Fluid Mechanics Modules

The Cooper Union for the Advancement of Science and Art

by
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The enclosed material has a number of fluid mechanics modules. The modules consist of a series of eleven self-contained sets of material that include:

● Theory
● Design examples
● Design homework problems
● Design homework projects
● Historical notes
● Experiments and demonstrations
● References

The theory consists of a brief overview of topics that generally make up one chapter in a fluid mechanics text. It is given as a guide to students; not to replace a standard fluid mechanics textbook. The design examples, homework, and projects are important in this revised fluid mechanics course because they show the students how fluid mechanics is used in engineering design.

The eleven modules attached are:

(1) Basic principles
(2) Fluid statics
(3) Kinematics
(4) Conservation of mass
(5) Conservation of momentum
(6) Equation of energy, turbulence, and pipe flow
(7) Dimensional analysis and similitude
(8) Navier Stokes equations
(9) Potential flow
(10)Boundary layer theory
(11)One-dimensional compressible flow

We are proposing a series of tabletop experiments that would be available in the studio/classroom to demonstrate certain points during lectures, and to let students experiment and discover for themselves some of the basic principles of the discipline. For this purpose, the equipment will also be accessible to the students during non-class time. The manuals to conduct the demonstrations/ experiments will be provided by the vendor "Tec Quipment Incorp."

The experiments and demonstrations have been keyed to the theory (lectures) by placing a letter next to the topic in the left hand margin. This letter refers to an experiment that can be used to demonstrate a fluid mechanic phenomena. In the experimental portion of the module the experiment is described.

A list of the proposed apparati is given below. These apparati will be used to demonstrate some of the following fluid mechanics principles:

Hele-Shaw: Potential flow, La Place equation
Impact of a Jet: Conservation of linear momentum
Losses in Pipe: Conservation of energy - Bernoulli's principle, pipe
              systems, turbulence, meter flow.

Flow Channel: Uniform, gradually varied, and rapid flow; hydraulic jump
              vs. shock wave.

Hydrostatic & Property of Fluids: Fluid properties and hydrostatics
Hydraulic Bench: Weir flow

The studio/classroom will have multimedia equipment readily available, since there is now an abundance of videotapes and CD-ROM's which illustrate principles of fluid mechanics, flow visualization, turbulence, etc.

The Cooper Union has acquired a series of reference video tapes demonstrating basic fluid mechancis principles:

Fluid Mechanics
Flow Visualization
Turbulence
Vortcity
Low Reynolds – Number Flow
The Fluid Dynamics of Drag
Eulerian and Lagrangian Description in Fluid Mechanics
Stratified Flow
Cavitation
Pressure Fields and Fluid Acceleration
Fundamentals of Boundary Layers
Boundary Layer Control

I. Basic Principles

(a)   • Fluid - A media that deforms continuously under a constant shear force.

•Units

$\gamma = \rho g \equiv$ Specific Weight

English: $\left(\dfrac{\#}{ft^3}\right) = \left(\dfrac{slugs}{ft^3}\right) \times \left(\dfrac{ft}{sec^2}\right)$

Metric: $\left(\dfrac{N}{m^3}\right) = \left(\dfrac{kg}{m^3}\right) \times \left(\dfrac{m}{sec^2}\right)$

Water at 60⁰F: $\gamma = 62.4 \ ^{lb}\!/_{ft^3} = 9806 \ ^{N}\!/_{m^3}$

$\rho = 1.938 \ ^{slugs}\!/_{ft^3} = 1000 \ ^{kg}\!/_{m^3}$

Mass = $\dfrac{FT^2}{L}$ = slug = 32.2 lbm where lbm = Pound Mass

Mass = (kg)

Earth Surface g = $32.174 \ ^{ft}\!/_{s^2}$

1 kg force = 1 kg mass at g = $9.806 \ ^{m}\!/_{s^2}$

(b)   • Coefficient of Viscosity (resistance to shear)

$v = ^{\mu}\!/_{\rho}$

μ (dynamic)   ν (kinematic)

$\left(^{lbx\,sec}\!/_{ft^2}\right) = f(Temp)$ $\left(^{ft^2}\!/_{sec}\right)$

$\left(^{slug}\!/_{ftx\,sec}\right)$ $\left(^{m^2}\!/_{sec}\right)$
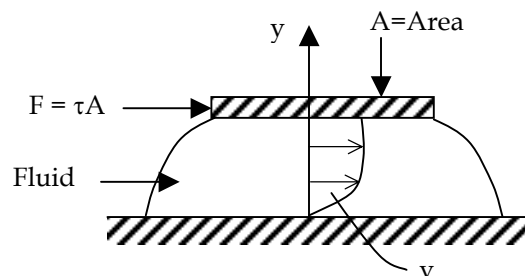
-     Causes of Viscosity (μ = μ(Temp))
      Liquid: Cohesion; Temperature increases, cohesion decreases
              and so does μ
      Gas: Momentum transfer; Temperature increases, motion of
              molecules increases and so does μ

• Newtonian Fluid

Shear Stress, $\tau = \mu \dfrac{\partial v}{\partial y}$

1) Elastic: Deform proportional to F but not continuously at a defined rate
2) Fluid: Deform proportional to F and <u>continuously</u> at a defined rate
3) Vacuum: Deform proportional to F and at an ever increasing rate (no resistance)

(c) • Perfect Gas

Perfect elastic collision of molecules

Equation of State→$pv = RT$ where $v = (1/\rho)$

p=Absolute Pressure; R = Gas Constant, depends only on the molecular weight of fluid

$$p \equiv \left(\frac{lb}{ft^2}\right), \left(\frac{N}{m^2}\right), (Pascal)$$

Standard Atmosphere p = 101,325 Pascals = 14.7 psia

Gases near condensation conditions depart from a perfect gas; steam, ammonia

• Compressibility

Liquids are only slightly compressible under pressure, but may be important (ie. water hammer)

Coefficient of Compressibility $\beta = -\dfrac{1}{V}\left(\dfrac{\partial V}{\partial p}\right)_T$
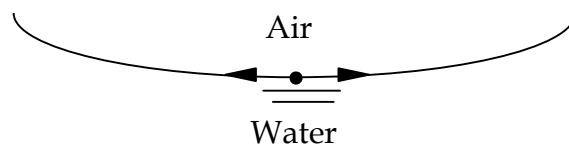
where T≡ Constant Temperature and V≡ Volume

Bulk Modulus $\kappa = \dfrac{1}{\beta}$

(d) •Surface Tension (σ)

Cohesion - Molecular attraction like molecule

Adhesion - Molecular attraction unlike molecule

Air

Water

Measured as a line load for water:

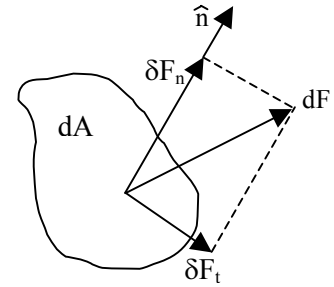$$\sigma = .073 \; N\!/\!m = 4.92 \times 10^{-3} \; lb\!/\!ft$$

- Stress Field

  Normal Stress

  $$\tau_n = \lim_{\delta A_n \to 0} \frac{\delta F_n}{\delta A_n}$$

  Shear Stress

  $$\tau_t = \lim_{\delta A_n \to 0} \frac{\delta F_t}{\delta A_n}$$

  The stationary or uniformly moving fluid

  $$\tau_{xx} = \tau_{yy} = \tau_{zz} \qquad \text{(Pascal's Law)}$$

  shear stress are zero

  Nonviscous fluid in motion

  shear stress are zero
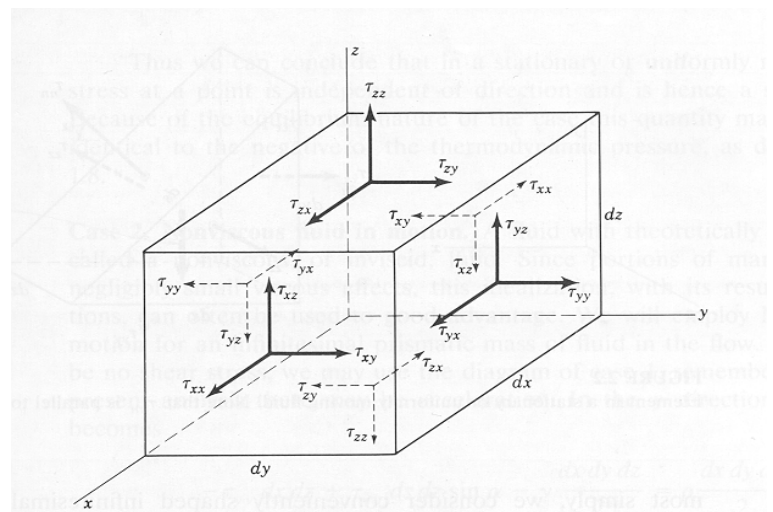
  $$\tau_{xx} = \tau_{yy} = \tau_{zz}$$

  Viscous fluid with no rotation
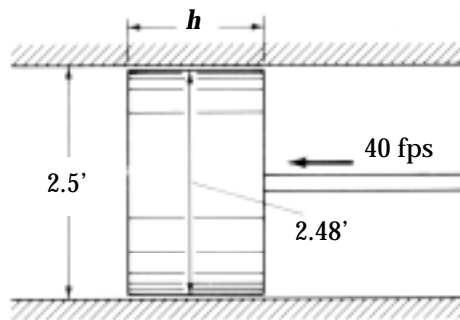
  $$\tau_{xy} = \tau_{yx}$$
  $$\tau_{yz} = \tau_{zy}$$
  $$\tau_{zx} = \tau_{xz}$$

6

II. Design Example

Design a plunger of width **h** that is moving at $40\,\frac{\text{ft}}{\text{sec}}$ through a cylinder. The force necessary to move this cylinder is 155.82 lb. and the viscosity of the film separating the plunger from the cylinder is .010 $\frac{\text{lb}\cdot\text{sec}}{\text{ft}^3}$.
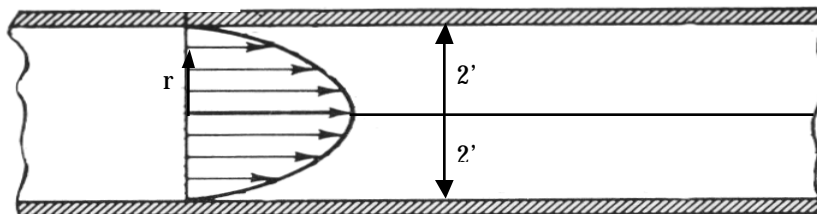

$$\therefore \quad F = \mu\left(\frac{V}{y}\right)A$$

$$155.82 = (.04)\left(\frac{40}{\left(\dfrac{2.5 - 2.48}{2\cdot 12}\right)}\right)\left(\frac{\pi \cdot 2.48 \cdot h}{144}\right)$$

$$\therefore \quad \textbf{h} = 1.5\text{"}$$

III. Design Homework Problem

To design a circular pipe, determine the wall shear stress. Assume the fluid is water at 60^0F. Also find the force on a 100' length of pipe.

$$v = 4 - r^2$$

Blaise Pascal clarified principles of the barometer, the hydraulic press, and pressure transmissibility in the seventeenth century. Isaac Newton (1642-1727) explored various aspects of fluid resistance (inertia, viscosity and waves) and discovered jet contraction.

V. Experiments/Demonstrations

- Hydrostatic and Properties of Fluids Apparatus (H 314)
    (a) Pressure Gauges (Bourden and Mercury Barometer): Operation and calibration of a Bourden pressure gauge and mercury barometer to determine pressure variation for different liquid and gas containers and vessels.
    (b) Falling Sphere Viscometer: A sphere will be placed in a fluid and the rate of fall will be timed. By a force balance equation the viscosity will be estimated and by the use of a viscometer. This experiment can be repeated using different liquids and spheres.
    (c) Determination of Fluid Density and Specific Gravity: To establish the specific gravity of a liquid when compared to water a Hares Tube Apparatus will be used.
    (d) Surface Tension Balance: For measuring the surface and interface tension of liquids.
- Video
    (e) Characteristics of Laminar and Turbulent Flow (22)

VI. References

Mechanics of Fluids, Shames: Sections 1.1 – 1.6, 1.8 – 1.10, 2.1 – 2.4, 2.6 – 2.8

Fundamentals of Fluid Mechanics, Munson, Young, Okiishi: Sections 1.2 – 1.9

Introduction to Fluid Mechanics, Fox, McDonald: Chapter 1, Sections 2-1, 2-4, 2-5

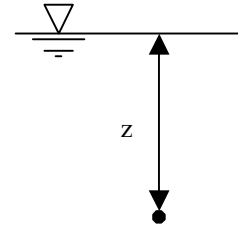Video Tape: Characteristics of Laminar and Turbulent Flow

I. Fluid Statics

(f)   • Pressure Variation - General

(h)   Standard Atmosphere $p = 2116.2 \; {}^{\#}\!/_{ft^2} = 101.3$ kPa

$$\frac{dp}{dz} = -\gamma \, ; \; p = \text{pressure}$$

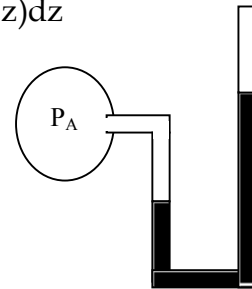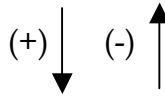$\gamma = \rho g$ where $\gamma =$ Specific Weight and $\rho =$ Density

z

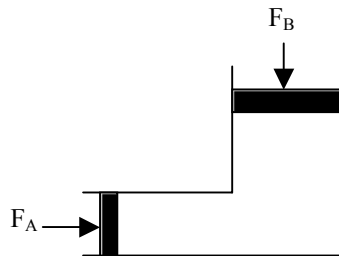(g)   •Pressure Variation - Incompressible $\Delta p = -\int \gamma(z)dz$

Manometer
$\gamma =$ Constant
$p = \gamma z$ (+) $\downarrow$ (-) $\uparrow$
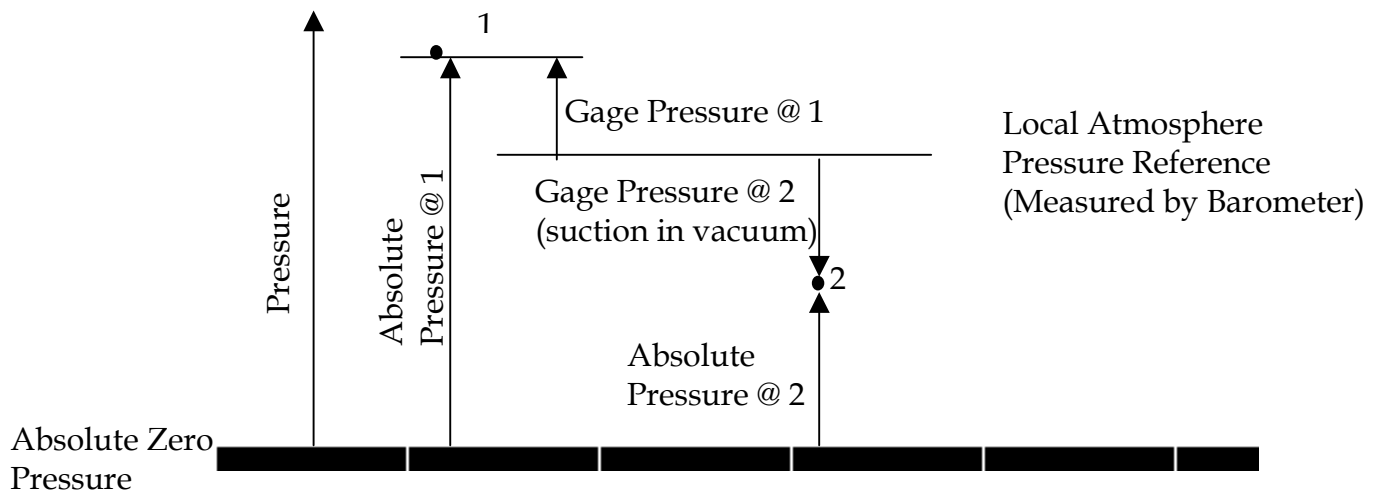
P_A

(h)   Hydraulic Jack

F_B

$F_A << F_B$

F_A

• Measure of Pressure

1

Gage Pressure @ 1

Gage Pressure @ 2
(suction in vacuum)

Local Atmosphere
Pressure Reference
(Measured by Barometer)

2

Absolute
Pressure @ 2

Pressure

Absolute Pressure @ 1

Absolute Zero
Pressure
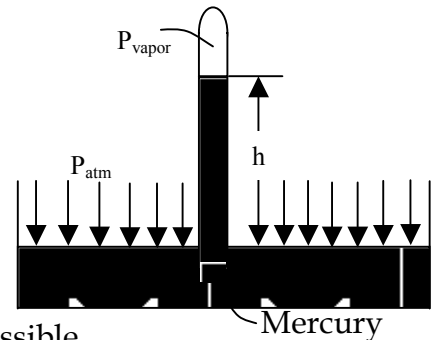
(a) • Mercury Barometer

$P_{atm} = \gamma h + P_{vapor}$

Standard Atmospheric Pressure @ 59^0F ⟶
$2116 \; \frac{lb}{ft^2} = 14.7 \; psi = 101{,}300 \; kPa$

—Mercury

(b) •Hydrostatic Force on a Plane Surface - Incompressible
(e)

Pascal's Law: No Motion ∴ No shear forces ∴ Pressure force perpendicular to the surface

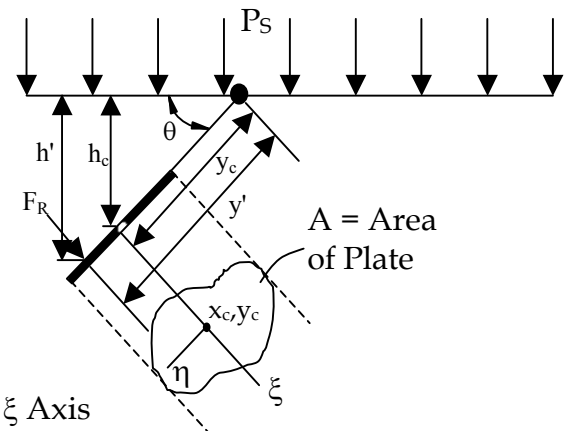Resultant Force = F_R
$F_R = (P_S + \gamma h_c)A = P_c A$

y' = Center of Pressure (F_R)
y_c = Center of Centroid

A = Area of Plate

$y' - y_c = \dfrac{\gamma \sin\theta I_{\xi\xi}}{P_c \, A}$

$I_{\xi\xi}$ = Second Moment of Area about ξ Axis
$P_c = \gamma \sin\theta \; y_c$
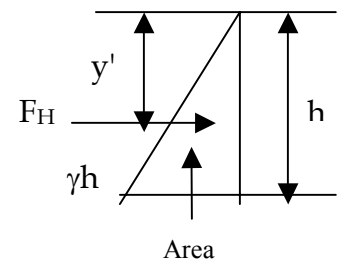
(e) •Pressure Prism

F_H = Volume Prism = $\gamma h A$
$y' \equiv$ Centroid of Prism (in the figure $\frac{2}{3}h$)

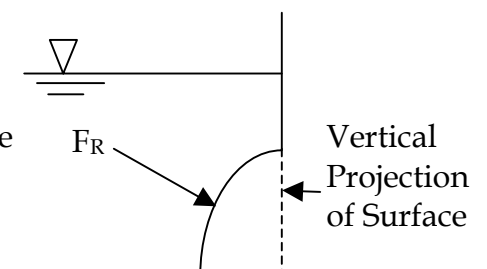•Hydrostatic Force on Curved Surfaces

$d\bar{F}_R = -pd\bar{A}$
$F_R = \sqrt{\left(F_H^2 + F_V^2\right)}$
F_H = Force on the Vertical Projection of the Surface
F_V = Weight of the Liquid above the Surface
y' = Find Center of Pressure for F_V and F_H
$\theta = \tan^{-1}\dfrac{F_V}{F_H}$

Vertical Projection of Surface

10
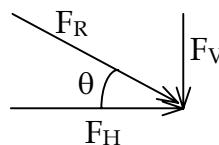
(c)   • Buoyancy

Archimedes Principle
F_B = WT where F_B = WT of Liquid Displaced
y' = Centroid of Displaced Liquid

WT

(Cross Hatched
Volume) F_B

(d)   • Flotation Stability

WT

c'

F_B

Restoring
Couple

WT

c

c'

F_B

Overturning
Couple
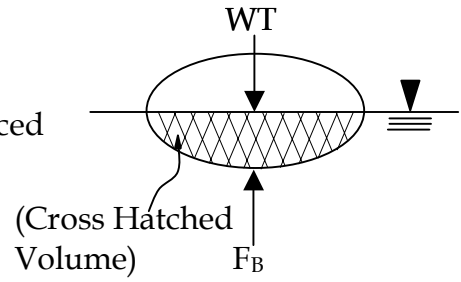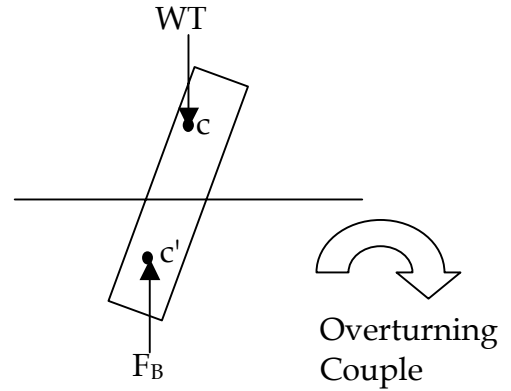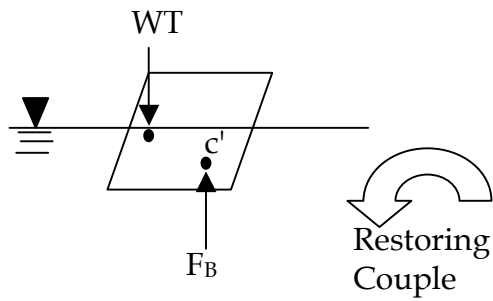
c' = Centroid of New Displaced Volume

II. Design Example

A designer is asked to determine the force on the block shown to hold the 5 ft by 4ft door closed.
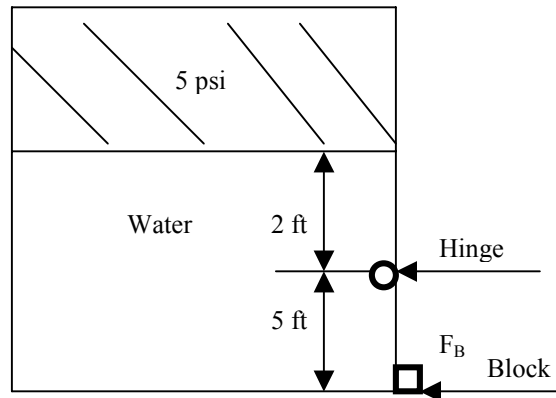
$$F = P_c A = (5 \times 144 + (2 + \frac{5}{2}) \times 62.4)(5 \times 4) = 20{,}016 \text{ lb}$$

$$y' - y_c = \frac{62.4(\frac{4 \times 5^3}{12})}{20{,}016} = 0.1299 \, ft$$

y' from bottom $= 2.5 - 0.1299 = 2.37$ ft

$$F_b \times 5 = 20{,}016 \times 2.37$$

$$\therefore F_b = 9.488 \text{ lb}$$

III. Design Homework Problem

Design the square rod AB shown, $\sigma_{\text{allowable}}$ = 20,000 psi, for the cofferdam shown. The shape and the size of the cofferdam is a 10′ cube and is in 8 feet of homogeneous mud media, S.G. = 1.5. Also find what the weight of the cofferdam must be so that it stays in place.

Design Homework Project

You are asked to check the design for an 11.875′ wide gate, water temperature 57.5⁰F. A series of nine piezometers are drilled in the gate to determine the horizontal force on the gate (nu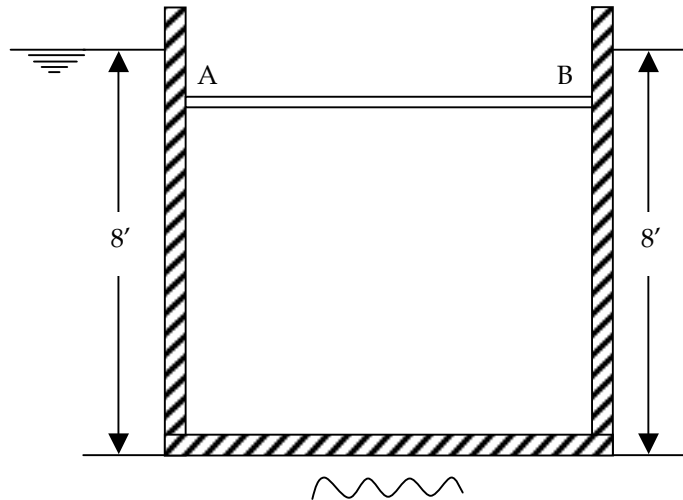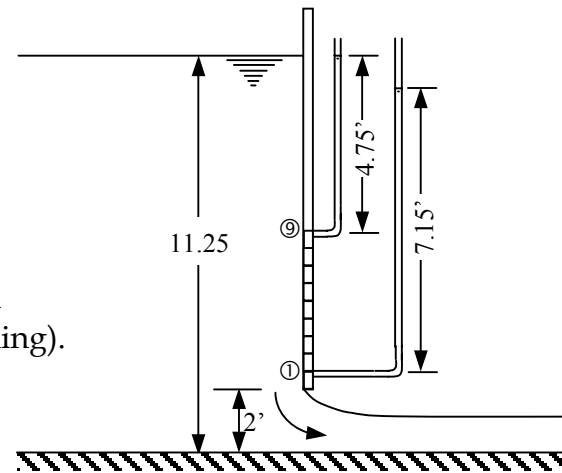mber 1 and number 9 are shown). The first piezometer starts .5′ from the bottom of the gate and the rest of the piezometers are spaced .5′ apart. A table of the height of the water in each piezometer is given (measured from the piezometer opening).

Piezometer	1	2	3	4	5	6	7	8	9
Height of Water (ft)	7.15	7.45	7.35	7	6.5	6.25	5.75	5.25	4.75

(a) Why can you use hydrostatics to determine the horizontal force on the gate?
(b) Plot the pressure on the gate.
(c) Determine the horizontal force on the gate.
(d) Plot the hydrostatic pressure on the gate and determine its force on the gate.
(e) Why is the force determined in (d) greater than the force determined in (c)?

IV. Historical Notes

The science of fluid mechanics began with the need to control water for irrigation in ancient Egypt, Mesopotamia, and India. Although these civilizations understood the nature of channel flow, there is no evidence that any quantitative relationships had been developed to guide them in their work. It was not until 250 B.C. that Archimedes discovered and recorded the principles of hydrostatics and flotation.

V. Experiments/Demonstrations

- Hydrostatic and Properties of Fluids Apparatus (H314)
    - (a) Hydrometer: Principle and use to determine specific gravity. Place the hydrometer in the liquid and record the specific gravity by reading the graduated stem.
    - (b) Pascal's Law: Determine stress at a point with no shear. Measure the pressure on different surfaces in different vessels.
    - (c) Archimedes' Law: Demonstration of flotation by liquid displacement. Different objects will be placed in water and direct measurements will be indicated by the buoyant force.
    - (d) Stability of a Floating Body: The metacentric height is measured directly off the scale.
    - (e) Force and Center Pressure on a Plane Surface: The pressure force and its location is read off of a gage. This test can be repeated for different surfaces in a variety of vessels.
    - (f) Pressure Gauges (Bourden and Mercury Barometer): Operation and calibration of a Bourden pressure gage and mercury barometer in a number of different vessels.
    - (g) Manometers (Fluid/Air): A series of U tubes is used to measure pressures: fluid/air and mercury in and underwater.
- Videos, CD's, etc.
    - (h) ASCE Fluid Mechanics Text

VI. References

Mechanics of Fluids, Shames: Sections 3.1 – 3.2, 3.4 – 3.10

Fundamentals of Fluid Mechanics, Munson, Young, Okiishi: Sections 2.1 – 2.11

Introduction to Fluid Mechanics, Fox, McDonald: Chapter 3

I. Kinematics

- Velocity Field $\bar{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

      $v_x = f(x,y,z,t)$
      $v_y = f(x,y,z,t)$
      $v_z = f(x,y,z,t)$

(a-g)   • Stream Lines $d\bar{s} \times \bar{v} = 0$

Lines are drawn parallel to velocity vector

    (Photo)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & dy & dz \\ v_x & v_y & v_z \end{vmatrix} = 0$$

streamtube
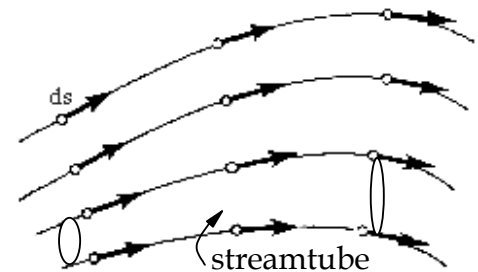
(a-g)   • Pathlines

$$v_x = \frac{dx}{dt}; \; v_y = \frac{dy}{dt}; \text{Eliminate t or solve in terms of t}$$

Lines tracing a particle's path for laminar steady flow the same as a streamline (Movie)

(a-g)   • Streaklines

All particles in a flow that have previously passed a common point. Locus of particles at t that previously (time less than t) passed through 0.

Streakline at fixed time passing 0

Pathlines

- Acceleration of a Flow Particle (Material Derivative)

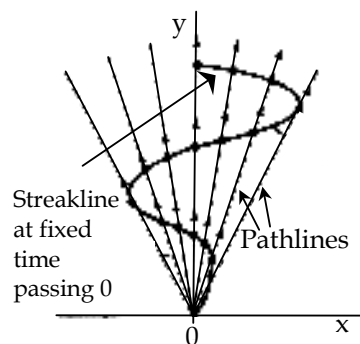$$\frac{DV}{Dt} = \left(\vec{V} \bullet \vec{\nabla}\right)\vec{V} + \frac{\partial \vec{V}}{\partial t}; \quad \frac{D}{Dt} = V \bullet \nabla + \frac{\partial}{\partial t} \quad \text{(Material Derivative)}$$

(d)

(e)   Steady flow $\frac{\partial V}{\partial t} = 0$ and $V \neq f(t)$

(e)   Uniform flow $(V \bullet \nabla)V = 0$

(e)
(b)
(f)

- One Dimensional flow
    All variables only a function of the dimension along the
    streamline.

    Constant across streamline
    $V_x = f(x)$ ⟶   x   (Note: x is direction of
    $V_y = g(x)$ streamline)
    $V_z = h(x)$

(g)   • Control system (La grangian)
    Collection of matter of fixed identity which may move, flow, and
    interact with its surroundings. (Direct description of the motion of
    the particle as a function of time)
    Material Derivative $\left(\frac{D}{Dt}\right)$ and path line follow system in time and
    space.

(g)   • Control Volume (Eulerian)
    Volume in space (a geometric entity, independent of mass) through
    which fluid may flow.
    Stipulating fixed coordinates as a function of time. Streamlines
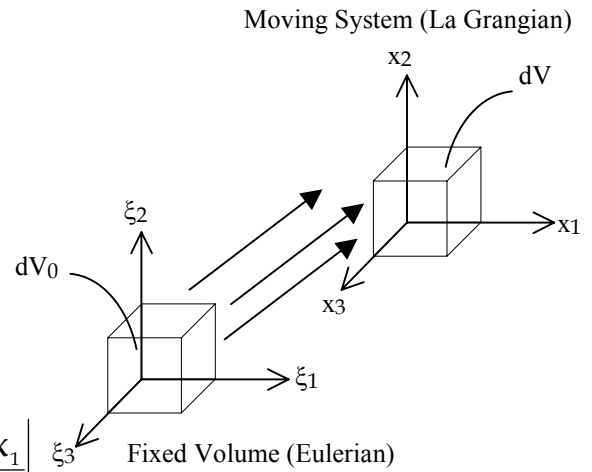
•Transformation Jacobian

$dV = JdV_0$

$dV = d_{x1}d_{x2}d_{x3}$

$dV_0 = d\xi_1 d\xi_2 d\xi_3$

$$J = \frac{\partial(x_1, x_2, x_3)}{\partial(\xi_1, \xi_2, \xi_3)} = \begin{vmatrix} \dfrac{\partial x_1}{\partial \xi_1} & \dfrac{\partial x_1}{\partial \xi_2} & \dfrac{\partial x_1}{\partial \xi_3} \\ \dfrac{\partial x_2}{\partial \xi_1} & \dfrac{\partial x_2}{\partial \xi_2} & \dfrac{\partial x_2}{\partial \xi_3} \\ \dfrac{\partial x_3}{\partial \xi_1} & \dfrac{\partial x_3}{\partial \xi_2} & \dfrac{\partial x_3}{\partial \xi_3} \end{vmatrix}$$

$$\frac{DJ}{Dt} = (\nabla \bullet v)J \qquad \text{Basically Conservation of Mass}$$

• Relation between the System and the Control Volume
  Reynold's Transport Equation

Control System   Control Volume
coor. (x_1, x_2, x_3) coor. (ξ_1, ξ_2, ξ_3)

$$\frac{D\bar{F}}{Dt} = \oiint f(\rho\bar{v} \bullet d\bar{A}) + \frac{\partial}{\partial t} \iiint f\rho dV_0$$

$$F = \iiint f dV_0$$

$\bar{F} = $ Fluid Parameter $\equiv$ Mass, Momentum, Energy

II. Design Example

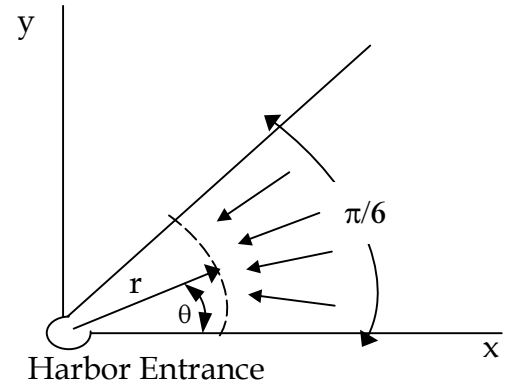An engineer wishes to determine the scour velocity and volume rate of flow (per unit length) produced by flow into the wedge shaped harbor entrance shown. Assume the potential function can be expressed by the following equation $\phi = -2\ln r$.

$$v_r = \frac{\partial\phi}{\partial r} = -\frac{2}{r}; \; v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = 0$$

Note! The velocity increases to the harbor entrance.

$$q = \int_0^{\pi/6} v_r d\theta = -\int_0^{\pi/6}\left(\frac{2}{r}\right) r d\theta = -\frac{\pi}{3} = -1.05\,\text{ft}^2\!\big/\!\text{sec}$$

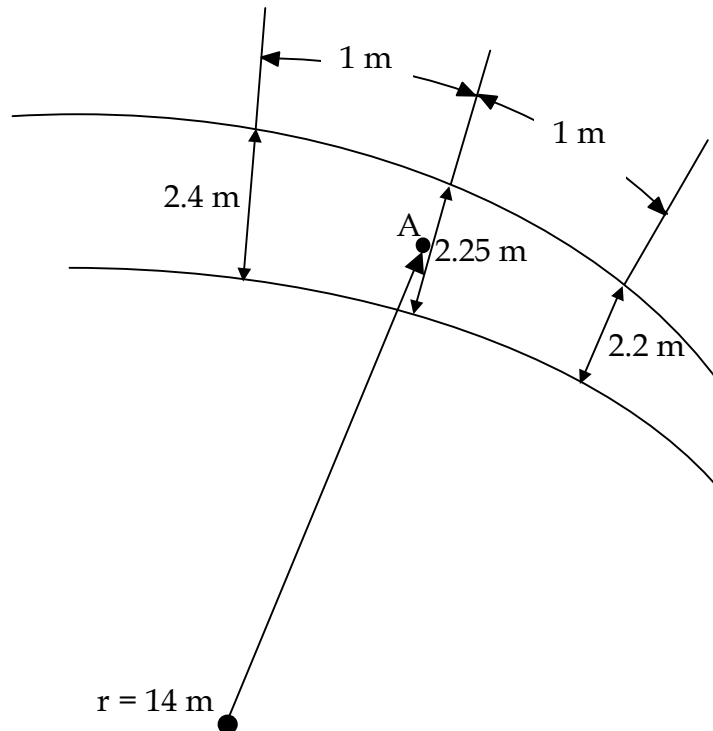The negative sign indicates that the flow is toward the harbor entrance.

III. Design Homework Problem

You are asked to determine the movement of sediment in the stream shown at point A. As a first step find the acceleration at point A. Assume two dimensional flow and $Q = 40\,\dfrac{\left(\text{m}^3\big/\text{sec}\right)}{\text{m}}$.

$$a_n = \frac{v^2}{r} \quad\text{and}\quad a_t = v\frac{\partial v}{\partial s}$$

IV. Historical Notes

Beginning with the Renaissance period (about the fifteenth century), a continuous series of contributions began that form the basis of the science of fluid mechanics. Leonardo da Vinci (1452-1519) described through sketches and writings many different types of flow phenomenon. The work of Galileo Galilei (1564-1642) marked the beginning of experimental mechanics.

V. Experiments/Demonstrations

- Hele-Shaw Apparatus (H 9)
    - (a) Source and Sink: Observation of streamlines from a regulated flow of dye solution. The students can vary the positions and place different objects in the flow to determine a variety of streamline patterns.
    - (b) Source in uniform flow: same as (a)
    - (c) Doublet in uniform flow: same as (a)
    - (d) Flow around a cylinder: same as (a)

- 2.5 Metre Flow Channel (H 23)
    - (e) Dye Study of uniform flow (vel. Profile): Streamlining and pathlining by dropping dye in the channel for both steady, unsteady and uniform and non-uniform flows. These conditions can be obtained by placing objects (weirs, channel spaces, etc.) in the flow and also removing them abruptly and inserting dye streams.

- Videos-
    - (f) Flow Visualization (11)
    - (g) Eulerian and Lagrangian Description in Fluid Mechanics (10)

VI. References

Mechanics of Fluids, Shames: Sections 4.1 – 4.3, 4.6 – 4.10

Fundamentals of Fluid Mechanics, Munson, Young, Okiishi: Sections 4.1 – 4.4

Introduction to Fluid Mechanics, Fox, McDonald: Sections 2-2, 4-2, 5-3

Video Tape: Flow Visualization
              Eulerian and Lagrangian Description to Fluid Mechanics

I. Conservation of Mass

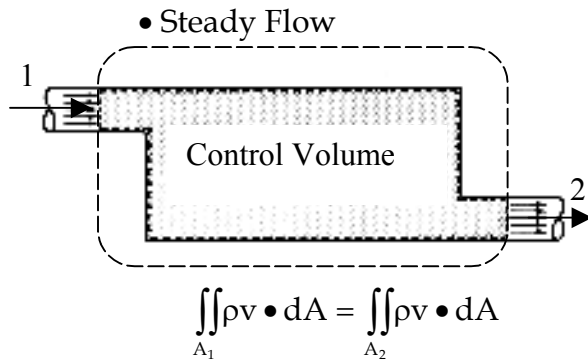$\bullet \dfrac{DM}{Dt} = 0$ (Control System)

$$M = \iiint \rho dVol$$

$$\oiint \rho v \bullet dA \;=\; -\dfrac{\partial}{\partial t} \iiint \rho dVol \quad \text{(Control Volume)}$$

$$\nabla \bullet (\rho v) \;=\; -\dfrac{\partial \rho}{\partial t}$$

Steady Flow $\oiint (\rho v \bullet dA) = 0$

(c) $\bullet$ Steady Incompressible Flow

$$\oiint v \bullet dA = 0 \longrightarrow \nabla \bullet v = 0$$

$\bullet$ Steady Flow

$$\iint_{A_1} \rho v \bullet dA = \iint_{A_2} \rho v \bullet dA$$

$\bullet$ Steady One Dimensional Flow

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2 = \dot{m} \equiv \text{Mass Flow} \quad \left(\dfrac{slug}{sec}\right) \text{or} \left(\dfrac{kgm}{sec}\right)$$

(a)

(b) $\bullet$ Steady, Incompressible, One Dimensional Flow

(c)

$$v_1 A_1 = v_2 A_2 = Q \equiv \text{Discharge} \quad \text{(cfs) or (cms)}$$
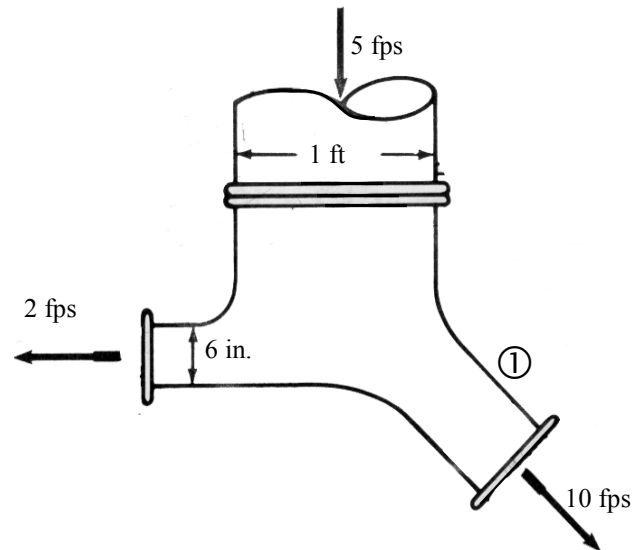
II. Design Example

To assure that there will be no settling of sewage waste, a minimum velocity of $10 \; \frac{ft}{sec}$ is required in pipe ①. Determine the diameter of this pipe.

$Q_{IN} - Q_{OUT} = 0$

$$\left(\frac{\pi}{4}\right)\left(1^2\right)5 - \left(\frac{\pi}{4}\right)\left(.5^2\right)2 - \left(\frac{\pi}{4}\right)\left(D^2\right)10 = 0$$

D = 0.6704 ft or 8.044 in

Take D = 8 in

5 fps

1 ft

2 fps

6 in.

①

10 fps

III. Design Homework Problem

You are asked to design a lubrication system that has a total flow of $2 \; \frac{m^3}{sec}$ and a S.G. of 0.95 from the inflow shown. Determine the S. G. of the oil stream.

Water

$Q_1 = 1 \; m^3/sec$

S. G. = .95

$Q = 2 \; m^3/sec$

Oil ↑ Q_2
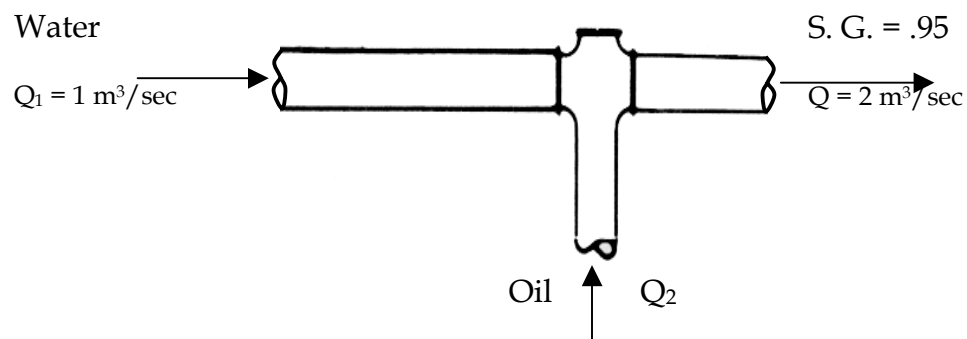
IV. Historical Notes

From about 200 B.C. to 100 A.D., the Romans developed elaborate water supply systems throughout their empire. Sextus Julius Frontinus (40-103 A.D.) described these systems. For Rome itself, the usual practice was to convey water from springs to an aqueduct and then to cisterns throughout the city from which water was delivered to consumers through lead and baked-clay pipes. Eleven aqueducts supplied Rome with about 200 million gallons of water daily.

V. Experiments/Demonstrations

- Volumetric Hydraulic Bench (H 10)
    (a) Volumetric Flow Experiment: Time and measured the volume of water to determine its mass flow and discharge. The flows can be valved to change the discharge rate for different experiments.

- 2.5 Metre Flow Channel (H 23)
    (b) Pitot Tube Experiment: Determine the velocity profile and integrate the area under the profile to determine the discharge. The cross sections of the channel can be changed by placing reducers in the flow and the experiment repeated at the reduced sections. In this way the students can observe that in steady flow the discharged is constant even though the velocity may change.

- Video-
    (c) Pressure Fields and Fluid Acceleration (1)

VI. References

Mechanics of Fluids, Shames: Sections 5.1, 5.2, 7.3

Fundamentals of Fluid Mechanics, Munson, Young, Okiishi: Sections 5.1, 6.2

Introduction to Fluid Mechanics, Fox, McDonald: Sections 4-1.1, 4-3
Video Tape: Pressure Fields and Fluid Acceleration

I. Momentum

(A) Linear Momentum
(B) Moment of Momentum (Angular)

(A) Linear Momentum

- $\sum F_{BODY} + \sum F_{SURFACE} = \dfrac{DP}{Dt}$

$$P = \iiint \rho v\, dVol$$

- $\displaystyle \iiint_{CV} B\rho\, dV + \oiint TdA = \oiint v(\rho v \bullet dA) + \frac{\partial}{\partial t}\iint v(\rho dv)$

  T = Surface Traction (Pressure and Shear)
  B = Body Force/Mass $= -g\hat{k}$

(a)   • One Dimensional Steady Incompressible Flow
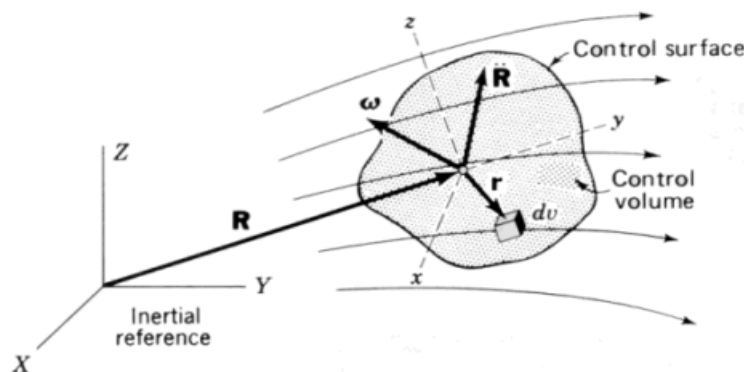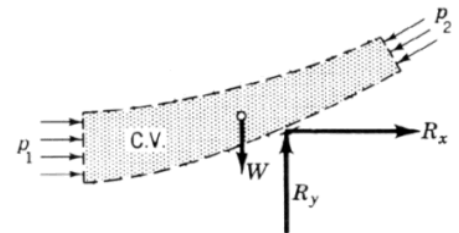
(d) $\sum F_B + \sum F_S = \rho Q \Delta v$

   R = Reactions

   • Non-Inertial Control Volume

$$\iiint_{CV} B\rho\, dV + \oiint TdA = \oiint v_{xyz}(\rho v_{xyz} \bullet dA) + \frac{\partial}{\partial t_{xyz}}\iint v_{xyz}(\rho dv) +$$

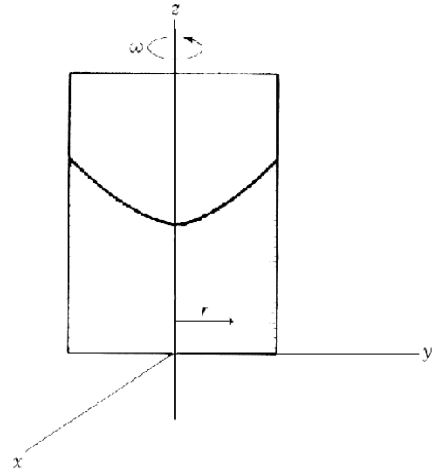$$\iiint \left[R'' + 2\omega \times v_{xyz} + \omega' \times r + \omega \times (\omega \times r) \right] \rho\, dV$$

- Euler's Equation (Differential Form of Momentum)

(c)
$$-\frac{1}{\rho}\nabla p - g\nabla z = (v \bullet \nabla)v + \frac{\partial v}{\partial t} = a$$

Example
$$\left(\frac{\nabla p}{\rho} + g\nabla + a\right) \bullet r = 0$$

$$\frac{P_2 - P_1}{\rho} = \omega^2(r_2^2 - r_1^2)$$

(B) Moment of Momentum (Angular Momentum)

(b)
$$\bullet \sum M_{BODY} + \sum M_{SURFACE} = \frac{DH}{Dt}$$

$$H = \iiint r \times \rho v\, dVol$$

$$\oiint r \times T dA + \iiint r \times B\rho dVol = \oiint (r \times v)(\rho v \bullet dA) + \frac{\partial}{\partial t} \iiint (r \times v)(\rho dVol)$$

- Pumps and Turbines

$$P_{atm}(A)_x - R_x = -Q\rho V_i + Q\rho[- (V_j - \omega r)\cos\beta + \omega r]$$

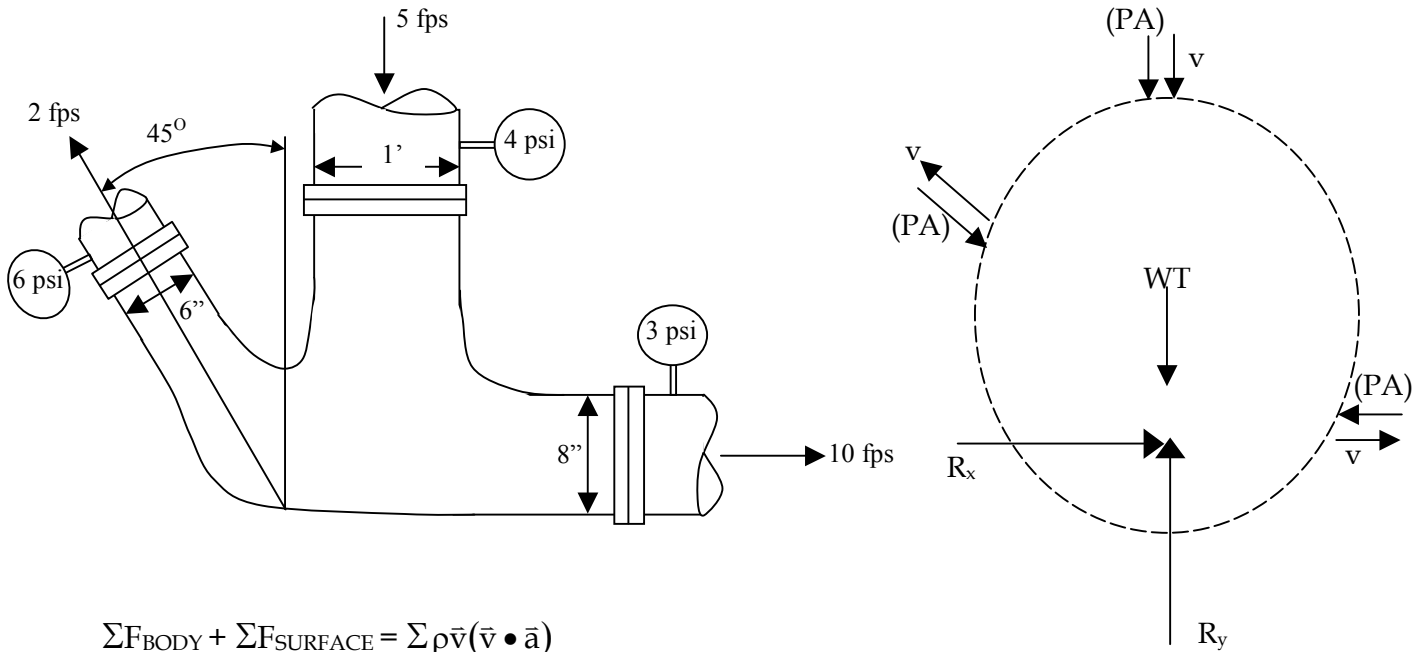- Non-Inertial Control Volume (See Figure Page 2)

$$\sum M_s + \sum M_B = \oiint (r \times v_{xyz})(\rho v_{xyz} \bullet dA) + \frac{\partial}{\partial t_{xyz}} \iiint (r \times v_{xyz})\rho dV +$$

$$\iiint \left[r \times \left(R'' + 2\omega \times v_{xyz} + \omega' \times r + \omega \times (\omega \times r)\right)\right]\rho dV$$

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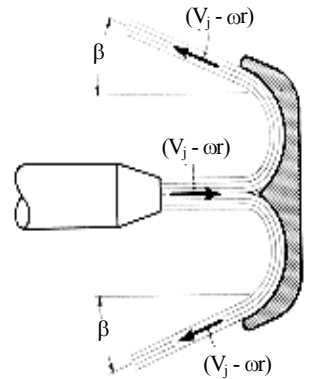
II. Design Example

An engineer must design the support system to hold the pipe junction shown. Determine the reactions to design this system. Take the weight of the junction and the water as 250 lb.

$$\Sigma F_{BODY} + \Sigma F_{SURFACE} = \Sigma \rho \vec{v}(\vec{v} \bullet \vec{a})$$

Horizontal Forces

$$R_x - \left(\frac{\pi}{4}\right)\left(\frac{8}{12}\right)^2 (3 \times 144) + \left(\frac{\pi}{4}\right)\left(\frac{1}{2}\right)^2 (6 \times 144)\cos 45^o =$$

$$1.94\,(10)\left[(10)\left(\frac{\pi}{4}\right)\left(\frac{8}{12}\right)^2\right] - 1.94\,(2\cos 45^o)\left[(2)\left(\frac{\pi}{4}\right)\left(\frac{1}{2}\right)^2\right]$$

$$\therefore R_x = 97.5 \text{ lb.}$$

Vertical Forces
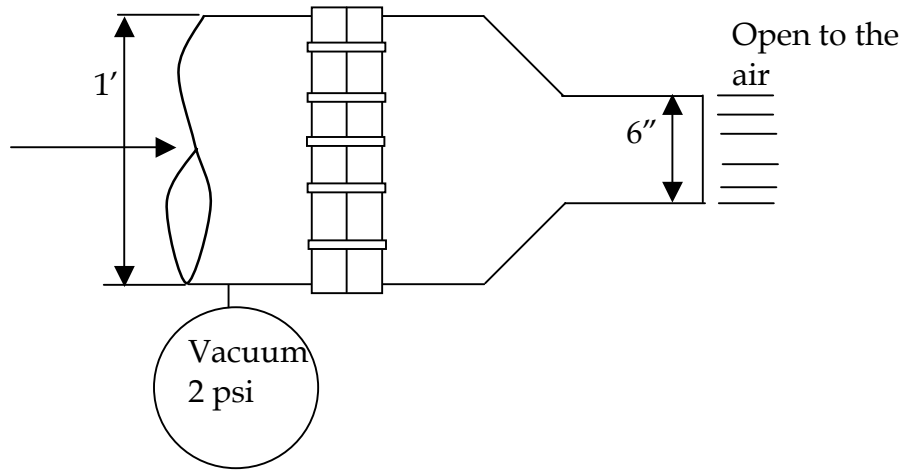
$$R_y - 250 - \left(\frac{\pi}{4}\right)(1)^2 (4 \times 144) - \left(\frac{\pi}{4}\right)\left(\frac{1}{2}\right)^2 (6 \times 144)\sin 45^o =$$

$$1.94\,(-5)\left[(-5)\left(\frac{\pi}{4}\right)(1)^2\right] + 1.94\,(2\sin 45^o)\left[(2)\left(\frac{\pi}{4}\right)\left(\frac{1}{2}\right)^2\right]$$

$$\therefore R_y = 862.6 \text{ lb.}$$

III. Design Homework Problem

Determine the number of bolts necessary to hold the reducing section shown for a flow of 4 cfs. Each bolt is designed to take a compression or tension force of 45 lb.

Design Homework Project

For the reducer shown, the total reaction must not exceed 4,000 lb. The pump is places in the system to increase the head. The pump characteristics are given in the table where h is the increase in head across the pump. Assume the water temperature is 50^0F and the weight of the reducer is 1,000 lb. including the water. Will this pump work in the system?

Pump Characteristics

Head (feet)	Flow Rate (cfs)
100	0
80	18
60	25
40	30
20	34
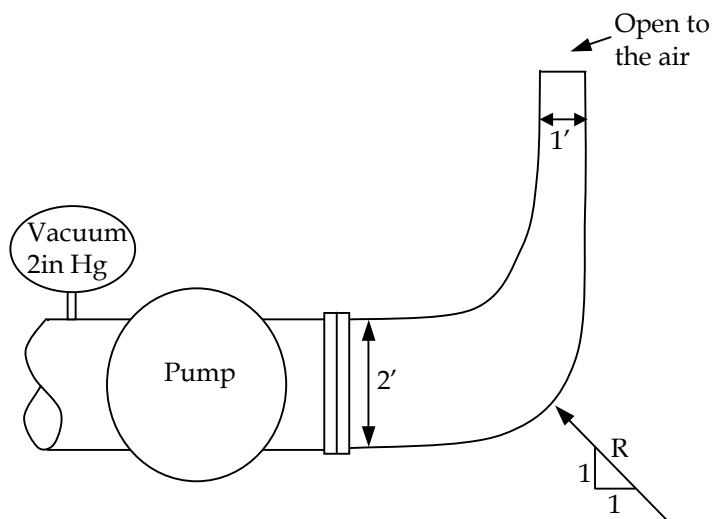
IV. Historical Notes

The early pioneers were Toricelli and Bernoulli, whose investigations were due to the hydraulic requirements of Italian ornamental landscape gardening. Great steps were taken by D'Alembert and Euler, in the eighteenth century, who successfully applied dynamical principles to the subject, and thereby discovered the general equation of motion of a perfect fluid, and placed the subject of Fluid Mechanics on a satisfactory basis.

V. Experiments/Demonstrations

- Impact of a Jet (H 8)
    (a) Measurement of Impact force on plates (Flat): Demonstrates the use of the momentum equation by directly measuring the force on the plate from the jet. The experiment can be repeated for different flow rates and therefore, different forces on the plate.
    (b) Measurement of Impact force on plates (inclined): By directly measuring the force on the plate the control volume can be demonstrated to determine forces at an incline. The flow rate can also be varied here to obtain different dynamic conditions.
    (c) Video-
        Pressure Fields and Fluid Acceleration (1)
    (d) ASCE Textbook on CD

VI. References

Mechanics of Fluids, Shames: Sections 5.3 – 5.8, 5.10 – 5.11, 7.3 – 7.4

Fundamentals of Fluid Mechanics, Munson, Young, Okiishi: Sections 5.2, 6.4

Introduction to Fluid Mechanics, Fox, McDonald: Sections 2-3, 4-1.2, 4-1.3, 4-4, 4-7

I. Equation of Energy for Finite Systems and Finite Control Volumes: The First Law of Thermodynamics

- $$\frac{DE}{Dt} = \frac{dQ}{dt} - \frac{dW_k}{dt}$$

  E = Stored Energy = Potential + Kinetic + Internal
  Q = Net Heat Added
  W_k = Work Done by System
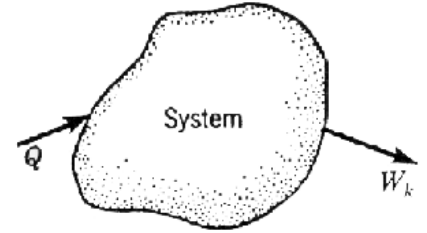
- $$\frac{dQ}{dt} - \frac{dW_S}{dt} + \left(\iiint_{CV} B \bullet v \rho dVol \right) =$$
  $$\oiint_{CS} \left(\frac{v^2}{2} + gz + u + \frac{p}{\rho} \right)(\rho v \bullet dA) + \frac{\partial}{\partial t}\left(\iiint_{CV}\left(\frac{v^2}{2} + gz + u \right)(\rho dVol) \right)$$

(c)   • One Dimensional System

$$\left[\frac{v_1^2}{2} + gz_1 + h_1\right] + \frac{dQ}{dm} = \left[\frac{v_2^2}{2} + gz_2 + h_2\right] + \frac{dW}{dm}$$

Specific Enthalpy $h = u + \dfrac{p}{\rho}$

Steady, Incompressible, 1D, No friction, $\dfrac{dQ}{dt} = 0$, u = Constant

(c)   • Bernoulli's Equation, 1D, Incompressible, Steady, Isothermal, Flow along a streamline or irrotational flow
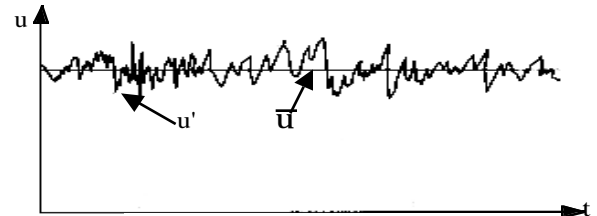
$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2 = Constant$$

(f)   • Turbulence

$$u = \bar{u} + u'$$
$$\bar{u} = (1/\Delta t)\int_t^{t+\Delta t} u\, dt$$
u' = Velocity Fluctuation Superimposed on $\bar{u}$

(g)   • Laminar Flow u' =0

Pipe Flow $\qquad v = \left(\dfrac{p_1 - p_2}{4\mu L}\right)\left(\dfrac{D^4}{4} - r^2\right)$

Reynolds Number $\qquad R = \dfrac{\rho v D}{\mu} = \dfrac{vD}{\nu}$

- $R < 3 \times 10^6$

Pipe flow; $\quad \dfrac{\overline{v}}{v_{max}} = \left(\dfrac{y}{D\big/2}\right)^{1/n}$; $6 \le n \le 10$ n=7 often employed

$\therefore \frac{1}{7}$ Power Law
y measured from the pipe wall

- $R > 3 \times 10^6$

Apparent Stress $\tau_{xy} = A\dfrac{\partial \overline{V}_x}{\partial y}$; $A = \rho\varepsilon$ = Eddy Viscosity

(x along the pipe axis)

$\tau_{xyapp} = -\left(\rho\overline{V_x'V_y'}\right) = \rho l^2\left(\dfrac{dV_x}{dy}\right)^2$ where $l \equiv$ mixing length

$\dfrac{\left(\overline{V}_x\right)_{max} - \overline{V}_x}{V_*} = -\dfrac{1}{0.4}\ln\dfrac{y}{R}$; R = Pipe Radius

- (A) Smooth Pipe

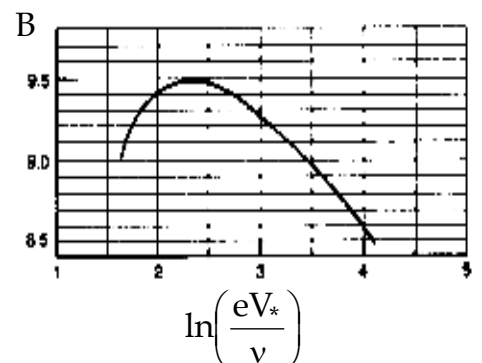$\dfrac{\overline{V}_x}{V_*} = 2.5\ln\dfrac{yV_*}{\nu} + 5.5$; $V_* = \sqrt{\left(\dfrac{\tau_0}{\rho}\right)}$ = Shear Stress Velocity

(d) $\qquad$ - (B) Rough Pipe

$\dfrac{\overline{V}_x}{V_*} = 2.5\ln\dfrac{yV_*}{\nu} + B$

(g) $\qquad$ i. Smooth Pipe

$\dfrac{eV_*}{\nu} \langle\, 5$; $B = 5.5$

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ii. Smooth-Rough Transition

$$5 \langle \frac{eV_*}{\nu} \langle 70 \text{ use figure to select B}$$

iii. Rough Pipe

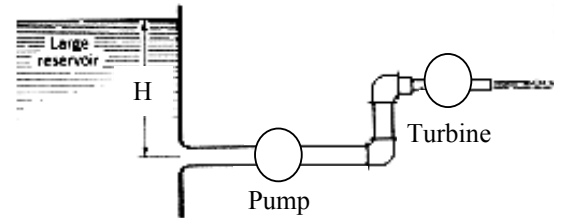$$\frac{eV_*}{\nu} \rangle 70 \qquad\qquad B = 8.5$$

- Wall Shear $\tau_w = \frac{f}{8}(\rho v^2)$; Valid for all Reynolds Numbers

- Pipe Flow (Pressure Conduits)

(c)
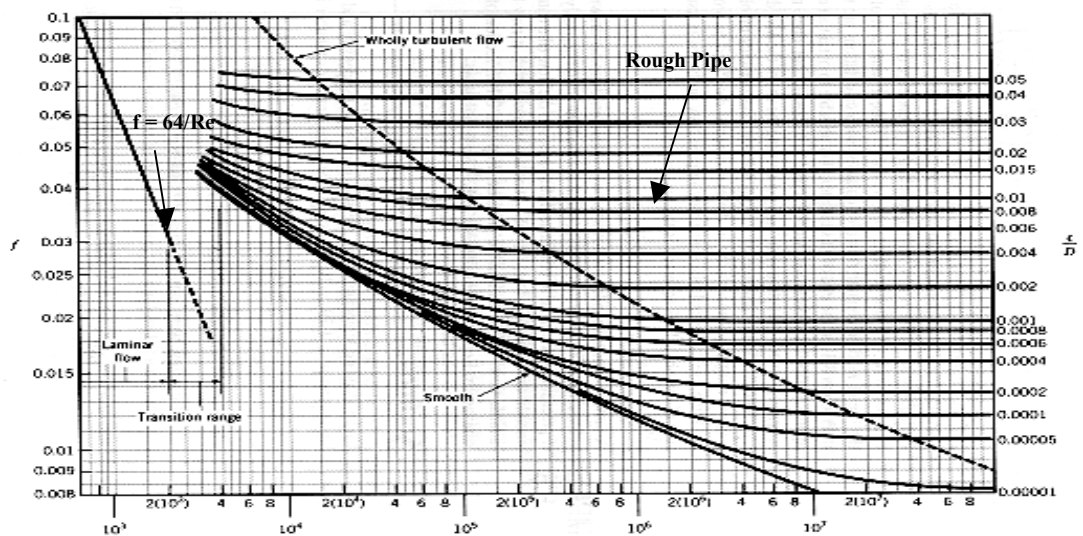(g)
Bernoulli's Equation

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 + H_{PUMP} = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + H_{TURB} + \text{Losses}$$

(b)
- Losses = $\sum_{Friction} \left(\frac{fL}{D}\right)\left(\frac{v^2}{2g}\right) + \sum_{Local} K\left(\frac{v^2}{2g}\right)$ where D = Pipe Diameter

(d)
- f= function($\frac{e}{D}$, R) $\equiv$ Moody Diagram

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• Piping Systems

(b) (c)   (a) Series

(g)
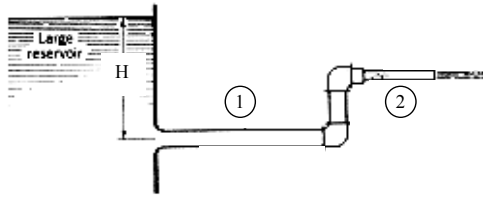
Q₁=Q₂

(b)   (b) Parallel

$h_{loss1} = h_{loss2}$

$Q = Q_1 + Q_2$
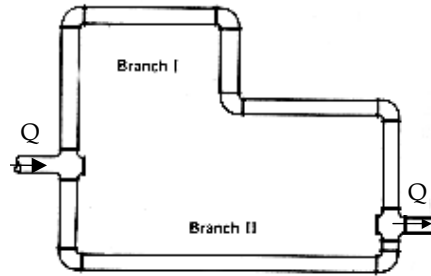
(c) Branching

at Junction, $Q_{In} = Q_{Out}$

Balance HGL A to P, B to P, P to C

Hydraulic Grade Line (HGL)

$$HGL = \frac{P}{\gamma} + z$$

Energy Grade Line (EGL)

$$EGL = \frac{P}{\gamma} + z + \frac{v^2}{2g}$$
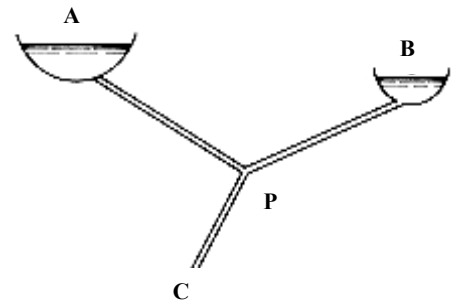
II. Design Example

You are asked to determine if the given piping system exceeds a scour velocity of $20 \: \text{ft}/\text{sec}$ and make recommendations.
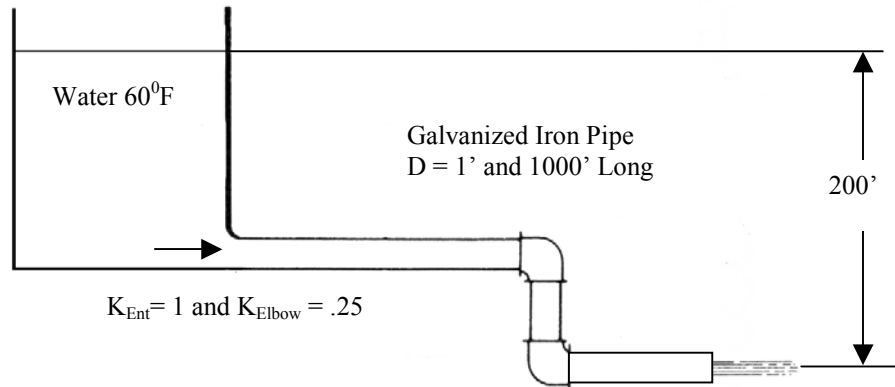
$$200 = \frac{v^2}{2g}\left[\frac{fl}{D}+1+\Sigma\left(1+2(.25)\right)\right]$$

f = .0167

$$\therefore v = 25.9 \: \text{ft}/\text{sec} \qquad \text{Check Re} = \frac{25.9(1)}{1.217 \times 10^{-5}} = 2.13 \times 10^6$$

f = .0167 O.K.

$\therefore$ Velocity too high, reduce pipe diameter to about .5′ to increase friction losses and reduce v!

III. Design Homework Problem

Design a venturi meter for a flow rate of 2 cfs. Find the flow rate for your venturi meter for a drop of 1" in the manometer shown.

$D_2 = (D_1/2)$
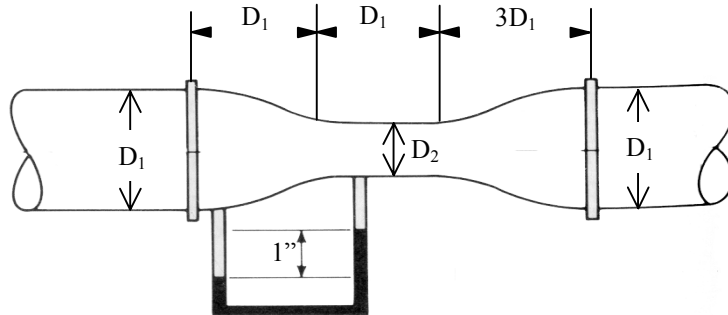
Design Homework Project

Design a piping network using cast iron pipe, starting from a reservoir at point A with a surface elevation of 200 feet and a water temperature of 60^0F. Note, the pipes can only run in the xy direction and take $K_{Elbow}=.5$, $K_{Entrance}=.5$, and $K_{Reducer}=1$. Velocity Range 2 to 12 fps and Pressure range 20 to 100 psi. Also draw the EGL and HGL.
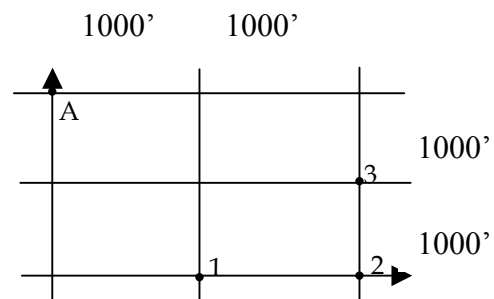
Point	Elevation (ft)	Supply Q (cfs)(min.)	Pressure (psi)
A	200	-	0
1	0	10	-
2	50	10	-
3	100	10	30

IV. Historical Notes

Osborn Reynolds (1842-1912) described experiments in many fields- cavitation, river model similarity, pipe resistance – and devised two parameters for viscous flow and adapted equations of motion of a viscous fluid to mean conditions of turbulent flow.

V. Experiments/Demonstrations

- Fluid Friction Apparatus (H 408)
    - (a) Laminar, Transitional and Turbulent: Observe dye at different Reynolds numbers to obtain the transition from a smooth pattern to a random pattern. This experiment can be repeated at different locations in the network like elbows and pipe reductions.
    - (b) Energy losses in bends, fitting, etc.: Measure the elevation of the water in the piezometer above the pipe to determine the HGL. This is a visual experiment since the losses across the fittings can be directly related to changes in water height measured in the piezometers.
    - (c) Pitot Tube: Measure velocity profiles at different locations to determine flows and EGL by using (b). Similar to (b) this is also a visual experiment.
    - (d) Smooth VS. Rough Pipe Flow: Combing (a) and (c) to observe different flow regions.
    - (e) Flow measurement using Venturi/Orifice Meter: The equations of continuity and energy are demonstrated by measuring the elevations in the piezometers and determining the EGL across these meters.

- Videos-
    - (f) Turbulence (2)
    - (g) Pressure Fields and Acceleration (1)

VI. References

<u>Mechanics of Fluids</u>, Shames: Sections 6.1 – 6.7, 9.1 – 9.16, 9.18

<u>Fundamentals of Fluid Mechanics</u>, Munson, Young, Okiishi: Sections 6.4, 8.1 – 8.5

<u>Introduction to Fluid Mechanics</u>, Fox, McDonald: Sections 2-5.2, 4-1.4, 4-8, 6-3, 6-4, Chapter 8

Video Tape: Pressure Fields and Fluid Acceleration
              Turbulence
              Flow Visualization

I. Dimensional Analysis

- Dimensional Grouping

$$\frac{F}{\rho v^2 D} = g\!\left(\frac{\rho v D}{\mu}\right)$$

- Buckingham's Π Theorem

Number of independent dimensional groups $\equiv j$
$j = n - r$
n = Number of variables (p, v, ρ, etc.)
r = Number of basic dimensions (M, L, T)

Example

$$\frac{\Delta p}{L} = f(\mu, D, Q) \qquad\qquad Q: L^3T^{-1}$$

$$\frac{M}{L^2T^2} = \left(\frac{M}{LT}\right)^a (L)^b \left(\frac{L^3}{T}\right)^c \qquad \frac{\Delta p}{L}: ML^{-2}T^{-2}$$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ D: L
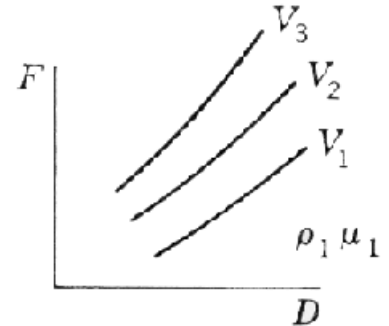
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ μ: ML^{-1}T^{-1}

M: 1=a
L: -2= -a+b+3c $\qquad \therefore \qquad$ a = 1; c = 1; b = -4
T: -2=-a-c

$$\therefore \quad \frac{\Delta p}{L} = \Pi(\mu D^{-4} Q) \quad \text{or} \quad \Pi = \frac{\dfrac{\Delta p}{L} D^4}{\mu Q}$$
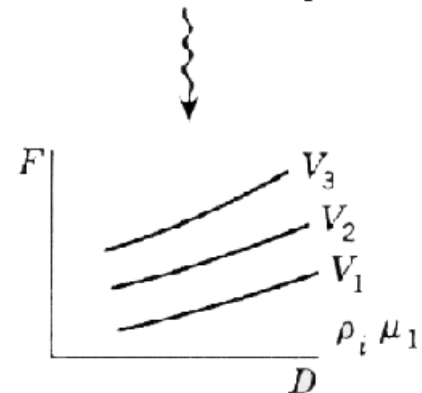
I. Similtude

- Geometric Similarity – Same shape
  Kinematic Similarity – Same streamline
  Dynamic Similarity – Forces and mass distribution the same

  $\therefore$ For similarity between Model and Prototype all these similarities must be true.
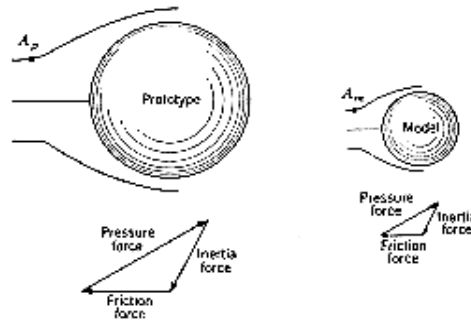
$$\frac{(\text{Press Force})_m}{(\text{Press Force})_p} = \frac{(\text{Friction Force})_m}{(\text{Friction Force})_p} = \frac{(\text{Inertia Force})_m}{(\text{Inertia Force})_p}$$

and for all parameters

$$\frac{|V_p|}{|V_m|} = \frac{|dV|_p}{|dV|_m}$$

Therefore, you can replace derivative with the variable.

	DIMENSIONAL GROUP	NAME	FORCE RATIO	APPLICATION
(a) (b) (c)	$\dfrac{\rho V \ell}{\mu}$	Reynolds Number	$\dfrac{\text{Inertia F}}{\text{Viscous F}}$	All Fluid Dynamics, -pipe flow -shear flow
(d) (h)	$\dfrac{V}{\sqrt{g\ell}}$	Froude Number	$\dfrac{\text{Inertia F}}{\text{Gravity F}}$	Free surface -open channel, free surface flow
(e) (g)	$\dfrac{P}{\rho V^{2}}$	Euler Number	$\dfrac{\text{Pressure F}}{\text{Inertia F}}$	Press. Difference -Drag, Press. Difference
(h)	$\dfrac{V}{C}$	Mach Number	$\dfrac{\text{Inertia F}}{\text{Compress F}}$	Compressibility -compressible flow
(f)	$\dfrac{\rho V^{2}\ell}{\sigma}$	Weber Number	$\dfrac{\text{Inertia F}}{\text{Surface Tension F}}$	Surface Tension

For a given fluid dynamic problem select the correct number.

II. Design Example

An ocean surface discharge is to be modeled to make sure that its plume does not effect the fish. The ocean discharge has the following parameters: 4′ by 4′, Q = 360 cfs, discharge temperature is 60°F (same as the ocean). Try a $\frac{1}{40}$ scale model.

This is a surface discharge so only check the Froude number.

$$\left(\frac{v}{\sqrt{gy}}\right)_{Model} = \left(\frac{\left(\frac{360}{16}\right)}{\sqrt{g4}}\right) = 1.98 = F_{Model}$$

But you must check the Reynolds Number to make sure that you are in the turbulent range.

$$\text{Prototype Reynolds \#} = \frac{vD}{\nu} = \frac{\left(\frac{360}{16}\right) \times 4}{1.217 \times 10^{-5}} = 7.39 \times 10^6$$

∴ Turbulent in Prototype and must be Turbulent in model.

$$\frac{v_{Model}}{\sqrt{g\left(\frac{4}{40}\right)}} = 1.98 \therefore v_{Model} = 3.6 \;^{ft}\!/_{sec}$$

and $Re_{Model} = \dfrac{3.6(.1)}{1.217 \times 10^{-5}} = 2.96 \times 10^4$

∴ $\frac{1}{40}$ scale O.K.

III. Design Homework Problem

A series of circular wires are holding a pole. Vortices can develop on the downward side of the wire that are shed in a regular fashion. Therefore, it is important to determine the shedding frequency. Using a $\frac{1}{4}$ scale model, a wind velocity of 25 $\frac{ft}{sec}$, and a temperature of 50^0F, determine the prototype frequency. The model is in water at 50^0F and the measured frequency was 50 Hz.

IV. Historical Notes

Theodor Von Karman is one of the recognized leaders of the twentieth century fluid mechanics. He provided major contributions to our understanding of surface resistance, turbulence, and wake phenomenon.

V. Experiments/Demonstrations

- Fluid Friction Apparatus (H 408)
    (a) Laminar; Transitional and Turbulent: Observe dye at different Reynolds numbers to see the transition from a smooth pattern to a random pattern. This experiment can be repeated at different locations in the pipe network to observe the effect at different settings.
    (b) Smooth VS. Rough Pipe Flow: Combing (a) and (c) to observe differences in flow regions. This experiment can also be repeated at different locations in the pipe network.
    (c) Pitot Tube: Measure velocity profiles at different pipe locations to observe variation in Reynold numbers versus location.

- 2.5 Metre Flow Channel (H 23)
    (d) Uniform-Gradually Varied Flow: Vary the flows over weirs, etc, and place dye in the flow to observe the flow patterns and to calculate the Froude Number.

- Hydrostatic and Properties of Fluids Apparatus (H 314)
    - (e) Pressure Gauges: Determine Δp and calculate Euler's number for different setups.
    - (f) Surface Tension Balance: Determine σ and calculate Weber number.
    - (g) Fall Sphere Method: From the viscometer determine the viscous force and Reynold and Euler numbers by a force balance. This experiment can be repeated for different fluids and spheres.

- Video-
    - (h) Effects of Fluid Compressibility (24)

VI. References

<u>Mechanics of Fluids</u>, Shames: Sections 8.1 – 8.11

<u>Fundamentals of Fluid Mechanics</u>, Munson, Young, Okiishi: Sections 7.1 – 7.10

<u>Introduction to Fluid Mechanics</u>, Fox, McDonald: Chapter 7

Video Tape: Effects of Fluid Compressibility

I. Navier Stokes Equations

I.   Equation of Momentum

   - Surface Force and Body Force

   $$\oiint T \bullet da + \iiint B dV_{OL} = \iiint \rho \frac{Dv}{Dt} dV_{OL}$$

   $$\oiint T \bullet dA = \iiint \nabla \bullet T \, da$$

   $$\nabla \bullet T = \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \bullet \begin{pmatrix} \hat{i}\sigma_{xx}\hat{i} & \hat{i}\tau_{xy}\hat{j} & \hat{i}\tau_{xz}\hat{k} \\ \hat{j}\tau_{yx}\hat{i} & \hat{j}\sigma_{yy}\hat{j} & \hat{j}\tau_{yz}\hat{k} \\ \hat{k}\tau_{zx}\hat{i} & \hat{k}\tau_{zy}\hat{j} & \hat{k}\sigma_{zz}\hat{k} \end{pmatrix}$$

   - Hooks Law

   $$\epsilon_{ii} = \frac{1}{E} \left[\tau_{ii} - \nu \left(\sigma_{jj} + \sigma_{kk} \right) \right]$$

   $$G = \frac{E}{2(1+\nu)}$$

   - Stokes Viscosity Law

   $$\tau = Gv = \mu \frac{\partial \gamma}{\partial t}$$

   $$\therefore$$

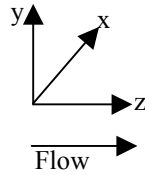   - Navier Stokes Equations

   $$\rho \frac{D\vec{v}}{Dt} = \rho\vec{B} - \vec{\nabla}P + \mu \left(\frac{1}{3} \vec{\nabla}\left(\vec{\nabla} \bullet v\right) + \nabla^2 \vec{v} \right)$$

   For Incompressible Flow $\vec{\nabla} \bullet \vec{v} = 0$

- Parallel Flow

$$P = \overline{P} + Pg \quad \text{where } \overline{P} = \text{Dynamic Pressure}$$
$$Pg = \text{Static Pressure}$$

$$\therefore \quad \frac{\partial \overline{P}}{\partial y} = 0$$
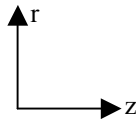
$$\frac{\partial P_y}{\partial y} = \gamma$$

Air or high pressure liquid flow $\tau_{xx} = \tau_{yy} = \tau_{zz} = -p$

$\therefore$ Pressure does not vary perpendicular to the streamlines for these conditions

- Flow in a Circular Pipe – Parallel Flow
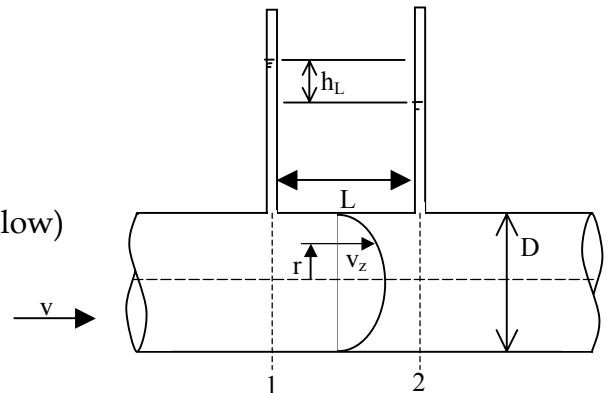
(a) (b) (c)
(d)

N. S. $\quad \dfrac{\partial p}{\partial z} = \mu\left(\dfrac{\partial^2 v_z}{\partial r} + \dfrac{1}{r}\dfrac{\partial v_z}{\partial r} \right) = -\beta = \text{Constant}$

$$\therefore v_z = \frac{P_1 - P_2}{4\mu L}\left(\frac{D^2}{4} - r^2 \right)$$

Define $h_L = \dfrac{P_1 - P_2}{\rho} = \dfrac{32 v L \mu}{\rho D^2}$ (Laminar Flow)

$$v = \frac{Q}{\pi \dfrac{D^2}{4}} = \text{Average Velocity}$$

II. Design Example

In a processing plant, you must make sure that there is a laminar flow velocity profile. Find the discharge for this 2′ diameter pipe with a pressure drop along a horizontal section of 5 x 10⁻⁶ PSF/ft, a viscosity of 3 x 10⁻⁵ $\mathrm{lb \cdot sec}/\mathrm{ft^2}$ and a density of 1.3 $\mathrm{slug}/\mathrm{ft^3}$.

Select laminar profile.

$$v = \frac{\Delta p}{4\mu L}\left(\left(\frac{D^2}{4}\right) - r^2\right)$$

$$\frac{\Delta p}{L} = .000005 \, \mathrm{PSF}/\mathrm{ft}$$

$$v = \frac{.000005}{4\left(3 \times 10^{-5}\right)}\left(\left(\frac{4}{4}\right) - r^2\right)$$

$$v = .0417 \, (1 - r^2)$$

$$Q = \int_0^1 v2\pi r\,dr = 2\pi \int_0^1 .0417\left(1 - r^2\right)r\,dr = 2\pi(.0417)\left[\frac{r^2}{2} - \frac{r^4}{4}\right]_0^1 = .2617\left[\frac{1}{2} - \frac{1}{4}\right]$$

$$= .0654 \, \mathrm{cfs}$$

Check Reynolds Number

$$v = \frac{3 \times 10^{-5}}{1.3} = 2.3 \times 10^{-5}$$

$$\mathrm{Re \#} = \frac{\left(\dfrac{.0654}{\pi \dfrac{2^2}{4}}\right)2}{2.3 \times 10^{-5}} = 1810 < 2000 \text{ Checks.}$$

III. Design Homework Problem

You are asked to make sure that the design of an annulus with an inner radius of 1' and an outer radius of 2' can carry at least 300 cfs of water at 50^0F. Assume laminar flow. Take a pressure drop of 10 PSF over 5000 ft.

IV. Historical Notes

In 1845, Professor Stokes published his well-known theory of the motion of a viscous liquid, in which he endeavored to account for the frictional action which exists in all known liquids and which causes the motion to gradually subside by converting the kinetic energy into heat. This paper was followed in 1850 by another in which he solved various problems relating the motion of spheres and cylinders in a viscous liquid. Previously to this paper, no problem relating to the motion of a solid body in a liquid had ever been solved, in which the viscosity had been taken into account.

V. Experiments/Demonstrations

- 2.5 Metre Flow Channel
    - (a) Pitot Tube: Measure the velocity profile in an uniform flowing channel and compare it to the Navier Stokes Solution.
    - (b) Dye Experiment: same as (a) but use dye.

- Fluid Friction Apparatus
    - (c) Pitot Static Tube: same as (a) but in a pipe.

- Videos-
    - (d) Characteristics of Laminar and Turbulent Flow (22)

VI. References

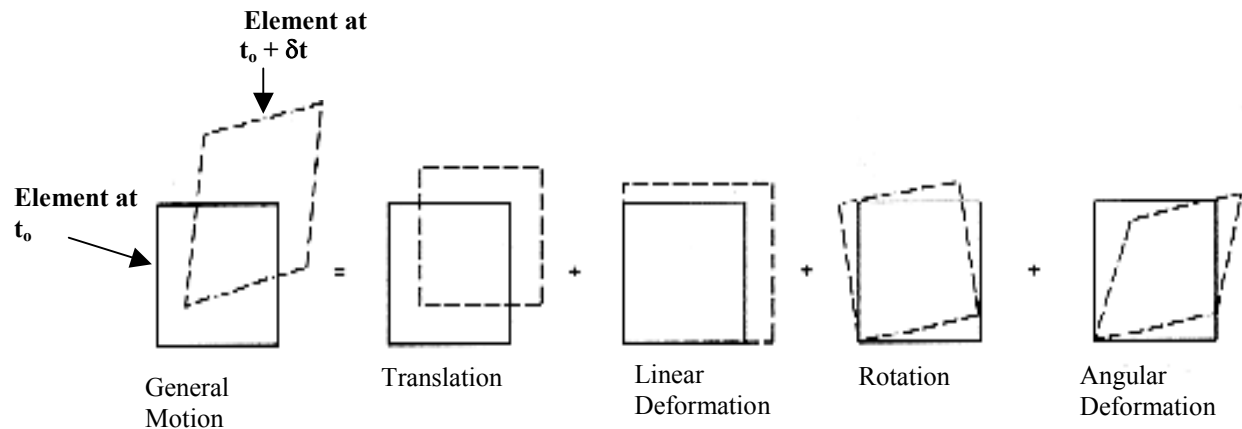Mechanics of Fluids, Shames: Sections 7.7, 7.8, 10.1 – 10.6

Fundamentals of Fluid Mechanics, Munson, Young, Okiishi: Sections 6.8, 6.9

Introduction to Fluid Mechanics, Fox, McDonald: Sections 5-1, 5-4, 8-1, 8-2, 8-3

Video Tape: Characteristics of Laminar and Turbulent Flow

I. Potential Flow

A fluid elemant moving in a time interval δt can undergo the following
deformations and movements.

Element at
t₀ + δt

Element at
t₀

General
Motion

Translation

Linear
Deformation

Rotation

Angular
Deformation

We are interested in rotation in this section.

- Incompressible, Irrotational, Steady Flow

- Curl $\quad \omega = \dfrac{1}{2} \nabla \times v$
  Irrotational Flow $\nabla \times v = 0$

- Circulation $\quad \Gamma = \oint v \bullet ds = \iint \nabla \times v \bullet dA$

- $v = \nabla \phi \qquad \phi \equiv$ Velocity Potential

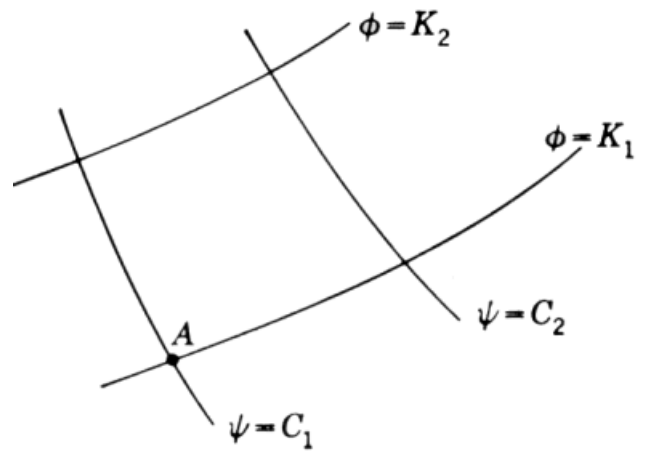(e) $\quad$ • Stream Function ψ
  Discharge q = ψ
  $$v_x = \frac{\partial \psi}{\partial y} \qquad v_y = -\frac{\partial \psi}{\partial x}$$

- Flow Nets
  Cauchy-Rieman Equation

$$\frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x}$$

$$\frac{\partial \psi}{\partial x} = -\frac{\partial \phi}{\partial y}$$

- Basic Laws

    - Conservation of Mass $\nabla \bullet v = 0, \nabla^2 \phi = 0, \nabla^2 \psi = 0$ La Place Eq.

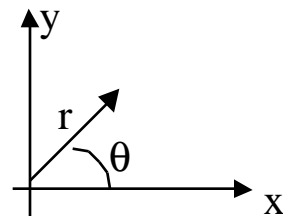   - Energy (Bernoulli) $\dfrac{v^2}{2g} + \dfrac{p}{\gamma} + z = Cons\tan t$

    (along a streamline or irrotational)

(a) (b)

	ϕ	ψ	Γ	Velocity
Uniform Flow	$V_0 x$	$V_0 y$	0	$V_x = V_o$, $V_y = 0$
Source	$\dfrac{\Lambda}{2\pi}\ln r$	$\dfrac{\Lambda\theta}{2\pi}$	0	$V_r = \dfrac{\Lambda}{2\pi r}, V_\theta = 0$
Vortex	$\dfrac{\Lambda\theta}{2\pi}$	$\dfrac{-\Lambda}{2\pi}\ln r$	Λ	$V_r = 0,\ V_\theta = \dfrac{\Lambda}{2\pi r}$
Doublet	$\dfrac{x\cos\theta}{r}$	$\dfrac{-x\sin\theta}{r}$	0	$V_r = \dfrac{-x\cos\theta}{r^2}$, $V_\theta = \dfrac{-x\sin\theta}{r^2}$

2a = spacing between source and sink
as a$\rightarrow$ 0 and $\Lambda \rightarrow$ 0

$$\frac{a\Lambda}{\pi} \rightarrow x$$

(c)   • Superposition
(f)   -Uniform flow + Doublet = Flow about a Cylinder

$$V_r = V_0 \cos\theta - \frac{x\cos\theta}{r^2}$$

$$V_\vartheta = -V_0 \sin\theta - \frac{x\sin\theta}{r^2}$$

(c)   • Drag = Force Parallel to the streamlines (D)
(f)   Lift = Force Perpendicular to the streamlines (L)

Note! For flow around a cylinder:

$$D = -\int_0^{2\pi} P_b \cos\theta \left(\frac{x}{V_0}\right)^5 d\theta = 0$$

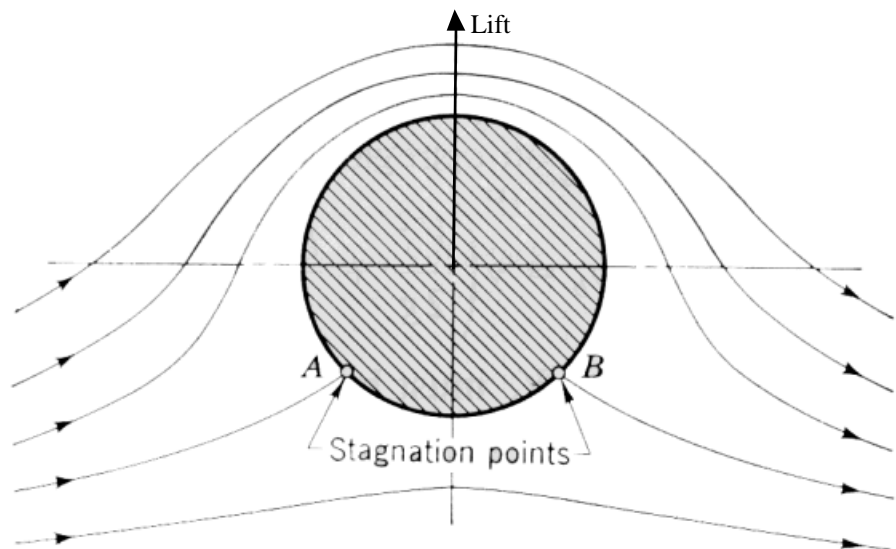$$L = -\int_0^{2\pi} P_b \sin\theta\, r d\theta = 0$$

(f)   • Rotating Cylinder

Uniform Flow + Doublet + Vortex

$$V_r = V_0 \cos\theta - \frac{x\cos\theta}{r^2}$$

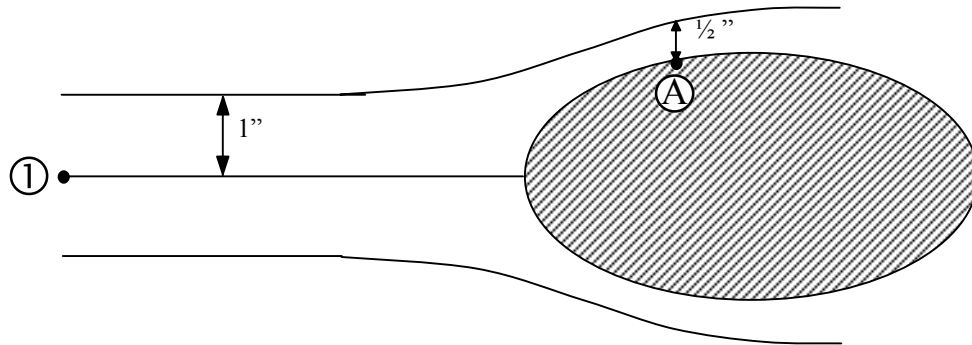$$V_\theta = -V_0 \sin\theta - \frac{x\sin\theta}{r^2} - \frac{\Lambda}{2\pi r}$$

$$D = 0$$

$$L = \rho V_o \Gamma$$

Stagnation points

51

II. Design Example

A small vessel is moving in fresh water at 10 $\frac{ft}{sec}$ and a depth of 10′. Find the pressure at point A shown in the streamline diagram.

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} = \frac{v_A^2}{2g} + \frac{p_A}{\gamma}$$

$v_1\,(1)\,L = v_2\,(.5)\,L \quad \therefore \quad v_2 = 2v_1$

$\frac{p_1}{\gamma} = 10?$ and $v_1 = 10\ \frac{ft}{sec}$

$$\frac{v_1^2}{2g} + \frac{10\gamma}{\gamma} = \frac{4v_1^2}{2g} + \frac{p_A}{\gamma}$$

$\therefore\ p_A = \dfrac{-3(10^2)\rho}{2} + 10\gamma = 333\ \text{PSF} = 2.31\ \text{psi}$

III. Design Homework Problem

A harbor has the shape of the 90° corner shown. You are asked to determine the pressure variation along the harbor wall for development. Assume a two dimensional flow of a nonviscous fluid is valid and take the stream function as $\Psi = 2r^2 \sin 2\theta$. Find an equation for the pressure difference ($p_2 - p_1$) along the harbor wall.
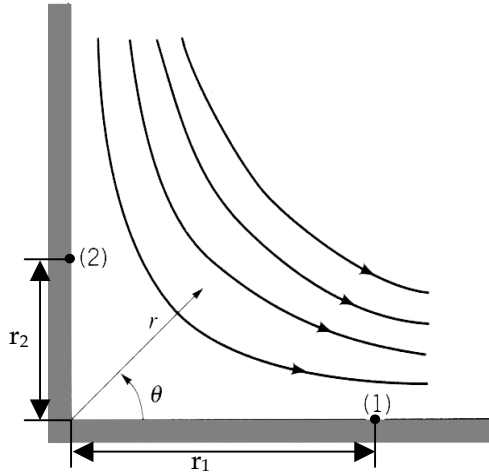
Design Homework Project

Design a wing of an airplane and test the wing in the Hele-Shaw apparatus at 0, 10,and 20 degrees from the horizontal. From the streamline pattern, determine the lift and drag on the wing.

IV. Historical Notes

In 1858, Helmholtz gave a complete investigation on the peculiarities of rotational motion. This work is in his celebrated memoir on Vortex Motion.

V. Experiments/Demonstrations

- Hele-Shaw Apparatus (H 9)
    - (a) Source and Sink in a uniform stream: Demonstrate this flow pattern from the dye streaks. The students can vary the positions of the source and the sink and place different objects in the flow to determine a variety of flow patterns.
    - (b) Doublet in a uniform flow: same as (a)
    - (c) Flow around a circular cylinder: same as (a)
    - (d) Flow around an airfoil: same as (a)

- Videos-
    - (e) Vorticity (3I)
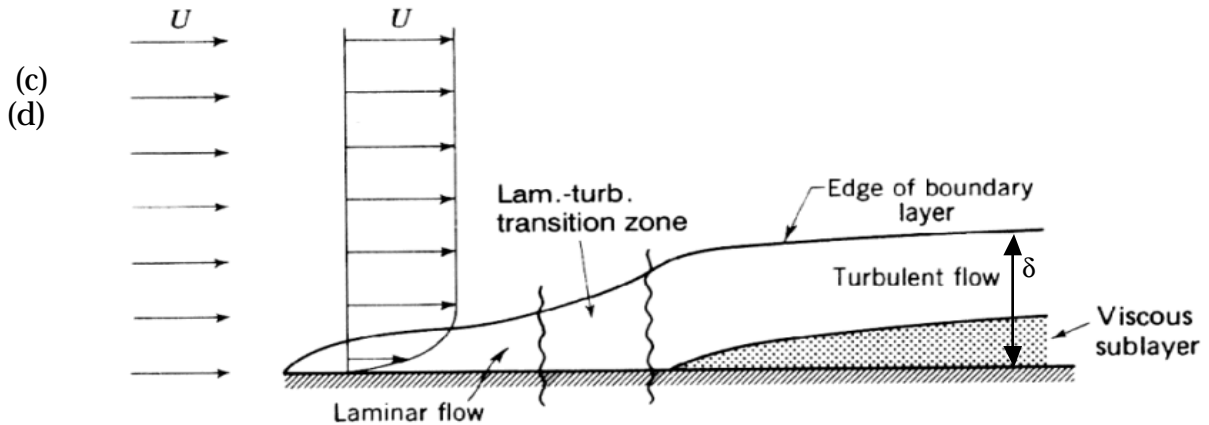   - (f) Form, Drag, Lift and Propulsion (23)

VI. References

Mechanics of Fluids, Shames: Sections 4.4, 4.5, 12.1, 12.25

Fundamentals of Fluid Mechanics, Munson, Young, Okiishi: Sections 6.4 – 6.6

Introduction to Fluid Mechanics, Fox, McDonald: Sections 5-2, 5-3, 6-6

Video Tape: Form, Drag, Lift and Propulsion
Vorticity

I. Boundary Layer Theory
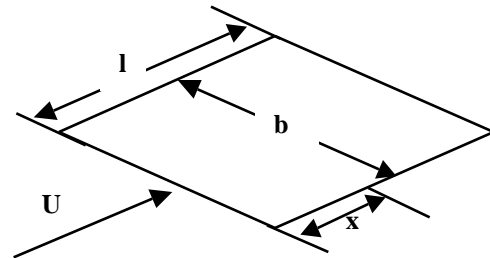

- δ Boundary Layer Thickness

    From Navier Stokes and Continuity Equation

    $$\frac{\delta}{x} = 4.96(\text{Re}_x)^{-.5}$$

    Reynolds Number

    $$\text{Re}_x = \frac{Ux}{v}$$

    Also from Blasius Equation

- Von Karman Integral Momentum Equation
    Equation Momentum and Continuity Equation

    $$\tau_w = -\frac{d}{dx}\left[\int_0^\delta \rho\left(u^2 - Uv\right)dy\right]$$

(c)   • Skin Friction (DRAG)

    $$C_f = \frac{\tau_w}{.5\rho U^2}$$

    $$\tau = \mu\frac{\partial u}{\partial y}; \qquad u = \text{From velocity profile Laminar Flow}$$

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$\therefore$ Laminar Skin Friction $\qquad Re_L \le 5 \times 10^5$

$$C_f = \frac{1.328}{\sqrt{Re_L}} \; ; \qquad Re_L = \frac{U_L}{\nu}$$

(c) $\qquad$ Drag $D = C_f \dfrac{\rho}{2} U^2 A \; ; \qquad\qquad A = bl$

(d)

- Turbulent Skin Friction (Drag)
  Smooth
  $$5 \times 10^5 \le Re_L \le 10^7$$

  Blasius
  $$\tau_w = 0.0225 \, \rho u^2 \left(\frac{U}{\nu \delta} \right)^{\frac{1}{4}}$$

  $$\frac{\delta}{x} = 0.37 \, Re_X^{-\frac{1}{5}}$$

  $$C_f = \frac{0.074}{[Re_L]^{\frac{1}{5}}} \qquad \text{(Turbulent B.L. over entire plate)}$$

  $$C_f = \frac{0.074}{Re_L^{\frac{1}{5}}} - \frac{a}{[Re_L]} \qquad \text{(with leading edge laminar B.L.)}$$

Re_{cr}	3×10^5	5×10^5	10^6	3×10^6
a	1050	1700	3300	8700

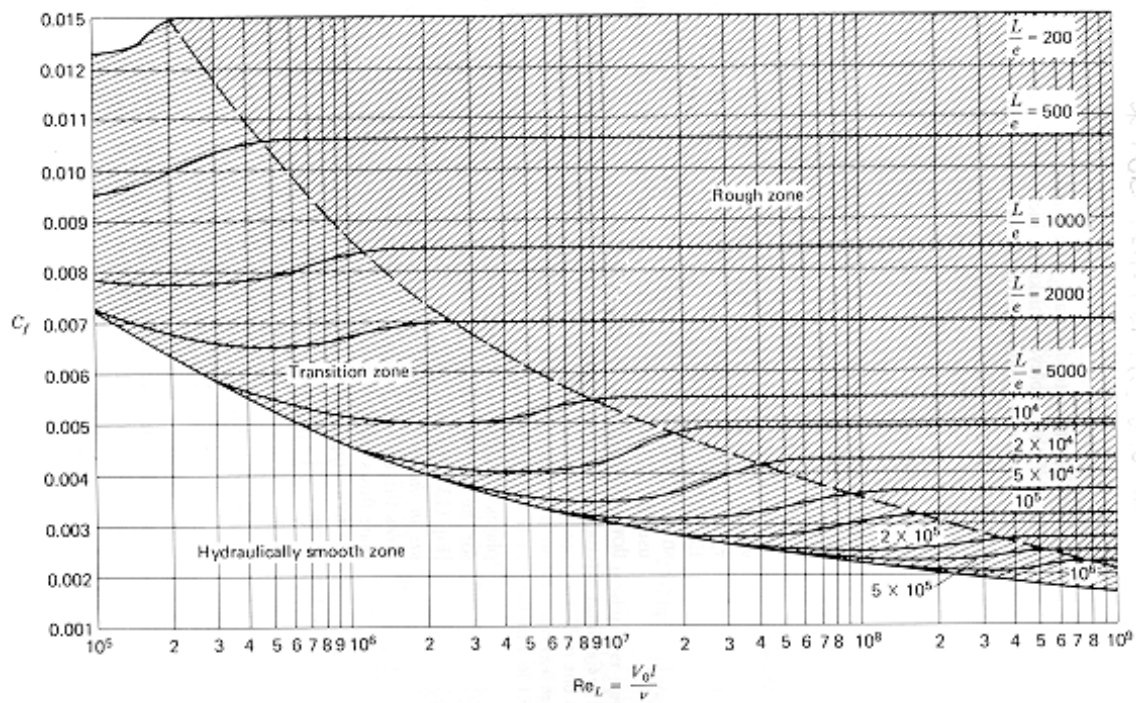$Re_{cr} \equiv$ Critical Reynold Number for transition

$R > 10^7$

$$C_f = \frac{0.455}{(\log Re_L)^{2.58}}$$

or

$$C_f = \frac{0.455}{(\log Re_L)^{2.58}} - \frac{a}{Re_L}$$

- Turbulent Skin Friction
  Rough Plates
  Similar to pipes-

e = mean height of plate protuberances
      (similar to pipe e)

For hydraulically smooth pipe flow
$$\frac{v_* e}{\gamma} < 5$$
where
$$v_* = \sqrt{\frac{\tau_\omega}{\rho}}$$
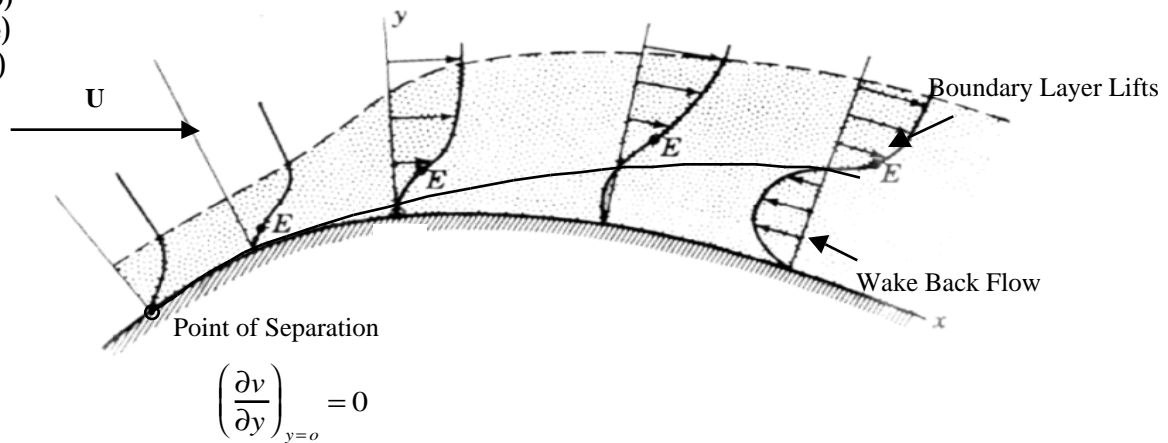
$$C_f = \left(1.89 + 1.62\log\frac{l}{e}\right)^{-2.5}$$

(a)   • Pressure Drag
(b)
(e)
(f)

U

Boundary Layer Lifts

E

E

E

E

Wake Back Flow

Point of Separation

$$\left(\frac{\partial v}{\partial y}\right)_{y=o} = 0$$

$$D = C_D A \frac{\rho U^2}{2} = \text{Total Drag (Pressure + Skin)}$$

A = Projection of Wetted Area in the Direction of Motion

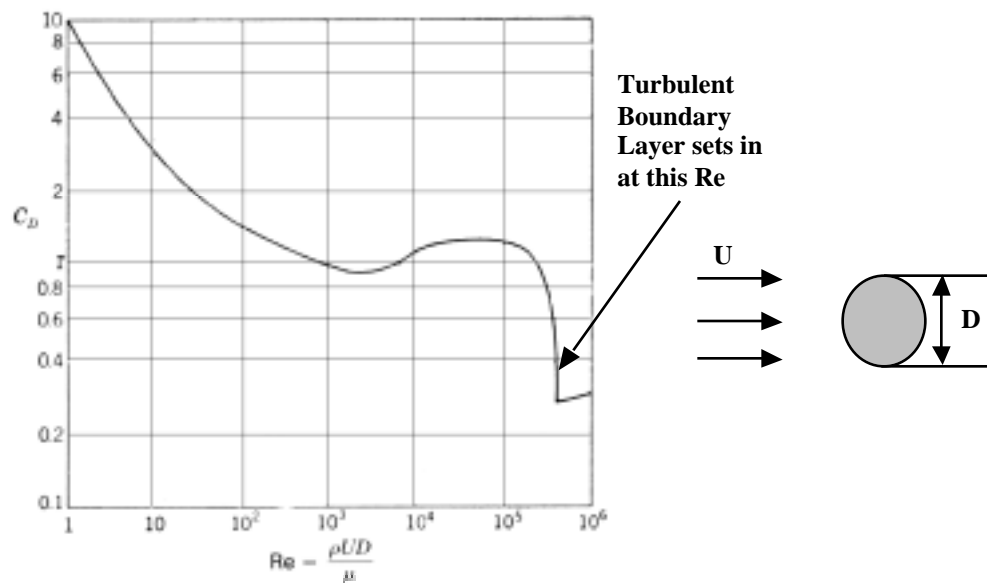(Project cross section into flow perpendicular to streamline)
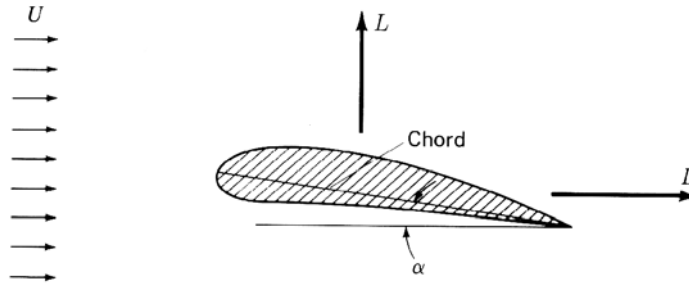
C_D from tables

(a)
(e)
(f)

Turbulent
Boundary
Layer sets in
at this Re

U

D

(b)

(c)

$$Re = \frac{\rho U D}{\mu}$$

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(b)   • Lift $L = \dfrac{C_L}{2} \rho U^2 A$

(c)

$C_L = $ Coefficient of Lift

• Added Topics
    Stall and Stall velocity: An airfoil has a maximum coefficient of lift.
    This implies that there is a minimum speed, called the stall speed,
    for an aircraft when a plane is at the maximum coefficient of lift
    and is supporting its deadweight (minimum landing speed).

    Van Karmon Vortex Street and Strouhal number $\dfrac{\omega D}{U}$

Vortex at fixed l & h for given flow

(c)
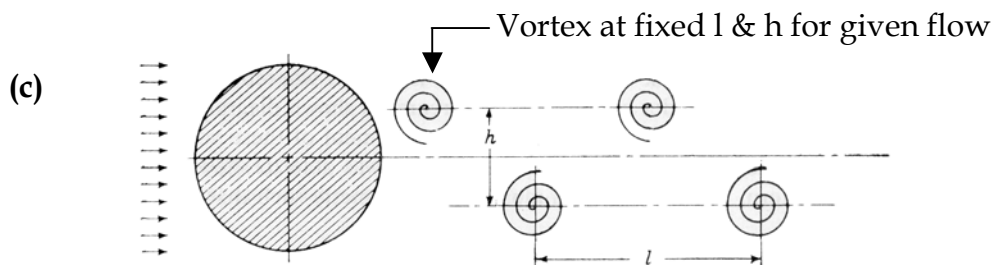
II. Design Example

An engineer must design a cable system consisting of 100′ of 1″ diameter supporting rods. The structure that the cable is supporting is shown in the figure. Determine the wind force on the system for a maximum wind of 50 mph. Take the wind at 50⁰F.
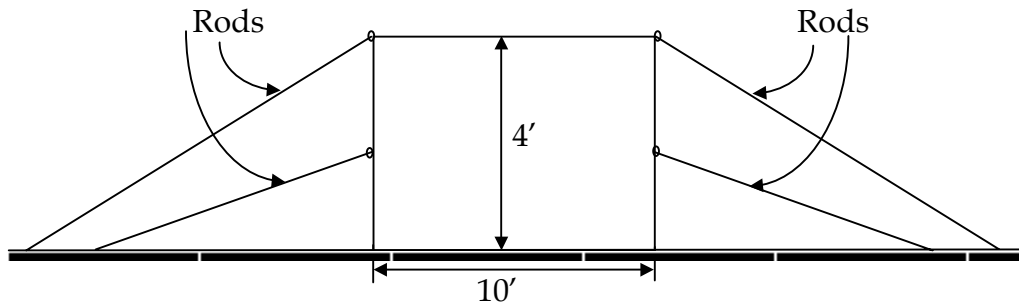
$$\text{Drag} = \Sigma C_D \frac{\rho U^2}{2} A$$

$C_D = 0.30$ Rods (Turbulent) and 1.2 (Rectangle)
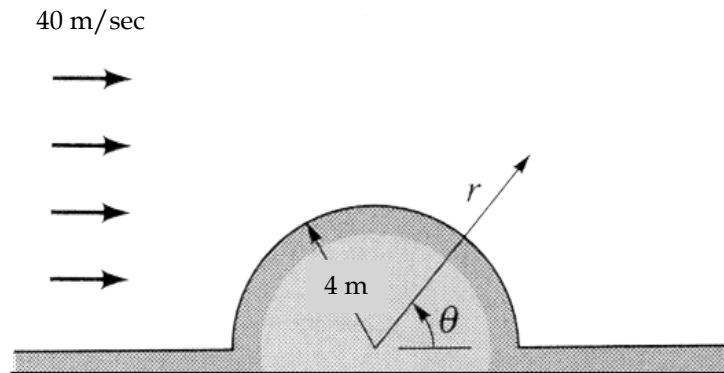
$U = 1.4667(50) = 73.3 \; \text{ft}\big/\text{sec}$

$\rho_{Air} = .002378 \; \dfrac{\text{slug}}{\text{ft}^3} \; ; \; \nu = 1.8 \times 10^{-4} \; \text{ft}^2\big/\text{sec}$

$$\text{Drag Force} = 0.3(.002378)\frac{(73.3)^2}{2}\left(100 \times \frac{1}{12}\right) + 1.2(.002378)\frac{(73.3)^2}{2}(4 \times 10)$$

Drag Force = 323 lb.

III. Design Homework Problem

A cold water station is in the shape of a half sphere. Determine the force on this station for a maximum velocity of $40\,\frac{m}{sec}$ and take the temperature at 0^0C.


40 m/sec

4 m

r

θ

Design Homework Project

To determine the drag force on a vessel, boundary layer tests were conducted. A pitot tube was used in the test shown. The results of these tests are given in the table.

      (a) Calculate and plot the boundary layer velocity profile.
      (b) Estimate the boundary layer thickness.

y (in.)	.02	.035	.044	.06	.093	.11	.138	.178	.23	.27	.322
h (in.)	.3	.7	.8	1.4	1.8	2.6	2.9	3.3	3.9	4.0	4.0

h

v

y

vessel
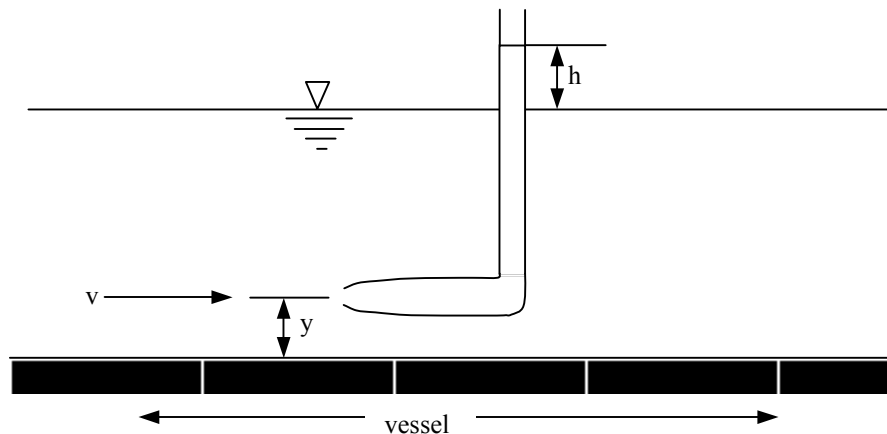
IV. Historical Notes

After the turn of the century, Ludwig Prandtl (1905) proposed the concept of the boundary layer. This concept not only paved the way to sophisticated analysis needed in the development of the airplane, but also resolved many of the paradoxes involved in the flow of a low-viscosity fluid.

V. Experiments/Demonstrations

- Hydrostatic and Properties of Fluid Apparatus
  (a) Falling sphere Experiment: Drop a sphere in a graduated cylinder and time the ratio of fall. From a force balance equation the drag can be determined. The experiment can be repeated for different liquid and spheres.

- Hele-Shaw
  (b) Flow around a cylinder and airfoil: From dye streaks the flow patterns around a cylinder and airfoil can be determined.

- Videos-
  (c) Fundamentals of Boundary Layers (5)
  (d) Fluid Dynamics of Drag (7III)
  (e) Fluid Mechanics of Drag (7I)
  (f) Form, Drag, Lift, and Propulsion (23)

VI. References

Mechanics of Fluids, Shames: Sections 13.1 – 13.2, 13.4, 13.6 – 13.12, 13.16

Fundamentals of Fluid Mechanics, Munson, Young, Okiishi: Sections 9.1 – 9.4

Introduction to Fluid Mechanics, Fox, McDonald: Chapter 9

Video Tapes: Fluid Dynamics of Drag
             Fluid Mechanics of Drag
             Form, Drag, Lift, and Propulsion
             Fundamentals of Boundary Layers

I. One Dimensional Compressible Flow

- Subsonic $.4 < M < 1.0$; $M \equiv$ Mach Number
- Transonic $M \approx 1.0$

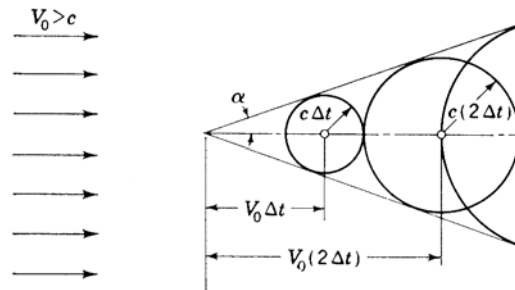(a) (c)   Supersonic $1 < M < 3$
- Hypersonic $M > 1$

(b)   • Elastic Wave   C = acoustic celerity (speed of sound)

$$C^2 = \frac{dP}{d\rho}$$

$$C = \sqrt{\frac{kP}{\rho}}$$

(c)   • Mach Cone $M = \frac{V}{C}$

     Compare M to Froude #

$$\sin \alpha = \frac{C}{V} = \frac{1}{M}$$

- Isentropic Flow

    S = constant; S = entropy ,   0 stagnation

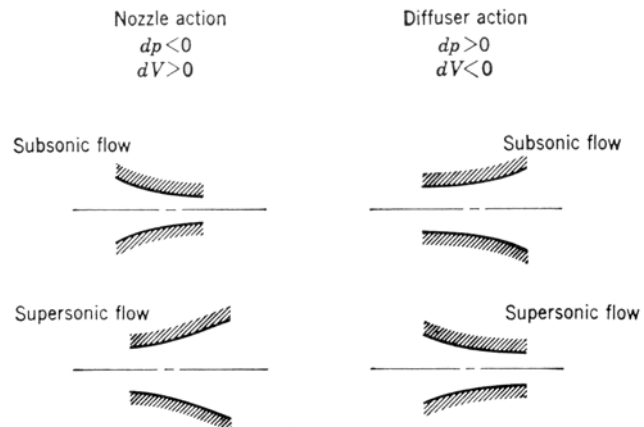$$h_0 = h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2}; \quad h = u + \frac{P}{\rho}$$

$$S_0 = S$$

$$\rho v A = \text{cons} \tan t; \quad R = \text{Friction Drag}$$

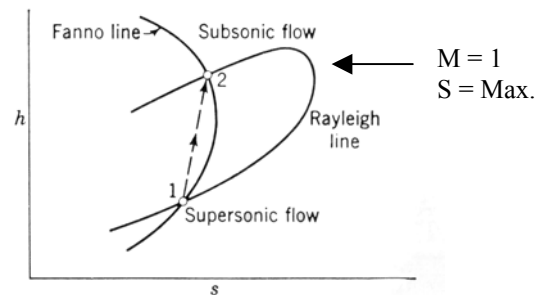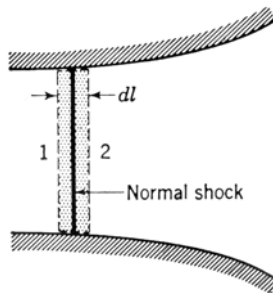$$P_1 A_1 - P_2 A_2 + R = \rho_2 v_2^2 A_2 - \rho_1 v_1^2 A_1$$

$$h = h(S, P), \quad \rho = \rho(S, P) \quad \text{Mollier Chart}$$

$$\frac{dA}{A} = \frac{dP}{\rho v^2}\left(1 - M^2\right)$$

- Isentropic Flow of a Perfect Gas
  Use One Dimensional Isentropic tables
  -equations: Energy, State, Continuity

(a) • Normal Shock
     Compare Normal Shock to Hydraulic Jump


Fanno line ⇒ adiabatic conditions Q = constant
Rayleigh line ⇒ Friction (Drag) R = constant
Flow can be considered as constant area through the control
volume, and boundary-layer friction and heat transfer can be
disregarded for the chosen control volume.

(c) • Normal-Shock Relations for a perfect Gas
     Use Normal Shock Tables
     -equations: Energy, State, Continuity, Momentum

     • Oblique Shocks

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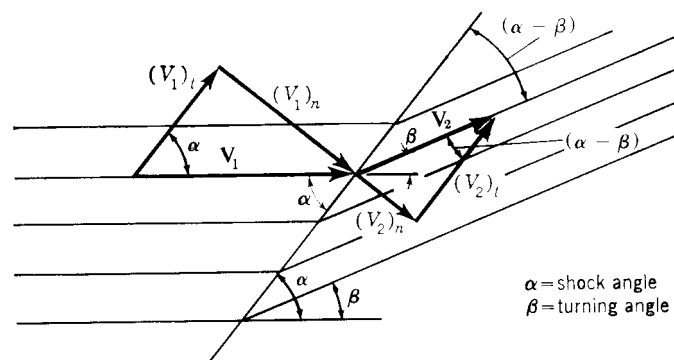
II. Design Example

A convergent-divergent nozzle with a throat area of .001 m² and an exit area of .00201 m² is connected to a tank where air is kept at a pressure of 500,000 Pa absolute and a temperature of 20⁰C. If the nozzle is operating at design conditions, what should be the ambient pressure outside and the mass flow? Neglect friction.

T_o= 20⁰C
ρ_o=500,000 Pa
$A^* = .001$ m²
$A = .00201$ m²
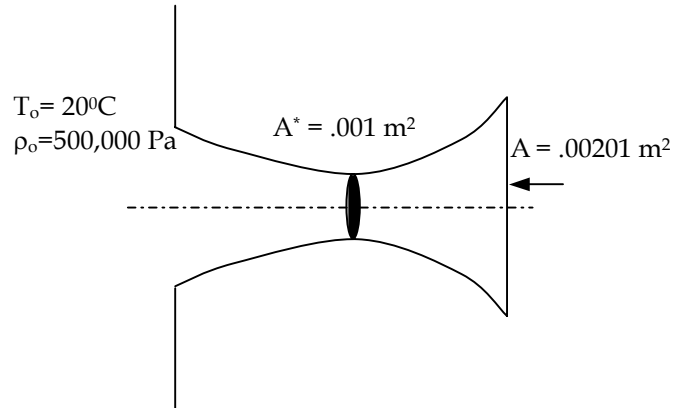
For design operation can say:

$$\frac{A_e}{A_o} = \frac{.00201}{.001} = 2.01$$

From the isentropic table:

$$\frac{\rho_e}{\rho_o} = 0.094 \therefore \rho_e = .094(500,000) = 47,000 \; Pa = 1.009 \; kg/m^3$$

$$\frac{T_e}{T_o} = .508 \therefore T_e = .508(293) = 149 \; K$$

$$M_e = 2.2$$

$$v_e = 2.2\sqrt{(1.4)(287)(149)} = 538 \; m/sec$$

$$\rho_e = \frac{47,000}{(287)(149)} = 1.099 \; kg/m^3$$

$\therefore$ m= (1.099)(538)(.00201) = 1.19 kg/sec

III. Design Homework Problem

Design a nozzle for an ideal rocket that is to operate at 15,000-m altitude in a standard atmosphere of 10 kPa and is to give a 5 kN thrust when the chamber pressure is 1000 kPa and the chamber temperature is 3000⁰C. What are the throat and exit areas and the exit velocity and temperature? Take k=1.4 and $R = 355 \; N\cdot m/(kg\cdot K)$ for this calculation. Take the exit pressure to be at ambient temperature.

IV. Historical Notes

Gas dynamics is a relatively new field, in that the earliest works on it did not appear until the nineteenth century. Riemann published his paper on compression (shock) waves in 1860, and 20 years later Mach observed such waves in supersonic projectiles.

V. Experiments/Demonstrations

- 2.5 Metre Flow Channel
    (a) Hydraulic Jump Experiment: Compare the hydraulic jump to the shock wave and Froude number to the Mach number.

- Hydrostatic and Properties of Fluid Apparatus
    (b) Pressure Gauges: Determine pressure variation in a gas and study of atmospheric pressures.

- Videos-
    (c) Effects of Compressibility (24)

VI. References

Mechanics of Fluids, Shames: Sections 11.1 – 11.9, 11.11 – 11.15

Fundamentals of Fluid Mechanics, Munson, Young, Okiishi: Sections 11.1 – 11.7

Introduction to Fluid Mechanics, Fox, McDonald: Chapter 12, Chapter 13

Video Tape: Effects of Compressibility