The Notions and Roles of Theory in Mathematics Education Research

Team Chair:
Teresa Assude, University of Provence, France, t.assude@aix-mrs.iufm.fr

Team Members:
Paolo Boero, University of Genova, Italy
Patricio Herbst, University of Michigan, USA
Stephen Lerman, London South Bank University, U.K.
Luis Radford, Laurentienne University, Canada

The rationale for this Survey Team (ST), commanded by the International Program Committee of ICME 11, is that:

> Notions and concepts of theory play key roles in mathematics education research, as they do in any scholarly or scientific discipline. On closer inspection, the notion, concept, and nature of what is termed "theory" in such research are very varied indeed, as are the roles, uses and implications of theories employed in mathematics education research. In other words, the term "theory" does not have one universal meaning in our field. Moreover, concrete theories put to use with regard to mathematics education originate in several different disciplines, many of which are external to mathematics education research itself. The task of this ST is to identify, survey, and analyse different notions and roles of "theory" in mathematics education research, as well the origin, nature, uses, and implications of specific theories pertaining to different types of such research.

This task defines a very important problematique in mathematics education research but, even if this problematique is clear, its treatment is problematic. Of course, the investigation of this problematique can be different and we can produce different answers.

In this paper, to carry out this task, we will consider three levels corresponding to some questions. The first level is a preliminary interrogation about: how to do a survey? What are the data? What are the tools for doing this survey? What are the criteria? Are these criteria theoretical or empirical? Have we common or different tools for doing this task? What are our assumptions about this task? This level is a methodological level but it is too an epistemological one: our practice and assumptions of mathematics education research found what we do in order to achieve this task.

A second level is a results level. We produce different surveys and we identify and analyse different roles and functions of "theory" in mathematics education research. We must point out different results of these surveys and these results are depending on the data and tools used in this work.

A third level is a reflexive level. We want to compare our different methodologies and assumptions in doing this task. What are the different types of theory? What is a theory in mathematics education research? What is the role of theory in the autonomy and identity of mathematics education as a scientific domain?

These three levels organise our text in three parts and we conclude by some "open questions". We organized the ST from preliminary individual work. We prepared five papers and this common paper is the result of the collective work. Three of us tended to work especially in the first and second level, and the other two in the third level but this is just a trend. Sometimes for further developments we will make reference to these preliminary papers because we cannot include their full content in this paper.

1. FIRST LEVEL: DATA, METHODOLOGIES, TOOLS, ASSUMPTIONS

In this chapter, we will make explicit our different data, methodologies and tools. We want to note that some of these surveys are not exhaustive and the results depend on the choice of data, methodologies and tools for analysing these data.

Lerman, Herbst and Assude each analysed a sub-section of the research literature in the field of mathematics education. Each researcher developed a set of categories for that analysis, hence producing a theoretical framework in interaction with an empirical set. We will present here extracts from each of these three papers in which the authors describe the methods, motives and categories used in each of these papers.

Lerman surveyed how researchers in the mathematics education research community work with theories, both in terms of which theories and how they work with them. In carrying out the survey he sampled research carried out between 1991 and 2003 on 12 years of the publications in *Educational Studies in Mathematics* (*ESM*), *Journal for Research in Mathematics Education* (*JRME*), *and Proceedings of the International Group for the Psychology of Mathematics Education* (*PME*).

In his[1] research he developed a tool, in interaction with the data, for analysing a whole range of aspects of the research productions of the community as evidenced in a sampling of published articles. In this paper he focused on just two elements of the analysis, those of use of theory & orientation. By orientation he meant to theoretical or empirical inquiry; whether the theories used have changed over time; whether researchers revisit the theories used in their studies; the relationships established between the theoretical and the empirical; and the focus and methodology of the studies.

By 'theories' he intended learning theories, perhaps set in the context of philosophical orientations, perhaps informed by psychology, or sociology or other fields. It is our expectation that such theories guide the design of a research study and the analysis, or perhaps are used retrospectively as lenses through which to interpret a set of findings. This approach focuses on theories as resources to help towards the achievement of those desired outcomes.

Herbst wants to complement the contribution made in the chapter "Theory in mathematics education scholarship" (Silver and Herbst, 2007) with some data gathered from a superficial inspection of the 39 articles published in the *Journal for Research in Mathematics Education* from January 2005 to January 2008.

His main objective has been to describe whether and how authors of research articles use the word theory (or its cognates such as theorizing, theorization, theoretical) in relation to the pursuit of their research. One question has been

[1] The project, entitled "The Production and Use of Theories of Teaching and Learning Mathematics" and funded by the Economic and Social Research Council in the UK, project No. R000 22 3610. The full text of the project proposal and the research papers published from it are at http://www.lsbu.ac.uk/~lermans/ESRCProjectHOMEPAGE.html

to describe the extent to which the articles in this corpus identify themselves as theory building, theory using, or otherwise make no appeal to theory. Subsidiary questions are, in the first case, whether the articles contribute to building local theories, middle range theories, or grand theories. In the second case, whether the articles use theory to describe, explain, predict, or prescribe practices, or whether they prescribe research operations. Simultaneously, he's been attentive to the particular practices aimed at by articles that use theory and by articles that build theory.

The methodology used for this survey included the following procedures. To constitute the corpus he extracted all research articles from all issues of JRME starting in January 2005—this means that he did not include editorials, brief reports, research commentaries, book reviews, telegraphic book reviews, or announcements in the sample. Other than that all articles were included, totalling as noted above 39 articles. JRME publishes 5 issues per year and each of those issues tends to include 3 articles. Once the text of each article was available electronically we produced three word searches after "theor," "framework," and "construct." He second-guessed the idea of looking only at places where authors had used the word "theory" and its cognates based on some of the reasons noted in Silver and Herbst (2007) that might propel people to shy away from its use.

The word search heuristic based on those three words (theor, framework, construct) was useful inasmuch as it allowed to find intellectual tools that researchers have used to do a number of operations in their work. He specifically attended to the operations of describing, explaining, and predicting phenomena, prescribing educational practices, and prescribing research operations as examples of the ways in which theory might help researchers connect research to practice and to the problems of practice. These tools are used the earlier work by Silver and Herbst (2007).

In this survey, theory assists the triadic relationships between research, problems, and practices. Drawing on the distinction between local theories (e.g., what levels of development exist in students' learning of fractions?), middle range theories (e.g., what is classroom mathematics instruction), or grand theories (e.g., what is the mathematics education field) he identified those articles that had a theoretical aim and noted what that aim was.

Assude wants to identify the roles and functions of "theory" in mathematics education research taking a corpus formed by the papers published in the review "Recherches en didactique des mathématiques". This review is an important tool for the researchers' community, especially the French speaking one: it is one of the main tools to disseminate the researchers' work in this domain in France (or among French speaking researchers).

Her data are formed by all the papers published in RDM between 2000 and 2006. RDM publishes 3 issues per year and 3 papers per issue or so. There are 59 papers, 8 in Spanish, 2 in English and 49 in French.

For analysing these data, she needs to precise what is theory in this context. In her opinion, theory in mathematics education deals with teaching and learning mathematics from two points of view. First a structural point of view: theory is an organised and coherent system of concepts and notions in the mathematics education field. Second a functional point of view: a theory is a system of tools that permit a "speculation" about some reality. This "speculation" is an active one because these tools can allow to observe, analyse, interpret a teaching and learning reality (or practices), and can produce new knowledge about this reality. According of this double point of view, she can take a theory as a tool and a theory as an object. Finally she will take other indicators like: internal /external theory in mathematics education if theory is produced or not within this domain; local/global theory if the theory concerns a study of a problem or a study of a domain; the effective theoretical elements used in the work; the functions of these elements (for example, a theory can be a tool to conceive a didactical engineering).

She will use this preliminary grid for analysing our data and she wants to point out that some functions and roles of theory are not specified of one theory, but different theories can assumed the same functions even if the knowledge produced by their uses are different.

Radford developed an analytical tool which can be applied to any of the theories that are used in mathematics education research. He presents the elements of the tool and then exemplifies it by the analysis of three theories; the theory of didactic situations; constructivism; and sociocultural theories. Radford will deal with the question of the types of theories used in mathematics education research (Radford's paper[2]). His goal is to contribute to clarify

[2] The full version of the paper ("Theories in Mathematics Education: A Brief Inquiry into their Conceptual Differences") can be retrieved from the Publication section of http://www.laurentian.ca/educ/lradford/

one of the two central themes around which our Survey Team revolves, namely the investigation of the notion of theory in mathematics education research, as stipulated in the appointing official letter. How will he proceed? He could proceed by giving a definition, T, of the term "theory" and by choosing some differentiating criteria c_1, c_2, etc. Theories, then, could be distinguished in terms of whether or not they include the criteria c_1, c_2, etc. Although interesting, he will take a different path. In the first part of his paper, he will focus on a few "well-known" theories in Mathematics Education (constructivism, theory of didactical situations, social cultural theory) and attempt to locate their differences at the theoretical level, that is, he will discuss their differences in terms of their theoretical stances.

Boero carries out a study of the relationship between key theories in the field and the ways in which external frameworks are drawn into the field. His analysis will be presented in the third level.

2. SECOND LEVEL: SOME RESULTS

In this level we are presenting some results of our surveys. Sometimes we use the results of the authors' works before the work in the ST.

2.1. *Uses of Theory and orientation: theory as a tool*

Lerman's analysis showed, for the period from 1990 to 2001, that 70.1% of all articles in ESM have an orientation towards the empirical, with a further 8.5% moving from the theoretical to the empirical, and 21.5% presenting theoretical papers. This changed little over those years. Most of the papers used theory (92.7%), and more than four-fifths (86.4%) were explicit about the theories they used in the research reported in the project. Again this has not varied across the years. Similarly, 86.2% of all articles in the journal JRME had an orientation towards the empirical, with a further 2.2% moving from the theoretical to the empirical, and 11.6% presenting theoretical papers. This changed little over the years. Most of the papers used theory (83.3%), with a relatively higher percentage of papers that did not use any theory, compared to the other two journals considered here. Three-quarters (75.4%) were explicit about the theories they were used in the research reported in the articles. Again this has not varied across the years. Finally, 84.5% of all papers in the PME proceedings had an orientation towards the empirical, with a further 6.8% moving from the theoretical to the empirical, and 8.8% staying in the theoretical. This has changed little over the years. Furthermore 89.9% of the papers used theory, with 10.1% not using any theory, and more than four-fifth (82.4%) were explicit about the theories they are using in the research reported in the article. Again this has not varied across the years.

Regarding the relationship between the theory and the empirical study, in 65.5% of articles in ESM the theory informs the empirical, in 2.3% the empirical informs the theoretical and in a further 4.0% we determined that the relationship is dialectical. 7.3% did not refer to a theory either explicitly or implicitly. In JRME, in 71.7% of articles the theory informs the empirical, in 0.7% the empirical informs the theoretical but there are no cases in which we determine that the relationship is dialectical. 16.7% did not refer to a theory either explicitly or implicitly. In PME proceedings, in 79.1% of articles the theory informs the empirical, in 4.7% the empirical informs the theoretical and in a further 0.7% we determine that the relationship is dialectical. 10.1% did not refer to a theory either explicitly or implicitly.

A result of this survey is that the uses of theory is important in mathematics education research but the empirical orientation prevails. The role of theory is especially a tool.

2.2. *Types of Theory: external or internal?*

In Lerman' analyses, some interesting changes have been depicted concerning the item 'theory type'. The predominant theories throughout the period examined for all three types of text were traditional psychological and mathematics theories, but there is an expanding range of theories used from other fields. The psycho-social theories, including re-emerging ones, and the sociological and socio-cultural theories are increasing. The predominant theories were external theories in mathematics education as a scientific domain.

This result is not verified in the Assude's analysis about papers published in the journal RDM. In this case, the predominant theories are internal theories in mathematics education research: these theories are constructed within this domain.

This difference has perhaps a link with the global project of building a new scientific field – mathematics education research – with some autonomy regarding to other neighbouring fields like psychology or sociology. Silver and Herbst (2007) show that David Johnson (the first editor of JRME) point out the lack of theory in mathematics education in 1980 and he suggests to the researchers:

> "first investigate the adaptability of various psychological theories… to the learning and teaching mathematics, and [only] in the event such adaptation is not feasible, move the creation of a new theory" (in Silver and Herbst 2007, p.43).

This position – adaptation to mathematics education of theories existing in other fields – is a common position yet now. The Herbst's analyses about 39 articles published in JRME from 2005 to 2008, confirm these results since they show that there is no paper dealing with the construction of a "grand theory" (e.g. what is the mathematics education field). But 10 articles are involved in theory making to produce a local or a middle range theory while 24 papers are involved only in theory using and 5 articles don't use theory. Here we can say again the predominant role of theory as a tool.

2.3. Functions of Theory as a Tool

The Assude's analyse (Assude's paper) identifies some functions of theory in the researchers' work (see table 6 for some examples of papers):

- *conception of didactical engineering or didactical device*: for example, theory can allow to define some didactical variables to produce a didactical engineering;
- *methodological development*: for example an a priori analysis is a methodology based on a theory;
- *didactical analysis*: the analysis can be very different according to the reality (an observation of a classroom, an observation of a pupil's work, a curriculum, etc.). Different operations as describing, explaining, interpreting, justifying can be identified;
- *definition of a research problematique*: some practical problems in the educational system are not research problems. It is necessary to transform these problems in a research problem (for example doing some hypothesis or doing some categorisations);
- *study of a research problem*: theory can be a tool for defining different steps in the study of a problem;
- *production of knowledge*: theory is a tool to identify some didactical phenomena, some new knowledge about some reality.

In Herbst's analysis about papers published in JRME, he identifies some functions of theory as a tool to describe, explain, prescribe and we precise the different object these functions are dealing with such as activity or curriculum. Silver and Herbst (2007) analyse the uses of theory in mathematics education scholarship and propose to consider theory as mediator between problems, practices and research. In this work, the authors identify some functions of theory in the role of mediator between:

- *research and problems:* interpretation results; analysing data; producing results of research on a problem; giving closure to the corpus of data to study a problem; transforming a commonsensical problem into a researchable problem; generator of researchable problem; organization of a corpus of research on a problem;
- *research and practice:* prescription; understanding; description; explanation; prediction; generalisation;
- *practice and problems:* solution to a problem of practice; comparison; designing new practices; justifying choices

There is a great variety of functions for theory as a tool and it concerns all researchers' activities. These functions are not specific to a particular theory.

2.4 – *Functions of a Theory as an Object*

We suppose that theory can have two roles: as a tool and as an object. We want to give explicit some of the functions of theory as an object. Theory is not something static but dynamic: the evolution of theories in a scientific field is a means to understand the evolution of this field.

Lerman looked at whether, after the research, the researchers have revisited the theory and modified it, expressed dissatisfaction with the theory, or expressed support for the theory as it stands, he concluded that authors may not revisit the theory at all; content to apply it in their study.

The role of theory as a tool is predominant but some works exists where theory making is one of the goals. In Herbst's survey, he distinguishes three types of theories: local theories (e.g., what levels of development exist in students' learning of fractions?), middle range theories (e.g., what is classroom mathematics instruction?), or grand theories (e.g., what is the mathematics education field?). Ten papers are concerned with theory building: 7 for local theories, 3 for middle range theories and none for grand theories.

In Assude's survey (Assude's paper), she identifies that some authors use a theory for putting to the test the theory or some concepts or relations in this theory. This "theory testing" is a way to produce new theoretical developments. These are some functions for this "theory testing":

- decontextualisation, transposition and generalisation of theory in other contexts;
- relations with contingency;
- new interpretations of a phenomena;
- verifying the domain of validity of a theory;

The development of a theory is one of the functions of theory as an object: sometimes there is just one theory, sometimes two or more theories exist, and the development of a local or middle range theory is done by articulating or juxtaposing some elements of different theories.

We can quote Silver and Herbst' work for complementing this list:

> "the role of theory [is] not so much as a mediator of relationships among practices, problems and research,(...) but rather (or also) as the collector, beneficiary, or target of that interplay in a fundamentally academic theory-making exercise".

Theory-making (especially internal theories) has a role in the constitution of a mathematics education research as a specific field with an identity different from other fields as psychology. This project of constitution is present in the beginnings of this domain in some countries: for example Brousseau'work was based in the piagetian psychology but it had a theoretical ambition to become relatively independent. This idea is developed in Silver & Herbst (2007) too and we are going to develop some ideas about the autonomy and identity of mathematics education research in the 3th part.

2.5. *Conceptual Differences about Theories in Mathematics Education*

In the Radford's analysis, his goal is to contribute to clarify one of the two central themes around which our Survey Team revolves, namely the investigation of the notion of theory in mathematics education research. His choice of theories has been guided by what may be termed their "historical impact" in the constitution of mathematics education as a research field. By "historical impact" he does not mean the amount of results that a certain theory produced in a certain span of time. Although important, what he has in mind here is rather something related to the foundational principles of a theory:

> The foundational principles of a theory determine the research questions and the way to tackle them within a certain research field, helping thereby to shape the form and determine the content of the research field itself.

For him, to ask the question about the types of theories in our field is to ask for their differences and, more importantly, for that what accounts for these differences. Our argument is that these differences are better understood in terms of theoretical suppositions. Sriraman and English (2006) argued that the variety of frameworks in mathematics education is directly related to differences in their epistemological perspectives. He wants to suggest that, in addition to the underpinning corresponding epistemologies, differences can also be captured by taking into account the cognitive and ontological principles that theories in mathematics education adopt.

Radford gives three examples in his paper for the survey: constructivism, the theory of didactic situations (TDS) and the sociocultural approaches. It is not possible to present here this work but we will take just an example.

For constructivism and the TDS the autonomy of the cognizing subject vis-à-vis the teacher is a prerequisite for knowledge acquisition. For sociocultural approaches, autonomy is not the prerequisite of knowledge acquisition. Autonomy is, in fact, its result. This is one of the central ideas of Vygotsky's concept of zone of proximal development.

The ontological principle of the sociocultural approaches is that knowledge is historically generated during the course of the mathematical activity of individuals. The epistemological principle of these approaches is that the production of knowledge does not respond to an adaptive drive but is embedded in historical-cultural forms of thinking entangled with a symbolic and material reality that provides the basis for interpreting, understanding and transforming the world of the individuals and the concepts and ideas they form about it (Radford, 1997). The cognitive principle of these approaches is that learning is the reaching of a culturally-objective piece of knowledge that the students attain through a social process of objectification mediated by signs, language, artifacts and social interaction as the students engage in cultural forms of reflecting and acting. Learning, from a sociocultural perspective, is the result of an active engagement and self-critical, reflexive, attitude towards what is being learned. Learning is also a process of transformation of existing knowledge. And perhaps more importantly, learning is a process of the formation of subjectivities, a process of agency and the constitution of the self (Radford, 2008b).

3. THIRD LEVEL: THEORIES, AUTONOMY, IDENTITY

In our different surveys, we have not used the same categories and methodologies. These choices depend on our research practices and our assumptions about what a theory is and which is the role of theory for giving autonomy and identity to mathematics education field. This level is a reflexive level. We choice here to think about the relationships between the uses of theories in mathematics education and the autonomy and identity of this field.

If mathematics education aims at growing as a scientific discipline, it must develop theoretical work in order to deal with teaching and learning problems in a systematic, scientific way. Now this is a rather obvious, widely shared position. The problem is that the ways of developing theoretical work, and its autonomy or dependence from theories elaborated in other disciplines, have been rather controversial since the birth of mathematics education as a scientific discipline, in the seventieths. We have seen above the differences of theories in terms of theoretical suppositions and we have seen that these theories are not completely independent from theories in other fields. Then what is the autonomy and identity of mathematics education field?

3.1. *Permeability and the illusion of a complete autonomy*

In Boero's reflexion, mathematics education as a scientific discipline should neither work in a completely autonomous, autarchic way, nor transpose paradigms and results of other disciplines in its specific field of investigation. According to him, we should look instead to the possibility of an autonomous specific theoretical work mainly intended as selection, adoption or re-elaboration of tools coming from other disciplines, possibly integrated with the construction of other tools needed according to the specificity of the content to be taught (Boero & Radnai Szendrei, 1998; Kilpatrick & Sierpinska, 1998).

Among the disciplines that could be relevant for scientific work in mathematics education (history of mathematics, epistemology, psychology, sociology, anthropology, etc.), Boero focus on the relationships with epistemology and psychology. This choice depends on three reasons: first, in his opinion these disciplines have played a major role in influencing important changes in the teaching of mathematics during the last century; second, they can assume a crucial role in the development of mathematics education as a scientific discipline because they concern the "what" and the "how" teachers teach and students learn; third, they challenge autonomy of mathematics education as a scientific discipline because research in our field cannot ignore the fact that many results of those disciplines concern mathematics as a paradigmatic subject.

Psychological and epistemological investigations do not work (as their main aim) for a better learning of mathematics and for a better understanding of what is learning and teaching mathematics. When they deal with mathematics, epistemological theories are aimed at describing and framing some aspects of that discipline; most psychological theories dealing with learning of mathematics try to describe, interpret and, possibly, predict learners' laboratory behaviour on a given area of paradigmatic mathematical tasks. However, in the reality of the school teaching of mathematics, what comes from mathematics, epistemology and psychology is filtered and frequently deformed when it meets the complex school culture (textbooks, materials, tradition, programs…). In general, processes in the noosphere are sensitive to external influences (coming from politics, culture, etc) but they develop with a relative autonomy and inertia. What is the role of mathematics educators in those processes?

Some members of the noosphere that have special responsibilities in teachers' preparation and curriculum development (in particular, researchers in mathematics education) frequently act as if some epistemological and psychological theories would carry the truth about what mathematics is, and how students learn it. Frequently they assume an important role in "transposing" those theories in the school system, in particular through teachers' training. Other mathematics educators adapt and interpret ideas coming from epistemology and psychology by trying to match them with existing teaching devices and habits.

Boero says that mathematics educators frequently adopt ideas coming from the exterior (in particular, epistemology and psychology) to promote more or less coherent and radical changes in the school teaching of mathematics. In most cases they do not move from the identification of teaching and learning problems to the choice of theoretical tools suitable for tackling them. In those cases we can say that mathematics education mainly develops as a subaltern discipline. On the other hand, mathematics educators can not (and should not) develop a completely autonomous and autarchic science (or technology) of the teaching of mathematics in school. This is an illusion for two reasons: on one side, teachers come from a given school or university mathematics culture and are embedded in a given cultural environment, and mathematics educators are prepared in given cultural institutions; thus it is not possible to ignore what teachers and mathematics educators know and think about the teaching and learning of mathematics, and their scientific preparation. On the other, if mathematics educators want to go beyond mere descriptions of what happens in the mathematics classroom they need to consider what mathematics is, and how mathematics is appropriated by student; thus they need to deal with scientific results coming from epistemology and psychology. The unavoidable reference to epistemology and psychology can be denied or underestimated, but in that case what usually happens is that implicit assumptions are made, or explicit assumptions are assumed as unquestionable truth.

We can think different positions to develop mathematics education as a relatively autonomous scientific discipline, i.e. a research space where tackle teaching and learning mathematics problems with its own theoretical tools as well as adapted theoretical tools coming from other disciplines, critically considering their potential and limits, and their consequences on the solution of those problems.

3.2. *Towards a relative autonomy: adaptation and development*

The first position is the use of theories existing in other fields but we need to adapt these tools: these adaptations is part of the field autonomy. Boero argues that the problem is what choices to make and how to move on from those choices, keeping into account the variety of results and perspectives provided, in particular, by epistemology and psychology. The task of mathematics educators is not to choose an epistemological position or a psychological theory as an "all purpose" and universal reference (each outstanding epistemological position being culturally situated, each psychological theory having a limited domain of validity). What mathematics educators can do is to identify important teaching and learning problems, consider different existing theories and try to understand the potential and limitations of the tools provided by those theories, possibly adapted to the specific problems in order to tackle them. However this statement is still vague for two reasons. First, to identify important teaching and learning problems requires some preliminary theoretical assumptions regarding the importance and nature of the concerned competence and the way to ascertain related learning difficulties. Second, it is necessary to adopt some preliminary keys (suggested by epistemological and psychological analyses) to avoid a disperse view of the whole panorama of the teaching and learning of mathematics. A dialectic process should be developed: our epistemological and psychological culture together with our knowledge of what happens in school suggest to consider specific educational problems; in order to tackle those problems we need to identify and adapt appropriate tools from epistemology and psychology (and, in some cases, history of mathematic, sociology, etc.). It may happen that such tools oblige us to re-formulate the original educational problems, or to identify further related problems. When dealing with specific mathematics teaching and learning problems, we must recognize that in many cases existing tools elaborated by epistemology, psychology, sociology, etc. need to be adapted and re-elaborated. Cobb (2006) says:

> Mathematics educators should view the various theoretical perspectives as sources of ideas to be appropriated and adapted to their purposes. Cobb (2006)

The proliferation of theories can be a problem. In his recent article (2006) Cobb outlines two criteria through which to facilitate a conversation concerning what researchers should do when faced by a proliferation of theoretical perspectives. His first criterion is to focus on the types of questions that can be asked within each perspective about "the learning and teaching of mathematics, and thus the nature of the phenomena that are investigated and the forms of knowledge produced." His second criterion is that of usefulness:

> The usefulness criterion focuses on the extent to which different theoretical perspectives might contribute to the collective enterprise of developing, testing, and revising designs for supporting learning. This second criterion reflects the view that the choice of theoretical perspective requires pragmatic justification whereas the first focuses on the questions asked and the phenomena investigated. (Cobb 2006)

3.3. *Towards a relative autonomy: production*

The second position is the production of a new specific theoretical tools for tackling the specific problems of mathematics education domain. In spite of the eclecticism in terms of theory adopted for research, given that the goal is usefulness, or what works, elsewhere Cobb argues strongly for the importance of theory, but in the sense of the production of theory as a key part of the job of the design scientist. He illustrates this in DiSessa and Cobb (2004) by offering one category of theory production, that of 'ontological innovation', seen as the production of new objects, emerging from design experiments, that then prove useful as objects for study. Interestingly, one of the two examples offered in that paper is a retrospective look at the early work Cobb carried out with Erna Yackel and Terry Wood, a long term project based firmly within a constructivist paradigm. Nevertheless, the notions of social norms and socio-mathematical norms are presented as examples of ontological innovations that emerged from those studies, which themselves are re-interpreted retrospectively as design experiments.

3.4. *Towards a relative autonomy: reorganisation*

The third position is the reorganisation of the theoretical field. This reorganisation can be done by different forms. One example of this reorganization, of a new trend has observed in the Fifth Congress of the European Society for Research in Mathematics Education (CERME-5, 2007). The European Society for Research in Mathematics Education organizes biannual conferences that are designed to encourage an exchange of ideas through thematic working groups. One of the recurring CERME working groups is the one devoted to theories in mathematics education. The goal of this working group was not just to understand differences, but to seek new forms of linking and connecting current theories. More specifically, the idea was to discuss and investigate theoretical and practical forms of *networking* theories. Most of the papers presented at the meetings of working group 11 appeared in volume 40(2) of the journal *ZDM - The International Journal on Mathematics Education*. As we mention in the commentary paper written for this ZDM issue (Radford, 2008a), this new trend consisting of investigating ways of connecting theories is explained to a large extent by the rapid contemporary growth of forms of communication, increasing international scientific cooperation, and the attenuation of political and economical barriers in some parts of the world, a clear example of which being, of course, the European Community.

This new trend is leading to an inquiry about the possibilities and limits of using several theories and approaches in mathematics education in a meaningful way. The papers presented at the conference provided an interesting array of possibilities.

Depending on the goal, connections may take several forms. Prediger, Bikner-Ahsbahs, and Arzarello (2008) identify some of them, like “comparing” and “contrasting” and define them as follows. In “comparing” the goal is finding out similarities and differences between theories, while in “contrasting” the goal is “stressing big differences”. Cerulli, Georget, Maracci, Psycharis, & Trgalova (2008) is an example of comparing theories, while Rodríguez, Bosch, and Gascón (2008) is an example of contrasting theories. These forms of connectivity are distinguished from others like “coordinating” and “combining”. In coordinating theories, elements from different theories are chosen and put together in a more or less harmonious way to investigate a certain research problem. Halverscheid’s paper (2008) is a clear example of an attempt at coordinating theories, in that, the goal is to study a particular educational problem (the problem of modelling a physical situation) through the use of elements from two different theories (a modeling theory and a cognitive one). In combining theories, the chosen elements do not necessarily show the coherence that can be observed in coordinating connections. It is rather a “juxtaposition” of theories (Prediger, Bikner-Ahsbahs, and Arzarello’s paper (2008)). Maracci (2008) and Bergsten (2008) furnish examples of combining theories.

At least in principle, “comparing” and “contrasting” theories are always possible: given two mathematics education theories, it is possible to seek out their similarities and/or differences. In contrast, to “coordinate” or to “integrate” theories, which is another possible form of connection (Prediger, Bikner-Ahsbahs, and Arzarello’s paper (2008), seems to be a more delicate task.

Connecting theories can, in sum, be accomplished at different levels (principles, methodology, research questions), with different levels of intensity. Sometimes the connection can be strong, sometimes weak. It is still too early to make prognostics of how this new trend will evolve.

What is clear, in contrast, is that the investigation of integration of theories and their differentiation is likely to lead to a better understanding of theories and richer solutions to practical and theoretical problems surrounding the teaching and learning of mathematics.

REFERENCES

Bergsten, C. (2008).On the influence of theory on research in mathematics education: the case of teaching and learning limits of functions. ZDM - The International Journal on Mathematics Education, 39(2), 189-199.

Boero, P. & Radnai Szendrei, J. (1998). Research and Results in Mathematics Education: Some Contradictory Aspects. In J. Kilpatrick J. & A. Sierpinska A. (eds.), Mathematics Education as a Research Domain: A Search for Identity (pp.197-212). Boston: Kluwer

Brousseau, G. (1997b). Theory of Didactical Situations in Mathematics. Dordrecht: Kluwer.

Brown, A. L. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. Journal of the Learning Sciences, 2, 141–178.

Cerulli, M., Georget, J. P., Maracci, M., Psycharis, G., & Trgalova, J. (2008). Comparing Theoretical Frameworks Enacted in Experimental Research: Telma Experience. ZDM - The International Journal on Mathematics Education, 39(2), 201-213

Clements, D. H. (2007). Curriculum research: Toward a framework for "Research-based curricula". Journal for Research in Mathematics Education, 38(1), 35-70.

Cobb, P. (1988). The tension between theories of learning and instruction in mathematics education. Educational Psychologist, 23(2), 87-103.

Cobb, P. (1994). Where Is the Mind? Constructivist and Sociocultural Perspectives on Mathematical Development. Educational Researcher, 23(7), 13-23.

Cobb, P. (2006) Supporting a discourse about incommensurable theoretical perspectives in mathematics education. Philosophy of Mathematics Education Journal, 19 http://www.people.ex.ac.uk/PErnest/pome19/index.htm (Accessed 31/1/2008)

Cobb, P., Boifi, A., McClain, K., & Whitenack, J. (1997). Reflective Discourse and Collective Reflection. Journal for Research in Mathematics Education, 28(3), 258-277.

Cobb, P., & Yackel, E. (1996). Constructivist, Emergent, and Sociocultural Perspectives in the Context of Developmental Research. Educational Psychologist, 31(3/4), 175-190.

Cobb, P., Yackel, E., & Wood, T. (1992). A Constructivist Alternative to the Representational View in Mathematics Education. Journal for Research in Mathematics Education, 23(1), 2-33.

DiSessa, A. A. & Cobb, P. (2004). Ontological innovation and the role of theory in design experiments Journal of the Learning Sciences, 13(1), 77-103.

Goldin, G. A. (1990). Epistemology, Constructivism, and Discovery Learning in Mathematics. Journal for Research in Mathematics Education. Monograph 4, 31-47.

Gravemeijer, K. (1998). Educational Development and Developmental Research in Mathematics Education. Journal for Research in Mathematics Education 25, 443-471.

Gravemeijer, K. (1994). Developmental Research in Mathematics Education. Journal for Research in Mathematics Education 25, 443-471.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics Mathematical Thinking & Learning, 1(2), 155-177.

Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. Mathematical Thinking & Learning, 6(2), 105-128.

Halverscheid, S. (2008). Building a local conceptual framework for epistemic actions in a modelling environment with experiments. ZDM - The International Journal on Mathematics Education, 39(2), 225-234

Kelly, A.E., & Lesh, R. (2000). Handbook of research design in mathematics and science education. New Jersey: Lawrence Erlbaum & Associates.

Kilpatrick J. & Sierpinska A. (eds.). Mathematics Education as a Research Domain: A Search for Identity. Boston: Kluwer.

Lerman, S. (1996). Intersubjectivity in Mathematics Learning: A Challenge to the Radical Constructivist Paradigm? Journal for Research in Mathematics Education, 27(2), 133-150.

Lerman, S. (2001). The Function of Discourse in Teaching and Learning Mathematics: A Research Perspective. Educational Studies in Mathematics, 46, 87-113.

Lesh, R. & Sriraman, B. (2005). Mathematics education as a design science. ZDM, 37(6), 490-505.

Lesh, R., & English, L (2005). Trends in the evolution of the models-modelling perspective on mathematical learning and problem solving. International Reviews on Mathematical Education, 37(6), 487-489.

Maracci, M. (2008). Combining different theoretical perspectives for analyzing students' difficulties in Vector Spaces Theory. ZDM - The International Journal on Mathematics Education, 39(2), 265-276.

Prediger, S., Bikner-Ahsbahs, A., & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches: first steps towards a conceptual framework. ZDM - The International Journal on Mathematics Education. 40(2), 165-178.

Rodríguez, E.., Bosch, M., & Gascón, J. (2008). A networking method to compare theories: metacognition in problem solving reformulated within the Anthropological Theory of the Didactic. ZDM - The International Journal on Mathematics Education, 39(2), 287-301.

Radford, L. (1997). On Psychology, Historical Epistemology and the Teaching of Mathematics: Towards a Socio-Cultural History of Mathematics. For the Learning of Mathematics, 17(1), 26-33.

Radford, L. (2008a). Connecting Theories in Mathematics Education: Challenges and Possibilities. ZDM - The International Journal on Mathematics Education, 40(2), 317-327.

Radford, L. (2008b). Culture and Cognition: Towards an Anthropology of Mathematical Thinking. In L. English (Ed.), Handbook of International Research in Mathematics Education (2nd Edition) (pp.439-464). New York: Routledge, Taylor and Francis.

Silver, E. and Herbst, P. (2007). The role of theory in mathematics education scholarship. In F. Lester (Ed.), Second Handbook of Research in Mathematics Teaching and Learning (pp. 39-67). New York: Information Age.

Sriraman, B. and L. English (2005). Theories of Mathematics Education: A global survey of theoretical frameworks/trends in mathematics education research. Zentralblatt für Didaktik der Mathematik, 37(6), 450-456.

von Glasersfeld, E. (1995). Radical Constructivism: A Way of Knowing and Learning. London: The Falmer Press.

Wittmann, E. (1998). Mathematics Education as a 'Design Science'. In A. Sierpinska & P. Kilpatrick (eds.) Mathematics Education as a Research Domain: A Search for Identity (pp. 87-103). Dordrecht: Kluwer Academic Publishers.