Teaching Multiplication and Division Facts to the Whole- to -Part, Visual Learner:

A Guide to Developing Fluency With Math Facts.

For additional support, please go to the web site http://cwoodinmathfacts.tripod.com.

Christopher L. Woodin, Ed. M. 2008

Efficient Fact Learning

• Fluency Involves Accuracy and Efficiency.

Fluency is achieved through the accurate repetition of knowledge, in this case math fact "sentences." It is very important to drill for fluency only after the student is able to produce a math fact accurately. The student should never be placed in a situation that forces them to guess and produce a fact error. Verbalizing inaccurate fact sentences compromises the learning process and leads to frustration. Students should always have the ability to either refer to the correct answer and read it, use a learned strategy to create the answer, or say, "pass" and have the correct answer supplied before expressing the entire fact sentence. A great deal of time is spent in math class developing strategies that promote awareness of fact family patterns, and empower students to construct accurate fact sentences. Progress in efficiency cannot be achieved without accuracy.

• Provide Wait Time For Verbal Expression.

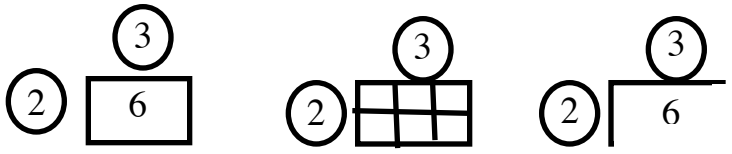
Many students with language-based learning disabilities need ample time to formulate and express their responses. Learned strategies that help your child may involve visualizing a graphic organizer, or referring to a pattern inherent to the fact family. This active reasoning demands processing time. Allow the student to make a response or say "pass" before attempting to cue or prompt a response. Responses may take 15 seconds to formulate. Try to be patient and not let your impatience or frustration rub off on your child. It doesn't help- it just adds to their anxiety level. Time a 15 second period of silence prior to working with your child. It seems like an eternity, but time spent processing in a thoughtful manner will eventually produce an accurate, and increasingly efficient response. In this manner, the efficient expression of math facts may be integrated with the visual, kinesthetic, or semantic (meaning-based) patterns that have been associated with that fact family during math class.

• Multiplication Facts Should Be Practiced in Conjunction With Their Division Counterparts.

Learning facts in a relational context that prompts the student to recognize and express them in both multiplication and division formats is very important. Facts may be expressed with four fact sentences: two multiplication, and two division (with the exception of perfect squares). The entire fact family is modeled with a rectangular matrix diagram that dovetails visually with the area model of multiplication, as well as traditional division notation (see Figure1 below). Gaining exposure to the dynamic expression of fact knowledge allows students to develop necessary flexibility with their knowledge base. Creating these related math sentences from a completed diagram provides a relatively error free opportunity to practice. It also involves processing the three fact elements rather than merely repeating them. Time used to hold the fact information in short term memory while formulating the related facts may facilitate transfer of the fact to long term memory. An initial strategy to facilitate this integration involves the alteration of traditional multiplication syntax (number/word order). Prompt multiplication sentence production with the product. This serves to activate an image of the fact as a whole, as well as related background semantic knowledge. For instance, "Picture a six pack. How is it made?" "6 = 2 x 3."

Figure1.

Rectangular Matrix Diagram | Area Model | Division Notation | Related Facts



Initially:
6 = 2 x 3
6 = 3 x 2
6 ÷ 2 = 3
6 ÷ 3 = 2

Then...
2 x 3 = 6
3 x 2 = 6

Altering the syntax fosters the integration of multiplication and division facts as the three elements of each fact are activated simultaneously, then articulated and stored as a complete unit.

From: Arithmetic for the Whole to Part Learner (in press) 2008, Christopher Woodin, Ed.M

Order Of Fact Instruction

• Cadenced facts: x2, x10, x5, x1

These facts are the most important, and most frequently required facts. They are called cadenced facts as the products may be skip counted with a metered beat- in the manner of a drum cadence. These four fact families comprise 64% of the 100 multiplication facts. They are necessary for multidigit computation, telling time, developing base ten relationships, and fraction simplification. They also lend themselves to many semantic associations that may be modeled and expressed through word problems. Skip counting helps familiarize the student with the products; however, it is not a reliable or efficient means to produce facts. The process of defining factors of products by skip counting may place overwhelming demands on the student's auditory processing abilities. Students often miscount the number of beats of the cadence, or arrive at the correct product without ever rehearsing the entire fact sentence. As a result, they become rigidly dependent on this counting strategy and fail to make progress toward recognizing and retrieving the entire multiplication fact sentence, or a division correlate.

Time spent producing the cadenced facts in an effortless, fluent manner has a marked impact on math learning. These are the fact families that need regular drill and practice to develop fluency. Time spent at home working on these fact families will have the greatest positive impact on your child.

Divisibility Rules

The x2, x10 x5, and x1 fact families have products that are easily recognizable. As a result, divisibility rules have been developed to define them. These rules are helpful in that they provide an efficient means to recognize and accept, or exclude and reject a product from each family. Prior to practicing a multiplication fact family, it is helpful to review its accompanying divisibility rule. The rule will help to constrain the student's products to a pool of acceptable answers. See Figure 2 for a visual depiction of the following rules. Highlighting each fact family on a 1-100 chart provides an opportunity for the student to see these patterns.

Prior to working on a specific fact family, highlight the multiples. Work on one fact family at a time. Achieve fluency with that family before moving to the next. Examine the pattern. Cadenced facts will be represented on the 1-100 chart as columns. Guide the student to detect the following patterns, then develop the following divisibility rules. Find the 1-100 chart on the following page.

• Highlight multiples of two with yellow. They have a 0,2,4,6, or 8 in their one's place. Even numbers.

• Highlight multiples of 5 with red. Even multiples of five always end in zero while odd multiples end in five.

• Multiples of ten will be orange after the 2x and 5x families have been highlighted.. These multiples always have a zero in their one's place.

Figure2

Highlight with yellow.					Highlight with red.		These become orange as a result.
Multiples of 2					Multiples of 5		Multiples of 10
2	4	6	8	10	5	10	10
12	14	16	18	20	15	20	20
					25	30	30
					35	40	40
					45	50	50...

From: Arithmetic for the Whole to Part Learner (in press) 2008, Christopher Woodin, Ed.M

Use the 1 - 100 Chart to Explore Divisibility

Explore each fact family visually using the 1-100 chart. Prior to working on a specific fact family, highlight the multiples. Work on one fact family at a time. Achieve fluency with that family before moving to the next. Guide the student to to generate their divisibility rules based on these patterns. Continue to use the same chart as you explore each fact family in it's proper sequence.

Cadenced Facts: The 2x, 5x, and 10x families create columns on the 1 - 100 chart. These products have similar, recognizable digits in their one's place.

Two Times Family: Color all of the 2x fact products yellow.
The student may skip count to do this. Make sure that the student starts with 2!
Pattern: The 2x products will emerge as 5 (vertical) columns of yellow.
Divisibility rule to generate, then write: Multiples of two have 0,2,4,6, or 8 in their one's place.

Five Times Family: Color all of the 5x fact products red.
Pattern: The 5x products will emerge as 2 (vertical) columns of red.
Divisibility rule: Multiples of five have 0 or 5 in their one's place.

Ten Times Family: The 10x products become orange as they have already been highlighted yellow and red.
Pattern: The 10x products will emerge as 1 (vertical) column of orange.
Divisibility rule: Multiples of ten have a 0 in their one's place.

Other Patterns: The 9x, 3x, 6x families create diagonal patterns that produce divisibility rules.

Nine Times Family: Color all of the 9x fact products dark blue.
Pattern: The 9x products will emerge as a diagonal pattern.
Divisibility rule: Multiples of nine have digits that add to 9, or a multiple of nine (like 18: 1 +8 = 9).

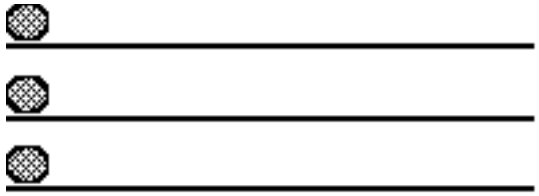
Three Times Family: Color all of the 3x fact products light blue- some will already be dark blue.
Pattern: The 3x products will emerge as a diagonal pattern.
Divisibility rule: Multiples of three have digits that add to 3, 6 or 9, or a multiple of thereof. Each diagonal will add to the same sum of 3, 6 or 9.

Six Times Family: The 6x products become green as they have already been highlighted yellow and blue.
Pattern: The 6x products will emerge as a "checkerboard" diagonal pattern.
Divisibility rule: Multiples of six are even multiples of three, therefore, they must pass the divisibility rules for both 2 and 3.

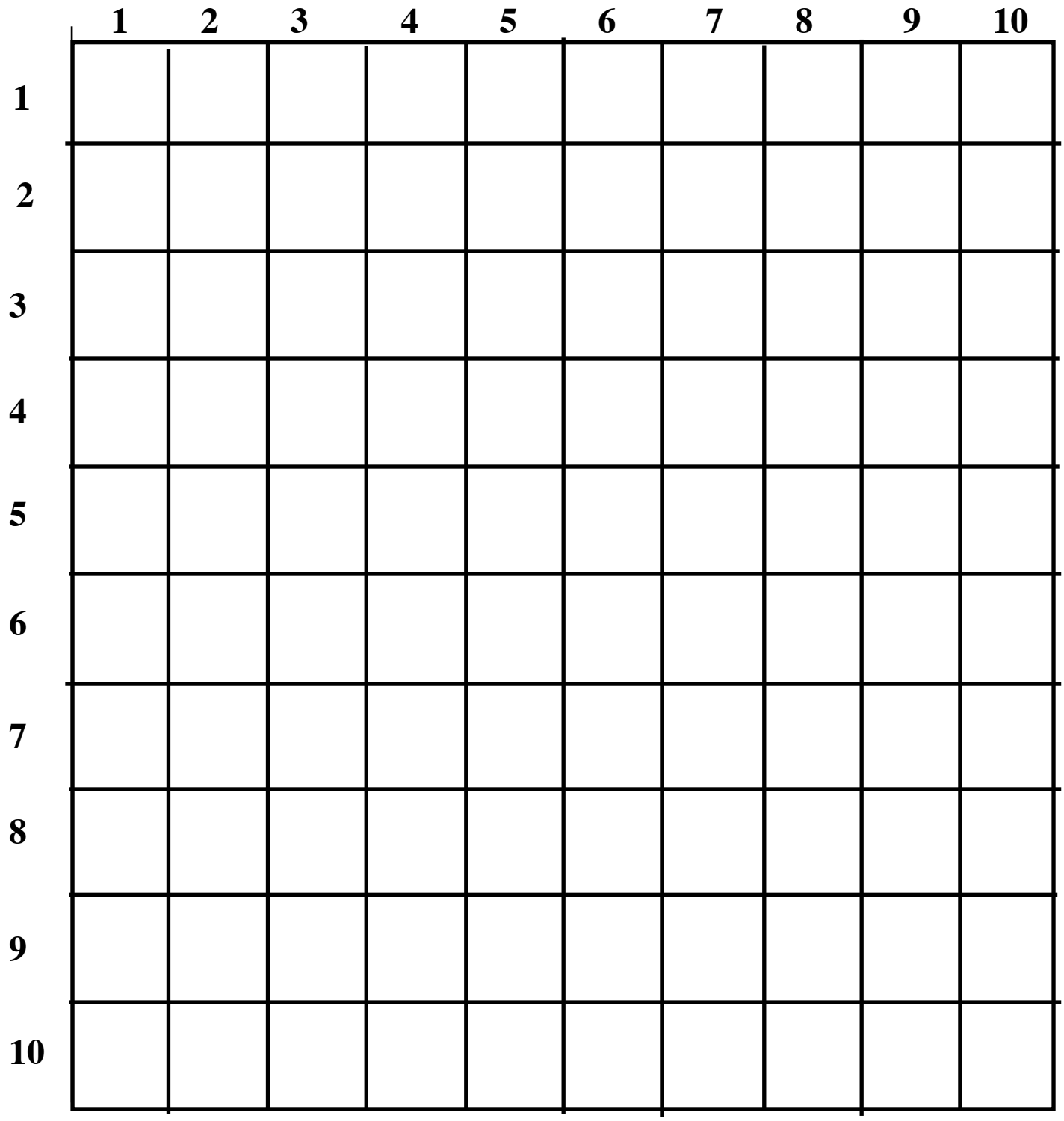
Time spent developing these divisibility rules will have a later benefit. These rules may be used to check multi-digit multiplication products, simplify fractions, and factor numbers. Time should be spent on developing these related skills in conjunction with each math fact family.

1 - 100 Chart

Multiples of: 2 = Yellow, 5 = Red; 10 = Orange
9 = Dark Blue, 3 = Light blue; 6 = green

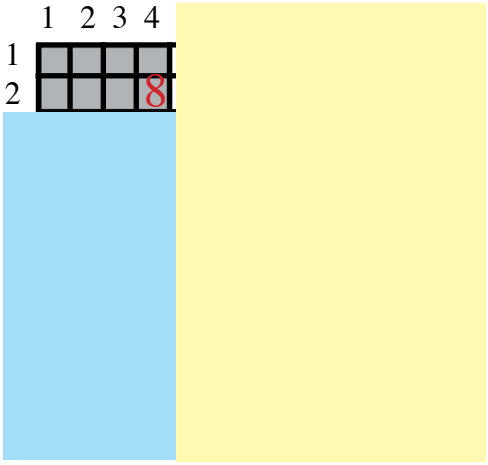


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Multiplication Table Area Model

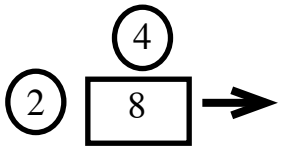
Multiplication Table



Use the multiplication table like a geoboard to show area. Mask the right side and bottom of the multiplication table with two pieces of paper. This will define a rectangular area of the chart that will correspond to a multiplication or division fact.

For instance- note the eight square unit area defined by the 2 x 4 grid shown to the right. This would prompt the student to say, "8 = 2 x 4, or 8÷2 = 4." Start using terminology relating to the concept of area to relate facts with this application. Note that this rectangle also relates to the rectangular matrix diagram that is pictured below.

Rectangular Matrix Diagram



Related Facts

8 = 2 x 4
8 = 4 x 2
8 ÷ 2 = 4
8 ÷ 4 = 2

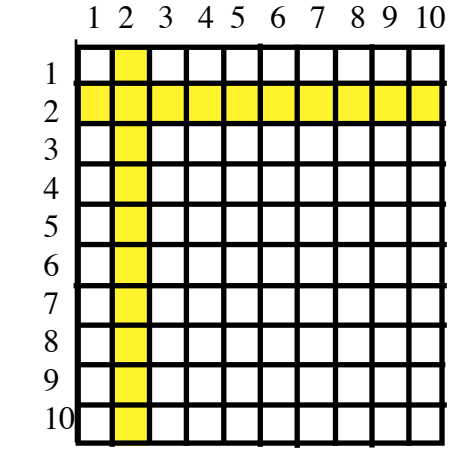
Then...

2 x 4 = 8
4 x 2 = 8

Each fact should be verbalized, and/or written in it's entirety so that it is stored as an intact verbal string. Use visual cues to prompt verbal responses. Use verbal cues to prompt the student to produce a diagram. This cross-modal processing fosters integration between the auditory, fine motor, and visual processing systems.

Avoid asking the student to trace or copy diagrams, or "parrot" facts as they can do so with minimal novel processing. Active processing- the act of building structure serves to prolong the amount of time that the act is held in short term memory. This facilitates the long-term retention of the fact.

Multiplication Grid With Highlighted 2x Facts



Use the multiplication table in conjunction with the 1-100 chart activities that have already been described.. Highlight the appropriate row and column of that specific fact family after identifying these multiples on the 1-100 chart.

After highlighting the row and column, write the products in pencil while working on that fact family. Have the student ink them in when they can be expressed independently.

Highlighting the multiplication chart in this manner provides a great deal of motivation for the student and justifies the suggested order of fact instruction.

Learning the related facts of the most predictable fact families first provides a way for the student to learn many facts belonging to less predictable fact families. For instance, learning 2x7 =14 within the context of the two times family will also teach the student 7x2=14. Soon most of the chart becomes highlighted as facts are assimilated.

- After learning the two times family, 19 facts are learned, 81 remain unfamiliar.
- After learning the five times family, 17 additional facts are learned, 64 remain unfamiliar.
- After learning the ten times family, 15 additional facts are learned, 49 remain unfamiliar. Over 1/2 way!
- After learning the one times family, 13 additional facts are learned, 36 remain unfamiliar.
- After learning the nine times family, 11 additional facts are learned, 25 remain unfamiliar.
- After learning the six times family, 9 additional facts are learned, only 16 remain unfamiliar.
- After learning the perfect squares, 4 additional facts are learned, only 12: 6 pairs are left to memorize.

Use Hands to Model Two-Times Facts and Place Value

Have students hold out their left hand with fingers extended.
* Make sure that their hand is within their visual field- so that they can see it.
Teacher: “You are showing me five fingers, one time.”

The teacher should model this by mirroring their image - extending his right hand as he faces the student.
“High five” the student to make a “ clap.”
Teacher: “ This is five, two times or one ten.”

Teacher’s right hand

“ Clap! ”

Student’s left hand

The teacher should ask the student to keep
the student’s left fingers extended.
Teacher: “Now show me seven fingers by sticking out two more fingers
(on your right hand).”
Teacher: “You are showing me seven one time.”
As the teacher faces the student,
the teacher models this by mirroring their image:
The teacher extends the five fingers on his right hand,
as well as two from his left hand.
Teacher: “ This is fourteen. It is seven two times. Two times seven is fourteen”

Teacher’s hands:

“ Clap! ”

Student’s hands:

This hand model represents the product 14 within the students primary reference frame. This provides the student with a internally-based place-value template to help him encode the digits in their proper order (14, not 41).

1
One clap:
one ten.

4
2 fingers
+ 2 fingers
4 fingers

Shootout Game

Start the game in the ready position: The teacher is extending all five of the fingers on his right hand. This hand should hang at belt level- like an old west gun fighter about to “draw” his sidearm.
The student should assume a similar posture -have his five left hand fingers extended with his left hand hanging at his side.
The teacher should call out a number from 5 to 9. That is the signal to “draw.”
Both teacher and student rush to display the quantity of fingers to match the number called-out- pointing their hands at one another.

The pairs of hands will create a two times fact that should be verbalized by the winner of this finger drawing contest. The other person should state a related fact.
For example: The teacher calls out, “seven.” Both teacher and student “shoot-out” seven fingers as pictured above. The teacher and student should move toward each other, clap their five-fingered hands, and identify the number of additional fingers that will comprise the one’s digit of the product (in this case 2+2=4)
If the student gets his seven extended first, he must verbalize that 2x fact: “14 = 2 x 7, ” or “ 7 x 2 = 14.”
The teacher should respond with a related division fact, “ 14 ÷ 2 = 7.”

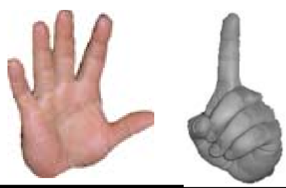
2x Facts: Cut out these flash cards on the dotted lines.
Students mirror the fingers to produce “two times” the quantity of fingers, then verbalize the four related facts.

2

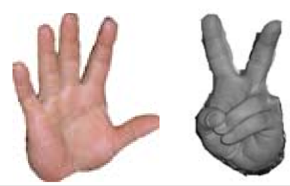
2

2

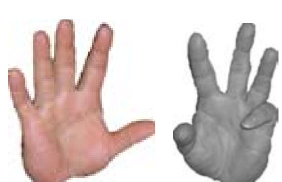
2



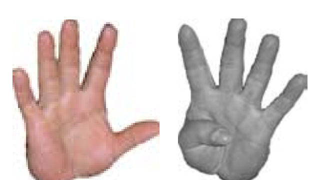
2



2



2



2

More Activities to Learn 2x Facts

Fact Twister

This activity reinforces number facts as well as directionality. It can be played individually or in small groups. Write the products of the fact family you are addressing on paper plates. Arrange the paper plates on the floor in front of the student. Have the students answer your fact questions by touching the correct plate with the appendage you name.

• Example

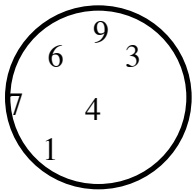
Teacher, "Put you right foot on the product of two times five."
When a student correctly performs the task, signal success by asking for the parroted fact,
2x5 = 10; the commuted multiplication fact, 5 x 2 = 10; or one of the two related division facts,
10 ÷5 = 2 or 10 ÷ 2 = 5.

Rad Minute

After the student writes his name, date and day on the Rad Minute sheet, have him fold the paper along the dotted line. The first order of business is to have the student fill-in the products from top to bottom. This may be achieved by skip counting by twos: " 2, 4, 6..." etc. Next, have the student check all of the products using the 2x divisibility rule:

"All of the products must have a 0,2,4,6 or 8 in their one's place."

The teacher should also check the products for accuracy within each fact. For instance, be on the lookout for errors like 2x7 =18. A check for divisibility will not pick up this type of error. After that, have the student copy each math fact in its entirety. Each fact should be written from left to right like a sentence. Do not allow the student to write in columns: all of the first digits of the facts, then all of the "x" signs, etc. The object is to have the student rehearse each fact as a unit so that it may be stored, then later retrieved from auditory memory.
See the Rad Minute sheet on the following page.



Fact Ball

After practicing a multiplication fact family using flash cards or another production task, play a game of fact ball. Write numbers on a soft, light ball - perhaps a tennis ball. Toss the ball to the student. The student should use the first number that he sees on the ball to begin the fact sentence. Initially, write only those factors that are most familiar, perhaps 1,2,5,10. Add more factors as the student gains confidence with harder facts.
For example, the teacher tosses the ball to Johnny who catches it with one hand. Have Johnny read the number from the ball that is closest to his thumb. " Six," says Johnny. "Ok," says the teacher, "say a 2 times sentence starting with 6...."
Johnny says, " Six times two is 12." The teacher responds with a related fact, "Two times six is 12."
The teacher may also choose to model one of the two related division facts: 12÷2=6, or 12 ÷6 =2. Johnny then tosses or rolls the ball to the teacher and the roles are reversed.

Make sure that the student has the ability to look up each correct math fact if necessary. The student should never be forced to guess at an answer. If unsure of a fact, prompt the student to either wait and think, look it up, or say "pass"- and have the answer supplied by the teacher. The student should then rehearse the fact by saying it in its entirety.

• Constant Time Delay

This process involves showing the student a flash card, then having the student raise his hand or point to an answer when he is able. After waiting four seconds, clap your hands to prompt the student to respond with the missing factor or product. If his answer is correct, have him then recite a complete multiplication sentence and a related division sentence. The imposed delay encourages the student to internally recite the fact. Research has shown this procedure to facilitate long term retention of facts in some children

Use Gross Motor Kinesthetic Practice to Drive Integrated Fact Production

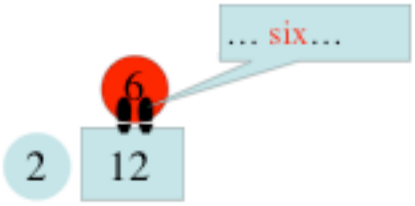
Many students have fine motor processing delays or impediments. These issues are often evidenced by relatively low score on the processing Speed Index score of the W.I.S.C. These students may have fine motor planning issues that may impact their handwriting output, or fine motor problems associated with their verbal articulation. Their slow production rate, exacerbated perhaps by inefficient sensory-motor or visual feedback loops prevent them from refining their rate or quality of production. Gross motor-kinesthetic practice may have an immediate positive effect on their ability to integrate visualand auditory processing with motor skills.

Describe the gross motor exercise here

Work on fluency by verbalizing four facts while hopping on the three elements of the fact diagram.

1

The student must maintain a consistent reference frame (facing straight ahead).



2

3

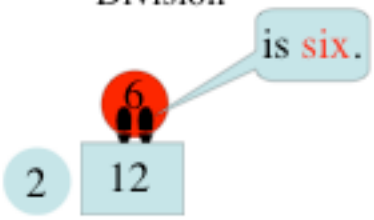
Division

4

Division

5

Division



6

RAD MINUTE

1) Fill in blanks.
2) Check with divisibility rules.
3) Copy each number sentence.

Correct
/20

Warm -up

Copy:

1 x 2 =

---

2 x 2 =

3 x 2 =

4 x 2 =

5 x 2 =

6 x 2 =

7 x 2 =

8 x 2 =

9 x 2 =

10 x 2 =

One Minute Quiz

3 x 2 =

8 x 2 =

2 x 2 =

6 x 2 =

9 x 2 =

4 x 2 =

1 x 2 =

7 x 2 =

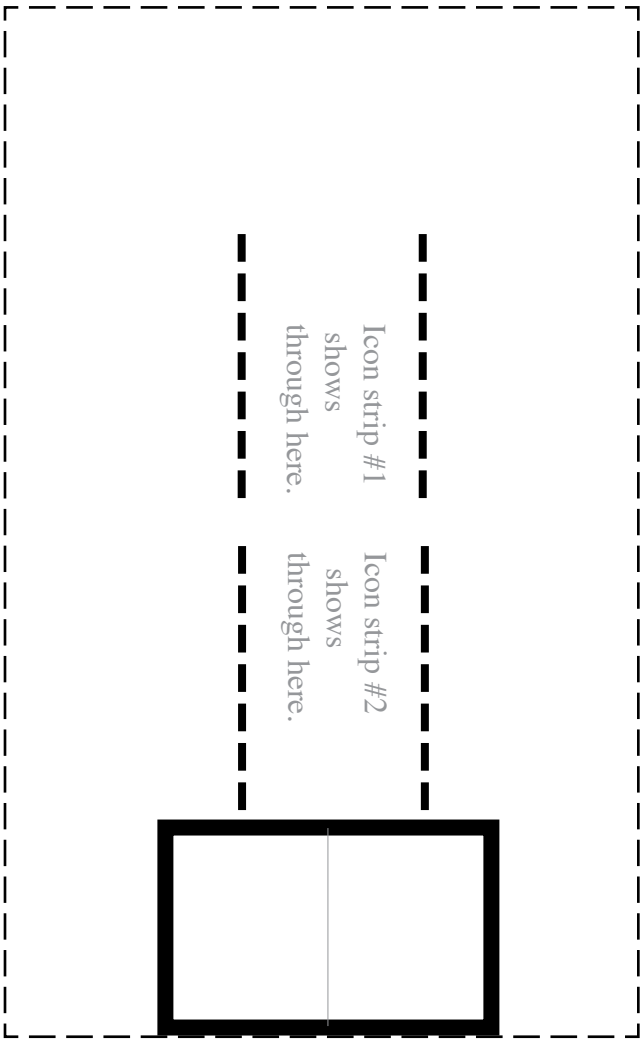
10 x 2 =

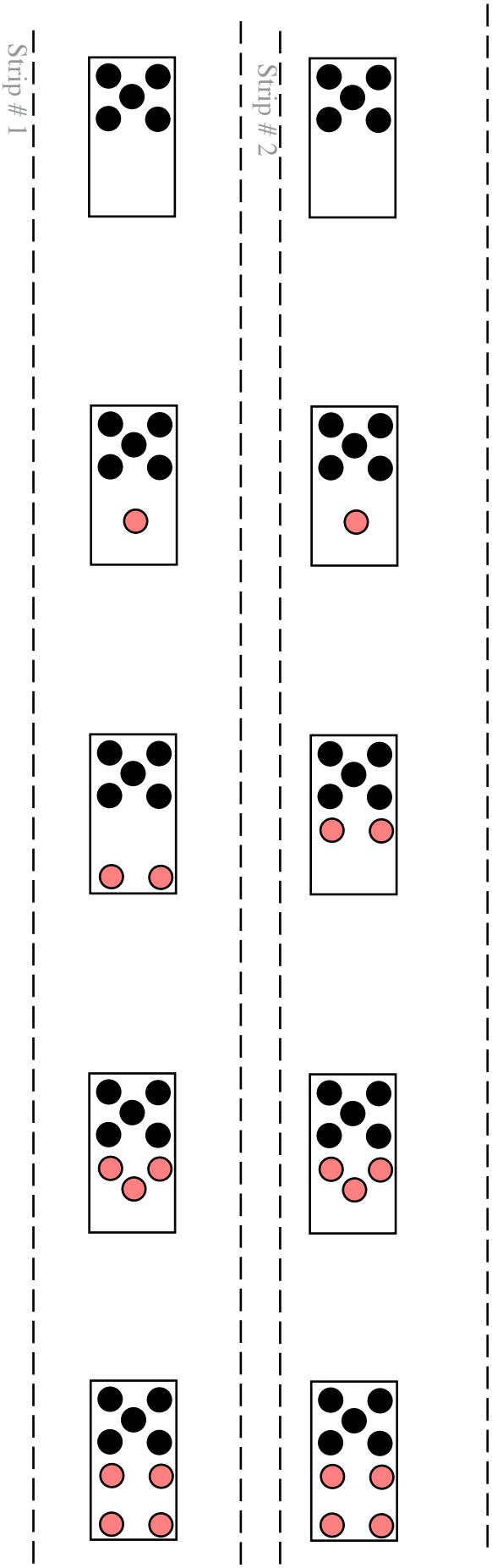
5 x 2 =

Adding with the Icon Slide Rule

• Cut -out the two number line strips.
•Tape strip #1 to the student's desk in a horizontal format.
• Place the second number line strip under the taped strip.
• Move the second strip right or left to align the two icons to be added.

Option #2 -Make a slide rule out of the two icon number line strips.
Reinforce the strips and holder by putting tape on their backs before cutting them out. Cut out the holder, then put a knife slit in along each dark line. Insert the strips into the tachistoscope (holder) below.
• Move the strips so that the Icons to be added are aligned between the slits.





Fact Mosaics

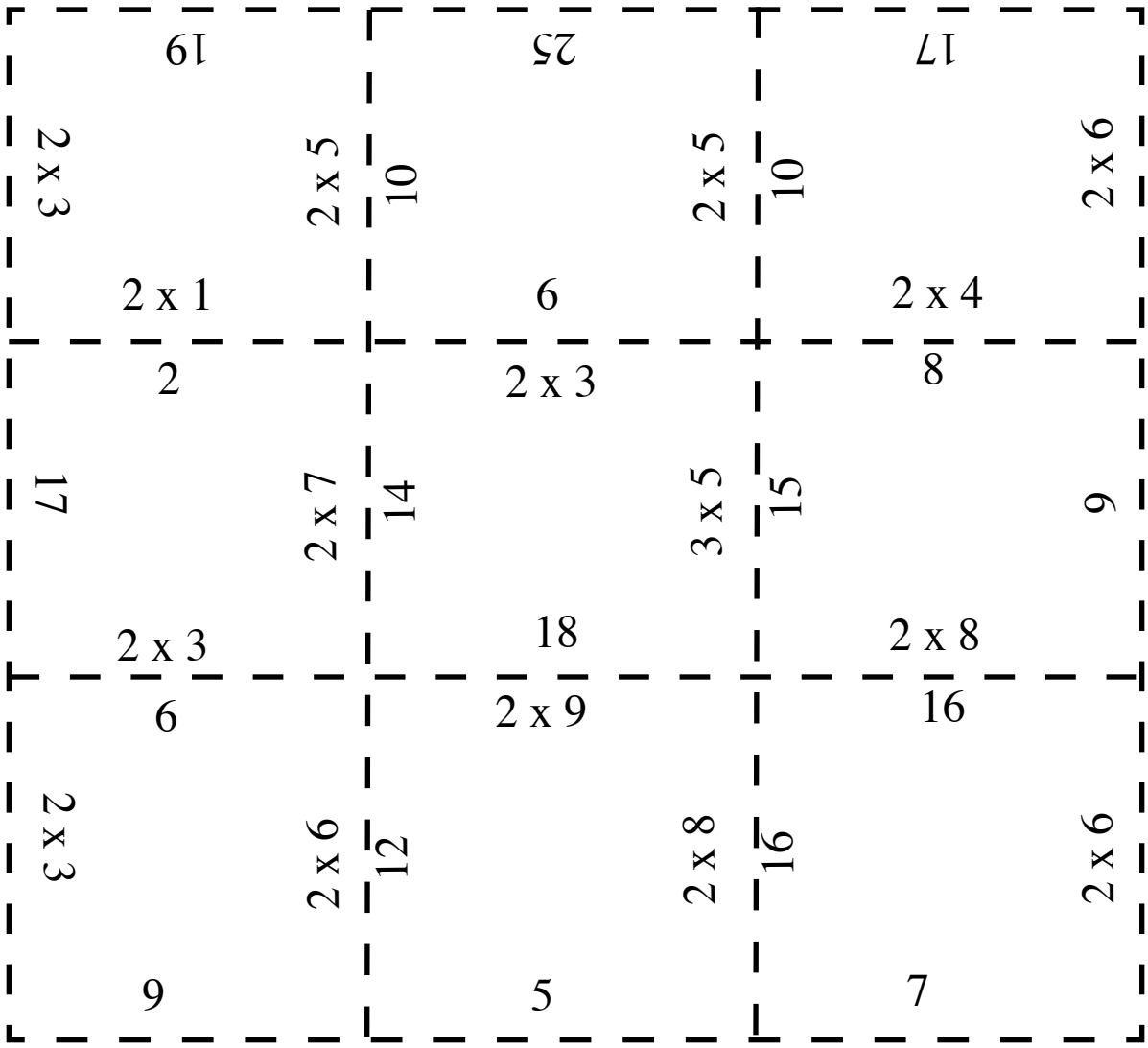
This jigsaw-type puzzle reinforces number facts, the following of sequential directions, directionality, problem-solving, and perception of multiple reference frames. It is played either individually or in small groups. Start by having each student complete a Rad Minute for the fact family you want to address. Rad Minute templates can be found on the website http://cwoodinmathfacts.tripod.com. Correct the worksheets to ensure correct number sentences. These number facts are the only clues for reassembling the puzzle once it is cut into pieces.

Give each student a square piece of paper divided into nine equal squares, like a tic-tac-toe grid. Draw the lines carefully so the nine pieces are identical. In the future, students can make the grids following microunited instructions.

Using the corrected Rad Minute sheets, have the students write the product (six) on one side of a grid line and the partial fact statement (2 x 3) on the other. Again, check to see that the products or quotients are accurate. When students have labeled all grid lines with a statement and opposing product, turn the sheet over. Have students draw a simple design that covers the entire back of the sheet. Cut the paper along the grid lines to produce a mosaic of nine pieces.

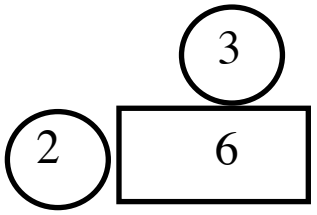
Ask students to reassemble this jigsaw-type puzzle using the front side. They can check the puzzle by flipping it over to the design side. Once students reassemble their own mosaic they can trade mosaics with another student. This activity is a good way to reinforce any type of short question and answer facts.

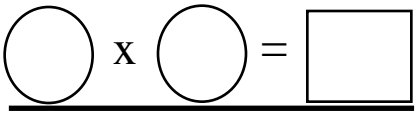
2x Math Mosaic: Cut along the dotted lines to make a jig saw puzzle to reassemble.

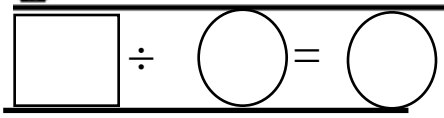


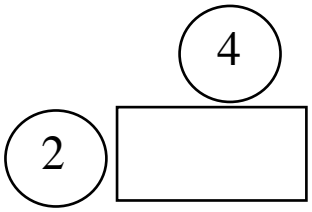
Diagrams that Generate Multiplication and Division Facts

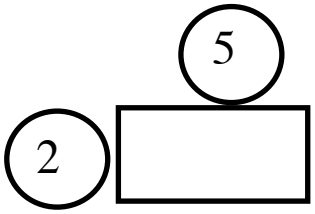
- Complete the diagrams.
- After they are checked, write four related multiplication and division sentences for each diagram.

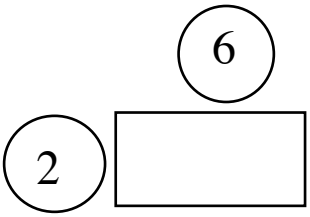


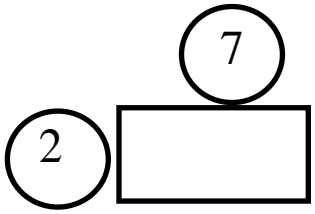


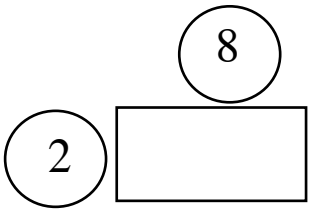


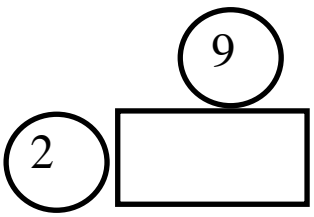






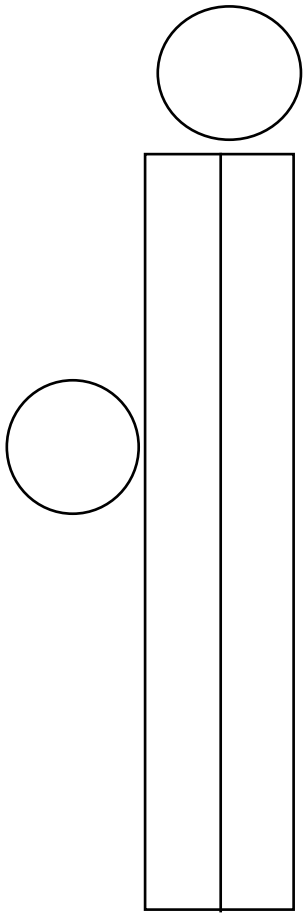




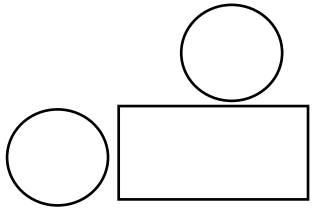


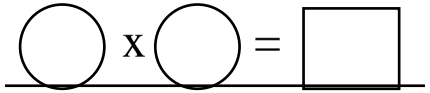
Learning 10X Facts Using a Matrix Diagram of Base 10 Rods

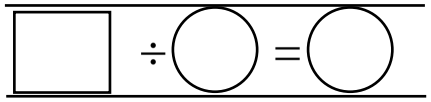
Use Base-Ten "Sticks" to help produce facts.

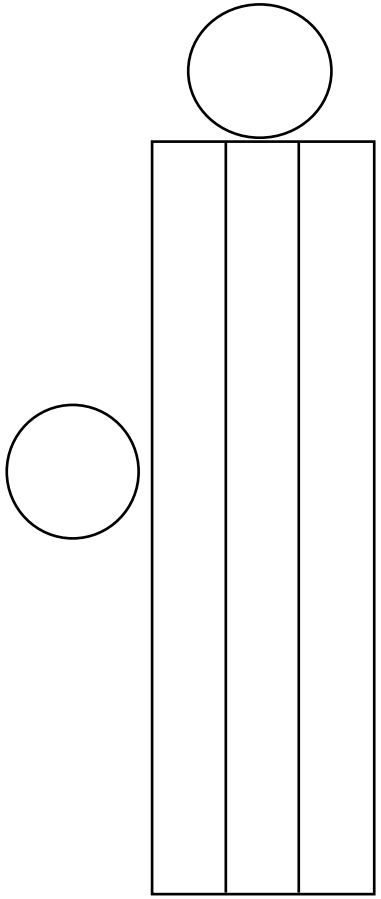


Diagram

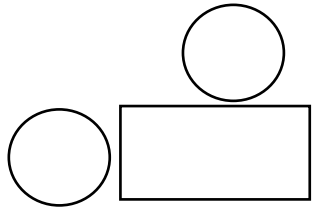






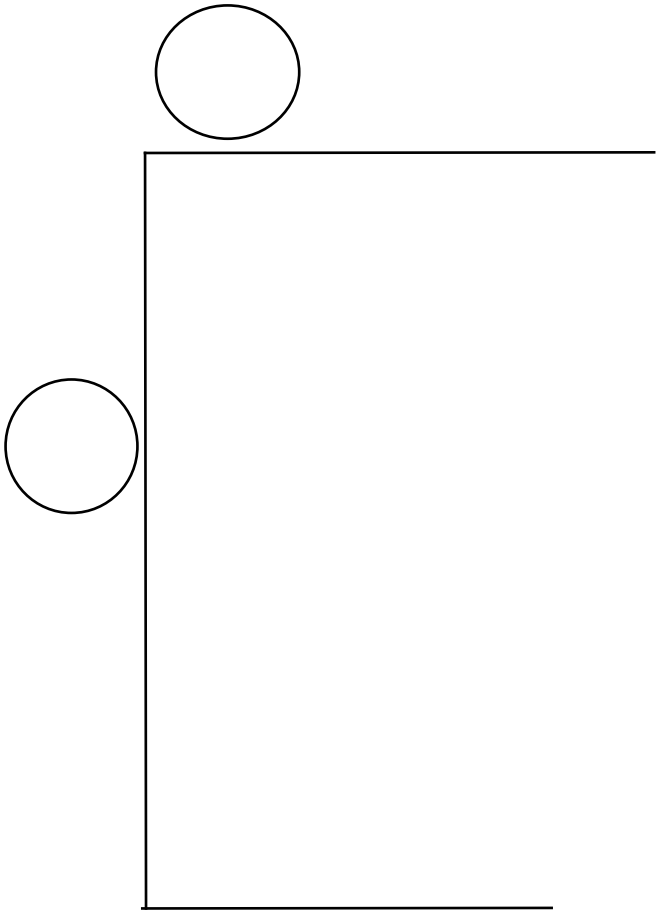


Diagram

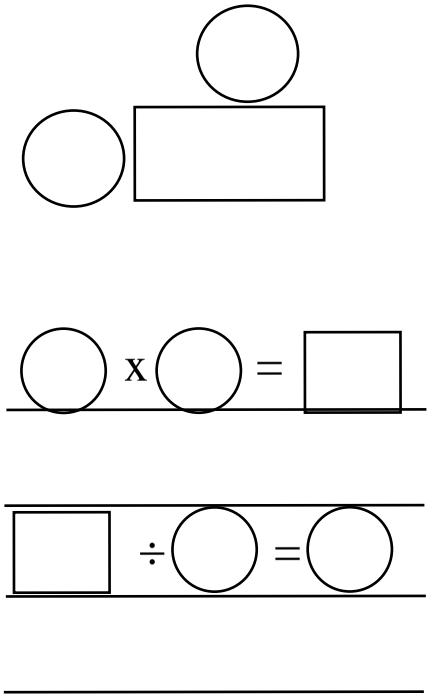


Learning 10X Facts Using a Matrix Diagram of Base 10 Rods

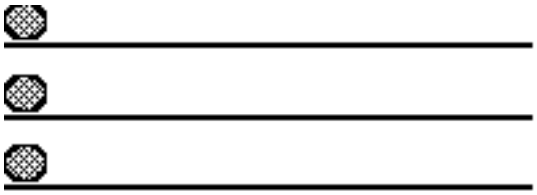
Use Base-Ten "Sticks" to help produce facts.



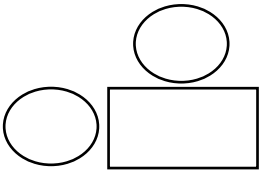
Diagram



Value # Dimes

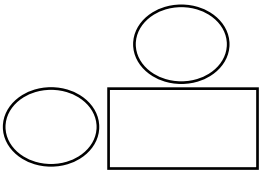


_____ cents = _____ x 10



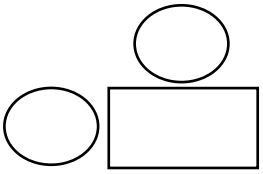
Value # Dimes

_____ cents = _____ x 10



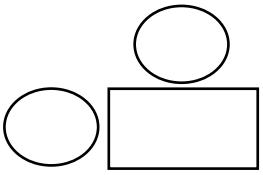
Value # Dimes

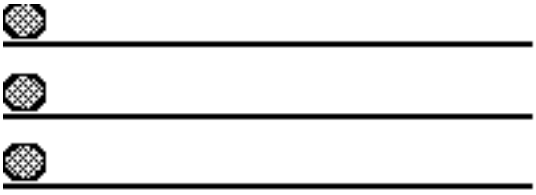
_____ cents = _____ x 10



Value # Dimes

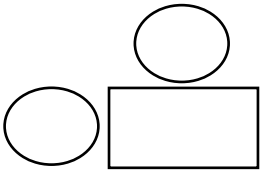
_____ cents = _____ x 10





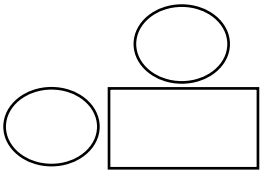
Value # Dimes

_____ cents = _____ x 10



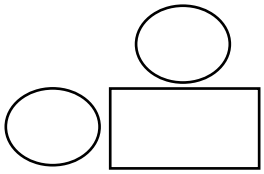
Value # Dimes

_____ cents = _____ x 10



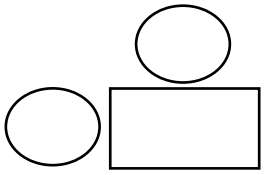
Value # Dimes

_____ cents = _____ x 10



Value # Dimes

_____ cents = _____ x 10



RAD MINUTE

1) Fill in blanks.
2) Check with divisibility rules.
3) Copy each number sentence.

Correct
/20

Warm -up

Copy:

1 x 10 =

2 x 10 =

3 x 10 =

4 x 10 =

5 x 10 =

6 x 10 =

7 x 10 =

8 x 10 =

9 x 10 =

10 x 10 =

One Minute Quiz

3 x 10 =

8 x 10 =

2 x 10 =

6 x 10 =

9 x 10 =

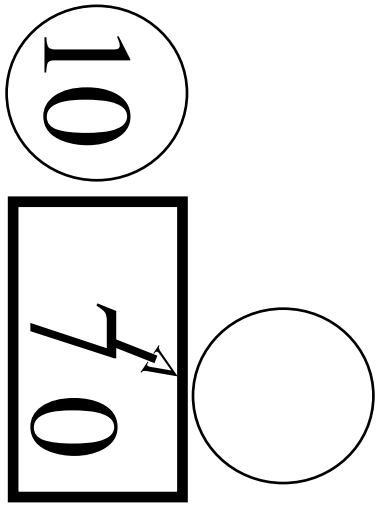
4 x 10 =

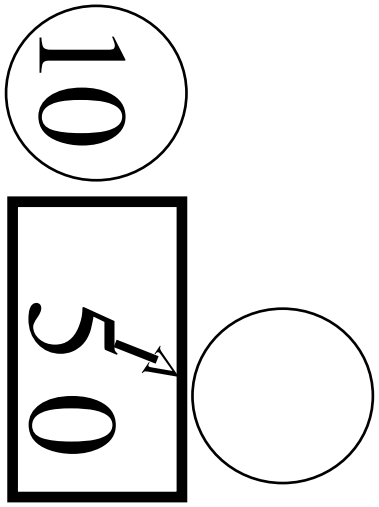
1 x 10 =

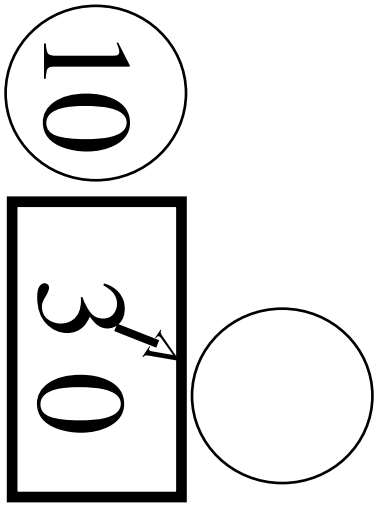
7 x 10 =

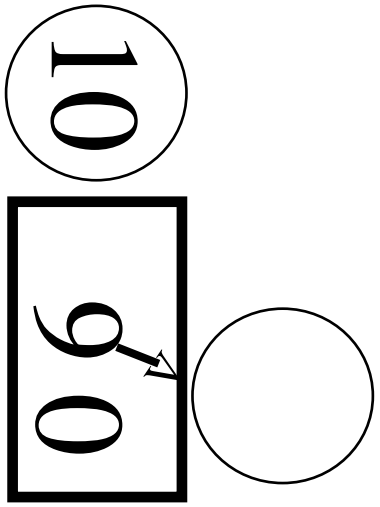
10 x 10 =

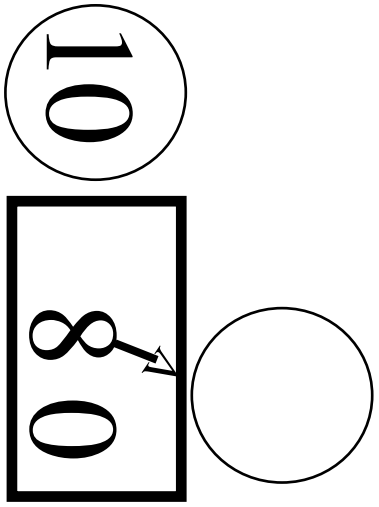
5 x 10 =

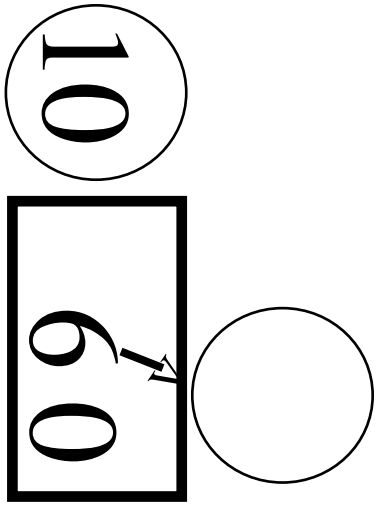


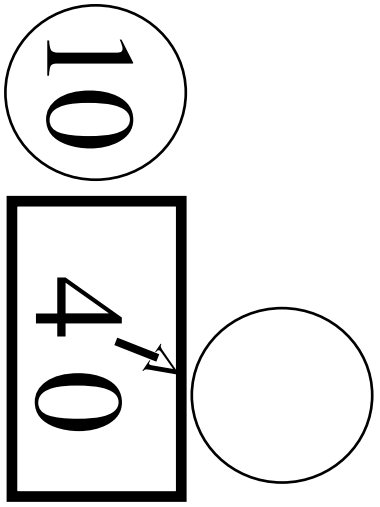


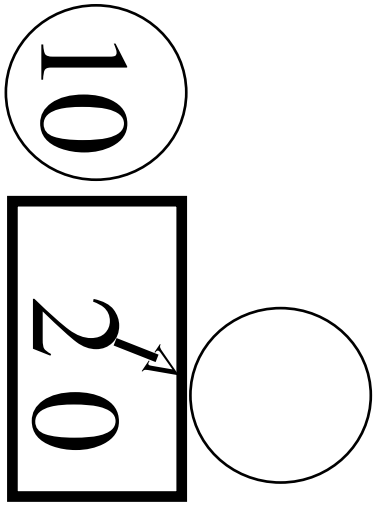












RAD MINUTE

1) Fill in blanks.
2) Check with divisibility rules.
3) Copy each number sentence.

Correct
/20

Warm -up

Copy:

1 x 5 =

2 x 5 =

3 x 5 =

4 x 5 =

5 x 5 =

6 x 5 =

7 x 5 =

8 x 5 =

9 x 5 =

10 x 5 =

One Minute Quiz

3 x 5 =

8 x 5 =

2 x 5 =

6 x 5 =

9 x 5 =

4 x 5 =

1 x 5 =

7 x 5 =

10 x 5 =

5 x 5 =

Iconic Grouping of Nickels and Multiplication

Circles: ○ represent nickels or 5 ones.

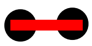
Coupled circles: ●● or ● represent groups of ten.

1 x 5 = ___ 5

How many nickels ?

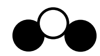
How many groups of one?

Link pairs of nickels (tens) with a line.

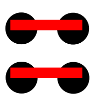


2 x 5 = 1 ___

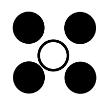
How many groups of ten?



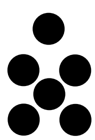
___ x 5 = ___ ___



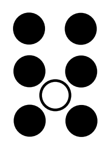
___ x 5 = ___ ___



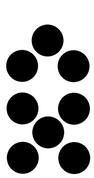
___ x 5 = ___ ___



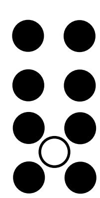
___ x 5 = ___ ___



___ x 5 = ___ ___



___ x 5 = ___ ___



___ x 5 = ___ ___

Directions for the Skip Counting Template Using Nickels (or Five Icons)

Add nickels, one by one. If the one's place is vacant, have the student fill the empty space with a nickel, then finger-write the number of tens, and the number of ones in the appropriate place-value boxes, and verbalize this number. If the one's place already has a nickel in it, have the student pick it up and combine it with the new nickel to fill one of the vacant ten spots. Have the student finger-write the number of tens, and the number of ones in the appropriate place-value boxes, and verbalize this number.

Examples:

- Give the student the first nickel. Ask the student what it is worth (5).

Ask, " How many ones?" Have him place the nickel on the vacant one's spot, then write it's value (5) in the one's place box.

- Give the student the second nickel. There is no spot to place the nickel in the one's place.

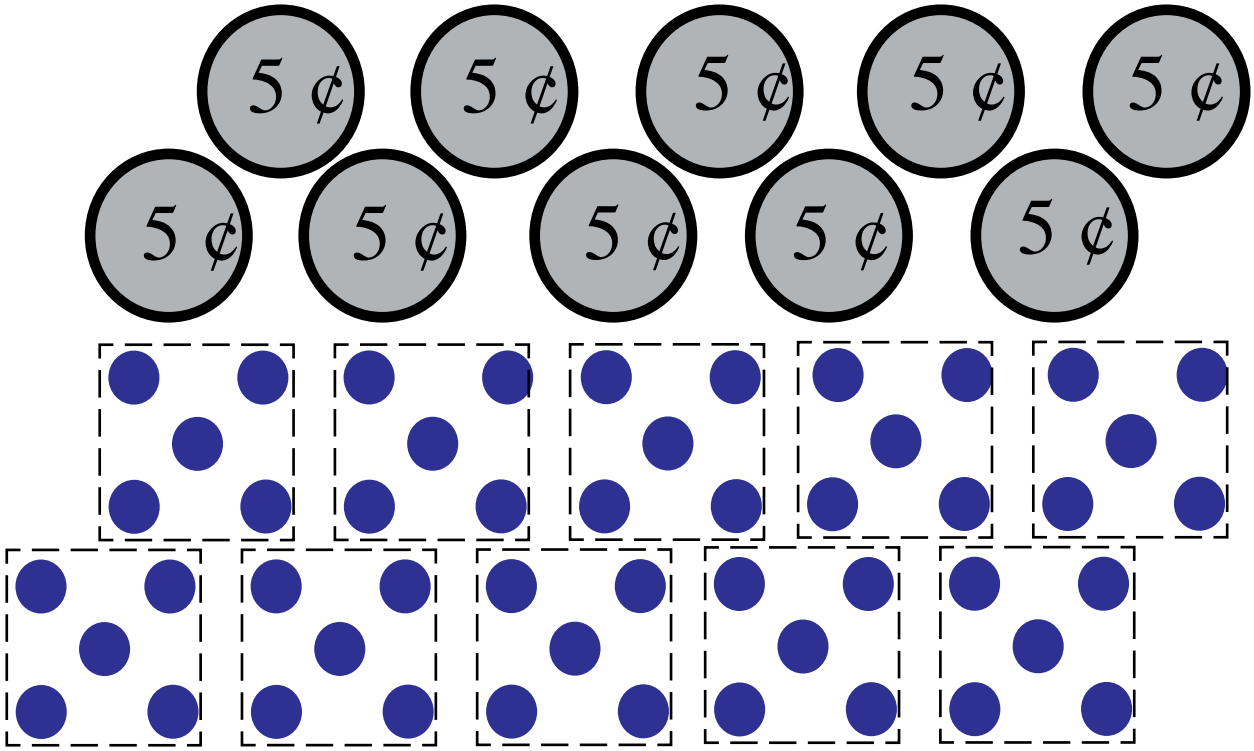
The student must move the first nickel and combine it with the new nickel to fill one entire ten's place.
Ask, " How many tens?" There is one ten. Have the student write the numeral 1 in the ten's place box. As you point to the vacant one's spot, ask, " How many additional ones are there?"
There are zero additional ones. Have the student write the numeral 0 in the one's place box.
Have the student rewrite the two digits: 1 ten and 0 ones. Now have the student verbalize the value of 1 ten and 0 ones (ten).

- Give the student the third nickel. Have him place the nickel on the vacant one's spot. Ask, " How many tens?" There is one ten. Have the student write the numeral 1 in the ten's place box. As you point to the recently filled one's spot, ask, " How many additional ones are there?"

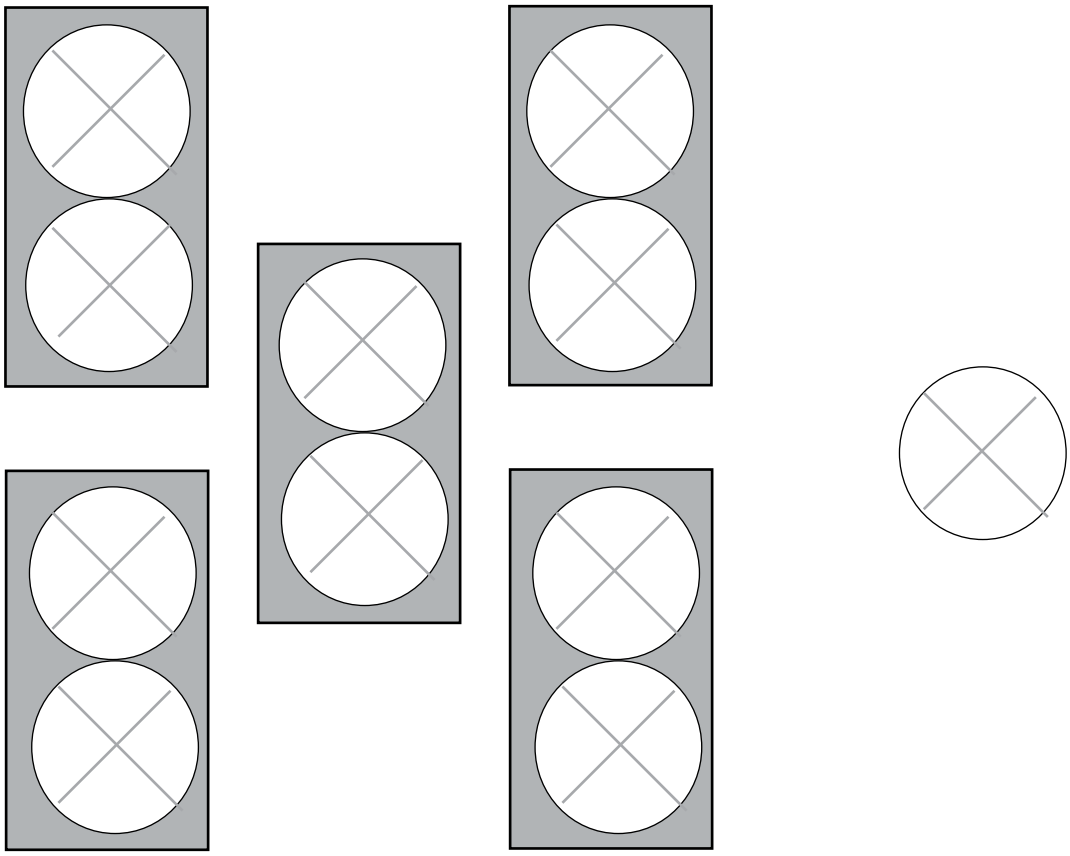
There are five additional ones. Have the student write the numeral 5 in the one's place box.
Have the student rewrite the two digits: 1 ten and 5 ones. Now have the student verbalize the value of 1 ten and 5 ones (fifteen).

- this activity can be extended to multiplication by asking the student to identify the number of *times* that he has placed a nickel on the sheet, then creating a "five times..." sentence to match.

" Nickels" and 5 Icons for the Skip Counting Template - Use real nickels if possible.



Skip Counting Template Using Nickels

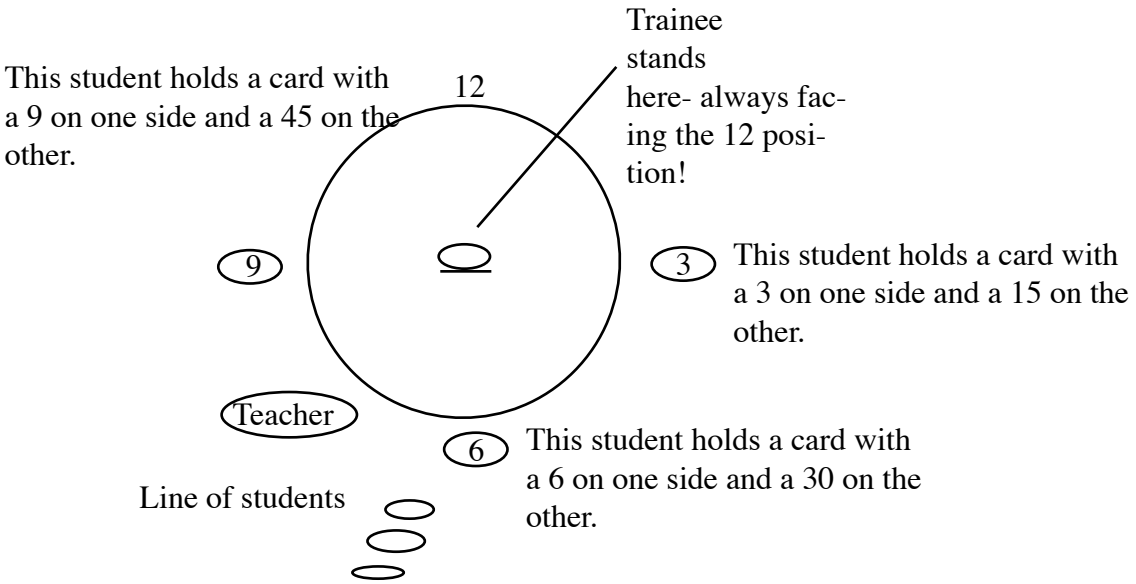


Number of Tens

Number of Ones

"Write" the digits with your index finger.

Gross Motor -Primary Reference Frame 5X facts



Level one... Teach the 4 benchmark locations of 3,6,9,12 on the analog clock dial.
a) Using a circle - the "round "KEY" portion of a basketball court, or a circular arrangement of icons painted on the ground, have your students form a line behind, and slightly to the left of the 6 position. If you are constructing the circle outside, have the 12 point toward the sun or uphill, or both. If inside, have a large artifact at the 12 position (basketball hoop-etc.). This will help maintain the correct reference frame.

Prepare four, "3 x 5" cards. Put a large "3" on one, 6, 9, and 12 on the other three. Place the 12 at the top of the circle. Hand one of the three remaining cards to the first person in line. Have that person go to the center of the clock face. Ask them to read their card. While facing the 12. E.g., "3." The teacher should again ask, look at the 12. The teacher should move to the 3 position and stamp their feet. Teacher, "I'm on the 3. Point to the 3." When the student points with their right arm, the teacher says, "Side step to the 3 -keep facing the 12." The student should go to the 3 position and stand there. Ask the next student to stand at the center. Ask the student standing on the 3 to stamp. Ask the student at the center to point to the stomping noise. The teacher should ask, "Where are you pointing?" "3." Teacher, "Sidestep to the three and take that person's card. Repeat for the remainder of the students- have the first person go a second time. Then repeat the drill for the 6 position -backing into position, then the 9 position -side stepping to the left.

b) On the next day, repeat the same drill. See how quickly the students can go to the three positions. Then, if the students appear to be doing well, move to the next activity. Give the first three students in line a card -one 3, the next 6 , the last one : 9...

Have the first student go to the center. Ask, “Where’s the 3?” Have the student point to the 3. If they are pointing to the wrong spot - the teacher goes to that spot and stamps her feet to draw attention to her position.

Once the student points to the correct position, ask them to go there- maintaining eye contact and body orientation towards the 12 position. Repeat this for the next two students with cards. The fourth person in line will not have a card. Have the student toe the line in the center.

Ask, ”Where is the 3?” Have the student point to the 3 - if unsure, ask the person standing on the 3 position to provide a cue by stamping their feet. Ask the student to point to all 3 positions twice, then on the last request, ask the student to point to the 3. Once the center student points to the 3 correctly, have the student side-step into position, then take the card of the student standing at that position. Continue, replacing the 6 and 9 positions as well. Try to run each student through twice.

Repeat this drill until all of the students can predictably point to the 3, 6, and 9 positions.

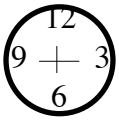
c) Learn the minute locations for 15, 30 and 45 minutes. Write these times on the backs of the corresponding 3,6,9 cards. Run the students through the previous drill to fill the three positions. As the three students take their positions, tell them to read the minute numbers backs of their cards. Next, Have the 4th student take the center. Have the student point to the 3. Ask the student the number of minutes for that position - have the person holding the 3 card face the “15” toward the center. Have the 3,6,9 people cue the minute times as necessary. On the second time through, interchange clock position numbers and the 15, 30, 45 minute positions.

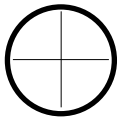
d) Repeat drill c until the students can point to the minute or number position without cueing.

e) Ask the center person to point to a benchmark location and have the student identify the correct minute amount for that location. Then, the teacher goes to that location and moves one (noisy deliberate) clockwise step. Teacher should then ask what number position that they are standing -and the time in minutes.

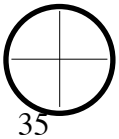
f) Put it all together -make 5x facts. Students are asked to point to a number or minute location , then say the corresponding number. Then prompt the students to say the 5x fact that relates the two quantities 45 = 9 x5. Have a student in line say the same fact - starting with a different number -Start with 9! 9x5=45 Start with 5!

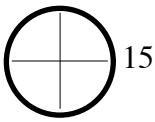
- Draw the minute hand to show the given minutes.
- Write the related 5x fact.

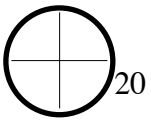


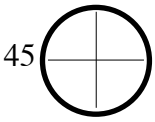


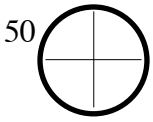
$5 \quad x$

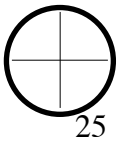


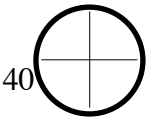


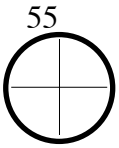


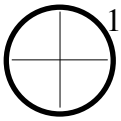












5 10

5 15

5 20

5 25

5 30

5 35

5 40

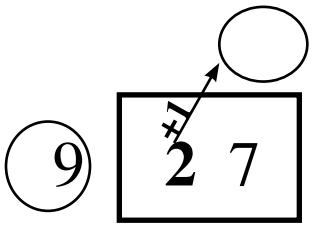
5 - 1 → 45

Digits add to 9.

Predictable Visual Patterns:

The nine times family and, even multiples of 6 add an additional 16% for 80% of the multiplication table. Both of these fact families have divisibility rules that involve adding the digits of the products. All of the x9 products on the multiplication table are comprised of digits that add to nine. For instance, 9x2=18 (the digits in this product: 1+8 add to 9). The 9x facts are highly predictible when flash cards present the division form of the fact. The missing factor will be one larger than the ten's digit of the product (see figure 3). When students are presented with a division flash card based on the 9x fact family, cue the student to identify the digit in the tens place of the product
(or dividend). Once they do this, they need only to add one to arrive at the missing factor. Have them recite the fact as a multiplication sentence that starts with this missing factor, then recite the other three related facts.

Figure 3



Add 1 to the ten's digit of the product:
2 + 1 = 3..... "3 x 9 = 27"

Have the student recite several related facts:

Prompts:
"Give me a division fact, start with 27." "27 ÷ 9 = 3."
"Give me a differnt division fact. Divide by 3." "27 ÷ 3 = 9."
"Give me a multiplication fact. Start with 27." " 27 = 3 x 9."
"Give me a differnt multiplication fact. Start with 9." " 9 x 3 = 27.
"Give me a multiplication fact. Start with 3." " 3 x 9 = 27.

Perfect Squares 3^2, 4^2, 7^2, 8^2 add an additional 4% for a total of 84% of the table.
8 facts remain to be memorized. They are: 3x4, 3x6, 3x7, 3x8, 4x7, 4x8, 7x6, 7x8

9 +1 1 8

9 +1 2 7

9 +1 3 6

9 +1 4 5

9 +1 5 4

9 +1 6 3

9 +1 7 2

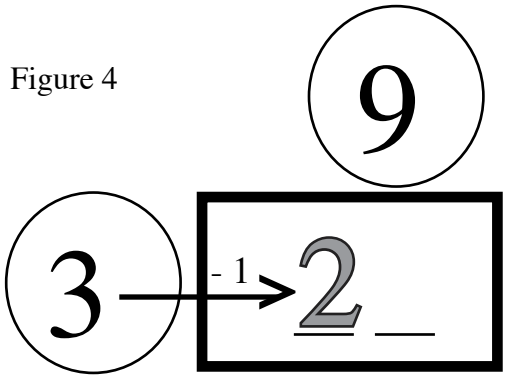
9 +1 8 1

When presenting the nine times family flash cards that present the factors and a missing product, use a similar strategy. The ten's digit in the missing product will be one smaller than the top factor. The student should subtract one from the top factor, then say the resulting number as a decade. This is usually enough to trigger the second digit of the product. If not, it may be computed by finding the missing addend to 9.

Start using these flash cards after the student has successfully practiced the flash cards that feature the product (number in the rectangle). These flash cards are most productive after the nine times products have become familiar through oral recitation.

Begin by using this template as a worksheet. Have the student write the products inside each rectangle. The "trick" is to first write the tens digit by subtracting one from the number in the left circle. This ten's digit should then said as a decade: In figure 4, the student writes 2, but is prompted to say "twenty." It is hoped that saying "twenty..." will prompt the remainder of the product. If it doesn't, the student probably needs more pratice with the flash cards that supply the product.

Figure 4

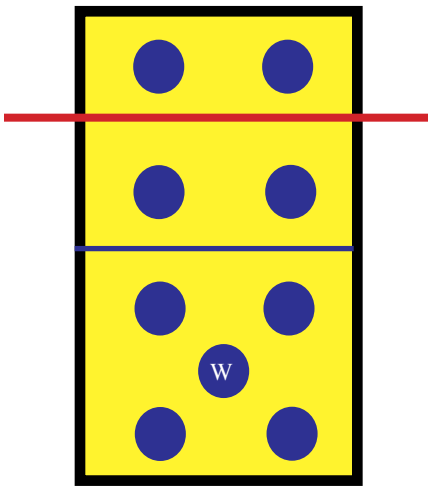


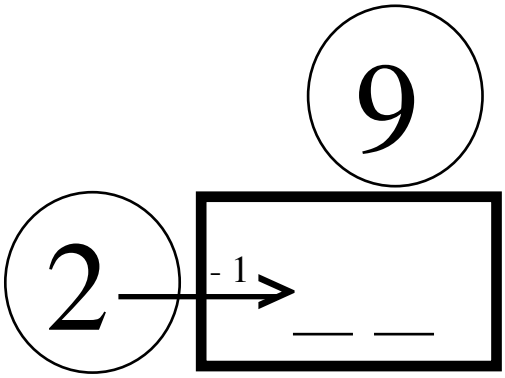
Digits add to 9.

One cueing method involves reminding the student that the digits of the product must add to 9. This is best done visually by supplying a 9 Icon - and separating two dots from the rest. Use the student's fingers to mask the top two dots, or separate them with a drawn line, piece of string, or pencil. See figure 5

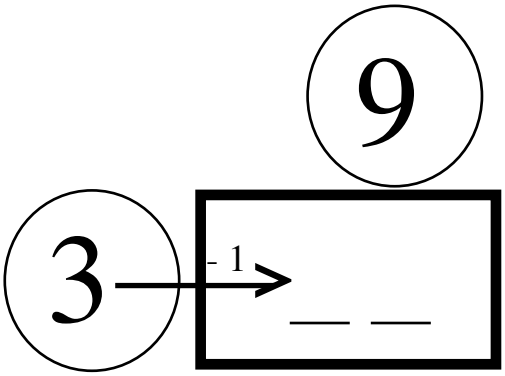
It should be noted that this is not really a trick at all. Nine times facts are similar, yet always smaller than a comparable ten times fact. For instance: 10 x 3 = 30. 9 x 3= (a little smaller than 30) Twenty... 7.

Figure 5

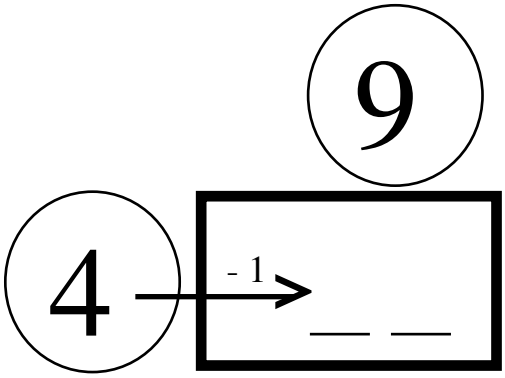




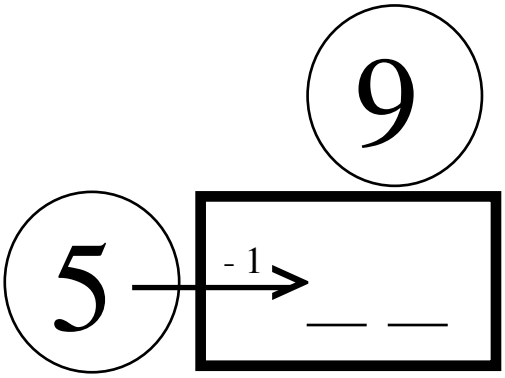
Digits add to 9.



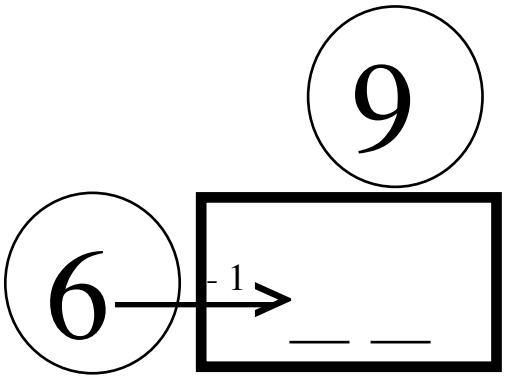
Digits add to 9.



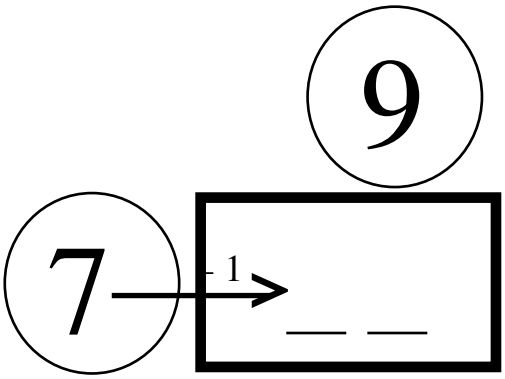
Digits add to 9.



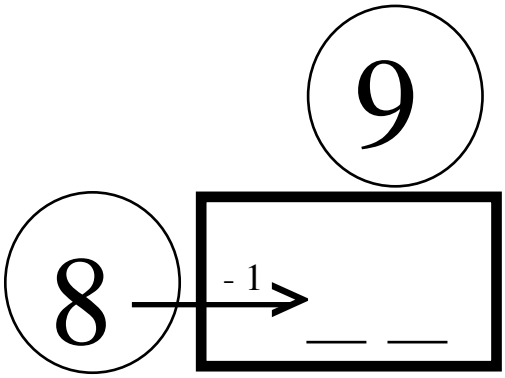
Digits add to 9.



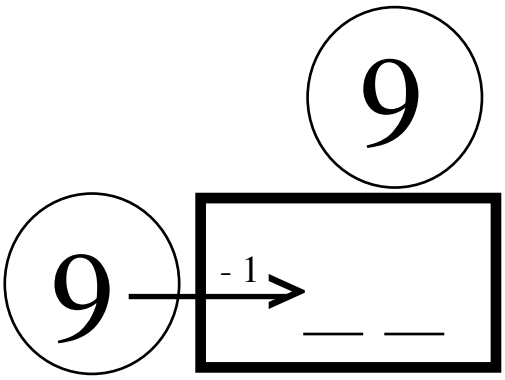
Digits add to 9.



Digits add to 9.

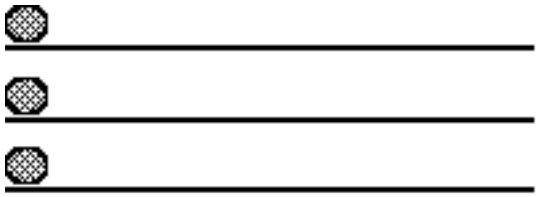


Digits add to 9.

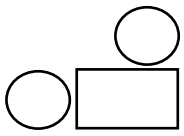


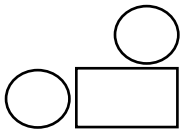
Digits add to 9.

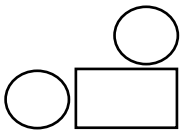
Multiplying by 7.
Distributive Property.

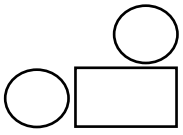


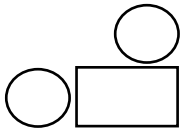
Create a diagram for each, then write 2x facts and 2 ÷ facts.

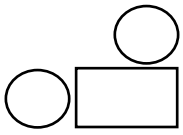
2 x (5 / 2) = = + 

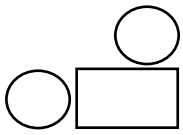
3 x (5 / 2) = + 

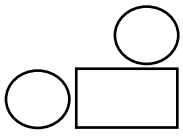
4 x (5 / 2) = + 

5 x (5 / 2) = + 

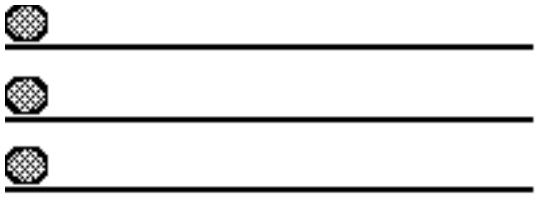
6 x (5 / 2) = + 

7 x (5 / 2) = + 

8 x (5 / 2) = + 

9 x (5 / 2) = + 

Multiplying by 7.
Distributive Property.



Write 7x facts using distributive property.

$2 \times (5 + 2) =$ _____
$= +$ _____

$2 (\quad + \quad) =$ ___+___
$2 (\ 7 \ ) =$ _____

$3 \times (5 + 2) =$ _____
$+$ _____

$3 (\quad + \quad) =$ ___+___
$3 (\ 7 \ ) =$

$4 \times (5 + 2) =$ _____
$+$ _____

$4 (\quad + \quad) =$ ___+___
$4 (\ 7 \ ) =$ _____

$5 \times (5 + 2) =$ _____
$+$ _____

$5 (\quad + \quad) =$ ___+___
$5 (\ 7 \ ) =$

$6 \times (5 + 2) =$ _____
$+$ _____

$6 (\quad + \quad) =$ ___+___
$6 (\ 7 \ ) =$ _____

$7 \times (5 + 2) =$ _____
$+$ _____

$7 (\quad + \quad) =$ ___+___
$7 (\ 7 \ ) =$

$8 \times (5 + 2) =$ _____
$+$ _____

$8 (\quad + \quad) =$ ___+___
$8 (\ 7 \ ) =$ _____

$9 \times (5 + 2) =$ _____
$+$ _____

$9 (\quad + \quad) =$ ___+___
$9 (\ 7 \ ) =$