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Comparison of Learning Theories in Mathematics Teaching Methods

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Abstract

The aim of this paper is to compare learning theories in mathematics teaching, and to study their influence on mathematics learning. Numerous studies have shown that students experience mathematics anxiety, which is a feeling of tension and fear that interferes with math learning. This may be attributed to the teaching methods utilized. Therefore, teaching methods must be re-examined, taking into account the three major learning theories: behaviorism, cognitivism, and constructivism. Behaviorism is a teacher-centered instruction, while cognitivism states learning is based upon how people mentally process stimuli encountered. Mathematics teachers following a constructivist approach favor extending class time to engage in varied activities associated with the discovery and construction of knowledge. Thus, the qualitative case study method was considered more feasible and appropriate to meet the study aim. Data were collected using observation and semi-structured interviews with teachers in secondary schools in Malaysia. It was observed that positive and negative reinforcement (behaviorism), problem solving (cognitivism) and discovery learning (constructivism) were practiced by the teachers. The findings reveal that students are more successful when systematic problem solving based on Polya's approach is incorporated into discovery learning. Consequently, there should be more emphasis on teaching methods which include less lecture, more student-directed classes and more discussion. The findings suggest that problem-solving and discovery-learning skills not only contribute to better mathematics learning but also enhance students' creativity to cope with life challenges.

Keywords: Learning theories, mathematics teaching methods, behaviorism, Cognitivism, constructivism

Introduction

Mathematics became the driving force for almost all technological and scientific developments in the nineteenth and twentieth centuries. It has a big influence on our professional and social daily life activities (Maasz & Schloeglmann, 2006). It plays a crucial role on students' success and nation building. Mathematics education has always been treated as an important section of general education and specifically science education.

It is believed that a theory is essential to any meaningful development effort. Different cultures and societies have different theories regarding education, specifically with respect to the teaching and learning of mathematics as illustrated in their curriculum. These variations of beliefs and values concerning mathematics learning may result in different mathematics educational systems. The role of teachers is to facilitate students' thinking and learning. Therefore, teachers should attempt to motivate their students to learn. To be aware of teaching practice activities done by teachers, we should have enough knowledge about learning theories and teaching methods. Different learning theories and teaching methods have been used in educational systems all over the world. Theories of learning are the main concern of this study, namely behaviorist theory, cognitive learning theory, and constructivist theory. These theories and their applications in the mathematics teaching methods will be explained more in the next section. The aim of this paper was to compare learning theories in mathematics teaching, and to study their influence on mathematics learning.

Literature Review

Learning Theories and Teaching Methods

Learning is one of the significant features of current psychology. Learning theories and teaching methods have been used in different educational systems around the world. Teaching methods involve the use of learning theories and each theory has different outcomes in mathematics education.

In order to succeed in teaching mathematics, teachers need to enhance their understanding of students' learning abilities, experiences, reasoning, and logical abilities. In doing so, they can employ this knowledge as a basis of their mathematics education strategies. The teachers strongly agreed that mathematics teaching and learning is an ongoing process through which students must develop a solid understanding of appropriate mathematics concepts and procedures at each academic level. These teachers ought to give students enough confidence to discover mathematics problems and to think critically to solve them as well as their lifelong problems. The following section discusses three learning theories, namely behaviorism, cognitivism, and constructivism.

Behaviorism

The two main creators of behaviorist approaches to learning were Skinner (1972) and Watson (1996). Watson stated that human behavior is a result of particular stimuli extracted from particular responses, while Skinner remarked that habits each of us develop stem from our distinctive operant learning experiences (Shaffer, 2000).

One of the most popular descriptions of learning is the one proposed by Kimble (1961, p. 6) stating "learning as a relatively permanent change in behavioral potentiality occurs as a result of reinforced practice." Kimble's definition has highlighted three aspects of learning. First, learning is manifested by a change in behavior. Second, this behavioral change is relatively permanent. Third, the change in behavior does not occur immediately following the learning experience (Hergenhahn & Olson, 2005).

In education, behaviorists apply rewards and punishments system in their classrooms effectively. They believe that rewards have significant roles in learning. The teaching methods based on behaviorism emphasize the claim that behavior can be shaped by

reinforcement through drill and practice. They set clear objectives to help students and teachers (Hergehahn & Olson, 2005).

In a mathematics class, using the behaviorist theory, the teacher reviews previous material and homework, and then demonstrates low-level problem solving followed by seatwork imitating the teacher's demonstration (Stonewater, 2005). This pedagogical approach of placing the primary focus on the teacher as a transmitter of knowledge (that is, teaching by telling) is representative of a behaviorist theory (Hackman, 2004). The common method of teaching mathematics using the behaviorists' theory is teacher-centered and giving lecture is the dominant practice.

Cognitivism

The cognitive approach is another important theory. Piaget (1936) was the first psychologist to make a systematic study of cognitive development. Cognitive development is a progressive reorganization of mental processes as a result of biological maturation and environmental experience. Cognitive learning theory suggests that learning is based upon how people mentally process stimuli encountered (Ormord, 1995). In reaction, as early as 1956, Benjamin Bloom created a taxonomy for cognitive skills that included knowledge, comprehension, application, analysis, synthesis, and evaluation, which he believed teachers should help students, so that they use and develop. The six stages are further elaborated and revised in the study by Anderson and Krathwohl (2001) as Remembering, Understanding, Applying, Analyzing, Evaluating, and Creating (Wilson, 2013).

A cognitive perspective implies that a behavioral analysis of instruction is often inadequate to explain the effects of instruction on learning. Both cognitive and behavioral approaches continue to be a part of educational psychology today. The main application of this theory in teaching of mathematics can be seen in the skill of problem solving.

Problem Solving

Problem solving is a significant element of mathematics education. In fact, problem solving in mathematics helps students to develop a wide range of complex mathematics structures and gains the capability of solving a variety of real-life problems (Tarmizi & Bayat, 2012). Moreover, the National Council of Teachers of Mathematics (NCTM, 1980) has emphasized that the mathematics teachers should focus on problem solving throughout their teaching-since it "encompasses skills and functions which are an important part of everyday life." Furthermore, problem solving helps people to adapt to changes and unexpected problems in their careers and other aspects of their lives. Problem solving lies beyond mathematics teaching dimensions so that students experience the influence of mathematics in the world around them (Taplin, 2011).

Concerning these required skills and approaches to problem solving, (Polya, 1945) stated a four-step approach to problem solving, including:

i. Understanding the problem: it is impossible to solve a problem if you do not know what the problem is. What is known or unknown? Is there enough information or is more information needed? What do the terms mean?;
ii. Devising a plan: the way we solve the problem. Possible strategies: (a) Draw pictures; (b) Use a variable and choose helpful names for variables or unknowns; (c) Be systematic; (d) Solve a simpler version of the problem; (e) Guess and check, Trial and error; Guess and test (Guessing is okay); (f) Look for a pattern or patterns; and (g) Make a list;
iii. Carrying out the plan: If the plan does not seem to be working, then start over and try another way. Often the first approach does not work. Do not worry just because an approach does not work. It does not mean you did it wrong. You actually

accomplished something, knowing a way does not work is part of the process of elimination; and

iv. Looking back: Did you answer the question? Is your result reasonable? Is there another way of doing the problem which may be easier?

In today's mathematics and science, problem solving does not only help to gain more skills and knowledge to solve the problem, but it also goes further in helping to increase reasoning skills among students (Hmelo, Guzdial, & Turns, 1998). Therefore, problem-solving methods are considered as essential factors to increase level of students' mathematics achievement. Also, it can be utilized to solve problems that arise in real life.

Constructivism

The constructivist approach is a learner-centered approach that emphasizes the importance of individuals actively constructing their knowledge and understanding through the guidance from the teacher. In the constructivist view, teachers should not attempt to simply pour information into children's minds. Rather, children ought to be given confidence to discover their world, find out knowledge, consider, and think critically with vigilant supervision and significant guidance of the teacher (Eby, Herrel & Jordan, 2005).

Nowadays, constructivism might involve a focus on collaborations; children working together strive to know and understand. Constructivism is identified as teaching that concentrates on the vigorous function of the learner in making sense of information and establishing knowledge (Woolfolk, 2008). Constructivists emphasize that students should form their own interpretation of evidence and submit it for review. Constructivist teachers encourage students to constantly assess how the activity is helping them gain understanding. By questioning themselves and their strategies, students in the constructivist classroom ideally become "expert learners."

The constructivist approach was proposed by William James' and John Dewey's philosophies of education and comes from many other people such as Jean Piaget, Maria Montessori, and Lev Vygotsky and from educational movements such as education, inquiry or discovery learning, open education and whole progressive language teaching (Gagnon & Collay, 2001).

Since the constructivist emphasizes that students form their own interpretation of evidences and submit them for review, within mathematics education, students have to build their own understanding of every concept of mathematics, so that the main responsibility of teaching is not explaining, lecturing, or attempting to convey mathematical knowledge, but creating situations for students that will promote students making the essential mental structures. Obviously, a lesson according to constructivism varies significantly with the conventional class type of teacher-as-lecturer (Hanley, 1994).

Constructivists believe that students are not passive recipients of knowledge but they create (construct) new mathematical knowledge by reflecting on their physical and mental actions. According to constructivists, learning reflects a social process in which children engage in dialogue and discussion among themselves as well as others as they develop intellectually (Hanley, 1994). This method is the best method to move away from the traditional method of teacher-centered practices, and more empowerment to the teacher and students both for better critical thinking and creating changes in old teaching methods (Sawada et al., 2002).

Hanley (1994) explains in more detail on the implementation of constructivist teaching which comprises the following procedures for teachers: seeking out and using students' questions and ideas, collaborating on and encouraging the use of alternative sources for information, encouraging students to challenge each other's conceptualizations and ideas, encouraging self-analysis, involving students in seeking information that can be applied in

solving real-life problems, emphasizing career awareness, especially those related to science and technology.

Discovery Learning

Discovery learning points to learning that occurs once students are required to find out something by themselves. For example, rather than telling students the value, the teacher asks them to measure spherical objects to find out the value themselves (Cruickshank, Bainer & Metcalf, 1995). Teachers use discovery learning to achieve three educational goals. Firstly, they would like learners to recognize how to find out things and think on their own. In fact, they would like them to be less dependent on getting knowledge from teachers and acknowledge the conclusion of others. Secondly, discovery learning users would like learners to see in what way knowledge is achieved. It indicates that teachers would like students to be enabled to learn by gathering, organizing, and analyzing information to achieve their own conclusion. Thirdly, the teachers would like learners to employ their higher order thinking skills. Among other things, they want students to be able to analyze, synthesize, and evaluate (Cruickshank et al., 1995).

One of the characteristics of discovery learning is that the role of the teacher is not imparting knowledge but rather creating and guiding classroom experiences in which learners are engaged to discover knowledge. The second characteristic of discovery learning is that while leaners are dealing with discovery, the teacher motivates them to think profoundly. The third feature is that learners acknowledge the challenge of realizing something for themselves rather than requiring the teacher to provide for them answers (Cruickshank et al., 1995).

Methodology

The qualitative case study method was considered more feasible and appropriate to meet the study aim. Data were collected using observation and semi-structured interviews with seven teachers, who were selected by the snowball method, in two secondary schools in different states of Malaysia. However, in this paper, we only bring data related to the three teachers including Teacher A, Teacher B, and Teacher C to examine the different teaching methods applied based on the different learning theories. The questions asked in the interviews and the criteria considered in the observations were related to the teaching methods based on the learning theories. The participants (the teachers) were interviewed and their performances in their classes were observed to analyze their teaching methods. On average, each teacher was interviewed for two hours and observation on their teaching was for six hours. In between the interviews and observations, the researchers had also made several non-formal conversations with the teachers, including discussion on their lesson plans.

Findings

As teachers have important roles in presenting the content of text books to their students, the researchers conducted some interviews with the mathematics teachers and observed their classes during their teaching hours. As examples for considering mathematics method of teaching by the teachers, based on different methods of teaching, we selected three of them teachers A, B, and C. Teacher A graduated in Mathematics Education and has taught for 25 years, while teacher B graduated in Computer Sciences (mathematics) and has taught for 20 years, and Teacher C graduated in Mathematics Education and has taught for 5 years.

Teacher A

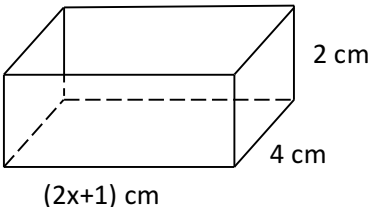
The observations supported that Teacher A used two kinds of teaching methods including the traditional based on the theory of behaviorism and problem solving. The observations confirmed that she paid attention to the students' ideas related to the course and provided opportunities for them to think about mathematics problems and subjects and allowed them to participate in the class activities and group discussions. She used the

blackboard and extra mathematics books in the class. Although the class was managed by more teacher-centered techniques, the students participated in the class through discussions on their understanding and reasoning. When asked about methods of teaching in the class during the interview, her response indicated that she was more inclined towards problem solving.

> "When I teach a topic, firstly I give examples and I solve them on board, then I give some other questions to my students and ask them to solve them, when they are solving the examples, I go around the class, and I can see whether they understood the topic. Sometimes I just ask a student who is in middle level to answer the question if they answer my questions, I know the other students understood it as well" (Interview with Teacher A, August 2011, p.8).

In her classes, she allowed her students to ask questions and discuss the topic and she gave her students extra tasks and exercises especially when they were weak in certain areas to enhance her students' understanding. She believed that students should take part in the teaching-learning processes, and use all their senses to optimize their understanding and to learn deeply.

Also, the observations were conducted when Teacher A was teaching linear equations and solving some mathematics problems. The observations showed that she was teaching the linear equation topic for Form 2 middle level students. The observations confirmed that she started the class based on her lesson plan and the main textbook. The example of linear equation was to find the value of x given the volume of the cuboid is 88cm^3. She used the four steps of problem solving and discovery learning to solve the problem.



Teacher A posed a mixed problem with two aims: geometry and algebra, and asked students to find the value of x when the volume v is 88 cm^3, to show how to solve a linear equation:

$v = 88cm^3$

$(2x + 1)\times 2\times 4 = 88 \rightarrow (2x + 1)\times 8 = 88 \rightarrow 2x + 1 = 11 \rightarrow 2x = 10 \rightarrow x = 5$

The observations showed that she used mixed methods of teaching and she was very flexible in her teaching methods and used problem solving and discovery learning activities, although the time was limited (around 65 minutes in one session).

Teacher B

The observation was conducted when she was teaching geometry. In her class, students were working together and she helped them while they were solving the problems. She also collected students' notebooks which they used for doing their homework. In another session when the researcher observed her class, she was teaching linear equations, and all of her students took part in the class and problem solving activities.

$$(1)\ \frac{3}{4}\mathrm{X} - 3 = \mathrm{X} + 4 \qquad (2)\ \frac{8\mathrm{m} + 7}{4\mathrm{m} - 3} = 3 \qquad (3)\ \frac{\mathrm{Y} - 6}{3} = \frac{6 - \mathrm{Y}}{2}$$

She had two purposes in presenting these exercises including teaching fraction and equations. In this way, she used the problem solving method to solve the problem by engaging the students in the problem solving process.

She gave an introduction about a new subject before starting it, but she did not review the previous lessons to relate the new lesson to the previous one. Therefore, the students may have some difficulties in linking the previous lesson to the new one. For example, she gave the students three exercises regarding linear equations as the following:

$$(1)\ \frac{3}{4}x - 3 = x + 4 \rightarrow \frac{3}{4}x = x + 3 + 4 \rightarrow \frac{3}{4}x = x + 7 \rightarrow 3x = 4(x + 7)$$

$$\rightarrow 3x = 4x + 28 \rightarrow 3x - 4x = 28 \rightarrow x = -28$$

$$(2)\ \frac{8m + 7}{4m - 3} = 3 \rightarrow 8m + 7 = 3(4m - 3) \rightarrow 8m + 7 = 12m - 9$$

$$\rightarrow 8m - 12m = -9 - 7 \rightarrow -4m = -16 \rightarrow m = 4$$

$$(3)\ \frac{y - 6}{3} = \frac{6 - y}{2} \rightarrow 2(y - 6) = 3(6 - y) \rightarrow 2y - 12 = 18 - 3y$$

$$\rightarrow 2y + 3y = 18 + 12 \rightarrow 5y = 30 \rightarrow y = \frac{30}{5} \rightarrow y = 6$$

While the students were doing the exercise, the teacher discussed with them and asked them what were differences between y-6 and 6-y and what happens in this exercise if the equation was: $\frac{Y-6}{3} = \frac{Y-6}{2}$. This discussion among them was interesting for students.

Teacher B taught low and middle level students in her classes. She had a schedule for her teaching instruction and tried to follow a student-centered method of teaching. In fact, it was not possible because there were around 40 students in her class. Also, she was using the problem-solving method of teaching. However, it depends on the topic and the situation of the class.

In the interview, she mentioned that she used diverse examples to describe the mathematics subject in her classes. Sometimes, she wanted her students to ask questions and discuss about them and also allowed them to share their ideas in the class. Regarding extra activities, she explained. *"Normally I give them homework to do at home and hand it in the next session; I also give them some exercises in the class to check whether they understand the topic" (September 2011, p. 40)*. Also, she explains her approach in dealing with the students' differences as follows:

> "We should know our students by name and respect them in the class and we also need to give them questions based on their comprehension level, we should give easier questions to weak students" (Interview with Teacher B, September 2011, p.39).

Teacher B believed that students should play a very important role in their learning processes. She provided students with the opportunities to participate in the class activities and group discussions.

Teacher C

In the observation, Teacher C used the method of discovery learning, despite sometimes using the traditional method for beginning the new lesson. In good spirits, she began to review the previous lesson and gave an introduction about the new subject before starting it. She had a lesson plan for teaching and paid attention to the students' ideas regarding the course. She allowed the students to participate in class activities, provided opportunities for them to think about the mathematic subject, and allowed group discussions in the class. She also used the whiteboard, PowerPoint, educational CDs, and extra mathematical books in the class. While doing exercises, she paid attention to students' ideas to clarify and to allow them to identify the problems.

In the interview about her methods of teaching, she believed in solving mathematics problems using discovery learning to increase activities and creative thinking of students and to encourage them to use skills of problem-solving approaches for solving mathematics problems.

> "We have many activities for students. In class, we do so many exercises and problem solving. We also have outdoor activities; in this school we take students for math, science and career festivals. These festivals encourage students to improve themselves in mathematics and science. Students should improve themselves to know what concepts in mathematics are related to other concepts in physics and so on" (Interview with teacher C, October 2011, p. 71).

An example that she wrote on the whiteboard was a linear equation with two variables $\frac{x}{y} = \frac{7}{2}$ and $x + y = 63$, with answers $x = 7$ and $y = 2$. And with the result $x + 7 = 9$ which was an interesting problem for discussion. Teacher C asked the students to solve the posed problem as $\frac{x}{y} = \frac{7}{2}$ and $x + y = 63$.

If $\frac{x}{y} = \frac{7}{2}$ and we knew that $(7, 2) = 1$, so we could say $x = 7$ and $y = 2$ which was the students answer in the mathematics classroom, then $x + y = 9$. However, in this case, there was another function $x + y = 63$, so we could not say $x = 7$ and $y = 2$ because $x + 7$ was not 63 so we have to solve the problem as two equations with two unknowns because we would want to find the solution of the equation by following:

$$(1)\ \frac{x}{y} = \frac{7}{2} \rightarrow 2x = 7y \rightarrow 2x - 7y = 0 \ \text{ and } \ (2)\ x + y = 63$$

$$\rightarrow y = 63 - x \rightarrow 2x - 7(63 - x) = 0 \rightarrow 2x - 441 + 7x = 0$$

$$\rightarrow 9x = 441 \rightarrow x = \frac{441}{9} = 49 \rightarrow y = 63 - 49 \rightarrow y = 14$$

Then the real answer was $x = 49$ and $y = 14 \rightarrow x + y = 63$.

She applied the method of discovery learning. The observations confirmed that she encouraged the students to express their ideas about the concept. She asked questions which were relevant to the students' level of learning and allowed them to ask questions and discuss

about the subject. Hence, she managed the class by the student-centered approach. Students were active during the teaching and learning activities, and the teacher had planned the activities well.

To sum up, our observation and interview results showed that learning theories including behaviorism, cognitivism and constructivism as well as teaching methods including traditional (teacher-centered), problem solving (teacher- and student-centered) and discovery learning (student-centered) were practiced by the teachers.

Discussion

This section aims to discuss the findings of interviews and observations of the three teachers regarding their teaching practices by considering the teaching methods utilized. Teaching practices have been analyzed from the response of the participants in the interviews as well as the observations done by the researchers. In addition to considering the aforementioned methods, the authors also paid attention to learning theories in general.

According to these interviews and observations and based on the authors' experiences in teaching mathematics, teachers usually tended to use problem-solving and discovery-learning methods in their teaching. It should be noted that it is not possible to exactly declare that the teachers use one method at all times. They applied other methods such as the traditional method, if necessary. However, due to limited teaching time and the large number of students in a class, teachers were more likely to apply the problem-solving method.

Teacher A used the mixed methods of the traditional (theory of behaviorism) and problem solving, Teacher B used the problem solving approach, and Teacher C used discovery learning. Based on observations in the classes and experiences of authors during many years of teaching mathematics, students in the classes that the teacher had used the traditional method had difficulties to understand mathematics and they had anxiety when performing class activities, discussions and exams. However, in classes in which the teachers had used other methods, the students were actively involved in class activities, group discussions with each other and with the teachers, and they had less or no anxiety to take exams.

Conclusion

The findings suggest that problem-solving and discovery-learning skills not only contribute to better mathematics learning but also enhance students' creativity to cope with life challenges. Since constructivist approaches give students the opportunity to think creatively, there should be more emphasis on teaching methods which include less lecture, more student-directed classes and more discussions. In classes that used the problem solving method, students are more active, they think better, and they have less anxiety for exams. In summary, the results indicate that students who learn mathematics by problem-solving and discovery-learning methods are more active in comparison with the students under the traditional teacher-centered method. These approaches, mainly, encourage students to think rationally in their daily life, and enhance their thinking, and reasoning power. The findings reveal that students are more successful and encouraged when systematic problem solving based on Polya's approach is incorporated in the lessons. These methods prepare students better in solving problems and facing discovery learning.

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