting}}$$

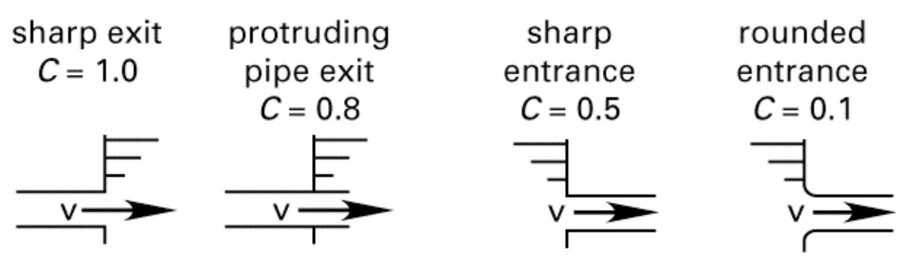
[U.S.] *24.30b*

$$h_{L,\text{fitting}} = C\left(\frac{v^2}{2g}\right)$$

24.31

Entrance and Exit Losses

- When entering or exiting a pipe, there will be pressure head loss described by the following loss coefficients:



Pump Power Equation

$$P = \dot{W} = \frac{Q\gamma h}{\eta} = \frac{Q\rho g h}{\eta} = \frac{\dot{m} g h}{\eta} \qquad 25.1$$

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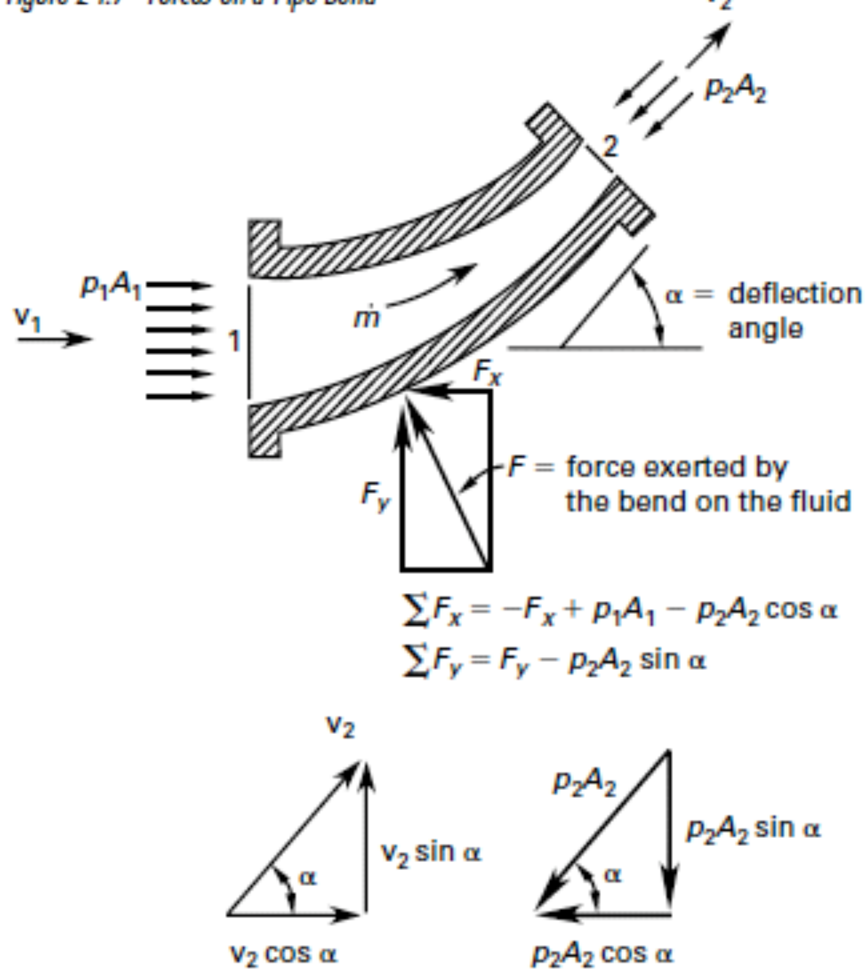
Impulse-Momentum Principle

$$\sum \mathbf{F} = Q_2\rho_2\mathbf{v}_2 - Q_1\rho_1\mathbf{v}_1 \quad \text{[SI]} \qquad 24.38a$$

Figure 24.7 Forces on a Pipe Bend



Pipe Bends, Enlargements, and Contractions

$$-F_x = p_2A_2\cos\alpha - p_1A_1$$

$$+ Q\rho(\mathrm{v}_2\cos\alpha - \mathrm{v}_1) \quad \text{[SI]} \qquad 24.39a$$

$$F_y = (p_2A_2 + Q\rho\mathrm{v}_2)\sin\alpha + m_{\text{fluid}}g \quad \text{[SI]} \qquad 24.40a$$

Fluid Mechanics 9-6b1

Impulse-Momentum Principle

Example (FEIM):

Water at 15.5˚C, 275 kPa, and 997 kg/m^3 enters a 0.3 m × 0.2 m reducing elbow at 3 m/s and is turned through 30˚. The elevation of the water is increased by 1 m. What is the resultant force exerted on the water by the elbow? Ignore the weight of the water.

$$r_1 = \frac{0.3 \text{ m}}{2} = 0.15 \text{ m}$$

$$r_2 = \frac{0.2 \text{ m}}{2} = 0.10 \text{ m}$$

$$A_1 = \pi r_1^2 = \pi(0.15 \text{ m})^2 = 0.0707 \text{ m}^2$$

$$A_2 = \pi r_2^2 = \pi(0.10 \text{ m})^2 = 0.0314 \text{ m}^2$$

By the continuity equation:

$$\text{v}_2 = \frac{\text{v}_1 A_1}{A_2} = \frac{\left(3\,\frac{\text{m}}{\text{s}}\right)(0.0707 \text{ m}^2)}{0.0314 \text{ m}^2} = 6.75 \text{ m/s}$$

Impulse-Momentum Principle

Use the Bernoulli equation to calculate ρ_2:

$$p_2 = \rho\left(-\frac{v_2^2}{2} + \frac{p_1}{\rho} + \frac{v_1^2}{2} + g(z_1 - z_2)\right)$$

$$= \left(997\,\frac{\text{kg}}{\text{m}^3}\right)\left(-\frac{\left(6.75\,\frac{\text{m}}{\text{s}}\right)^2}{2} + \frac{275000\ \text{Pa}}{997\,\frac{\text{kg}}{\text{m}^3}} + \frac{\left(3\,\frac{\text{m}}{\text{s}}\right)^2}{2} + \left(9.8\,\frac{\text{m}}{\text{s}^2}\right)(0\ \text{m} - 1\ \text{m})\right)$$

$$= 247000\ \text{Pa}\quad (247\ \text{kPa})$$

$$Q = \nu A$$

$$F_x = -Q\rho(v_2\cos\alpha - v_1) + P_1A_1 + P_2A_2\cos\alpha$$

$$= -(3)(0.0707)\left(997\,\frac{\text{kg}}{\text{m}^3}\right)\left(\left(6.75\,\frac{\text{m}}{\text{s}}\right)\cos 30^\circ - 3\,\frac{\text{m}}{\text{s}}\right) + (275\times 10^3\ \text{Pa})(0.0707)$$

$$+\ (247\times 10^3\ \text{Pa})(0.0314\ \text{m}^2)\cos 30^\circ$$

$$= 256\times 10^4\ \text{N}$$

Impulse-Momentum Principle

$$F_y = Qp(\mathrm{v}_2 \sin\alpha - 0) + P_2 A_2 \sin\alpha$$

$$= (3)(0.0707)\left(997\ \frac{\text{kg}}{\text{m}^3}\right)\left(\left(6.75\ \frac{\text{m}}{\text{s}}\right)\sin 30^\circ\right)$$

$$+(247\times 10^3\ \text{Pa})(0.0314\ \text{m}^2)\sin 30^\circ$$

$$= 4592\times 10^4\ \text{N}$$

$$R = \sqrt{F_x^2 + F_y^2} = \sqrt{(25600\ \text{kN})^2 + (4592\ \text{kN})^2} = 26008\ \text{kN}$$

Impulse-Momentum Principle

Initial Jet Velocity: $\text{v} = \sqrt{2gh}$ *24.41*

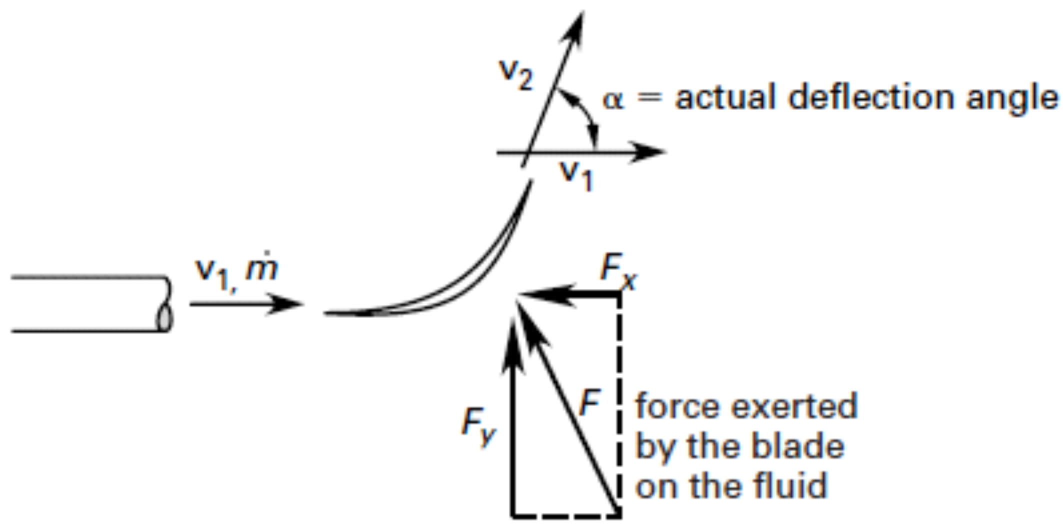
Jet Propulsion:

$$
\begin{aligned}
F &= \dot{m}(\text{v}_2 - \text{v}_1) \\
&= \dot{m}(\text{v}_2 - 0) \\
&= Q\rho\text{v}_2 \\
&= \text{v}_2 A_2 \rho \text{v}_2 \\
&= A_2 \rho \text{v}_2^2 \\
&= A_2 \rho \left(\sqrt{2gh}\right)^2 \\
&= 2g\rho h A_2 \\
&= 2\gamma h A_2 \qquad \textit{24.42}
\end{aligned}
$$

Impulse-Momentum Principle

Fixed Blades

Figure 24.9 Open Jet on a Stationary Blade



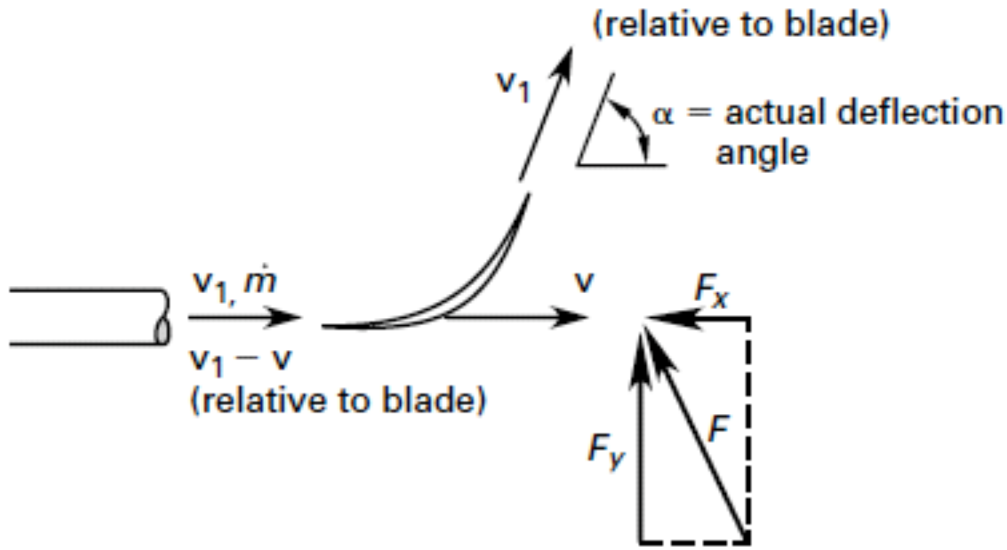
$$-F_x = Q\rho(\mathrm{v}_2 \cos\alpha - \mathrm{v}_1) \quad \text{[SI]} \qquad 24.43a$$

$$F_y = Q\rho\mathrm{v}_2 \sin\alpha \quad \text{[SI]} \qquad 24.44a$$

Impulse-Momentum Principle

Moving Blades

Figure 24.10 Open Jet on a Moving Blade



$$-F_x = -Q\rho(v_1 - v)(1 - \cos\alpha) \quad \text{[SI]} \qquad 24.45a$$

$$F_y = Q\rho(v_1 - v)\sin\alpha \quad \text{[SI]} \qquad 24.46a$$

Impulse-Momentum Principle

Impulse Turbine

Figure 24.11 Impulse Turbine

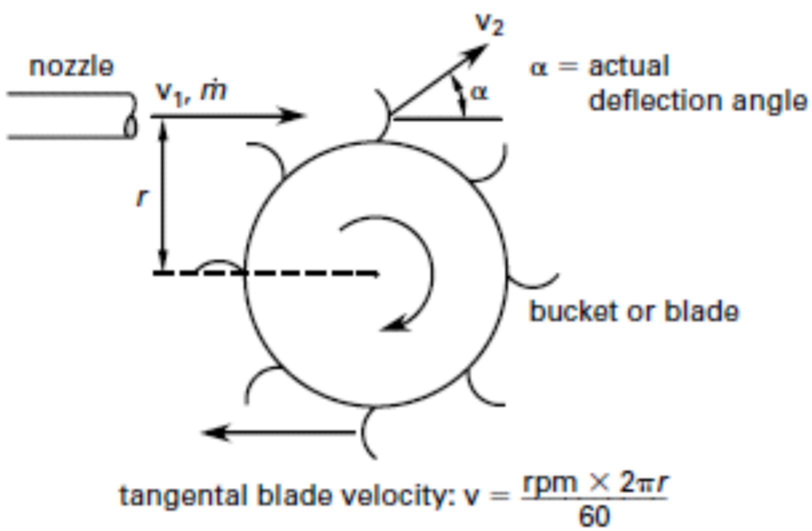
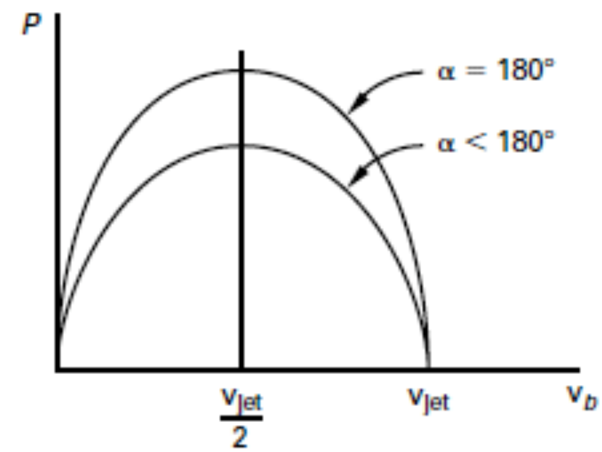


Figure 24.12 Turbine Power



$$P = Q\rho(v_1 - v)(1 - \cos\alpha)v \quad \text{[SI]} \qquad 24.47a$$

The maximum power possible is the kinetic energy in the flow.

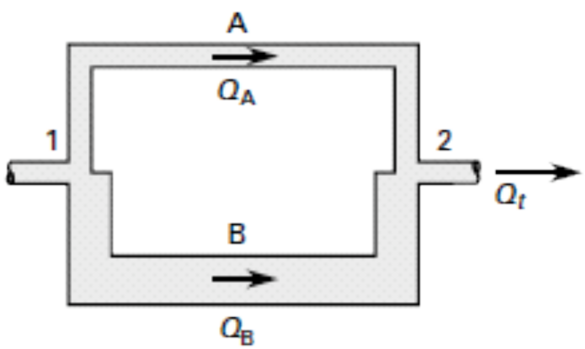
$$P_{max} = \frac{Q\rho v_1^2}{2} \quad \text{[SI]} \qquad 24.49a \qquad P_{max} = \frac{Q\gamma v_1^2}{2g} \quad \text{[U.S.]} \qquad 24.49b$$

The maximum power transferred to the turbine is the component in the direction of the flow.

$$P_{max} = Q\rho\left(\frac{v_1^2}{4}\right)(1 - \cos\alpha) \quad \text{[SI]} \qquad 24.48a$$

Multipath Pipelines

Figure 24.13 Parallel Pipe Loop System



- Mass must be conserved.

$$D^2 v = D_A^2 v_A + D_B^2 v_B$$

1) The flow divides as to make the head loss in each branch the same.

$$h_{f,A} = h_{f,B} \qquad 24.50$$

$$\frac{f_A L_A v_A^2}{2 D_A g} = \frac{f_B L_B v_B^2}{2 D_B g} \qquad 24.51$$

2) The head loss between the two junctions is the same as the head loss in each branch.

$$h_{f,1-2} = h_{f,A} = h_{f,B} \qquad 24.52$$

3) The total flow rate is the sum of the flow rate in the two branches.

$$\frac{\pi}{4} D_1^2 v_1 = \frac{\pi}{4} D_A^2 v_A + \frac{\pi}{4} D_B^2 v_B = \frac{\pi}{4} D_2^2 v_2 \qquad 24.54$$

Speed of Sound

In an ideal gas: $c = \sqrt{kRT}$ [SI] 26.48a

Mach Number: $\mathrm{M} = \frac{\mathrm{v}}{c}$ 26.49

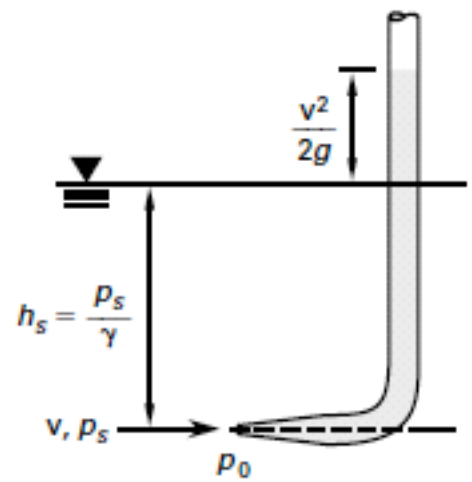
Example (FEIM):
What is the speed of sound in air at a temperature of 339K?
The heat capacity ratio is *k* = 1.4.

$$c = \sqrt{kRT} = \sqrt{(1.4)\left(286.7\ \frac{\mathrm{m}^2}{s^2\cdot\mathrm{K}}\right)(339\mathrm{K})} = 369\ \mathrm{m/s}$$

Fluid Measurements

Pitot Tube – measures flow velocity

Figure 25.1 Pitot Tube



- The static pressure of the fluid at the depth of the pitot tube (p_0) must be known. For incompressible fluids and compressible fluids with M ≤ 0.3,

$$v = \sqrt{\frac{2(p_0 - p_s)}{\rho}} \quad \text{[SI]} \qquad 25.11a$$

Fluid Measurements

Example (FEIM):
Air has a static pressure of 68.95 kPa and a density 1.2 kg/m³. A pitot tube indicates 0.52 m of mercury. Losses are insignificant. What is the velocity of the flow?

$$p_0 = \rho_{\text{mercury}} g h = \left(13\,560 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(0.52\ \text{m}) = 69\,380\ \text{Pa}$$

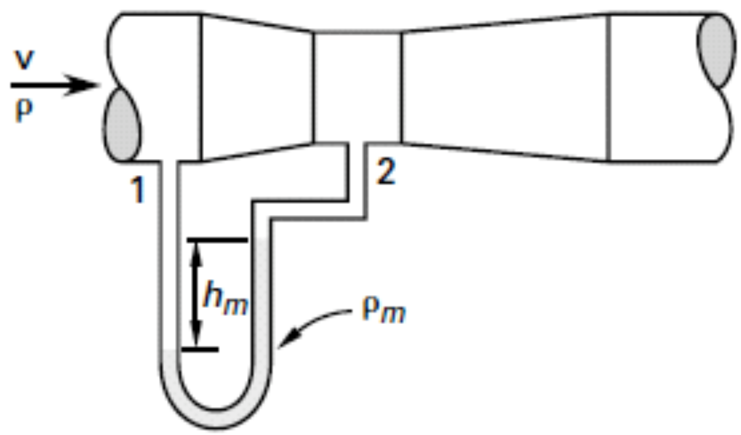
$$\text{v} = \sqrt{\frac{2(p_0 - p_s)}{\rho}} = \sqrt{\frac{(2)(69\,380\ \text{Pa} - 68\,950\ \text{Pa})}{1.2 \frac{\text{kg}}{\text{m}^3}}} = 26.8\ \text{m/s}$$

Fluid Measurements

Venturi Meters – measures the flow rate in a pipe system

- The changes in pressure and elevation determine the flow rate. In this diagram, $z_1 = z_2$, so there is no change in height.

Figure 25.2 Venturi Meter with Differential Manometer



$$Q = \left(\frac{C_v A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \right) \sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)}$$

[U.S.] *25.14b*

Fluid Measurements

Example (FEIM):
Pressure gauges in a venturi meter read 200 kPa at a 0.3 m diameter and 150 kPa at a 0.1 m diameter. What is the mass flow rate? There is no change in elevation through the venturi meter.
Assume $C_v = 1$ and $\rho = 1000 \text{ kg/m}^3$.

(A) 52 kg/s

(B) 61 kg/s

(C) 65 kg/s

(D) 79 kg/s

Fluid Measurements

$$Q = \left(\frac{C_v A_2}{\sqrt{1 - \left(\frac{A_2}{A_1} \right)^2}} \right) \sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2 \right)}$$

$$= \left(\frac{\pi (0.05 \text{ m}^2)^2}{\sqrt{1 - \left(\frac{0.05}{0.15} \right)^2}} \right) \sqrt{2 \left(\frac{200000 \text{ Pa} - 150000 \text{ Pa}}{1000 \frac{\text{kg}}{\text{m}^3}} \right)} = 0.079 \text{ m}^3/\text{s}$$

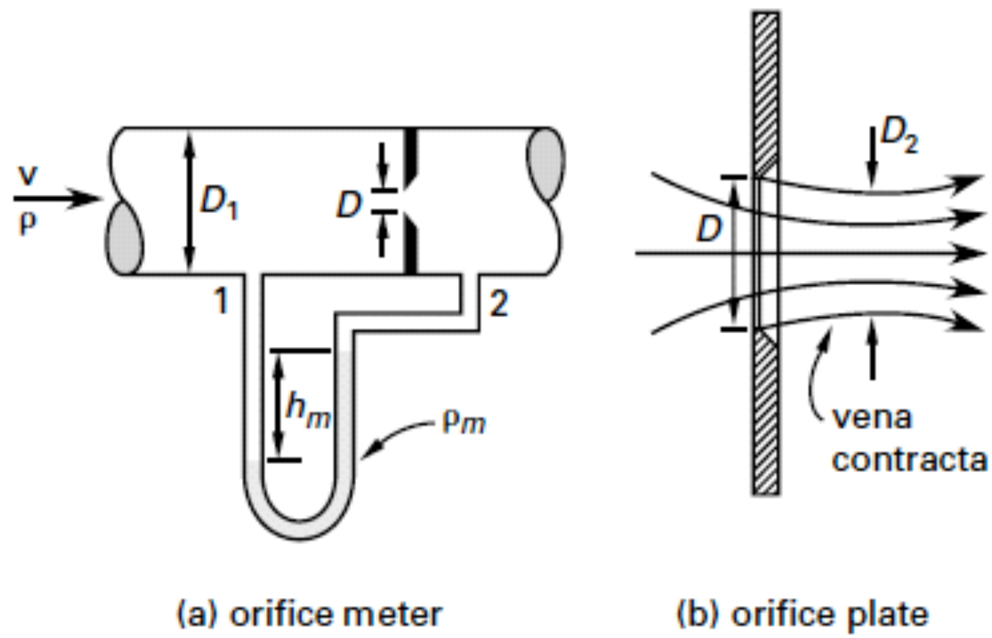
$$\dot{m} = \rho Q = \left(1000 \frac{\text{kg}}{\text{m}^3} \right) \left(0.079 \frac{\text{m}^3}{\text{s}} \right) = 79 \text{ kg/s}$$

Therefore, (D) is correct.

Fluid Measurements

Orifices

Figure 25.3 Orifice Meter with Differential Manometer

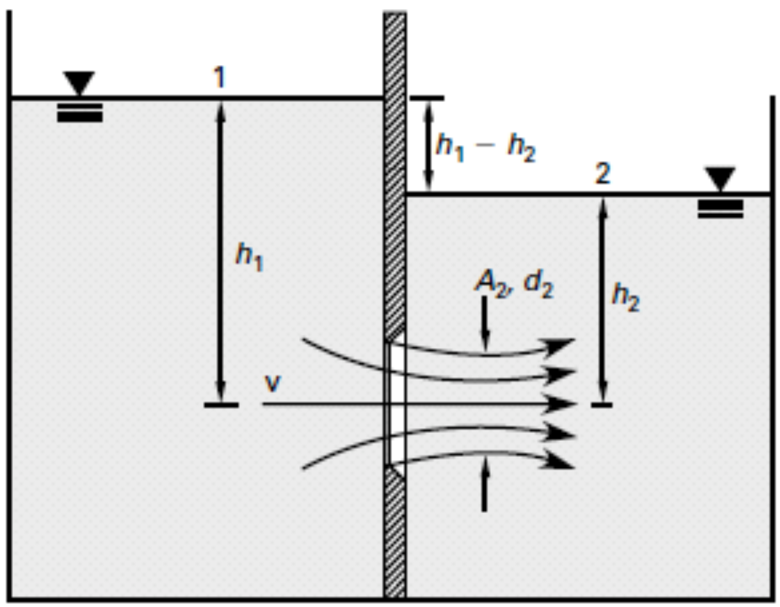


$$Q = CA\sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)} \quad \text{[U.S.]} \quad 25.17b$$

Fluid Measurements

Submerged Orifice

Figure 25.4 Submerged Orifice



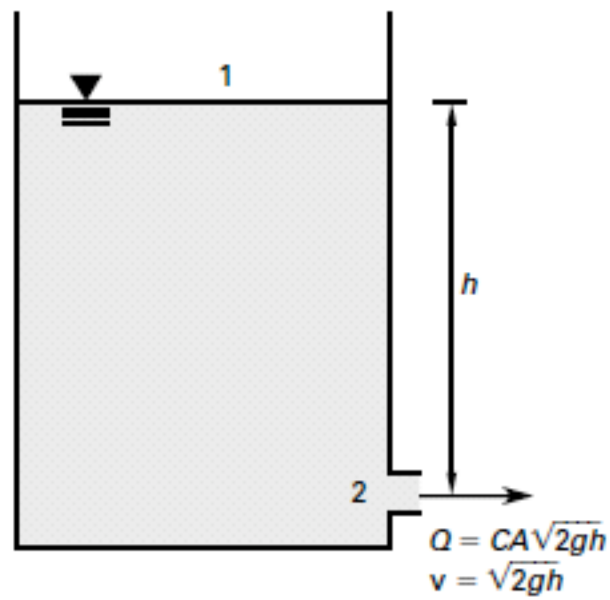
$$Q = A_2 v_2 = C_c C_v A\sqrt{2g(h_1 - h_2)} \qquad 25.18$$

$$C = C_c C_v \qquad 25.19$$

and C_c = coefficient of contraction

Orifice Discharging Freely into Atmosphere

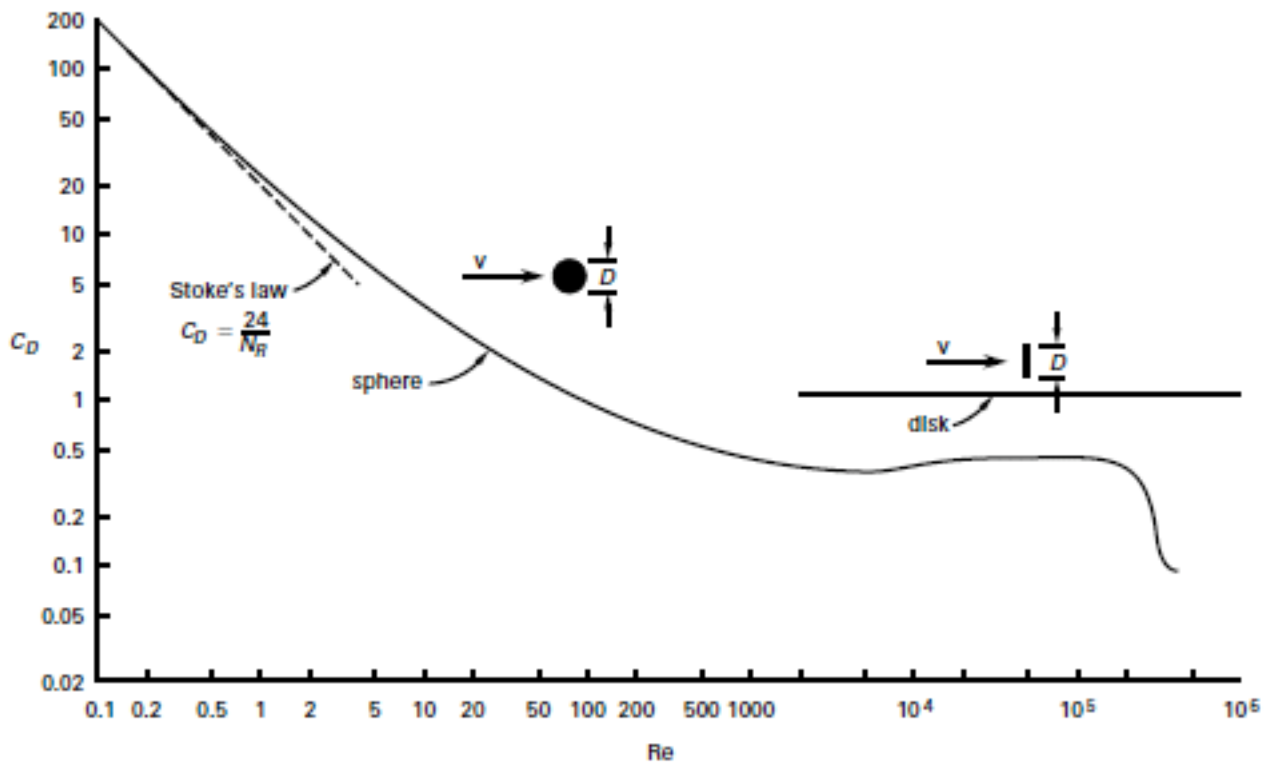
Figure 25.5 Orifice Discharging Freely into the Atmosphere



Fluid Measurements

Drag Coefficients for Spheres and Circular Flat Disks

Figure 24.14 Drag Coefficients for Spheres and Circular Flat Disks



$$F_D = \frac{C_D A \rho v^2}{2} \quad \text{[SI]} \qquad 24.55a$$