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On Searching Intuitionistic Fuzzy Shortest Path in a Network

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Abstract

This paper presents to find the shortest path in a directed graph which an intuitionistic fuzzy number, instead of a fuzzy number is assigned to each arc length. An existing algorithm is modified to find the optimal path. Inputs and outputs of the proposed algorithm are intuitionistic fuzzy numbers. Finally an illustrative numerical example is given to demonstrate the proposed approach.

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1. Introduction

The shortest path problem is the basic network problem, the real number is assigned to each edges. The Fuzzy shortest path problem was first analyzed by Dubois and Prade[2] using fuzzy number instead of a real number is assigned to each edges. In this paper we propose an intuitionistic fuzzy number instead of fuzzy number. An algorithm is based on the idea that from all the shortest paths from source node to destination node, an edge with shortest length is computed and the Euclidean distance is computed for all the paths with the edge of minimum distance is the shortest path for membership and non membership values. This paper is organized as follows: In section 2 provides preliminary concepts required for analysis. In section 3, an intuitionistic fuzzy shortest path length procedure is given. In section 4 and 5 intuitionistic fuzzy shortest path problem is explained using an illustrative example.

2. Preliminary and definitions

The concepts of an intuitionistic fuzzy set was introduced by Atanassov[1] to deal with vagueness, which can be defined as follows.

2.1 Intuitionistic Fuzzy Set

Let X be an universe of discourse, then an Intuitionistic Fuzzy Set (*IFS*) A in X is given by $A = \{ (x, \mu_A(x), \gamma_A(x)) \ / \ x \in X \}$ where the functions $\mu_A(x) : X \to [0,1]$ and $\gamma_A(x) : X \to [0,1]$ determine the degree of membership and non membership of the element $x \in X$, respectively and for every $x \in X$, $0 \le \mu_A(x) + \gamma_A(x) \le 1$.

2.2 Intuitionistic Fuzzy Graph

Let X be an universe, containing fixed graph vertices and let $V \subset X$ be a fixed set. Construct the *IFS* $V = \{ (x, \mu_v(x), \gamma_v(x)) \ / x \in X\}$ where the functions $\mu_v(x) : X \to [0,1]$ and $\gamma_v(x) : X \to [0,1]$ determine the degree of membership and non membership to set V of the element (*vertex*) $x \in X$, respectively and for every $x \in X$, such that $0 \le \mu_v(x) + \gamma_v(x) \le 1$.

2.3 Intuitionistic Fuzzy Number [7]

Let $A = \{(x, \mu_A(x), \gamma_A(x)) \ / \ x \in X \}$ be an *IFS*, then we call the pair $(\mu_A(x), \gamma_A(x))$ an intuitionistic fuzzy number. For convenience, we denote an intuitionistic fuzzy number by $(\langle a,b,c\rangle, \langle l,m,n\rangle)$. Where $\langle a,b,c\rangle \in F(I)$, $\langle l,m,n\rangle \in F(I)$, $I = [0,1]$, $0 \le c + n \le 1$.

2.4 Triangular Intuitionistic Fuzzy Number and its Arithmetic [4]

A triangular intuitionistic fuzzy number A is denoted by $A = \{(\mu_A, \gamma_A) \mid x \in R\}$, where μ_A and γ_A are triangular fuzzy numbers with $\gamma_A \le \mu_A^c$. So a triangular intuitionistic fuzzy number A is given by $A = (\langle a,b,c\rangle, \langle e,f,g\rangle)$ with $(\langle e,f,g\rangle \le \langle a,b,c\rangle^c)$ i.e., either $e \ge b$ *and* $f \ge c$ *or* $f \le a$ *and* $g \le b$ are membership and non membership fuzzy numbers of A.

An intuitionistic fuzzy number $(\langle a,b,c\rangle, \langle e,f,g\rangle)$ with $e \ge b$ *and* $f \ge c$ is shown in the following figure

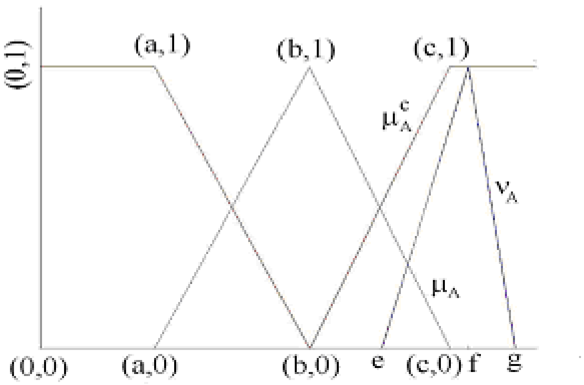

Figure-1 Triangular Intuitionistic Fuzzy Number $A = (\langle a,b,c\rangle, \langle e,f,g\rangle)$

The addition of two triangular intuitionistic fuzzy numbers are as follows [5] For two Triangular Intuitionistic Fuzzy Numbers $A = (\langle a_1,b_1,c_1\rangle : \mu_A, \langle e_1, f_1, g_1\rangle : \gamma_A)$ and $B = (\langle a_2,b_2,c_2\rangle : \mu_B, \langle e_2, f_2, g_2\rangle : \gamma_B)$ with $\mu_A \neq \mu_B$ and $\gamma_A \neq \gamma_B$, define

$$A + B = (\langle a_1 + a_2, b_1 + b_2, c_1 + c_2\rangle : Min(\mu_A, \mu_B)\ ,\ \langle e_1 + e_2, f_1 + f_2, g_1 + g_2\rangle : Max(\gamma_A, \gamma_B))$$

3. An Intuitionistic Fuzzy Shortest Path Length Procedure

In this paper the arc length in a network is considered to be an intuitionistic fuzzy number, namely triangular intuitionistic fuzzy number. The shortest path length procedure is based on the *Chuang* and *Kung* [6] method.

Step 1 Compute all the possible path lengths L_i from $i = 1,2,3\ldots,n$. Where

$$L_i = (\langle a_i', b_i', c_i'\rangle, \langle l_i', m_i', n_i'\rangle)$$

Step 2 Initialize $L_{Min} = (\langle a,b,c\rangle, \langle l,m,n\rangle) = L_1 = (\langle a_1', b_1', c_1'\rangle, \langle l_1', m_1', n_1'\rangle)$

Step 3 Initialize $i = 2$

Step 4 (a)Compute $\langle a,b,c\rangle$ for membership values

$$b = \begin{cases} b, & if \quad b \le a_i' \\ \dfrac{bb_i' - aa_i'}{(b + b_i') - (a + a_i')}, & if \quad b > a_i' \end{cases}$$

$$a = Min(a, a_i')$$

$$c = Min(c, b_i')$$

Step 4 (b)Compute $\langle l,m,n\rangle$ for non membership values

$$m = \begin{cases} m, & if \quad m \le l_i^{'} \\ \dfrac{mm_i^{'} - ll_i^{'}}{(m+m_i^{'})-(l+l_i^{'})}, & if \quad m > l_i^{'} \end{cases}$$

$$l = Min(l, l_i^{'})$$

$$n = Min(n, m_i^{'})$$

Step 5 Set $L_{Min} = (\langle a,b,c\rangle, \langle l,m,n\rangle)$ as calculated in step – 4

Step 6 $i = i + 1$

Step 7 if $i < n+1$, go to Step – 4

4. Numerical Example

In order to illustrate the above procedure consider a small network shown in figure, where each arc length is represented as a triangular intuitionistic fuzzy number.

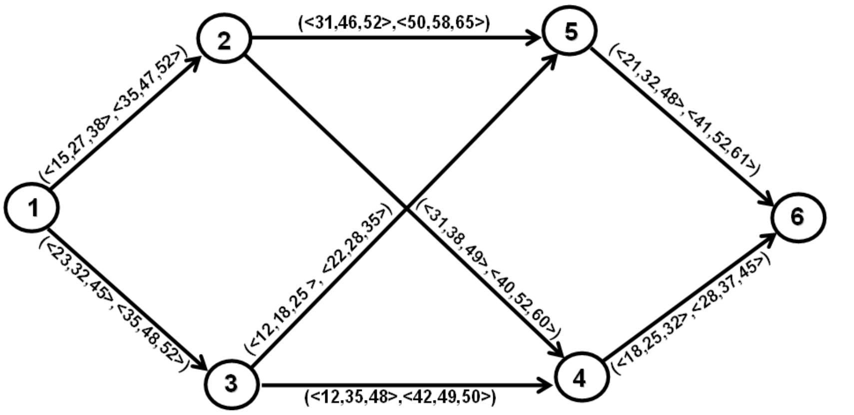

Figure - 2

Step 1 In the above network there are four possible paths (n = 4) from Source node 1 to destination node 6. The possible paths are as follows:

Path (1): 1→2→5→6 $L_1 = (\langle 67,105,138\rangle, \langle 126,157,178\rangle)$

Path (2): 1→3→4→6 $L_2 = (\langle 53,92,125\rangle, \langle 105,134,147\rangle)$

Path (3): 1→2→4→6 $L_3 = (\langle 64,90,119\rangle, \langle 103,136,157\rangle)$

Path (4): 1→3→5→6 $L_4 = (\langle 56,82,118\rangle, \langle 98,128,148\rangle)$

Step 2 Initialize:

$$L_{Min} = (\langle a,b,c\rangle, \langle l,m,n\rangle) = L_1 = (\langle a_1^{'},b_1^{'},c_1^{'}\rangle, \langle l_1^{'},m_1^{'},n_1^{'}\rangle) = (\langle 67,105,138\rangle, \langle 126,157,178\rangle)$$

Step 3 Initialize: $i = 2$

Step 4 (a)Compute $\langle a,b,c\rangle$ for membership values

$$if \quad b > a_2^{'} \quad then \quad b = \frac{bb_2^{'} - aa_2^{'}}{(b+b_2^{'})-(a+a_2^{'})} = \frac{105\times 92 - 67\times 53}{(105+92)-(67+53)} = 79.34$$

$$a = Min(a, a_2^{'}) = Min(67,53) = 53$$

$$c = Min(c, b_2^{'}) = Min(138,92) = 92$$

Step 4 (b)Compute $\langle l,m,n\rangle$ for non membership values

$$if \quad m > l_2^{'} \quad then \quad m = \frac{mm_2^{'} - ll_2^{'}}{(m+m_2^{'})-(l+l_2^{'})} = \frac{157\times 134 - 126\times 105}{(157+134)-(126+105)} = 130.13$$

$$l = Min(l, l_2^{'}) = Min(126,105) = 105$$

$$n = Min(n, m_2^{'}) = Min(178,134) = 134$$

Step 5 Set $L_{Min} = \left(\langle 53, 79.34, 92\rangle, \langle 105, 130.13, 134\rangle\right)$ as calculated in step – 4

Step 6 $i = i + 1 = 3$

Step 7 if $i < n+1(=5)$, go to Step – 4

Step 4 (a)Compute $\langle a,b,c\rangle$ for membership values

$$if \quad b > a_3^{'} \quad then \quad b = \frac{bb_3^{'} - aa_3^{'}}{(b+b_3^{'})-(a+a_3^{'})} = \frac{79.34\times 90 - 53\times 64}{(79.34+90)-(53+64)} = 71.62$$

$$a = Min(a, a_3^{'}) = Min(53,64) = 53$$

$$c = Min(c, b_3^{'}) = Min(92,90) = 11$$

Step 4 (b)Compute $\langle l,m,n\rangle$ for non membership values

$$if \quad m > l_3^{'} \quad then \quad m = \frac{mm_3^{'} - ll_3^{'}}{(m+m_3^{'})-(l+l_3^{'})} = \frac{130.13\times 136 - 105\times 103}{(130.13+136)-(105+103)} = 118.4$$

$$l = Min(l, l_3^{'}) = Min(105,103) = 103$$

$$n = Min(n, m_3^{'}) = Min(134,136) = 134$$

Step 5 Set $L_{Min} = \left(\langle 53, 71.62, 90\rangle, \langle 103, 118.4, 134\rangle\right)$ as calculated in step – 4

Step 6 $i = i + 1 = 4$

Step 7 if $i < n+1(=5)$, go to Step – 4

Step 4 (a)Compute $\langle a,b,c\rangle$ for membership values

$$if \quad b > a_4^{'} \quad then \quad b = \frac{bb_4^{'} - aa_4^{'}}{(b+b_4^{'})-(a+a_4^{'})} = \frac{71.62\times 82 - 53\times 56}{(71.62+82)-(53+56)} = 65.1$$

$$a = Min(a, a_4^{'}) = Min(53,56) = 53$$

$$c = Min(c, b_4^{'}) = Min(90,82) = 82$$

Step 4 (b)Compute $\langle l,m,n\rangle$ for non membership values

$$if \quad m > l_4^{'} \;\; then \;\; m = \frac{mm_4^{'} - ll_4^{'}}{(m+m_4^{'}) - (l+l_4^{'})} = \frac{118.4\times128 - 103\times98}{(118.4+128)-(103+98)} = 111.48$$

$l = Min(l, l_4^{'}) = Min(103, 98) = 98$

$n = Min(n, m_4^{'}) = Min(164, 128) = 128$

Step 5 Set $L_{Min} = \left(\langle 53, 65.1, 82\rangle, \langle 98, 111.48, 128\rangle\right)$ as calculated in step – 4

Step 6 *i = i + 1 = 5*

Step 7 if $i \geq n+1(=5)$, Stop the procedure

Finally we get an Intuitionistic Shortest Path Length $L_{Min} = \left(\langle 53, 65.1, 82\rangle, \langle 98, 111.48, 128\rangle\right)$

5. An Algorithm for Searching the Shortest Path

We aim at determining an intuitionistic fuzzy shortest path length L_{Min} and the shortest needed to traverse from source to destination. By combining an intuitionistic fuzzy shortest length method with similarity measure, an algorithm is as follows. An Algorithm for intuitionistic fuzzy shortest path [3]

Step 1 Find out all the possible paths from source node ***s*** to destination node ***d*** and compute the corresponding path lengths L_i, $i = 1,2,3,\ldots,n$.

Step 2 Compute L_{Min} by using an intuitionistic fuzzy shortest path length procedure

Step 3 Find the Euclidean distance d_i for $i=1,2,3,\ldots,n$ between all the possible path and L_{Min}.

Step 4 Decide the shortest path with the path having lowest Euclidean distance.

On executing the above algorithm on the example network of figure 2. The first two steps have been already calculated in *section – 4.*

The next step, we compute the similarity degree $S(L_{Min}, L_i)$ between L_{Min} and L_i for $i=1,2,3,\ldots,n$ by means of similarity measure $S(A,B) = \sum_{k=1}^{m} \frac{\left[1 - |A(x_k) - B(x_k)|\right]}{m}$.

In order to compute the accuracy in similarity degree, we should let a generic element *U* denoted by $\mu_t(i = 1,2,3,\ldots,n)$.

Step 1: From *Section – 4* we have four possible paths lengths in *Figure – 2* as

Path (1): 1→2→5→6 $L_1 = \left(\langle 67,105,138\rangle, \langle 126,157,178\rangle\right)$

Path (2): 1→3→4→6 $L_2 = \left(\langle 53,92,125\rangle, \langle 105,134,147\rangle\right)$

Path (3): 1→2→4→6 $L_3 = \left(\langle 64,90,119\rangle, \langle 103,136,157\rangle\right)$

Path (4): 1→3→5→6 $L_4 = \left(\langle 56,82,118\rangle, \langle 98,128,148\rangle\right)$

Step 2: From *Section – 4* we have calculated minimum path length is

$$L_{Min} = \left(\langle 53, 65.1, 82\rangle, \langle 98, 111.48, 128\rangle\right)$$

Step 3: Euclidean distance between all path lengths *Pi (i=1,2,3,4)* and L_{Min}

$$d(p_{1,}L_{Min}) = \left(\left\langle \sqrt{(67-53)^2 + (105-65.1)^2 + (138-82)^2} \right\rangle, \left\langle \sqrt{(126-98)^2 + (157-111.48)^2 + (178-128)^2} \right\rangle\right)$$
$$= \left(\langle 70.17\rangle, \langle 73.18\rangle\right)$$
$$d(p_{2,}L_{Min}) = \left(\left\langle \sqrt{(53-53)^2 + (92-65.1)^2 + (125-82)^2} \right\rangle, \left\langle \sqrt{(105-98)^2 + (134-111.48)^2 + (147-128)^2} \right\rangle\right)$$
$$= \left(\langle 50.72\rangle, \langle 30.28\rangle\right)$$
$$d(p_{3,}L_{Min}) = \left(\left\langle \sqrt{(64-53)^2 + (90-65.1)^2 + (119-82)^2} \right\rangle, \left\langle \sqrt{(103-98)^2 + (136-111.48)^2 + (157-128)^2} \right\rangle\right)$$
$$= \left(\langle 45.93\rangle, \langle 38.3\rangle\right)$$
$$d(p_{4,}L_{Min}) = \left(\left\langle \sqrt{(56-53)^2 + (82-65.1)^2 + (118-82)^2} \right\rangle, \left\langle \sqrt{(98-98)^2 + (128-111.48)^2 + (148-128)^2} \right\rangle\right)$$
$$= \left(\langle 39.88\rangle, \langle 25.94\rangle\right)$$

Step 4: Decide the shortest path with the path having lowest Euclidean distance for member and non member by examining the Euclidean distance *d* between L_{Min} and d_i for $i = 1,2,3,4$.

From the above calculations we can see that Path (4) 1→3→5→6 has the least Euclidean distance for membership and non membership. The shortest path from source node 1 to destination node 6 is 1→3→5→6.

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