8.3 Patterns on the Multiplication Table

ALGEBRA

Essential Question How can you use properties to explain patterns on the multiplication table?

the 5 Es ENGAGE

GO DIGITAL Lesson Opener

Making Connections

Invite students to tell you what they know about patterns.

What is a pattern? (something that repeats itself in an orderly way or according to a certain rule) **What kind of patterns have you seen? Have you ever made a pattern yourself?** Encourage students to think about different patterns in art, music, and mathematics.

Using the Digital Lesson

You may wish to use a number line to review the concept of patterns with students. Point out how each number increases by 1 in an orderly pattern.

Learning Task

What is the problem the students are trying to solve? Connect the story to the problem.

- **How many legs does each beetle have?** (6 legs)
- **How could you find how many legs 2 beetles have in total?** (Multiply 2×6 or add $6 + 6$)
- **How could you find how many legs 3 beetles have in total?** (Multiply 3×6 or add $6 + 6 + 6$)
- **How does the total number of legs change from 1 to 2 to 3 beetles? Does it get smaller, larger, or stay the same?** (Larger)

Literacy and Mathematics

Choose one or more of the following activities.

- Write the word *property* on the board. Have students discuss the different meanings they know for this word. (land someone owns; a rule or principle)
- Show or ask students to draw a picture of a beetle. Have students write a list of six adjectives that could describe beetles.

Texas Essential Knowledge and Skills

TEKS Number and Operations—3.4.E
Represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting *Also 3.5.B*

MATHEMATICAL PROCESSES
3.1.E Create and use representations
3.1.F Analyze mathematical relationships

Are You Ready?

Access Prior Knowledge

Use the *Are You Ready?* 8.3 in the *Assessment Guide* to assess students' understanding of the prerequisite skills for this lesson.

Vocabulary

GO DIGITAL Multimedia eGlossary at *thinkcentral.com*

GO DIGITAL Resources

For the student

- iSE **Interactive Student Edition** *provides students with an interactive learning environment!*
- Math on the Spot Video Tutor
- iTools Virtual Manipulatives
- Soar to Success Math Online Intervention

For the teacher

- **Digital Management Center** *organizes program resources by TEKS!*
- eTE eTeacher Edition
- Online Assessment System

Unlock the Problem

Explain that students will use patterns and properties as strategies to help them learn and remember multiplication facts.

Activity 1

Materials: Addition/Multiplication Table (see *eTeacher Resources*)

After they complete the products for the green squares, have students point to each product and say the corresponding multiplication sentence.

- Describe how the Commutative Property is shown on the table. Possible answer: the same product is shown twice in the green squares, but the factors are in a different order.

Math Talk Mathematical Processes

Use Math Talk to focus on students' understanding of how to use patterns in a multiplication table to find products.

Go Deeper

Distribute the multiplication tables. Demonstrate how to fold it in half along the diagonal from the upper left to the lower right and have students do the same.

Next, have students shade the products on the diagonal fold. Have them write a multiplication sentence for some of the shaded products.

- **What do you notice about the factors in each multiplication sentence?** The factors are the same numbers.

ELL English Language Learners

Leveled Activities	ELPS
Beginning: Activity 50	1.B.1, 2.E.3, 4.F.5
Intermediate: Activity 45	1.D, 2.C.3, 3.D.1
Advanced: Activity 5	1.D, 2.I.5, 3.H.3
Advanced High: Activity 29	3.E, 4.F.10, 4.G.4

GO DIGITAL Go to *thinkcentral.com* for the **ELL Activity Guide** containing these leveled activities.

Name _____

8.3 Patterns on the Multiplication Table

ALGEBRA

TEKS Number and Operations—3.4.E *Also 3.5.B*
MATHEMATICAL PROCESSES 3.1.E, 3.1.F

Essential Question How can you use properties to explain patterns on the multiplication table?

Unlock the Problem Real World

You can use a multiplication table to explore number patterns.

×	0	1	2	3	4	5	6	7	8	9	10
0		0				0	0		0		
1		1				5	6	7			
2		2			8	10	12				
3		3		9		15	18				
4		4	8		16	20	24				
5		5		15		25	30				
6	0	6	12			30	36				
7		7				35	42				
8	0	8				40	48				
9		9				45	54				
10		10				50	60				

Activity 1

Possible answers are given.

Materials ■ MathBoard

- Write the products for the green squares. What do you notice about the products?

I see products that are the same. When I reach the product in the middle, 16, the products change order.

Write the multiplication sentences for the products on your MathBoard. What do you notice about the factors?

They have the same factors, but the factors are in a different order.

- Will this be true in the yellow squares? **Explain** using a property you know.

yes, because of the Commutative Property of Multiplication

Write the products for the yellow squares.

- Complete the columns for 1, 5, and 6. Look across each row and compare the products. What do you notice?

The products for 6 are the sums of the products for 1 and 5.

What property does this show?

Distributive Property

Possible explanation: I can add the products for the columns for 1 and 6 to find the products for the column for 7.

Math Talk Mathematical Processes

Explain how you can use these patterns to find other products.

Differentiated Instruction

ELL Language Support Verbal / Linguistic Small Group

ELPS 1.A.1, 3.B.3

Strategy: Define

- Students can define words by using them in context and providing examples.
- Remind students that a *pattern* is an object or number that repeats.
- Write the following numbers on the board: 5, 10, 15, 20, 25.
- **What number comes next in the pattern?** 30
  How can you describe the pattern? Possible answer: the numbers skip count by five.
- Repeat with other number patterns.

Activity 2 Possible answers are given.

Materials ■ yellow and blue crayons

- Shade the rows for 0, 2, 4, 6, 8, and 10 yellow.
- What pattern do you notice about each shaded row? The products end in 0, 2, 4, 6, and 8; the products are even.
- Compare the rows for 2 and 4. What do you notice about the products? The products of 4 are the products of 2 doubled.
- Shade the columns for 1, 3, 5, 7, and 9 blue.
- What do you notice about the products for each shaded column? The products repeat–even, odd.

×	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Remember
Even numbers end in 0, 2, 4, 6, or 8. Odd numbers end in 1, 3, 5, 7, or 9.

Share and Show

MATH BOARD

1. Use the table to write the products for the row for 2.

0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

Describe a pattern you see. Possible answers: the ones digits repeat—0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

Possible answer: the product is always even.

Math Talk Mathematical Processes
What do you notice about the product of any number and 2?

Is the product even or odd? Write *even* or *odd*.

2. 5×8 even
3. 6×3 even
4. 3×5 odd
5. 4×4 even

Use the multiplication table. Describe a pattern you see. Possible patterns are given.

6. in the column for 10
The ones digit is always 0; each number is 10 more than the number above it.

7. in the column for 8
The ones digit repeat—0, 8, 6, 4, 2; the products are all even.

Enrich

Verbal / Linguistic
Partners

Materials: index cards

- Have students write the word *even* on three index cards and the word *odd* on three index cards. Mix the cards and place them facedown.
- One student turns over a card and reads the word. The partner then says a multiplication sentence that has a product that matches the word on the card.
- The first student writes the multiplication sentence and verifies that it is correct.
- Partners reverse roles and take turns.

GO DIGITAL Go to *thinkcentral.com* for additional enrichment activities in the **Enrich Activity Guide**.

Activity 2

Before students begin the activity, ask them to review the meaning of even and odd numbers. As students work on the activity, continue to focus on how patterns can help them remember multiplication facts.

- **How can you use patterns and properties to help you multiply by 0 and 1?** The product of 0 and a number is always 0. The product of 1 and a number is always that number.

After discussing any patterns they see, ask the following questions:

- **Are all the products for 6 even or odd?** even

Write the following on the board:
$(3 + 3) \times 5 = (3 \times 5) + (3 \times 5)$

- **What is the product?** 30 **Is the product even or odd?** even
- **What property did we use to show this pattern?** Distributive Property
- **How does this property explain the pattern?** Possible answer: I know that an odd number added to another odd number will always be an even number. So, any odd number multiplied by 6 will be even.

Have students use the Distributive Property to show that any even number multiplied by 6 will also have a product that is even. Students should provide an example.

Share and Show

The first problem connects to the learning model. Have students use the MathBoard to explain their thinking.

Use the checked exercises for **Quick Check**.

RtI Quick Check

IF a student misses the checked exercises

THEN **Differentiate Instruction** with RtI Tier 1 Lesson 35

Go Deeper

Write a pattern on the board such as 6, 12, 18, 22, 30, 36. Have students analyze the pattern and find the error. Analyzing information helps students check their understanding of multiplication patterns.

Problem Solving

Have students read Exercises 8 and 9. Ask them to explain how they will find a pattern for each table.

H.O.T. Problems

To explain Problem 10, students need to use the patterns in the ones digits they described in Exercises 8 and 9.

For Problem 11, have students compare Elijah's and Chloe's work. Elicit from students that Elijah's work shows 2 equal-sized groups and Chloe's work shows groups of 2.

As students analyze Elijah's work and Chloe's work, guide them to see that when an even number is multiplied by a number, the product can be grouped into two equal addends.

- **Look at Elijah's work and his drawing. How did Elijah show the product of 12 as two equal addends?** He showed 12 as 6 + 6. Both addends are 6, so they are equal addends.

Encourage students to use other multiplication sentences to reinforce this concept. Have them use drawings or counters to illustrate their findings.

COMMON ERRORS

Error Students confuse odd and even numbers.

Example Is the number 40 odd or even? Student answer: odd

Springboard to Learning Write the digits 0, 2, 4, 6, 8 on a strip of paper. Remind students that if a number ends with one of these digits, it is even.

Math on the Spot Video Tutor

Through the ***Math on the Spot Video Tutor***, students will be guided through an interactive solving of this type of H.O.T. problem. Use this video to also help students solve the H.O.T. problem in the Interactive Student Edition. With these videos and the H.O.T. problems, students will build skills needed in the TEXAS assessment.

GO DIGITAL **Math on the Spot** videos are in the Interactive Student Edition and at *thinkcentral.com*.

Name _____

Problem Solving

Complete the table. Then describe a pattern you see in the products.

8.

×	2	4	6	8	10
5	10	20	30	40	50

Possible answers: the ones digit is always zero; the products are even.

9.

×	1	3	5	7	9
5	5	15	25	35	45

Possible answers: the ones digit is always 5; the products are odd.

10. H.O.T. Write Math **Explain** how patterns of the ones digits in the products relate to the factors in Exercises 8 and 9.

Possible explanation: when an even factor is multiplied by 5, the ones digit is always zero; when an odd factor is multiplied by 5, the ones digits is always 5.

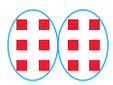
H.O.T. **Multi-Step Sense or Nonsense?**

11. Elijah says the product of an odd number and an even number is even. Chloe says the product of two even numbers is even. Whose statement makes sense? Whose statement is nonsense? **Use Math Language** to explain your reasoning.

Elijah's Work

odd ↓ × even ↓ = even ↓

3 × 4 = 12



I can circle 2 equal groups of 6 with no tiles left over. So, the product is even.

Chloe's Work

even ↓ × even ↓ = even ↓

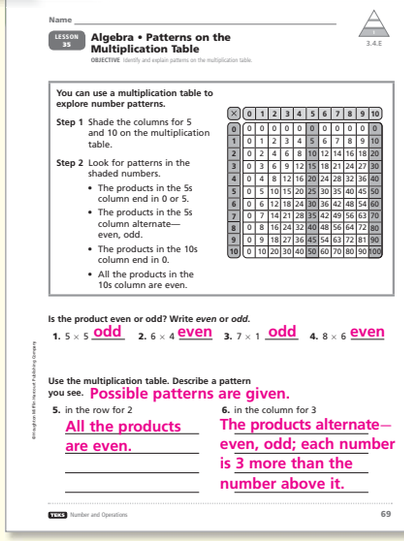
2 × 6 = 12

I can circle 6 pairs with no tiles left over. So, the product is even.

Possible explanation: both statements make sense. They both show how the product can be an even number.

Differentiated Instruction

RtI Tier I Lesson 35



Name _____

LESSON 35 **Algebra • Patterns on the Multiplication Table**

OBJECTIVE Identify and explain patterns on the multiplication table.

3.4.E

You can use a multiplication table to explore number patterns.

Step 1 Shade the columns for 5 and 10 on the multiplication table.

Step 2 Look for patterns in the shaded numbers.

- The products in the 5s column end in 0 or 5.
- The products in the 5s column alternate—even, odd.
- The products in the 10s column end in 0.
- All the products in the 10s column are even.

Is the product even or odd? Write *even* or *odd*.

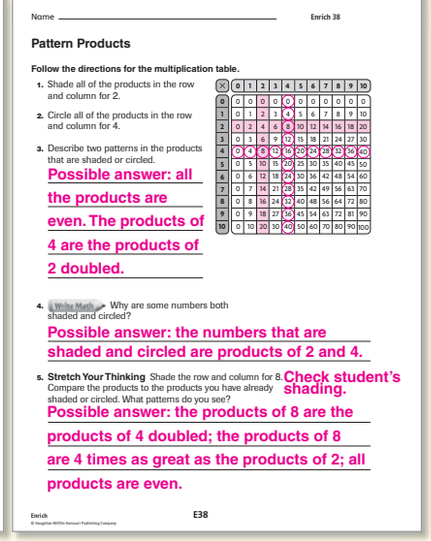
1. 5 × 5 odd
2. 6 × 4 even
3. 7 × 1 odd
4. 8 × 6 even

Use the multiplication table. Describe a pattern you see. Possible patterns are given.

5. in the row for 2: All the products are even.
6. in the column for 3: The products alternate—even, odd; each number is 3 more than the number above it.

TEKS Number and Operations 69

Enrich 38



Name _____ Enrich 38

Pattern Products

Follow the directions for the multiplication table.

1. Shade all of the products in the row and column for 2.
2. Circle all of the products in the row and column for 4.
3. Describe two patterns in the products that are shaded or circled. Possible answer: all the products are even. The products of 4 are the products of 2 doubled.
4. Write Math Why are some numbers both shaded and circled? Possible answer: the numbers that are shaded and circled are products of 2 and 4.
5. **Stretch Your Thinking** Shade the row and column for 8. Compare the products to the products you have already shaded and circled. What patterns do you see? Check student's shading. Possible answer: the products of 8 are the products of 4 doubled; the products of 8 are 4 times as great as the products of 2; all products are even.

Enrich E38

Mathematical Processes
Model • Reason • Communicate

Daily Assessment Task

Fill in the bubble for the correct answer choice.

12. A spider has 8 legs. Josie is using the multiplication table to find the number of legs on 1, 2, 3, 4, 5, and 6 spiders. Which pattern do the products show?

×	1	2	3	4	5	6
8	8	16	24	32	40	48

Ⓐ odd, then even, then repeat

Ⓑ even, then odd, then repeat

Ⓒ all odd

● all even

13. Xavier is counting groups of students at tables in the lunchroom. Which describes Xavier's pattern?

6, 12, 18, 24, 30, 36

Ⓐ Multiply by 2.

Ⓑ Multiply by 3.

● Add 6.

Ⓓ Add 12.

14. **Multi-Step** What are the next 2 numbers in this pattern?

7, 14, 21, 28, 35, ?, ?

Ⓐ 37, 39

● 42, 49

Ⓒ 70, 80

Ⓓ 49, 56

TEXAS Test Prep

15. Which of the following describes this pattern?

12, 16, 20, 24, 28

Ⓐ Multiply by 4.

● Add 4.

Ⓒ Multiply by 5.

Ⓓ Subtract 4.

Grab-and-Go!™ Ready-Made Independent Activities

Differentiated Centers Kit

Games
Multiplication Bingo
Students practice multiplication facts through 10.

Literature
Here's What I Do
Students read about using multiplication tables to win a computer game about multiplication.

Activities
Diamond Derby
Students complete purple Activity Card 15 by practicing multiplication facts through 10 by 10.

5 the 5 Es EVALUATE

Daily Assessment Task RtI

Can students use properties to explain patterns on the multiplication table?

IF	THEN
NO	• Soar to Success Math Warm-Up 12.38
YES	• Enrich 38 • Homework and Practice Lesson 8.3

TEXAS Test Prep Coach

Test Prep Coach helps teachers to identify common errors that students can make.

In the Test Prep exercise, if students selected:

A They multiplied by 4 instead of adding 4.

C They multiplied by 5 instead of adding 4.

D They subtracted instead of adding 4.

Essential Question

Write Math

How can you use properties to explain patterns on the multiplication table? I can use the Commutative Property to find patterns that show the same factors, but in a different order. I can also use the Associative Property and Distributive Property to find patterns for products that are even or odd.

Homework and Practice

TEKS Number and Operations—3.4.E
Also 3.5.B
MATHEMATICAL PROCESSES 3.1.E, 3.1.F

Name _____

8.3 Patterns on the Multiplication Table

ALGEBRA

Use the multiplication table. Describe a pattern you see. Possible patterns are given.

×	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

1. in the column for 4

   The ones digit repeat—0, 4, 8, 2, 6; the products are all even.

2. in the column for 5

   The ones digit repeat—0, 5, 0, 5, 0, 5; the products form an even/odd/even/odd pattern.

Complete the table. Then describe a pattern you see in the products.

3.

×	1	2	3	4	5
2	2	4	6	8	10

Possible answers: the products skip-count by 2; the products are even.

4.

×	2	3	4	5	6
3	6	9	12	15	18

Possible answers: the products skip-count by 3; the products form an even/odd/even/odd pattern.

Problem Solving Real World

5. Lani says that the product of an odd number and an odd number is an odd number. Draw a picture and write a number sentence that shows how Lani's statement makes sense.

   Check students' drawings. Possible answers are given.

   $3 \times 3 = 9$; $5 \times 5 = 25$; $3 \times 5 = 15$; $5 \times 7 = 35$

Lesson Check

TEXAS Test Prep

6. A store sells tricycles. Each tricycle has 3 wheels. Jin uses a multiplication table to find the total number of wheels. Which pattern do the products show?

×	2	4	6	8	10
3	6	12	18	24	30

Ⓐ all odd
● all even
Ⓒ odd, then even, then repeat
Ⓓ even, then odd, then repeat

7. Jin also uses a multiplication table to find the total number of wheels on the bicycles in the store. Which pattern do the products show?

×	1	3	5	7	9
2	2	6	10	14	18

Ⓐ all odd
● all even
Ⓒ odd, then even, then repeat
Ⓓ odd, even, odd, then repeat

8. Matt is counting groups of squares on a quilt he is making. Which describes Matt's pattern?

3, 6, 9, 12, 15, 18

Ⓐ Skip count by 2.
Ⓑ Multiply by 3.
● Add 3.
Ⓓ Add 6.

9. Cassie is counting groups of beads on a necklace. Which describes Cassie's pattern?

4, 9, 14, 19, 24, 29

Ⓐ Add 4.
Ⓑ Multiply by 4.
Ⓒ Multiply by 5.
● Add 5.

10. **Multi-Step** Jacy makes a pattern with triangles. How many triangles does Jacy need to place in the next two parts of his pattern?

4, 10, 16, 22, 28, ?, ?

Ⓐ 29, 30
● 34, 40
Ⓒ 32, 36
Ⓓ 30, 32

11. **Multi-Step** Vito makes a pattern with puzzle pieces. How many puzzle pieces does Vito have in the first part of the pattern?

?, ?, 8, 11, 14, 17, 20

Ⓐ 2, 4
Ⓑ 1, 3
● 2, 5
Ⓓ 3, 5

Homework and Practice

Use the Homework and Practice pages to provide students with more practice on the concepts and skills of this lesson.