Exercise 2.37 from *An Introduction to Mathematical Cryptography*
The multiplication table for the field $\mathbb{F}_2[x]/(x^3+x+1)$ is given in Table 2.5, but we have omitted fourteen entries. Fill in the missing entries. (This is the field described in Example 2.57. *Hint.* Once you compute $\mathbf{a}\cdot\mathbf{b}$, you've also computed $\mathbf{b}\cdot\mathbf{a}$.)

	0	1	x	x^2	$1+x$	$1+x^2$	$x+x^2$	$1+x+x^2$
0	0	0	0	0	0	0	0	0
1	0	1	x			$1+x^2$	$x+x^2$	$1+x+x^2$
x	0	x	x^2		$x+x^2$	1		$1+x^2$
x^2	0			$x+x^2$	$1+x+x^2$	x	$1+x^2$	1
$1+x$	0		$x+x^2$	$1+x+x^2$	$1+x^2$		1	x
$1+x^2$	0	$1+x^2$	1	x		$1+x+x^2$	$1+x$	
$x+x^2$	0	$x+x^2$		$1+x^2$	1	$1+x$	x	
$1+x+x^2$	0	$1+x+x^2$	$1+x^2$	1	x			$1+x$

Table 2.5: Multiplication table for the field $\mathbb{F}_2[x]/(x^3+x+1)$.