Times Tables 4U

Four Step Programme

The World's most Effective Method for mastering Times Tables and Giving your Son or Daughter the Mathematical Confidence they Deserve

– PARENT HANDBOOK –

With Alan Young

Important

Please, please, please do **not** just open up the student handbook at the practice table squares and give them to your son/daughter to tackle.

Please print this document, but **do** keep it away from your youngsters until you have read through the theory and four steps yourself and then take your time following the plan. This way, success is virtually guaranteed.

So, put the kettle on and with a cup of your favourite brew by your side, sit in your favourite armchair and read on....

What to print out from the Student Handbook:

You will have noticed that the Student Handbook is very long and if you have a slow printer or you are conscious of saving paper, you need only print out pages as and when you need them.

1. Pages 6, 22 and 50 are the initial assessment sheets (Levels 1, 2 and 3 respectively), so print out the one you wish to begin with.

2. Later, when you get to Step 4 of the programme, you can print out as many of the practice sheets as you wish to get started on the speed practice.

3. Then print out more of the practice sheets and some of the other exercises from page 86 that you might find useful.

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Foreword

I have taught mathematics for over thirty one years and pupils from seven years old to eighteen years. I have taught in the state sector and the private sector and I have taught many private pupils over the years.

All this experience has taught me two things:

Firstly, a good knowledge of times tables is necessary for far more work than is generally imagined (for a comprehensive list of concepts involving tables knowledge, see the appendix to this document).

Secondly, very few youngsters have a really good knowledge of tables and so their work is constantly being held up by their inability to multiply and divide.

And this sad state of affairs is not confined to just those of average ability or below. I have taught extremely bright GCSE students of 15/16 years old whose knowledge of tables is very poor and who instantly turn to their calculators to work such simple sums as 15 x 6.

It is important to remember that in virtually all external examinations these days at least one of the papers is a non-calculator paper and even in the calculator papers, precious seconds may be saved by being able to calculate mentally rather than having to pick up a calculator and press a few buttons.

So, to put it bluntly, the sooner your son or daughter masters multiplication tables, the easier they are going to find their future mathematical work and it may even get them an extra few marks in the GCSE examinations which could well push them up over the grade boundary to the next higher grade.

So, stick to the plan and see the smile on their faces grow as they really begin to master the calculation aspects of mathematics.

Good luck.

Alan Young

The Theory

When I say to teachers and parents that learning times tables has very little to do with chanting tables, singing songs or writing out number sequences, they nearly always look at me as though I am mad. Isn't that the way tables have always been learnt? Isn't that the way *they* learnt them at school? Aren't number sequences related directly to tables?

Well, that's the problem. Because they have been taught this way for generations, everyone assumes this is the best way to teach them.

But if these methods worked and gave the desired results quickly, why do very few of our youngsters know their tables well? My estimate is that fully 80% of our children do not know their tables well enough to tackle even the most basic arithmetic processes such as long multiplication and cancelling of fractions with ease. When you watch them tackling these problems (and many others that involve any sort of calculation) you see them struggling with tables knowledge.

So, if I am stating that these time honoured ways are quite wrong, you are probably feeling you need an explanation.

Why do we teach tables as number sequences in one form or another?

The first reason, I believe, is that children need to study number patterns when you are first teaching the concept of multiplication. It is very intuitive to say to a young child, 'Here's one group of five, so that's five. Now, here's two groups of five – how many is that? Now, here's three groups of five,' and so on.

This is a good method of introducing the multiplication concept and the temptation therefore is to continue this with the more difficult tables.

Secondly, as I have just stated above, most people have recollections of table chanting and singing from their own school days. Even if they did not sing or chant them, they probably wrote them out in one form or another as you might when studying number sequences.

But, when you watch children chanting their tables, they nearly always seem satisfied (and so do their parents and teachers) because they get them nearly all right. Watch a child reciting the six times tables, for instance, and you will see them get, say, eight of the individual facts correct. They may well hesitate with the other two (normally something like 'eight sixes' and 'seven sixes'). Because they have 'scored' 80% in that particular chanting, everyone is happy, but what is really happening is that they are just practising the ones they know over and over again and very little is ever done about the two they don't know!

Add these two unknown facts from the six times tables to a couple of others from the four, five, seven, eight and nine times tables and you have a bunch of multiplication facts that hold them up time and time again in all the number work they do. And still everyone thinks they are doing okay because they get most of them right.

So what should we do about it?

First of all I should say that there is a great deal of mathematics to be developed from the study of number sequences right up to A level (and a great deal of fun too), but this fact should not impede the learning of tables, which is a completely different issue.

So what we need to do is to treat the learning of tables as a separate issue and don't confuse it with number sequences by chanting or singing or writing out tables over and over again.

Once the multiplication concept is well understood (which is normally somewhere about seven or eight years old), the learning of tables should be by learning a lot of isolated facts. I know to many people this may sound counterintuitive, but think about a sum such as 368 x 7. The first thing we have to do is to multiply 8 by 7. A child that knows the isolated fact that ***seven eights is fifty six***, will be able to write down the six and carry the five straight away. Another child that has to recite to themselves that, 'five eights are forty, so six eights must be forty eight and therefore seven eights is fifty six,' is at a great disadvantage.

Let me give you an analogy. If you asked a child to tell you the capital of the UK, you would expect them to say straight away that it is London. You would laugh if they said something like, 'The capital of Germany is Berlin, the capital of France is Paris, so the capital of the UK must be London.'

Similarly, if you ask a youngster to spell the word 'cat', they don't have to go through the spelling of 'dog', 'horse' and 'mouse' before getting to 'cat'. All these facts are learnt in isolation as are the names of pop stars, the members of a football team, the names of dinosaurs and so on. So why should tables be any different?

If you think about it, by the time you, as a parent, realise that your son or daughter has a problem with tables, they will know quite a few already. They will know the one times tables, most of the two times tables, the five times tables, the ten times tables and many of the easier three, four and other tables. Of the one hundred multiplication facts they will eventually have to know, there aren't that many left, ***but is those few that cause all the problems.***

The Four Step Programme works by isolating the unknown bonds, learning them as independent facts, just as they would the names of dinosaurs or footballers, and then (and only then) working on speed of recall.

The exact method to be followed will be discussed in a moment, but let me say at this stage that you may need to be patient to begin with and help and encourage and praise. Think of it this way – it has taken years (often many years) to get to the position your son or daughter is now in of **not** having complete competency with tables, so a few more days or weeks isn't going to make a great deal of difference. And I am confident the results will amaze you as it has already amazed the parents who have used it with their youngsters.

Concepts

One idea that is often used in the learning of tables is the one that mathematicians call the **Commutative Law**, a big name for a very simple concept.

Many parents shy away from using big words with their children, but the irony is that children love trying to say them and finding out what they mean, so I would encourage you to introduce this term into your conversations about tables. If you want any proof of children's love of this type of terminology, you need look no further that the names of dinosaurs. Most children know Tyrannosaurus Rex and Triceratops. Surely 'Commutative Law' is no more difficult than these!

So, what is this Commutative Law? This is simply the law that says that 4 x 5 is the same as 5 x 4.

If we can swap the numbers over like this and still get the same answer, we say the operation (in this case 'multiplication') obeys the Commutative Law.

As an abbreviation, we often just say, '**Multiplication is Commutative**'.

The Commutative Law is used by many teachers to help in the learning of tables (although few actually use the term with the children) and we use it here too.

If a child knows '*six threes are eighteen*', but does not know what '*three sixes*' are, we often point out that it is the same the other way round and this helps. In fact, this is a very useful tool as it reduces the number of multiplication facts that have to be learnt in the initial phases. ***(But please remember that ultimately we are looking for instant recall of the number facts, so use of the Commutative Law is only a tool to get them up and running more quickly.)***

What is often omitted though is the reason that this works. Demonstrating this to your children and getting them to practise it is included in the explanation of the Four Step Programme', but for the moment I just want you to understand what the Commutative Law is and why we will be using it.

The British 'Powers That Be' have determined that:

> *by the time children reach eight years of age they should know the 1, 2, 5 and 10 times tables (I call this **LEVEL 1**)*
>
> *by the time they are 9 they should know the 1, 2, 3, 4, 5, 6 and 10 times tables (**LEVEL 2**)*
>
> *by the time children are 10 they should know all the tables up to 10 x 10. (**LEVEL 3**)*

Based on my experience of teaching mathematics, I know that **most children** (i.e. all except those that are known to have particular difficulties with mathematics) **can learn all the tables up to 10 x 10 much quicker than these targets suggest**. Tests confirm that most 9 year olds can be proficient by following the Four Step Programme.

Now here are two very surprising facts that most people do not know:

Fact No. 1 To go from knowing the 1, 2, 5 and 10 times tables to knowing the 1, 2, 3, 4, 5, 6 and 10 times tables, you only have to learn 15 new facts if you know the Commutative Law.

Fact No. 2 To go from knowing the 1, 2, 3, 4, 5, 6 and 10 times tables to knowing all the tables up to 10 x 10 you only have to learn 6 new facts if you know the Commutative Law.

Amazed? So was I the first time I discovered this, but this is why:

First of all, imagine that we are going from knowing the 1, 2, 3, 4, 5, 6, and 10 times table to learning the 7 times table.

Because of the Commutative Law, these facts are already known:

One seven
Two sevens
Three sevens
Four sevens
Five sevens
Six sevens
Ten sevens

Which leaves just three new facts to learn: **Seven sevens, eight sevens** and **nine sevens**.
Similarly, if we know all the tables up to the seven times table and the ten times table, there are only two new facts to remember when we learn the eight times tables and these are: **eight eights** and **nine eights**.

Lastly, if we know all these tables, there is only one new fact to remember when learning the nine times table and that is **nine nines**.

In summary, to go from knowing the 1, 2, 3, 4, 5, 6, and 10 times tables to knowing all the tables up to 10 x 10, we only need to learn the six new facts:

7 x 7 7 x 8 7 x 9 8 x 8 8 x 9 9 x 9 if we know the Commutative Law.

Using the same argument, we soon find that to go from knowing the 1, 2, 5, and 10 times tables to knowing the 1, 2, 3, 4, 5, 6, and 10 times tables, there are just fifteen new facts to learn:

3 x 3 = 9	**3 x 4 = 12**	**3 x 6 = 18**
4 x 4 = 16	**4 x 6 = 24**	**6 x 6 = 36,**
7 x 3 = 21	**7 x 4 = 28**	**7 x 6 = 42**
8 x 3 = 24	**8 x 4 = 32**	**8 x 6 = 48**
9 x 3 = 27	**9 x 4 = 36**	**9 x 6 = 54**

How cool is that? Now, please tell me why children have to spend hours and hours chanting tables. In the time it takes to chant the 6 times tables just once, a child could learn that 6 x 6 = 36!

So, enough of the theory and concepts involved. Let's get down to the nitty gritty of the Four Step Programme.

The Four Step Programme

I shall illustrate the Four Step Programme with the full set of tables up to 10 x 10 (LEVEL 3, as I call it), but if your son or daughter is too young yet to have covered all of these tables or if he/she is having a great deal of difficulty, try starting at a lower level (More on this on page 11).

STEP 1. Identifying the table facts that are causing problems.

One of the first pages at each level in the **Student Handbook** contains an **assessment table square** that we use to find the problem facts (see Index in the Student Handbook). Direct your son/daughter to this square, make sure they understand how to complete it and then give them as long as it takes to work out the hundred number facts.

Ask them to put a circle around any of the table facts that they filled in, but found difficult. The more honest they are about this, the more you and the Four Step Programme will be able to help them.

Give as much time as is necessary and **do not put any pressure on them to hurry up or to get the answers correct**. This will cause tension that will give a false result.

Be relaxed about the whole situation. If <u>you</u> are a little stressed, go off and make yourself a drink.

When the table is finished, make a note of any that they have circled. Then check the answers and put a circle around answers that are incorrect or squares that are blank as in this example:

	No. 1			Time:						
×	6	8	9	3	10	7	5	1	4	2
3	18	24	27	9	30	21	15	3	12	6
6	36	48	54	18	60	(48)	30	6	(26)	12
10	60	80	90	30	100	70	50	10	40	20
5	30	40	45	15	50	35	25	5	20	10
8	48	( )	(74)	24	80	56	40	8	32	16
4	24	32	36	12	40	28	20	4	16	8
9	(52)	72	(85)	27	90	(65)	45	9	36	18
7	(36)	(65)	(73)	21	70	49	35	7	28	14
1	6	8	9	3	10	7	5	1	4	2
2	12	16	18	6	20	14	10	2	8	4

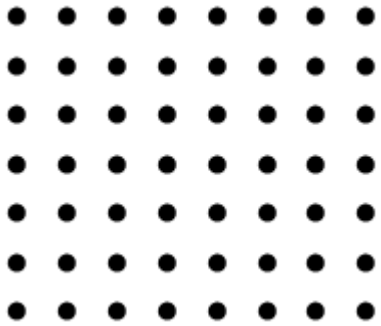
Make a list of the multiplication facts that are circled and you will have something like this:

6 x 4 6 x 7 7 x 6 7 x 8 7 x 9 8 x 8 8 x 9 9 x 6 9 x 7 9 x 9

<u>These are the ones that are probably holding up your son/daughter in all basic arithmetic and will continue to do so unless you take action.</u>

STEP 2. **Making sure the Commutative Law is well understood**.

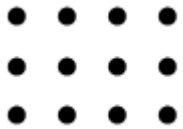
The Commutative Law may be demonstrated very easily by drawing a simple diagram.



This diagram contains **seven rows, each with eight dots**. The total number of dots is, of course, **56**. So, looked at from one point of view, **it represents seven eights**. Looked at from the other edge, as it were, **it also represents eight sevens**. Whichever way you look at it, the answer is always **56**.

<u>This is the Commutative Law in action</u>.

Get them to practise by asking questions such as, ' **Show me that three fours is the same as four threes**.' You will, of course, expect them to draw a diagram like this:



Some children will prefer to use crosses as they are easier to draw than dots. Younger children could use counters. It matters not.

You can even do this with tables they have not yet learnt. So, if they only know the 1, 2, 3, 4, 5, 6,and 10 times tables, you could still ask them to show you that *seven eights* is the same as *eight sevens*, thus preparing them for the next stage.

Most children soon catch on to this simple demonstration of the Commutative Law and so it is not often necessary to spend much time on it, but make sure you go through it before you move on.

STEP 3. **Learning the multiplication facts that you have identified as causing problems.**

Again, take your time with this step as there is no point in moving on to step 4 until all the facts in this level are known well.

Remember to treat these as simple facts just as you would the name of a particular football player in a team or the characters in a computer game. Youngsters can learn that *five sixes is thirty* without having to relate it to *four sixes are twenty four*.

It is very important not to rush this section. Remember that you are trying to solve a problem that has probably been niggling away for months, possibly years, and you don't want to blow it all now by rushing. You are creating an excellent foundation for your son's or daughter's future mathematical education and that deserves a little time, I am sure you will agree.

Get your youngster to learn the unknown table facts at the rate of **one per day** – never faster. With some children you may need to go even slower than this.

The plan is to introduce one new fact per day and revise the ones covered in the previous days. For example, using the list I had above, we would proceed as follows:

Day 1 Learn 6 x 4 = 24

Day 2 Learn 6 x 7 = 42
and practise 6 x 4 = 24

Day 3 Learn 7 x 6 = 42
and practise 6 x 4 = 24 and 6 x 7 = 42

If you sense that your child is struggling a little, have a couple of days when you just practise the ones you have already covered and don't introduce any new ones.

Most children will co-operate with the approach we are using because they see that at last they are going to beat a problem that has been worrying them for some while.

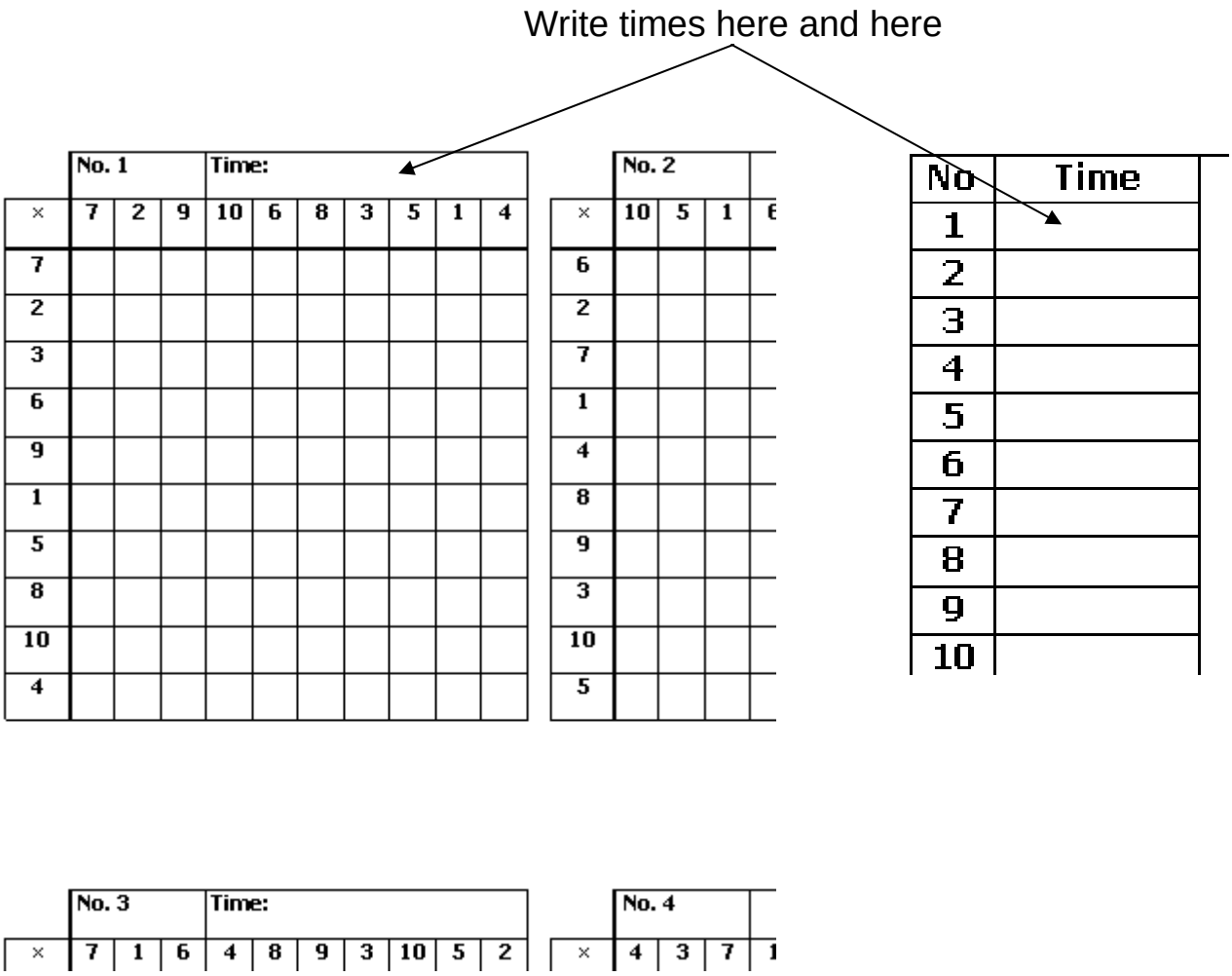
The secret is to keep asking them as often as possible, but in a gentle way, without putting them under too much pressure. You can add some fun by making it into a game – perhaps for every one you ask them, let them ask you one in return (that'll keep you on your toes!) Don't worry at this stage about the speed of recall, that's covered in the next stage.

Once you are happy that **all** the table facts at the current level are well know (which may take a couple of weeks), you can move safely on to step 4.

STEP 4. **Speed improvement.** It is time to improve the speed of recall. Now the difficult ones have been conquered, you will be surprised how soon the speed of recall improves.

Turn to the full set of practice squares at the appropriate level in the Student Handbook and without applying any pressure, time them to the nearest second to see how long it takes them to complete the first one. It is important not to rush them and not to interfere while they do the table square.

Once completed, ***write the time taken above the square and also in the record table for each level.***



Check through their answers and go over any incorrect answers as these are probably just minor errors. Emphasise that at this stage accuracy is more important than the time taken and don't even hint that they might have taken a long time . Twenty minutes is not unusual for the first attempt – just write it down without comment.

A 'Well done!' for completing the whole table will have the most positive effect of all.

> *Emphasise also that it is much quicker in the long run if they begin in the top left corner and work along each row in turn until the table is completed. Most children instinctively look for the easiest questions first and end up jumping all over the table, which wastes an awful lot of time as they will have to do them all eventually.*

So, let's review the progress so far. You have

1. isolated the table facts that were causing the problems and looked for patterns in this list

2. made sure the Commutative Law is fully understood for multiplication

3. taught the answers to the unknown facts and

4. tried a timed table square, emphasising the importance of accuracy.

All your child needs to do now is to practise until they can do a whole table square quickly and accurately.

To do this, give them just **one table square every day**.

There are just two targets:

a) Achieve 100% accuracy
b) Improve on yesterday's time

By recording the times as discussed above, you will be able to see the progress being made and you will be surprised how quickly the time drops.

There will come a time, of course, when the speed at which they complete the table squares is just about as fast as they will be able to achieve given their age and particular ability, therefore little progress will be made in the times from then on, but when this happens you can be sure they know their tables much better and are able to respond much quicker than other members of their class of similar mathematical ability and your hard efforts will be rewarded by the growing look of confidence when arithmetical operations are tackled. But don't finish there. Reduce the interval between each table square to every other day and eventually to just one per week to keep their hand in, especially during school holidays when things tend to slip a bit.

And that's it – Job Done!

However, if (and only if) you have a child that can cope with a little pressure, you can introduce an extra feature.

Try competing against them yourself and put 20p in a money box each time they beat you. Like all good parents, you may have to let them win a little at first to encourage them, but it won't be long before you might be losing on a regular basis. However, I am sure you will agree that 20p a day is a fair price to pay for such accurate and speedy recall of all the table facts up to 10 x 10! It could even be a whole family affair!

You will notice that in the Student Handbook after each set of table squares are some Multiplication Table Triangles to be completed. These are designed to show the relationship between multiplication and division. To complete them youngsters need to know the tables 'backwards' as well as forwards (i.e. 'What do you have to multiply 7 by to

get 56? etc). This skill is very useful in both short and long division and a host of other topics such as cancelling fractions and factor work.

At the end of the Student Handbook are a great many problems of all types based on tables work. Choose those that are suitable for your son or daughter. I should warn you that some of these are pretty difficult as they are designed for the older age groups, so please take a good look at them before encouraging your youngster to tackle them.

How to Proceed Now

Now you understand how the Four Step Programme works, I am sure you will be keen to put it into action. You will see that the practice element of the Student Handbook is divided into the three levels I discussed earlier:

LEVEL 1 *Tables 1, 2, 5 and 10*

LEVEL 2 *Tables 1, 2, 3, 4, 5, 6, and 10*

LEVEL 3 *All tables up to 10 x 10*

Choose your starting point firstly according to the age and ability of your son or daughter, but be prepared to go down a level if he/she is really struggling. It is much better to become proficient at a lower level and gradually move up than t begin at too high a level.

As a general rule, it is always much better to start at a level that is lower than you would expect, but is one which they can soon master than to start at a level that is too high and will make them more depressed by introducing more multiplication tables failure into their lives.

Whichever level you choose, follow the Four Step Programme. If you begin with Level 1, once this is mastered you will then need to practise just the fifteen facts needed to become proficient at Level 2 at the rate of one per day (slower if necessary). You will remember these are:

3 x 3 = 9	**3 x 4 = 12**	**3 x 6 = 18**
4 x 4 = 16	**4 x 6 = 24**	**6 x 6 = 36,**
7 x 3 = 21	**7 x 4 = 28**	**7 x 6 = 42**
8 x 3 = 24	**8 x 4 = 32**	**8 x 6 = 48**
9 x 3 = 27	**9 x 4 = 36**	**9 x 6 = 54**

If you begin with Level 2, when this is mastered you will then need to practise just the six facts to become proficient at Level 3 at the rate of one per day. These are:

7 x 7 7 x 8 7 x 9 8 x 8 8 x 9 9 x 9

Of course, if you begin at Level 3 and this is easily mastered, your job is complete and your son/daughter will be able to tackle any work given by their teachers that involve multiplication tables right up to GCSE level. Well done!

The next three pages give the answers to the test table squares so that you may check them easily and then follows the appendix.

Level 1 Assessment Table Square Answers

×	2	1	5	10
2	4	2	10	20
7	14	7	35	70
1	2	1	5	10
6	12	6	30	60
10	20	10	50	100
4	8	4	20	40
8	16	8	40	80
9	18	9	45	90
3	6	3	15	30
5	10	5	25	50

Level 2 Assessment Table Square Answers

×	5	2	10	1	3	6	4
2	10	4	20	2	6	12	8
7	35	14	70	7	21	42	28
1	5	2	10	1	3	6	4
6	30	12	60	6	18	36	24
10	50	20	100	10	30	60	40
3	15	6	30	3	9	18	12
8	40	16	80	8	24	48	32
4	20	8	40	4	12	24	16
5	25	10	50	5	15	30	20
9	45	18	90	9	27	54	36

Level 3 Assessment Table Square Answers

×	6	8	9	3	10	7	5	1	4	2
3	18	24	27	9	30	21	15	3	12	6
6	36	48	54	18	60	42	30	6	24	12
10	60	80	90	30	100	70	50	10	40	20
5	30	40	45	15	50	35	25	5	20	10
8	48	64	72	24	80	56	40	8	32	16
4	24	32	36	12	40	28	20	4	16	8
9	54	72	81	27	90	63	45	9	36	18
7	42	56	63	21	70	49	35	7	28	14
1	6	8	9	3	10	7	5	1	4	2
2	12	16	18	6	20	14	10	2	8	4

Appendix

This appendix details many of the concepts that involve the use of multiplication tables and shows how important it is to have an excellent grasp of these number facts if good progress is to made in this subject.

Tests with number facts:

5 x 7, 9 x 4 etc

Short multiplication:

247
X6

Long multiplication:

365 with decimals **4.78**
x46 **x 3.8**

Short division:

6)252

Long division:

23)5198

Cancelling fractions and equivalent fractions:

$$\frac{27}{36} = \frac{3}{4}$$

Adding and subtracting fractions:

$$\frac{5}{9} + \frac{7}{8} = \frac{40}{72} + \frac{63}{72} = \frac{103}{72} = 1\frac{31}{72}$$

Multiplying and dividing fractions:

$$\frac{4}{7} \times \frac{5}{9} = \frac{20}{63}$$

Finding multiples:

Give the first seven multiples of 8 (8,16, 24, 32, 40, 48, 56)

Finding factors:

What are the factors of 48? (1, 2, 3, 4, 6, 8, 12, 16, 24 and 48)

Word problems:

Mr Jones bought 14 tickets at £6.50 each. How much did he spend altogether?

The length of a rectangle is 16cm and its width is 9 cm. What is its area?

Find the product of 24 and 36

Find all the products you can make with the numbers 2, 5, 7 and 9

I have 250 cakes. Each box holds 9 cakes. How many boxes will I need to hold all the cakes?

72 cubes can be arranged to make a cuboid 2 x 3 x 12. What other cuboids can be made with 72 cubes?

Divisibility tests:

Is 563 divisible by 9? How can you tell?

Converting fractions to decimals:

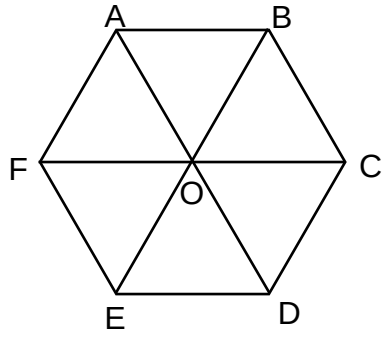
Convert 6/8 to a decimal. (This is done by dividing 6 by 8: 8)6.00)

Percentages:

What is 23% of £500?

Geometry:

What are the three angles of an equilateral triangle?



What is the angle BOC?

Sequences:

What are the next three terms in the following sequence and what is the general term:

10, 17, 24, 31, 38, …

Averages:

What is the mean of 5, 7, 9 and 11?

Algebra:

Solve the following equations:

$7x - 9 = 47$

$3x + 5y = 61$
$7x - 6y = 1$

$X^2 + 4x - 8 = 0$

Multiply $(3d - 9)(5d + 6)$

Area:

Taking π to be 3.14, find the area of a circle whose radius is 9 cm.

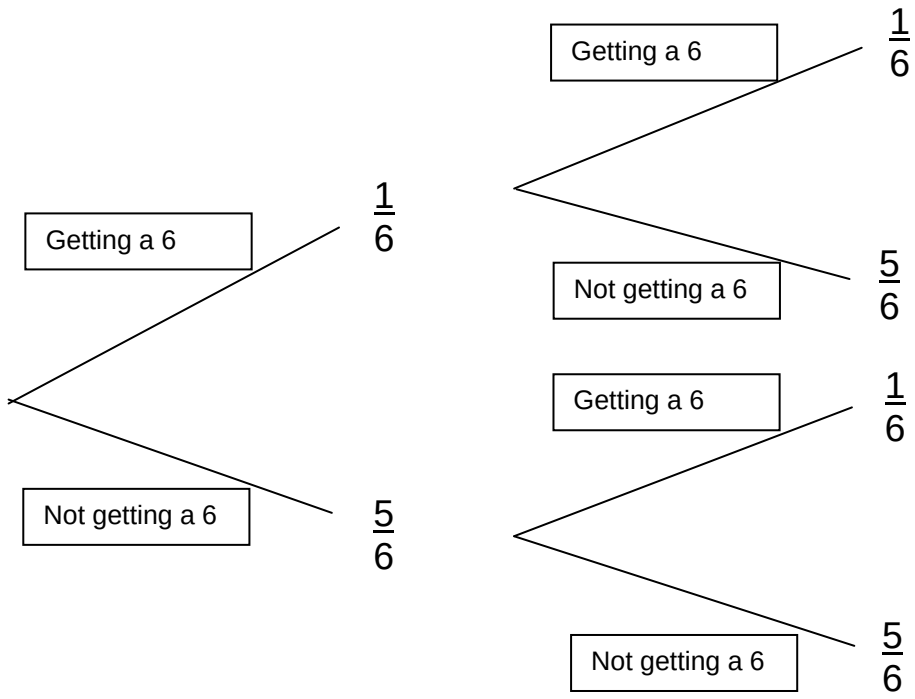
Standard form:

Simplify and write your answer in standard form:

$4 \times 10^6 \times 7 \times 10^5 \times 8 \times 10^4$

Probability:

Calculate the probabilities of the outcomes in this probability tree:



I hope this has shown you how important a good knowledge of multiplication tables is right up to GCSE level. Developing a good grasp of them now will see your child right for the next few years of their mathematical education.