TORSIONAL VIBRATION IN THE DIESEL ENGINE

By Frank M. Lewis, Member*

[Read at the thirty-third general meeting of the Society of Naval Architects and Marine Engineers, held in New York, November 12 and 13, 1925.]

I. GENERAL

1. With the increasing use of the Diesel engine the problem of synchronous torsional vibration has again been forced on the attention of marine engineers. Dr. Bauer, in 1900, was probably the first to show that dangerous torsional vibration may occur in the shafts of steamships. [See Bibliography (1), Art. (71).] In 1901 Gümbel and Frahm investigated the cause of several mysterious shafting failures and found that they were due to these synchronous vibrations at the so-called critical speeds (3, 4). Simultaneously with this work, it was shown by J. Frith and E. H. Lamb that the same form of vibration was present in reciprocating engine generator units, and that many breakages were due to that cause (2).

Torsional vibration was the cause of the repeated shafting failures that occurred on the *Connecticut* and other battleships of that class, and likewise on many reciprocating engine-driven destroyers (17). In addition to the many fractures that have actually occurred with the reciprocating steam engine, doubtless many vessels have been operated with torsional vibration that is not quite sufficiently powerful in itself to fracture the shafting, but, nevertheless, gives extremely high and dangerous stresses.

2. The Diesel engine, or any form of internal-combustion engine, is far more prone to torsional vibration than the reciprocating steam engine. This difference is due to the compression stroke and the higher initial pressure, both of which factors produce far greater irregularities in the turning effort of the engine than occur with steam.

Torsional vibration may occur with any form of reciprocating engine. It has caused trouble in direct-connected generator units, air craft, motor cars, etc., as well as in ships. While we deal in this paper with marine applications of the Diesel engine, the same treatment applies directly to any of these other fields.

3. It will be of interest to those entirely unfamiliar with the subject if we review briefly a few of its more fundamental aspects.

Vibration in the shafting is due, primarily, to periodic variations in the torque of the engine. In the Diesel engine these variations are of considerable magnitude. For instance, in a six-cylinder, four-cycle, single-acting engine the torque will vary, three times per revolution, from approximately -20 per cent to $+220$ per cent of its mean value. This negative torque means that, three times per revolution, the flywheel and other driven machinery must actually drive the engine. With a greater number of cylin-

*Professor of Mathematics, Webb Institute of Naval Architecture.

ders, or in two-cycle engines, the torque variation is less, but this does not help us appreciably so far as critical speeds are concerned.

4. If the engine shaft and all attached shafting were absolutely rigid in torsion, the only effect of this torque variation would be to cause a slight periodic speed variation in engine and shafting. One of the purposes of a flywheel is to reduce this speed variation to a minimum value.

Actually the shafting is far from being a rigid body, with the result that, under the influence of the torque variation, the shafting does not accelerate as a whole, but is thrown into a state of vibration. The entire shafting system, which comprises the engine shaft and pistons, flywheel, and all driven shafting and attached masses, can be considered as a torsionally elastic system. This elastic system has one or more modes of normal free vibration. These modes of normal vibration are distinguished by the number of nodes associated with each, nodes being points which, with respect to any particular vibration, have no motion. We may have 1 noded normal vibration, 2 noded, 3 noded, etc. In general only the first two or three forms are of practical importance. Associated with each of these normal modes of vibration is a natural frequency; the greater the number of nodes the higher the natural frequency.

Associated with each of these normal modes of vibration is a normal elastic curve, which is a curve whose ordinates for various points of the shaft line represent the relative amplitude of vibration of those points.

It should be noted in particular that this vibration is an angular, twisting vibration of the shaft and altogether different from the transverse vibration between bearings sometimes known as "whirling." Also that the torsional vibration problem is not related in any way to the balance of the engine.

5. When the shaft is turning at such a speed that the frequency of the impulses due to the torque variation coincides with the natural frequency of the shaft system, it is at a critical speed. Under these conditions the amplitude of vibration will build up to an extent determined by certain damping factors. The stresses due to this vibration may or may not be sufficiently great to fracture the shaft. Any elastic system may thus be set into violent vibration by applying a series of impulses at properly timed intervals.

6. The order of a critical speed is equal to the number of vibrations occurring per revolution of the engine.

We show later that, in theory at least, there exist in any 2-cycle engine critical speeds of every integral order: 1, 2, 3, 4, etc.; and that in any 4-cycle engine there are in theory critical speeds of every integral order and half order as well; for instance, orders 4½, 5, 5½, 6, etc.

A critical speed is distinguished by two quantities: (1) the number of nodes of its normal mode of vibration; (2) its order number. We may speak thus of a one-noded critical speed of 6th order, 2-noded critical of 4½ order, etc.

It will be noted that, since there are several modes of vibration and each of these has critical speeds of various orders, there may be, altogether, in a given installation, a large number of critical speeds. Fortunately, often many of these, and sometimes all, will be of such small amplitude that they can be disregarded.

7. We wish especially to emphasize the following points: The critical speeds depend, not on the engine alone, but on the engine and all machinery driven by it. For that reason it is quite impossible to speak of an engine as having certain critical speeds unless full details of all machinery driven by it are known. The successful performance of an

engine in one installation, so far as vibration is concerned, is no guarantee that it will be equally successful in another. Whether an engine does or does not develop serious torsional vibration on the test stand is no criterion of what its performance will be when driving its actual machinery.

8. The probability of a critical speed existing in any given installation may be said to be proportional to the range of speeds over which the engine is to be operated. If the engine is to be operated at a single speed or over a very narrow range of speeds, it is probable that there will be no critical speeds in the range. If the engine is to operate over a wide range of speeds, it is probable that there will be one or more critical speeds within that range. We describe in more detail later in the paper the possibilities of critical speeds in various types of installations.

If any changes prove necessary in an installation in order to avoid critical speeds, these changes must usually be made, not in the engine but in the driven machinery. Co-operation is therefore needed between engine builder and user, the user being willing to adapt his machinery so as to secure proper conditions for the engine.

The critical speeds of any given installation can be calculated, before construction, within a few revolutions. In the Torsiograph, an instrument invented by Dr. J. Geiger, we possess a practical means of determining the critical speeds that may exist in a completed installation (16). The instrument is driven by a belt from the shaft and draws a curve representing the variations in angular position from the mean. Critical speeds may be determined directly from this curve. Any installation in which there is a possibility of the existence of critical speeds should be so tested and the dangerous speeds, if any, avoided. It is quite unnecessary to break a shaft before discovering that serious vibration exists.

9. The methods and data given in this paper have been drawn from many sources. We have endeavored to acknowledge these, so far as possible, in the bibliography. We are indebted to the U. S. Navy Department for permission to publish information and data regarding submarine engines, dredges, and crankshaft tests; and to Wm. Cramp & Sons Ship and Engine Building Co. for data regarding the *Seekonk*.

II. THE CALCULATION OF THE NATURAL FREQUENCIES OF VIBRATION

10. Every torsionally elastic system has one or more modes of normal vibration. Associated with each of these modes of vibration is a natural frequency and a normal elastic curve. The values of these quantities depend solely upon the distribution, throughout the shaft, of mass polar moment of inertia, referred to hereafter merely as "mass," and elasticity. There have been devised several methods for the calculation of the natural frequency. In all of these, in order for the complex elastic systems found in practice to be amenable to mathematical treatment, it is necessary to make several simplifying assumptions; the purpose of these is to replace the actual complex system by a simpler one having nearly the same properties.

There are two methods of obtaining this simplification:

1. *Concentrated Masses.*—We may concentrate all the mass at a definite number of points of the shaft. For instance, in a Diesel driving flywheel and generator the masses would be concentrated at the cylinder center lines, flywheel and generator center line. The shafting between these points is considered as having elasticity, but no mass. Methods

for the calculation of the natural frequency of such an idealized system have been devised by Gümbel, Geiger, Holzer, and others (8, 15, 26).

2. *Distributed Masses.*—The author has devised a method of calculation based on the following assumptions. The shafting system is divided into so-called "steps," over each of which it is assumed that both mass and elasticity are distributed uniformly. We may assume a concentrated mass at any point of such a system or place elasticity between two steps, so that the method is very flexible and may be adjusted to the peculiarities of any given shafting arrangement. By this method of computation the entire engine shaft can be treated as a single unit in the calculations (20).

It is scarcely necessary to emphasize the value in practice of having independent methods of calculation which may be checked against each other.

11. When vibrating in one of the normal modes of vibration all points of the shaft vibrate in the same phase with simple harmonic motion. This may be expressed mathematically by writing:

$$\Theta = \Theta_0 \sin pt, \tag{1}$$

Θ being the angle of vibration at any point and Θ_0 the maximum value of Θ at the same point. t = time.

p, the frequency constant, is connected to the period T and the frequency N by the relations—

$$p = 2\pi N = \frac{2\pi}{T}. \tag{2}$$

All methods of calculation depend upon the principle that, at the instant of maximum amplitude, equilibrium exists between the acceleration forces of the masses and the elastic forces due to the torsion of the shaft.

12. *Concentrated Masses and Method of Calculation.*

Let J be the mass polar moment of inertia of any concentrated mass. If it is vibrating with a maximum amplitude Θ_0 the torque it exerts when Θ is a maximum is given by—

$$M = -\frac{J}{g}\frac{d^2\Theta}{dt^2} = \frac{J}{g}\Theta_0 p^2 \tag{3}$$

The successive masses will be denoted by subscripts as J_1, J_2, J_3, etc., the properties of the shaft connecting them by the double subscripts (1, 2) (2, 3), etc.

In any piece of shafting between two masses as (1, 2), the torque and the angle of twist are connected by the relation—

$$\Theta = \frac{M}{C} \tag{4}$$

C being a constant for that shaft section calculated from the known dimensions of the shaft. If the shaft is of uniform section we have—

$$C = \frac{M}{\Theta} = \frac{MGH}{Ml} = \frac{GH}{l} \tag{5}$$

G being the shearing modulus of elasticity, H the polar moment of inertia of the shaft, l the length between the adjacent masses.

If the shaft is not of uniform section we have—

$$C = \frac{M}{\Theta} = \frac{1}{\sum \frac{l}{GH}} \tag{6}$$

the summation being carried out between adjacent masses.

In Plate 85 is shown an arrangement of such masses, numbered consecutively.

Let us assume an amplitude of vibration of unity for mass 1. Then the torque it exerts is, by (3), $\frac{J_1}{g} p^2$, which is the torque in the shaft (1, 2).

The twist in the shaft (1, 2) will be, by (4), $\frac{J_1 p^2}{gC_{1,2}}$. The twist of mass 2 will therefore be

$$1 - \frac{J_1 p^2}{g C_{1,2}}$$

The torque of mass 2:

$$\frac{J_2 p^2}{g}\left(1 - \frac{J_1 p^2}{g C_{1,2}}\right)$$

The total torque in shaft (2, 3) will be:

$$\frac{J_2 p^2}{g}\left(1 - \frac{J_1 p^2}{g C_{1,2}}\right) + \frac{J_1 p^2}{g}$$

If the system comprises only two masses, the above expression is equated to 0, giving—

$$p^2 - \frac{g C_{1,2}(J_1 + J_2)}{J_1 J_2} = 0$$

or

$$N = \frac{1}{2\pi}\sqrt{\frac{g C_{1,2}(J_1 + J_2)}{J_1 J_2}} \tag{7}$$

This well-known formula may be applied to many of the cases that arise in practice.

If we have engine and flywheel amidships, a long line shaft, and propeller aft, let J_1 be the moment of inertia of engine and flywheel, J_2 that of propeller and $C_{1,2}$ the line shaft constant from propeller to flywheel. Then with fair accuracy N will give the one noded natural frequency. The two masses vibrate as shown in Fig. 5, Plate 84, the node being at their common center of gravity and their respective amplitudes of vibration inversely proportional to their moments of inertia, that is—

$$\frac{\Theta_1}{\Theta_2} = -\frac{J_2}{J_1}$$

13. If we had continued this process past the third mass, we would have obtained a quadratic in p^2 giving finally:

$$N = \frac{1}{2\pi}\sqrt{g}\sqrt{A \pm \sqrt{A^2 - B}}$$

where $$A = \frac{C_{1,2}(J_1 + J_2)}{2 J_1 J_2} + \frac{C_{2,3}(J_2 + J_3)}{2 J_2 J_3}$$

and $$B = \frac{(J_1 + J_2 + J_3) C_{1,2} C_{2,3}}{J_1 J_2 J_3} \tag{8}$$

The roots are the two natural frequencies of the system.

14. If the process is continued to n masses, we will obtain an equation for solution of the $(n - 1)$ order in p^2. Wydler (28) shows how this equation may be written and its roots obtained.

In practice, however, if there are more than three masses, it is far easier to resort to a process of trial and error. We assume a value of p and work through the entire system, using numerical values, and finding finally the torque remaining beyond the last mass. If this is 0 the value of p so assumed is one of the frequency constants. If a curve be plotted of this remaining torque for various values of p, the points where the curve cuts the axis will be the frequency constants of the given system.

15. The work may be most conveniently arranged in tabular form. In Plate 85 is given the tabulation by this method for the dredges *Dan C. Kingman*, etc. For $N = 15.3$ the excess torque is positive and for $N = 15.4$ negative, showing that the calculated natural frequency lies between these values. In column 1 are the masses divided by g. Column 4 gives the inertia torque of each mass, column 5 the total inertia torque of all masses up to and including the one calculated. Column 5, divided by the C of the shafting in which that torque occurs, gives the twist in that section in column 7. This twist, added to the twist of the last mass (algebraically), gives the twist in the next successive mass in column 3, and so to the end. The last figure in column 5 is the excess torque after the last mass.

There have been various modifications of the above process proposed, designed with a view to shortening the labor of computation, but we have found the method given to be the most practical, and it has the further advantage that it may be extended to the calculation of forced vibrations.

16. It is evident, after we have obtained the closing value of p, that in a table with this value we have at hand the relative amplitude of vibration of all the masses, that is, the normal elastic curve, and the vibration torque at any point of the shaft, for an amplitude at mass 1 of 1 radian.

For a critical speed of known amplitude the vibration torque at any point may be found by multiplying the tabular torque by the actual amplitude in radians at mass 1. From this torque the vibration stress can be calculated.

Distributed Masses

17. Consider a step of uniformly distributed mass and elasticity such as is shown in Fig. 1, Plate 84. The conventions of sign are shown in the figure and are of importance.

Let J_w be the mass polar moment of inertia per unit length.

Let H = stiffness moment of inertia.

If a length dx of this step is vibrating with a maximum amplitude Θ, the torque exerted by it will be, by (3), $\frac{J_w\, dx\Theta p^2}{g}$, which will be minus the increment of torque in the length dx. For a length dx we have

$$M = + GH \frac{d\Theta}{dx} \quad \text{or} \quad dM = GH \frac{d^2\Theta}{dx^2}\, dx \tag{9}$$

Therefore

$$\frac{d^2\Theta}{dx^2} = - \frac{J_w\, p^2\, \Theta}{gGH} = - m^2\, \Theta \tag{10}$$

which leads to

$$\Theta = a \cos (mx + \gamma) \tag{11}$$

as its general solution; and

$$M = -\ GHma \sin (mx + \gamma)$$

$$= \sqrt{\frac{CJ}{g}}\, pa \sin (mx + \gamma) \tag{12}$$

C being defined in the same way as for the concentrated mass method and J being the total mass moment of inertia of the step.

The solution in a system of any complexity must be by trial and error, and is accomplished in the same manner as the solution for concentrated masses; that is, starting at one end where the torque is 0 and the amplitude unity, we work through to the other, the condition that the chosen value of N satisfies being that there shall be no torque at that end. The form of the above expressions leads to a simple graphical construction.

18. Consider Fig. 2, Plate 84. At the beginning of any step we have, since $x = 0$,

$$\Theta = a \cos \gamma$$

$$M = \sqrt{\frac{CJ}{g}}\, pa \sin \gamma \qquad \text{or} \qquad \frac{M}{\sqrt{\frac{CJ}{g}}\, p} = a \sin \gamma$$

If, now, we know at the beginning of any step the angle Θ and torque M, we can locate a point P which represents that point of the shaft.

It is evident, then, from 11 and 12, that the step can be represented by an arc of a circle drawn through P and subtending an angle equal to ml. At the other end of the step

the torque will be equal to $QQ^1 \times p \sqrt{\frac{CJ}{g}}$ and the angle to OQ^1.

We may thus represent every point of the shaft lineup by a point in a diagram whose abscissa is equal to the amplitude of vibration at that point and whose ordinate is equal to a constant times the vibration torque at that point. This will be called the polar diagram of the shaft.

It will be convenient to modify slightly the above expression for ml, the angle subtended. We have in radians:

$$ml = \frac{\sqrt{J_w l p}}{\sqrt{gGH}} = \frac{2\pi N \sqrt{J}}{\sqrt{g \frac{GH}{l}}}$$

In degrees we have:

$$ml = \frac{360 \sqrt{J}}{\sqrt{gC}} N \quad \text{degrees.}$$

The part $\frac{360 \sqrt{J}}{\sqrt{gC}}$ can be calculated for each step and will be designated ϕ.

$$\phi = \frac{360 \sqrt{J}}{\sqrt{gC}} \tag{13}$$

19. If y be the ordinate of any point of the polar diagram, then the torque at the corresponding shaft point will be:

$$M = 2\,\pi N y \sqrt{\frac{CJ}{g}} \tag{14}$$

y is in radians and to the same scale as the abscissas.

If PQ in Fig. 2, Plate 84, represents one step of the system, then it is evident that the initial point of the next step must lie on QQ^1, since the amplitude at the end of one step must equal the amplitude at the beginning of the next.

Likewise, since the moments at end and beginning must be equal, we have:

$$2\pi N \sqrt{\frac{C_a J_a}{g}}\, QQ^1 = 2\pi \sqrt{\frac{C_b J_b}{g}}\, Q^1 R$$

or

$$\frac{Q^1 R}{Q Q^1} = \frac{y_b}{y_a} = \frac{\sqrt{C_a J_a}}{\sqrt{C_b J_b}} \tag{15}$$

from which the initial point R of the next step can be determined.

Concentrated Masses in a Step or between Two Steps

20. Let there be located between two steps or at the end of the system a concentrated mass of moment J, and let Θ be the amplitude of vibration of the mass; then it is easily seen, by equating (3) and (14), that the ordinate at the beginning of the next step is given by—

$$y_b = y_a \frac{\sqrt{C_a J_a}}{\sqrt{C_b J_b}} + \frac{2\pi J \Theta N}{\sqrt{g}\sqrt{J_b C_b}} \tag{16}$$

Due regard must be given in using this to the signs of Θ and y.

In general we can say that a concentrated mass in the polar diagram is represented by a vertical line of length

$$\frac{2\pi J \Theta N}{\sqrt{g}\sqrt{J_b C_b}}$$

21. If between two steps, a and b, there is located a shaft considered to be without mass, or an elastic coupling of constant C, then it is evident that this is represented in the polar diagram by a horizontal line. The amplitude at the beginning of the b step is given by:

$$\Theta_b = \Theta_a - \frac{2\pi\sqrt{J_a C_a}}{C\sqrt{g}}\, y_a N \tag{17}$$

and the ordinate y_b is given by (15).

22. The procedure of calculation is as follows:

The shaft is divided up into uniform steps and concentrated masses in such a manner as to produce a system having, as nearly as possible, the same characteristics as the original.

For each of these steps the quantity ϕ given by (13) and $\sqrt{CJ}$ is calculated. The step ratios $\frac{\sqrt{C_b J_b}}{\sqrt{C_a J_a}}$ between two steps are calculated. If there are any concentrated masses, the constants of (16) are evaluated for each.

Assuming a value for N, and an initial deflection OS (Fig. 2) the angle $SOU = \phi_1 N$ is laid off to represent the first step, the ordinate UV multiplied by the step ratio to obtain the initial point of the second step, etc. The vertical lines representing concentrated masses are laid in as we arrive at each. The diagram is thus constructed through from one end of the shaft line to the other.

If the polar diagram covers 180°, the value of N assumed is the 1-noded natural frequency; if it covers 360° it is the 2-noded natural frequency, etc. In the closing polar diagram the relative amplitudes of all points of the shaft are given directly and the moment at any point may be found by (14).

23. In Plate 86 are the constants for the dredge *Dan C. Kingman*, original condition, and the closing polar diagram for 1-noded vibration. There are three steps—main engine, engine to flywheel, and flywheel to generator center line; and three concentrated masses—air pump, flywheel, and generator.

Assuming $OA = 10$; AB is laid off equal to $.0027\Theta N = .0027 \times 15.21 \times 10 = .412$; $BC = \phi_1 N = 4.99 \times 15.21 = 76°$; ordinate D = ordinate $C \times 2.32$; $ED = \phi_2 N = 7.3°$; $EF = 1.205\ \Theta N = -1.205 \times 1.0 \times 15.21 = -18.2$; $GF = \phi_3 N = 3°$; $GH = -.228 \times 1.3 \times 15.21 = -4.4$.

Since the point H is on the horizontal line, the diagram has covered 180°, and 15.21 is the 1-noded natural frequency. By the concentrated mass method the calculated value of N is 15.31. The value found by torsiograph tests was about 15.0.

Computation Details

24. The division of the given shafting arrangement into concentrated masses and uniform steps is made so that the substitute system shall equal, as nearly as possible, the original arrangement. Beyond this it is difficult to state any rules. The examples given in the plates illustrate the divisions made in those cases.

The same units must be used for all quantities throughout the calculations. It is recommended, if the machinery is dimensioned in feet and inches, that inch, pound, second, units be used.

In general the computation of mass polar moment of inertia offers no difficulties. Two points, however, require special attention.

Propellers.—The water surrounding the propeller produces two effects. It dampens the vibration, and the mass of the propeller is increased. The amount of this increase is uncertain, but it has been customary to increase the moment of inertia about 25 per cent to allow for it. Model propeller experiments might be of value in this connection.

25. *Reciprocating Masses.*—The action of the reciprocating masses is highly complex in character and cannot be discussed fully in the limits of the present paper.

It may be shown that, if a periodic force of given frequency acts upon a rotating system containing reciprocating masses, the result is a series of vibrations, of the same frequency as the periodic force, these being the ones of greatest amplitude, and also vibrations of less amplitude but higher frequency. As regards the vibrations of the same frequency, the effect is the same as if the reciprocating masses were replaced by rotating masses having a moment of inertia equal to hwr^2, where w = reciprocating weight, r = crank radius, h = a constant.

The constant h varies slightly with the crank connecting rod ratio, but is very nearly equal to $\frac{1}{2}$, and may be taken equal to that for practical computations.

The connecting rod is divided up into a rotating and reciprocating part in the manner of engine balance; this is, if w = weight of rod, l = length, l_1 = distance crank pin center to C. of G. of entire rod; then

$$w\left(\frac{l-l_1}{l}\right) = \text{rotating part.}$$

and

$$w\,\frac{l_1}{l} = \text{reciprocating part.}$$

26. *Elasticity.*—The greater number of uncertainties in the calculations arise from assumptions that must be made regarding the elasticity of various parts. The main

crankshaft is the most important of these. Formulae of varying degrees of complexity, for the equivalent stiffness of a crankshaft have been proposed by Geiger (25), Holzer (26), Timoschenko (29). We consider it doubtful whether any formula for the stiffness of such a complex elastic structure as a crankshaft can be more than roughly empirical. The stiffness is dependent, to a certain extent, on the amount of restraint at the bearings. Some of the formulae take account of this. The oil film between shaft and bearing undoubtedly has the effect of increasing this restraint, but to what extent is problematical. On Plate 88 is given the simplest of the formulae, Geiger's. There are given also the results of twisting experiments made, at the New York Navy Yard, upon several crankshafts under various conditions of bearing restraint. The experimental results are expressed as the ratio of the stiffness of the crankshaft to that of a shaft of the same length and of diameter equal to the bearing diameter.

27. It is customary for many writers to use the term "equivalent length," this being the length of a shaft of a given section having the same elasticity as the original. Geiger's formula is given in this form. We consider it preferable to designate the stiffness of any part by means of the single constant "C."

The value of G for steel has been taken equal to 11,800,000, the mean of a number of experimental tests.

Care should be exerted that parts are given as much elasticity as they really possess. For instance, in way of flanged couplings it should be assumed that the shaft extends part way into the flanges and that the remaining flange has a diameter no greater than that of the bolt circle. In way of keyed couplings it should be assumed that the stiffness is that of the shaft for one-half the length and that of the collar for the remaining distance. It is impossible to give rules for all the special cases that may arise in practice. The accuracy of results must be dependent to a certain extent upon the skill and judgment of the calculator.

III. VIBRATION IN A MULTI-CYLINDER ENGINE

28. In the calculations for natural frequency no account has been taken either of the torque of the cylinders upon the shafting, or of any damping forces, the condition for the existence of a normal mode of vibration being that no external forces act upon the shaft system. We will now consider in detail the effect produced by these two sets of forces.

The forces acting upon the elastic system of the shafting are as follows: Acceleration forces, proportional to the acceleration; torque forces, proportional to the angle of twist; the force of the periodically varying cylinder torque; damping forces. The first two of these are linear, that is, are proportional to the cause producing them. For damping forces to be linear it is necessary that they be proportional to the velocity. It is convenient to assume for the present that all damping forces are linear. We shall see later that this is not strictly true, but that results obtained on this supposition may be properly modified.

29. The principle of linear superposition plays a very important role in the further study of the subject. For our purposes this principle may be stated as follows:

In any linear elastic system the motion produced by two or more sets of periodically varying forces, acting simultaneously, is equal to the sum of the motions which would be produced by the separate forces acting alone, due regard being given to phase relations between the respective components.

The application of this principle to a multi-cylinder engine is obvious.

First, all vibratory motions and forces are superimposed upon the mean constant torque and angular velocity of the shaft.

Second, the motion of the shaft is equal to the sum of the motions which would be produced by the varying torques of the single cylinders acting alone.

Third, the motion produced by a single cylinder is equal to the sum of the motions produced by the separate harmonic components of its turning effort curve.

As previously stated, due regard must be given to the phase relations between these components; that is, they must be considered as vector quantities.

The Torque Curve of a Single Cylinder

30. The torque produced by a single cylinder may be divided into two parts:

Torque produced by acceleration forces of the reciprocating masses.

Torque produced by the gas pressure. The harmonic components of these two torque curves will be found independently.

Analysis of Indicator Cards

31. In Fig. 1, Plate 89, are shown seven different indicator cards, covering a range of mean indicated pressure from 0 to 140 pounds per square inch. The curves given are identical with a set of actual indicator cards, taken on a 4-cycle, 1,750 brake horsepower Diesel, only slight irregularities having been faired.

In Fig. 2, Plate 89, are the turning effort curves constructed from these cards, for a cylinder of 1 square inch area and 1 inch crank radius. Curve 1, reversed, left to right and upside down, is the compression curve for all seven expansion curves. Over the remainder of the cycle, covering one revolution, the ordinates are 0 and have therefore not been drawn.

32. Consider any function, $y = f(x)$, which repeats itself in the intervals $(0, 2\pi)$, $(2\pi, 4\pi)$, $(4\pi, 6\pi)$, etc. We may call this a periodic function. The turning effort curve of an engine is such a function. For a single cylinder, in a two-cycle engine the interval of repetition is one revolution; in a four-cycle engine two revolutions. Fourier's theorem states that any such periodic function $y = f(x)$, whether it is given analytically or graphically, may be expanded as a series of the form:

$$f(x) = F_m + F_{c1} \cos x + F_{c2} \cos 2x + F_{c3} \cos 3x + \dots$$
$$+ F_{s1} \sin x + F_{s2} \sin 2x + \dots$$

The terms of the series are called the harmonic components of the given function. F_m is the mean height of the given curve.

We have made such a harmonic analysis for each of the above seven turning effort curves. The first 24 harmonic components have been obtained. The results are plotted in the form of cross curves on a base of mean indicated pressure. In Plates 92 to 95 are the individual sine and cosine components F_s and F_c. The order number on each curve refers to the number of oscillations per revolution and is half the number of the harmonic coefficient in the above equation. The above expansion may be written in the form:

$$f(x) = F_m + F_{r1} \sin (x + \gamma_1) + F_{r2} \sin (x + \gamma_2) + \dots$$

where $$F_r^2 = F_s^2 + F_c^2 \quad \text{and} \quad \tan \gamma = \frac{F_c}{F_s}$$

In Plates 90 and 91 are the resultant coefficients F_r. The above analysis is for a crank connecting-rod ratio of 4.25. The results will not be appreciably different for other ratios, however.

33. The above results have been calculated for a four-cycle single-acting engine. They can be utilized for other types as follows:

Two-cycle single-acting: No half order components present. Multiply components of other orders by 2.

We have not as yet calculated the components for the lower half of a double-acting engine. The following rules may be taken as being approximately correct for the lower orders of harmonics for the entire cylinder.

Two-cycle double-acting: Only even numbered components present. Multiply given coefficients by 4.

Four-cycle double-acting:

Orders 1, 3, 5, 7, etc., are 0 (or nearly 0).

Orders $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, etc., multiply coefficients by $\sqrt{2}$. (Phase relations altered for these orders.)

Orders 2, 4, 6, etc., multiply coefficients by 2.

Let A be the area of a single cylinder and r = crank radius. Then any harmonic component of the torque of a single cylinder is equal to $C \times A \times r$.

A similar analysis has been made by Wydler, using theoretical cards. The analysis has been carried to only the sixth order instead of the twelfth. The results to that point are in fair agreement with those given.

Harmonic Components Due to Acceleration Forces

34.

Let W be the reciprocating weight

r = crank radius

l = connecting rod length. $\frac{l}{r} = \lambda$

ω = angular velocity of crank (radians)

θ = crank angle from top center.

Then it may be shown that the turning moment due to the acceleration of the reciprocating masses is given by:

$$M = -\frac{W}{g}\omega^2 r^2 \left\{\cos\theta + \frac{\lambda^2 \cos 2\theta + \sin^4\theta}{(\lambda^2 - \sin^2\theta)^{3/2}}\right\}\left\{\sin\theta + \frac{\sin\theta\cos\theta}{\sqrt{\lambda^2 - \sin^2\theta}}\right\} \tag{19}$$

The above expression may be expanded directly as a series of the form, $\Sigma a_n \sin n\theta$, and the coefficient a_n, of each of these terms, will be the harmonic component of the turning effort curve of order n due to the acceleration of the reciprocating masses.

We may write:

$$\text{Harmonic component} = C\frac{W}{g}r^2\omega^2$$

The values of C for various orders of vibration, and value of λ have been calculated by Mr. F. Porter and are given in the table on Plate 94. Usually only the first, second, and third order components of the inertia torque will be of sufficient magnitude to need consideration.

The resultant harmonic torque is found by adding the components due to gas pressure and inertia forces. If M_s is the sum of the sine terms of the harmonic torque (gas pressure + inertia force) and M_c is the cosine component (gas pressure only), then the resultant is given by

$$M_r = \sqrt{M_s^2 + M_c^2}$$

and the phase angle γ by $\tan \gamma = \frac{M_c}{M_s}$. In adding the separate sine components attention must be given to sign. If the inertia components are so small that they can be neglected, the resultant coefficients of Plates 90 and 91 may be used to find M directly.

Vibration Produced by a Single Harmonic Component Acting at a Point of the Shaft. Forced Vibration

35. At speeds remote from the critical speed, damping forces have little effect and may be neglected. Under these conditions the external harmonic torque causes a forced vibration of the shaft system. The forced vibration is of the same phase as the harmonic torque. That is, if

$$M = M_0 \sin pt,$$

then

$$\Theta = \Theta_0 \sin pt.$$

The motion and torques in every part of the shaft system under the influence of the external harmonic torque may be easily found by means of the polar diagram. An example will illustrate this best.

Dredge *Kingman:* To find the amplitude of vibration at the forward end of the engine and at the generator, produced by the sixth order torque variation of No. 3 cylinder at 140 R.P.M.

For 140 R.P.M. $N = 14$ vibrations per second.
Cylinder diameter = 22″. Crank radius = 16″. M.I.P. = 95 lb.-in.^{2}
Harmonic coefficient (see Plate 91) = 4.68
Harmonic torque $= 4.68 \times 16 \times \frac{\pi}{4} \times 22^2 = 28{,}500$ in.-lbs.

Fig. 1, Plate 96, shows the polar diagram.

Starting at any convenient point H on the horizontal axis and for $N = 14$, we construct the polar diagram backwards through G, F, E, D, C to P, the point representing No. 3 cylinder. The step constants are on Plate 86. In doing this the engine step is split into two parts, CP being $3\frac{1}{2}/6$ of the total. Likewise starting at any point A, we construct the diagram up to P, representing the same cylinder, BP being $2\frac{1}{2}/6$ of the full step angle. Extend OP' and draw the vertical PQ. From Q continue the diagram to A'. Then $B, Q, P, C \ldots H$ is the polar diagram of forced vibration and PQ or y represents the harmonic torque of No. 3 cylinder. By equation (14):

$$PQ = y = \frac{M}{2\pi N \sqrt{\frac{CJ}{g}}} = \frac{28500 \times 19.63}{2\pi \times 14 \times 33100000} = .001923 \text{ radians.}$$

The amplitude at the forward end of the engine is then equal to

$$\frac{OA^1}{PQ} \times .001923 = \frac{13.7}{2.4} \times .001923 = .0110 \text{ radians.}$$

At the generator the amplitude is

$$\frac{OH}{PQ} \times .001923 = \frac{2}{2.4} \times .001923 = .001603 \text{ radians.}$$

The maximum torque is in the flywheel shaft at point R and is equal to

$$\frac{OR}{PQ} \times 28500 \times \frac{\sqrt{C_2 J_2}}{\sqrt{C_1 J_1}} = \frac{34.3}{2.4} \times 28500 \times \frac{1}{2.32} = 175{,}500 \text{ inch-lbs.},$$

which, it will be noted, is much greater than the cylinder torque causing it.

If a closure of the polar diagram cannot be obtained with H to the left of O, it may be obtained by starting to the right and drawing the diagram in the same manner in a clockwise direction from H and counterclockwise from A.

36. To find the motion produced by the entire group of cylinders we add the motions produced by the separate cylinders. A single polar diagram will suffice to find these. This diagram for the above case is shown in Fig. 2, Plate 96. It is evident from the construction of the last figure that the intercepts y_1, y_2, etc., will represent the harmonic torques of each cylinder. By the above example they are all of the length .001923 radians. The amplitude at the generator is therefore equal to

$$\sum \frac{HO}{y} \times .001923 = .001923\ HO \sum \frac{1}{y} = .001923 \times 2 \times 2.171 = .00835 \text{ radians.}$$

The amplitude at the forward end of the engine is

$$.001923 \sum \frac{a}{y} \times \frac{AO}{BO} = .001923 \times 28.67 \times .99 = .0546 \text{ radians.}$$

The harmonic torque at R

$$OR \times 28500 \times \frac{\sqrt{C_2 J_2}}{\sqrt{C_1 J_1}} \sum \frac{1}{y} = \frac{34.3 \times 2.171 \times 28500}{2.4} = 884{,}000 \text{ inch-lbs.}$$

The amplitude or torque at any other points may be found in the same manner. It will be sufficiently accurate in many cases to assume that the harmonic torque of all the cylinders is applied at the center plane of the engine. In the above example we would have obtained in this manner.

Amplitude at forward end of engine:

$$6 \times \frac{a_c}{y_c} \times .001923 = .0586 \text{ radians.}$$

Amplitude at generator:

$$6 \times \frac{OH}{y_c} \times .001923 = .00888 \text{ radians.}$$

Harmonic torque at R:

$$6 \times \frac{OR}{y_c} \times 28500 \frac{\sqrt{C_2 J_2}}{\sqrt{C_1 J_1}} = 972{,}000 \text{ inch-lbs.}$$

37. If the concentrated mass method of calculation has been used, the forced vibration produced by the cylinder torques may be found as follows. Let x be the motion at the forward end of the engine which is to be found. In Plate 85, instead of the motion at the forward end of the engine being assumed unity, let it be x. The table is then evaluated in the usual manner, carrying through the x and adding at each cylinder its har-

monic torque. Each term in the table will be of the form $ax + b$. The moment beyond the last mass is found as an expression of that form and equated to 0, giving the desired value of x. We will give an example illustrating this process later (Art. 70).

Critical Speeds

38. There must exist throughout the shaft a complete balance between all the forces, internal and external. The polar diagram illustrates this balance and shows how the forced vibration at all speeds assumes such a value as to balance the external torque of the cylinders. As N approaches the value which closes the polar diagram, the intercepts which represent the constant harmonic torque of the cylinders become smaller and smaller and the forced vibration becomes larger and larger, until, at the critical speed, it would be infinite. Near the critical speeds a change of condition takes place, however, and the damping forces, which so far have been neglected, become of importance; so that the amplitude will be limited.

In a single-cylinder engine we would have a critical speed corresponding to each of the harmonic components of its turning effort curve. There may exist also in a multi-cylinder engine a critical speed corresponding to each of the harmonic components of the single cylinder curve. In a four-cycle engine we may therefore have a critical speed of every integral order and half order, such as 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$; and in a two-cycle engine of every integral order as 4, 5, 6, etc. There is, of course, a succession of such critical speeds corresponding to each normal mode of vibration.

39. It is convenient to divide the critical speeds into two classes.

A major critical speed is one whose order is equal to, or a multiple of, the number of firing impulses per revolution, provided the firing intervals are equal. All others are called minor criticals. The general explanation of minor critical speeds is as follows:

Consider any given harmonic component. Each cylinder produces a vibration due to this component which is proportional to the ordinate of the normal elastic curve at that cylinder. The normal elastic curve, defined in article 4, is found by the same calculations that determine the natural frequency, its determination being an incidental part of those calculations.

There is a phase relation between the vibrations produced by the separate cylinders. This phase relation depends on the crank arrangement, firing order, and the particular order of critical under consideration.

For major criticals the phase relation is 0, that is, the vibrations given by the separate cylinders may be simply added together.

For minor criticals the vibration produced by each cylinder must be considered as a vector quantity, and the resultant vibration is the vector sum of the separate vibrations. This vector sum may be large or small, depending on how nearly the vibrations of the separate cylinders come to cancelling each other. It thus happens that in a given engine some of the minor criticals may be large and others negligible. These minor criticals exist not only in theory, but they may in some cases be sufficiently powerful to break crankshafts.

Amplitude of Vibration in the Critical Speed Produced by a Single Harmonic Component for a Single Cylinder

40. Consider a shaft as shown in Fig. 3, Plate 84, either uniform or irregular as to mass and elasticity distribution. Let M represent the harmonic force applied at any

point and let a damping force be applied at any other point. The motion of every point of the shaft is harmonic. At the damping point let it be given by $\Theta = \epsilon \sin pt$ and at any other point by $\Theta = \Theta_n \sin pt + \Theta_d \cos pt$.

Both Θ_n and Θ_d are functions of x, shaft length. The damping force is given by

$$M = -k\frac{d\Theta}{dt} = -k\epsilon p \cos pt.$$

At synchronism, since all forces of the normal phase balance, the harmonic torque P must be of the same phase as the damping force $-k\frac{d\Theta}{dt}$, that is, of the phase $\cos pt$.

The component of the motion $\Theta_d \cos pt$ is of this same phase and is called the damping component of the motion. The component of the motion $\Theta_n \sin pt$ is called the normal phase of the motion and at any point of the shaft is the ordinate of the normal elastic curve. We note that Θ_d is small compared with Θ_n, being of the same order of magnitude as the stresses produced by the ordinary forces of the pistons, while Θ_n is of the order of magnitude of the stresses produced by a critical speed.

41. The calculations for the amplitude of vibration in a critical speed are based on the following principle:

The energy given to the shaft during one vibration by the harmonic forces must equal the energy absorbed by the damping forces.

42. Let the motion at the cylinder be given by $\Theta = \beta \sin pt + \beta_d \cos pt$; and the harmonic torque be given by $M_1 = M \cos pt$, M being the maximum value.

Then the work done at the cylinder in one vibration is

$$K = \int_0^{2\pi} M \cos pt \, d\Theta = \int_0^{t=\frac{2\pi}{p}} \cos pt \, (\beta \cos pt - \beta_d \sin pt) \, dt$$

or integrating:

$$K = \beta M \pi \tag{20}$$

We note in particular that there is no work done due to the damping phase of the motion. It may also be easily shown that there is no work done due either to the uniform rotation of the shaft or its vibration with any other frequency than that of the harmonic force.

43. The work absorbed at the damping point is given by

$$K = \int_0^{2\pi} k \frac{d\Theta}{dt} \, d\Theta$$

if

$$\Theta = \epsilon \sin pt$$

we find

$$K = k\epsilon^2 p\pi \tag{21}$$

44. Let α be the amplitude of vibration at the forward end of the engine. Let α, β, ϵ, etc., represent the actual amplitudes of vibration, and $\bar{\alpha}$, $\bar{\beta}$, $\bar{\epsilon}$ the amplitude in inches in the normal elastic curve at the corresponding points.

$$\text{Since } \frac{\beta}{\alpha} = \frac{\bar{\beta}}{\bar{\alpha}} \quad \text{and} \quad \frac{\epsilon}{\alpha} = \frac{\bar{\epsilon}}{\bar{\alpha}} \tag{22}$$

we find, for equation (20)

$$K = \alpha \frac{\bar{\beta}}{\bar{\alpha}} M \pi \tag{23}$$

and for equation (21)

$$K = \alpha^2 \frac{\bar{\epsilon}^2}{\bar{\alpha}^2} k p \pi \tag{24}$$

45. Equating (23) and (24), the amplitude at the forward end of the engine is obtained:

$$\alpha = \frac{M}{k p} \frac{\bar{\alpha}}{\bar{\epsilon}^2} \bar{\beta} \tag{25}$$

It should be noted that the motion α is 90 degrees in phase behind the force M, as shown in Fig. 4, Plate 84.

Extension to a Multi-cylinder Engine

46. We show first how the phase relations between the torques given by the different cylinders may be obtained. As an example we take the shaft of an eight-cylinder four-cycle engine.

For each order of critical a phase diagram may be constructed, which represents the phase relation between the torques of the various cylinders; 360 degrees in these diagrams represents one complete vibration. For critical of order ½ there will be one complete vibration in two revolutions. Therefore 90 degrees in the shaft diagram is represented by 45 degrees in the phase diagram.

Referring to Plate 97, there is shown first the crank arrangement. For critical of order ½ it is evident that the cylinders come into action 45 degrees apart and in the same order as the firing order. The phase diagram for order ½ is shown in (*a*). For order 1, a cylinder must pass over double the phase diagram angle to be at top dead center that it did for order ½. We therefore construct the diagram for order 1 by doubling the angles of diagram (*a*). Diagrams *c*, *d*, *e*, etc., for the successive orders are constructed in the same manner by multiplying the ½ order diagram angles, by 3, 4, 5, etc. After reaching the fourth order, the diagrams repeat. That is, order 4½ is the same as ½; 5½ as 1½; etc. We note that for orders 4, 8, 12, etc., the phase relation is 0, as before stated; that is, the cylinders act simultaneously.

We may construct in the same way the phase diagrams for any crankshaft arrangement and firing order, on any type of engine. These phase diagrams represent, fundamentally, the phase relation between the harmonic torques of the various cylinders, but since the motion of each cylinder is 90 degrees in phase behind its torque the diagram may also be taken as representing the phase relations between the vibrations given by the separate cylinders.

Motion Given by the Entire Group of Cylinders

47. For each cylinder number n the motion at the forward end is given by equation (25)

$$\alpha_n = \frac{M}{k p} \frac{\bar{\alpha}}{\bar{\epsilon}^2} \overline{\beta_n}$$

For the entire group we may therefore write

$$\sum \alpha_n = \alpha = \sum \frac{M}{k\,p} \frac{\bar{\alpha}}{\bar{\epsilon}^2} \overline{\beta_n} = \frac{M}{k\,p} \frac{\bar{\alpha}}{\bar{\epsilon}^2} \sum \overline{\beta_n} \tag{26}$$

By the summation sign Σ we mean the vector sum, each cylinder being given its position according to the phase diagram.

Since the vibration α_n produced by each cylinder is 90 degrees behind its harmonic torque in phase and is proportional to $\overline{\beta_n}$, it follows that the resultant motion α will be 90 degrees in phase behind $\Sigma\overline{\beta_n}$, if we give to $\overline{\beta_n}$ for each cylinder the phase of its harmonic torque.

From the above it follows that for linear damping forces the phase of the resultant motion is so fixed with respect to that of the harmonic moments of the cylinders that the work done by these moments is a maximum.

Equation (23) gives the work done by a single cylinder. In consequence of the above we may write for the entire group:

$$K = M\pi \frac{\alpha}{\bar{\alpha}} \Sigma\ \bar{\beta} \tag{27}$$

α in this equation is in radians. It will be convenient to express the equation with α in degrees.

$$K = \frac{M\pi^2}{180} \frac{\alpha}{\bar{\alpha}} \Sigma\ \bar{\beta} \tag{27a}$$

K is the energy given to the shaft during one vibration by the harmonic forces of all the cylinders. $\Sigma\bar{\beta}$ is the vector sum of the amplitudes of vibration in the normal elastic curve, each cylinder being given the angular position determined by the phase diagram.

It will be noted that, strictly speaking, the above is true only for a single linear damping force, but we will assume equation (27) to hold both for nonlinear and distributed damping forces. These assumptions make a practical solution possible and the error involved in them is very slight.

Damping Forces

48. Damping forces may be divided into two classes: External forces such as the damping effect of the propeller or generator due to its torque variation with varying speed. Engine friction and friction in shafting, etc., also air friction; damping due to vibration of engine.

Internal forces: Elastic hysteresis in the shafting; working in the shaft couplings and clutch, etc.; damping in elastic couplings; play between crank pins and rods.

49. *Damping Factor of a Marine Propeller.*

Let Q be the propeller torque.

Then the damping factor k is determined by the equation $k = \frac{dQ}{d\omega}$, ω being the angular velocity of the shaft in radians per second.

In Fig. 6, Plate 84, is shown a torque revolutions curve, which may be drawn for a given ship, either from its known propulsive data or by assuming that Q varies as R^2. We wish to evaluate k at a point such as P. The value of k cannot be determined directly from this curve, however, since, in obtaining it, the ship speed has been assumed variable and the slip constant; while for the small speed variations that take place when the shaft vibrates, the ship speed will be constant and the slip variable.

The value of $\frac{dQ}{d\omega}$ for constant ship speed and variable slip may be found by utilizing the model propeller experiments of D. W. Taylor, given in "Speed and Power of Ships."

Let s = slip; p = pitch; R = revolutions per minute; V = speed in knots.

Let the m subscripts denote the corresponding properties of the model of the propeller.

We have
$$\frac{\frac{\partial Q}{\partial s}}{Q} = \frac{\frac{\partial Q_m}{\partial s}}{Q_m} \tag{a}$$

the slip being the same for each.

$$pR = \frac{101\frac{1}{3}v}{1-s} \quad \text{therefore} \quad \frac{\partial R}{\partial s} = \frac{R}{1-s}$$

$$\frac{\partial Q}{\partial s} = \frac{\partial Q}{\partial R} \cdot \frac{\partial R}{\partial s} = \frac{\partial Q}{\partial R} \cdot \frac{R}{1-s} \quad \text{also} \quad R = \frac{60}{2\pi}\omega \tag{c}$$

Substituting (c) in (a) we find:

$$k = \frac{\partial Q}{\partial \omega} = \frac{60}{2\pi} \frac{Q(1-s)}{R} \cdot \frac{\frac{\partial Q_m}{\partial s}}{Q_m} \tag{28}$$

In Figures 185 to 208 of "The Speed and Power of Ships" are given curves of Q and s for model propellers. Taking a model propeller similar to the ship's propeller, we evaluate from the proper curve $\frac{\partial Q_m}{\partial s} \Big/ Q_m$ and then determine by equation (28) the factor k.

This analysis takes no account of purely frictional effects, which will be small compared with the above.

50. *Damping Factor of a Generator:*

Let E = total E.M.F. of generator (terminal voltage + internal drop).

E_c = counter E.M.F. of driven machinery (motor or storage battery, but not an external resistance).

I_a = current.

R = revolutions per second.

Then it may be readily shown that as an approximation,

$$\frac{dQ}{d\omega} = k = \frac{1.409}{2\pi} \frac{E^2 I_a}{(E - E_c) R^2} \tag{29}$$

inch, lb., second units.

from which k may be evaluated if the necessary data are at hand.

Frictional Damping

51. The friction of engine and shafting may be divided into two parts: A constant part independent of velocity; a variable portion which may be considered as being a function of the velocity.

The portion of the friction which is independent of the velocity can have no damping effect. This point is important. The constant engine friction is balanced by a portion of the constant mean torque. The result is obvious likewise when we consider that the applied torque is a harmonic force and that this must be balanced by harmonic forces. A constant force cannot balance a harmonic force.

It is extremely difficult to determine whether the friction varies with the engine speed. An analysis of test results indicates that the variation is slight, if any. We have made calculations which indicate that, even if as much as one-half the total friction is variable, the damping effect produced by this is only a small part of the necessary total to produce the observed amplitudes of vibration. We have accordingly neglected engine friction in all calculations for amplitude. The air friction effect is likewise very small and can be neglected.

Elastic Hysteresis

52. By elastic hysteresis is meant the absorption of energy that occurs when a metal is taken through a cycle of stress changes below its elastic limit. This absorption is due to the fact that Hooks law is not exactly true, and that the stress-strain curve of a bar taken through a cycle of stress changes is a closed loop. The area of this loop is the energy absorbed per cycle.

The elastic hysteresis of steel has been investigated by several experimenters, but the results which can be utilized for our purposes are very meager. The most important are given in a paper by Rowett (14).

The experiments of Rowett were made on thin walled mild steel tubes, stressed alternately in torsion to a point below the elastic limit. It was found that the hysteresis energy loss varied as a power of the stress range between the square and the cube.

In Plate 98 are plotted the results of Rowett, converted to English units for convenience. Up to about 8,000 pounds per square inch, the data are fitted fairly well by the curve:

$$K = \frac{1.37}{10^{10}} S^{2.3} = fS^{2.3} \tag{30}$$

S being the stress amplitude or one-half the stress range.

Beyond 8,000 pounds per square inch the hysteresis increased at a much faster rate than given by the equation, so that for higher stresses we can only estimate the losses. The experiments of Rowett were made with but a single quality of mild steel. It is not known how the hysteresis will vary with the composition or heat treatment. Results based on this data must therefore be accepted with due caution until further experimental information on the subject has been obtained.

Calculation of the Hysteresis Loss in an Actual Shaft System

53. Let r_1 be the outer and r_2 the inner radius of a hollow shaft. Then the hysteresis loss, per unit length, assuming equation (30), will be found by integration to be:

$$K = \frac{2\pi}{4.3} fS^{2.3} \frac{r_1^{4.3} - r_2^{4.3}}{r_1^{2.3}} \tag{31}$$

f being the hysteresis constant $\frac{1.37}{10^{10}}$

If we find the hysteresis loss in the shafting for 1 degree amplitude at the forward end of the engine, and if this is K_1, then the energy absorbed for any other amplitude α will be

$$K_1 \alpha^{2.3} \tag{31a}$$

Let α be the abscissa of the forward end of the shaft in the polar diagram and $\bar{y}$ the ordinate of any point. Then if the shaft vibrates with 1 degree amplitude at the forward end, we find from (14) that the moment at any point is given by

$$M = \frac{\bar{y}}{\bar{a}} \left(\frac{2\pi^2 N}{180} \sqrt{\frac{CJ}{g}} \right) \tag{32}$$

by means of which the moment at every point of the shafting can be determined. The stress at every point is then determined and by equation (31) the hysteresis loss in inch-pounds, per inch of length, a curve of which quantity is drawn. The area of this curve is the total hysteresis loss per cycle, for 1 degree amplitude.

Other Damping Losses

54. There is a hysteresis loss due to the alternating vibration stress in the pistons and connecting rods. Our estimates indicate that this is quite small, and accordingly we have neglected it. Another source of loss which might be of quite appreciable magnitude would be due to any slight play in flanged or keyed couplings, clutch parts, etc. Since the torsional vibration moment is of high magnitude, even an extremely small slip could absorb considerable energy. Such a loss should be practically constant for all amplitudes. There is at present no rational method of estimating its magnitude. We do not believe that the loss due to any play between crank pins and connecting rods is of appreciable magnitude. It would appear probable that during these rapid vibrations the oil film acts as a solid and will not permit motion.

Calculation of the Amplitude of Vibration in Any Critical Speed

55. In any critical speed the vibration attains such an amplitude that the outgo of energy from the various damping factors equals the input from the harmonic forces. We may plot a curve of the total damping losses in terms of α, the amplitude at the forward end of the engine, or find the equation of such a curve. Where this curve cuts the straight line for energy input given by equation (27a) is the calculated value of α. If there is only one type of loss present, the amplitude may be found by formula. For propeller or generator damping it is given by (26). For hysteresis loss only, let K_1 be the hysteresis energy loss for 1 degree amplitude, then for any amplitude α we have by (31a):

$$K = K_1 \, \alpha^{2.3}$$

If K_2 is the energy for 1 degree amplitude given by (27a), then equating we have:

$$\alpha = \sqrt[1.3]{\frac{K_2}{K_1}} \tag{33}$$

Calculations for amplitude of vibration in the critical speeds cannot be made with the same degree of accuracy as those for position of the critical. The calculated amplitudes come somewhat too large, indicating that all sources of energy loss have not been taken into account. When more complete and accurate data are obtained regarding the hysteresis, creep, and other losses, it is to be expected that better agreement will obtain between the calculated and observed amplitudes.

56. In Plates 99 and 100 are the calculations for the amplitude of the criticals of the two-noded vibration, surface condition, of a submarine engine.

In Plate 99 are shown the polar diagram, shaft arrangement, normal elastic curve, torsional moment curve and hysteresis curve. In this arrangement the propeller amplitude is small in comparison with the amplitude at the engine, with the result that propeller damping is very small and can be neglected. The only remaining source of loss which has been taken into account is the hysteresis loss.

On Plate 100 are the calculations. In columns 1 and 2 are given the orders of critical and revolutions per minute corresponding. Column 3 is the estimated propeller torque at each of these speeds. Column 4 is obtained by adding the frictional torque. In column 7 are the harmonic coefficients obtained from Plates 90 and 91. The vector sums $\Sigma\bar{\beta}$ are obtained from the phase diagrams of Plate 97 by giving to each vector of the diagram a length $\bar{\beta}$ found from the polar diagram. The vector diagrams have not been actually drawn, but the resultants have been obtained by taking horizontal and vertical components.

Since there is only one source of energy loss, the hysteresis, we may obtain the amplitudes given in column 11 by formula (33). In column 12 are the amplitudes observed on torsiograph test. The minor criticals of 5½ and 6½ order and the 8th order major are the only ones observed, and it will be noted that these are the greatest ones by calculation and are all in about the same ratio to the observed values.

The observed amplitudes are in all cases less than the calculated ones. This is undoubtedly due to all sources of energy loss not being taken into account. Of these unaccounted losses those due to working in couplings or clutch are probably the largest, although a part may also be due to engine friction.

Effect of the Firing Order and Crank Arrangement upon the Amplitude of Vibration

57. Reference to the phase diagrams of Plate 94 shows that the vector sum $\Sigma\bar{\beta}$ and consequently the amplitude of certain of the minor criticals are affected by the crank arrangement and firing order.

In the eight-cylinder, four-cycle engine we have discussed, the half-order criticals are affected. With the given crank arrangement there are eight possible firing orders. The phase diagrams for these eight orders are shown in Plate 101, and $\Sigma\bar{\beta}$ is marked on each. It will be noted that as regards the 5½ and 6½ order vibrations, arrangement 7, the standard firing order, is the worst possible, and arrangement 4 the best.

If not only the firing order but the crank arrangement can be chosen, much wider possibilities exist for the reduction of minor critical speeds. In a two-cycle engine, since there is no choice of firing order, this is the only course open.

We note that in a six-cylinder, four-cycle engine there is possible one balanced crank arrangement and four firing orders; in an eight-cylinder engine four balanced crank arrangements, each having eight firing orders; and in a ten-cylinder engine twelve balanced crank arrangements, each having sixteen firing orders. Of course, many of these crank arrangements or firing orders would be unsuitable for other reasons, such as groups of cylinders firing in succession, high inertia moments in framing, etc. In the eight-cylinder engine discussed, the 5½ and 6½ order criticals could have been reduced to about 15 per cent of their original value by the use of a somewhat different crank arrangement as well as firing order.

Range of the Critical Speeds

58. We have shown how the amplitude of forced vibration can be calculated at speeds removed from the critical speeds and the amplitude in the synchronous speeds estimated. The serious vibration does not occur at a single point but may be spread over a considerable range on both sides of the true synchronous speed. This range of the critical speed

may be from a few revolutions to twenty or more. By an extension of the foregoing methods the amplitude at any speed, and consequently the range of the critical, may in theory be calculated, but this analysis is not as yet in a satisfactory state. In avoiding critical speeds, allowance should be made not only for the range of the critical but for possible errors in its calculated position.

IV. TYPES OF INSTALLATION AND THEIR VIBRATION CHARACTERISTICS

59. So far as their torsional vibration characteristics are concerned the majority of Diesel engine installations fall into one of the following classes:

(1) Installations in vessels with engines amidships and a long shafting line to the propeller; with slow or medium speed engines of long stroke.

(2) The same class of vessel with engines aft or with comparatively short shaft lines.

(3) "Short connected" arrangements such as engine-driving generator, centrifugal pump, etc., with little shafting between engine and driven unit. These engines are usually of the medium or high-speed type.

(4) Geared drive and other miscellaneous applications.

60. The machinery of the *Seekonk* may be considered as being typical of an installation of the first type. The vessel was described in a paper by Mr. J. C. Shaw before the society last year.

The engine is of the Burmeister and Wain, four-cycle, single-acting, long-stroke type, with 6 cylinders $29\frac{1}{8}$-inch bore by 59-inch stroke; and develops 2,300 indicated horsepower at 85 revolutions per minute. Behind the engine is a $9\frac{1}{2}$-foot diameter by 24-inch face flywheel, and 140 feet of shafting to the propeller. We estimate as follows:

Moment of inertia of flywheel......................	90,500,000 in.^{2} lbs.
Moment of inertia of engine........................	89,000,000 in.^{2} lbs.
Moment of inertia of propeller plus water allowance...	25,000,000 in.^{2} lbs.
Moment of inertia of line shaft....................	1,360,000 in.^{2} lbs.
C for shafting, flywheel to propeller................	22,700,000 in.^{2} lbs.
(stiffness equivalent to 1,580 in. of $13\frac{1}{4}$ in. shafting)	
C for engine shaft................................	267,000,000 in.-lbs.

The 1-noded natural frequency may be easily found by equation (7), considering engine and flywheel masses as J_1, propeller as J_2, and elasticity only in line shafting we have:

$$N = \frac{1}{2\pi}\sqrt{\frac{384 \times 22700000 \times 204500000}{25000000 \times 179500000}} = 3.18$$

The third order critical speed will therefore be at

$$3.18 \times \frac{60}{3} = 63.6 \text{ R.P.M.}$$

This speed is not far removed from the operating speed. It must be gone through in starting and stopping, and the engine is likely to be operated at that point in slow-speed running. It is therefore of interest to determine the amplitude of the critical at 62 revolutions per minute.

The mean engine torque at 85 revolutions per minute is:

$$\frac{63030 \times 2300}{85} = 1,707,000 \text{ in.-lbs.}$$

The propeller torque at 62 revolutions per minute approximately:

$$1707000 \times \left(\frac{62}{85}\right)^2 \times .75 = 872{,}000 \text{ in.-lbs.}$$

The resultant harmonic coefficient for 90 pounds M.I.P. is 22.5; cylinder area, 660 square inches.

The harmonic torque, 3rd order, entire engine, is

$$22.5 \times 660 \times 29.5 \times 6 = 2{,}630{,}000 \text{ in.-lbs.}$$

For the propeller damping factor use formula (28).

We find $\dfrac{\frac{\partial Q_m}{\partial S}}{Q_m} = 4$; therefore

$$k = \frac{60}{2\pi} \times \frac{872000}{62} \times .9 \times 4 = 483{,}000$$

$$\frac{\bar{\alpha}}{\bar{\epsilon}} = \frac{\bar{\beta}}{\bar{\epsilon}} = \frac{25000000}{179500000} = \frac{1}{7.17}$$

Using equation (25) for the engine amplitude, we have:

$$\alpha = \frac{2630000}{483000 \times 2\pi \times 3.1} \times \frac{1}{7.17^2} = .00525 \text{ radians.}$$

The propeller amplitude will be $.00525 \times 7.17 = .0377$ radians.

The total shaft twist $.0377 + .0052 = .043$ radians.

The torsional moment due to this is $.043 \times 22{,}700{,}000 = 977{,}000$ inch-lbs.

Comparing this with the mean engine torque we see that this critical is of very small amplitude. We may run directly in it without occasioning any noticeable vibration or unusual stresses in the shafting. Consideration of the foregoing calculations will show that this small amplitude is due to the relatively large moment of inertia of the engine and flywheel compared to that of the propeller. In consequence, the amplitude at the propeller is large compared with that at the engine. The result of this is that an arrangement of this type, provided the engine and flywheel masses are several times the propeller masses, is inherently self-damping so far as 1-noded vibrations are concerned. It would be as well, nevertheless, to design such an installation so that the principal operating speed is not directly at a critical.

To find the 2-noded natural frequency we will need to use a more refined method of calculation. We take engine shaft as step 1, flywheel concentrated; line shaft as step 2, propeller concentrated. The constants for this arrangement, and the 1 and 2 noded polar diagrams, are given on Plate 102. For 1-noded vibration $N = 3.1$, agreeing with the value previously obtained. For 2-noded vibration $N = 10.8$. The 3d order critical speed is therefore $10.8 \times \frac{60}{3} = 216$ R.P.M.; 6th order 108 R.P.M.; and 9th order 72 R.P.M.

Inspection of the polar diagram shows that the propeller amplitude is small compared with the engine amplitude. The critical speed is practically that of engine and flywheel alone, and is affected little by propeller masses. There will be only hysteresis damping present, and we would therefore expect any critical speeds to be of large amplitude. We would not expect the 9th order critical to be serious, however, although if the 6th had been slightly lower it would probably have caused trouble. Due to a shorter stroke, the majority of Diesel engines would have their 2-noded criticals higher than that of the example given, and any dangerous criticals of this type would therefore be above the operating range. They may be avoided, in any case, by keeping the flywheel as close as possible to the engine and making the crankshaft of sufficient diameter to keep dangerous 2-noded criticals above the operating range.

61. As an example of the second type of installation we will consider the above example, altered by moving the engine aft 101 feet, that amount of line shafting being removed. The "C" of the remaining shafting is therefore

$$22{,}700{,}000 \times \frac{1580}{368} = 97{,}600{,}000.$$

The 1-noded natural frequency will be approximately:

$$3.18 \times \sqrt{\frac{97600000}{22700000}} = 6.60 \text{ vibrations per second.}$$

The 3d order critical will be at $6.6 \times \frac{60}{3} = 132$ R.P.M. and the 6th order at 66 R.P.M.

Provided the masses of the engine and propeller are of the proper relative amount, this type of installation has, for its 1-noded criticals, the same self-damping quality as the first type discussed. But, due to the shorter shaft line in which to take up the twist, a critical of given amplitude will mean much higher shafting stresses than it would in the first type. Care should be taken, therefore, that at least the 1st and 2nd major criticals are not near the usual operating speed.

The 2-noded vibration in the second type of installation is practically identical with that of the first, and the same remarks therefore apply.

Minor criticals will usually be of small importance in either the first or second classes of installation. There can be no 1-noded minors, since the cylinders all vibrate with equal amplitude and the vector diagram therefore cancels. Any serious 2-noded minors will usually be above the operating range.

62. It is in the third type of installation that the majority of the difficulties due to torsional vibration in the Diesel engine have been experienced. A characteristic of this type is that it has little of the self-damping feature of the first two, the damping in many cases being practically limited to hysteresis in the shaft. In consequence, any criticals present are likely to be of large amplitude. The first major critical will usually be above the operating speed but majors of even high order may cause trouble. Criticals of as high as the twenty-fourth order have been found on torsiograph records. Due to the unequal action of the cylinders, minor criticals will be present. It is possible for minors, especially those of low order, to be of greater amplitude than majors. Due to the relative short shaft line, a critical of given amplitude results in a greater shafting stress than in the first two types. Any installation of this type, unless identical so far as mass and elasticity distribution are concerned, with a known successful installation, may be considered as being liable to serious torsional vibration.

We have already given two examples of installations of this type, the dredge *Dan C. Kingman* and others, and the submarine engines. The dredge installation consists of a 6-cylinder 1,000 horsepower four-cycle McIntosh & Seymour engine driving a flywheel and generator. The generator is designed to operate at 150 revolutions per minute, and the engine is held to that speed by a governor. After several shafts had failed in service, by fracture in the journal nearest the flywheel, torsiograph tests were made and it was found that a severe 1-noded, 6th-order critical existed at exactly the operating speed. In order to raise this critical speed above the operating range the following changes have been made in the installation:

Crankshaft diameter increased from 13 to 14 inches; generator shaft increased from 15 to 18 inches; flywheel shifted 44½ inches toward engine. The altered arrangement and its polar diagram are shown on Plate 87. The 6th-order critical has been raised 35 revolutions per minute by these changes, and the engine now runs very smoothly and is entirely free from dangerous critical speeds.

Geared Drives

63. The geared Diesel drive is a comparatively recent development in marine engineering, and there are as yet but few installations of this type in operation. When one recalls the numerous difficulties experienced with the geared turbine, a machine whose torque is practically constant, it will be seen that the application of gears to the Diesel engine, whose torque is highly irregular, is not an easy problem. There are certain special dynamic conditions that must be met in a geared drive, and, unless they are satisfied, failure will result. Failure will almost inevitably result from stringing together a combination of engine, flywheel, gears and driven machinery in haphazard fashion without regard to the dynamic problems they may involve.

Just as in the steam turbine, the first condition of success may be said to lie in having good gears, accurately cut, and of sufficient size to carry the load.

The second condition for the operation of a geared Diesel drive may be stated as being "The Condition of Positive Torque." It is essential that the torque at the gear faces shall not pass through zero, otherwise the teeth will separate and pound. It is doubtful whether even the best of gears will stand up long when the teeth are first separating and then knocking together several times per revolution. To meet the condition of positive torque means, in effect, that we must avoid all critical speeds, even many of those that in a solidly connected drive would be considered as being of very small amplitude. It means, furthermore, that we must avoid these criticals with a very wide margin.

64. The calculation of the natural frequency or torque relations of a geared drive offers no special difficulties. Either the concentrated mass or polar diagram method of computation may be used. Assume we are working from engine to propeller and that the gear reduction ratio is three to one. Having arrived at the point in the calculations or diagram represented by the gear face, it is merely necessary to multiply the torque M by 3 and to divide the angle θ by 3, and continue to the other end of the shaft from this new point.

The torque variation at the gear face may be found by the methods of articles 35 to 38 for determining the forced vibration. It will usually be possible to consider the arrangement as a three-mass system, mass one being engine and flywheel, mass two gears and extra flywheel, if any, and mass three, propeller or other driven machinery. This method will give the 1 and 2-noded forms of vibration; the 3-noded form is often nearly that of engine and flywheel alone and is found most easily by means of the polar diagram.

We must consider the critical speeds and torque variations of each of the several orders of vibration in each of the several modes. The condition of positive torque states that the torque variation must be less than the mean torque. We must therefore be certain that the sum of the variation due to all these orders of vibration will be less than the mean torque at any speed within the operating range. In an actual case it will be found that the work is not so complicated as the above general rule would make it appear, for many of the critical speeds will be remote from the operating range.

65. It will be found to be extremely difficult in many cases, while using connecting shafting of ordinary dimensions and keeping the criticals above the operating range, to satisfy the condition of positive torque. This difficulty has led to the use of the so-called "quill shaft drive." The device has been previously used for geared turbines. The pinion shaft is bored hollow and extends aft alongside the line shaft. The reduced diameter quill shaft is fastened to the engine aft of the flywheel, passes through the hollow pinion shaft, and is connected to it by an arrangement permitting slight longitudinal motion at their after extremities. It is generally necessary to fix an extra flywheel to the pinion shaft. The effect of this arrangement is that the critical speeds are reduced far below the operating range. The engine and flywheel can then respond to the torque variations by variations in the flywheel speed. The twist is taken up in the quill shaft, and the gears can run at a nearly constant speed and torque. This arrangement has met with complete success when applied to a single engine-driving gear. Examples of such a drive are the motor ships *Havelland* and *Munsterland* (36). A quill-shaft drive involving the use of step-up gears to a centrifugal pump has been recently constructed and is now in operation on the dredge *Norfolk* of the Atlantic, Gulf and Pacific Co.

The purpose of the quill shaft can be accomplished in some cases by the use of a torsionally elastic coupling (30). A coupling could be used in all cases were it practical to build one with sufficient torsional elasticity. As it so happens, more elasticity can be introduced by means of the quill shaft than in any other way. An attempt has been made on the *Monte Sarmiento* to drive a single gear by two engines through quill shafts, but this arrangement proved unsuccessful from unexpected causes, and the vessel is now being operated with the engines directly connected to the gears (34, 35, 36, 38).

V. ELIMINATION OF VIBRATION

66. It is difficult to state any general rules for the elimination of critical speeds. Even slight differences in arrangement may demand a widely different treatment, and there are so many practical considerations that enter the problem that each case must be considered strictly on its own merits. It is needless to emphasize that changes should be made while the arrangement is on the draughting board, and not after construction, when they will cause delay and expense.

There are three general methods of eliminating or reducing a given critical speed: (*a*) Shifting the critical speed either above or below the operating range; (*b*) damping devices, or damping by shifting the mass distribution; (*c*) eliminating or counteracting the exciting cause, that is, the harmonic torque.

The first method has been applied in the great majority of cases. A critical speed may be raised by increasing stiffness, which generally means increased shaft diameter, or decreasing masses; and is lowered by the opposite changes. All critical speeds belonging to the same mode of vibration are altered in the same ratio. If all parts of the line up are changed in the same proportion, the variation of all critical speeds is as the square root of that change. The property of the line up "changed," is, of course, either stiffness, polar moment of inertia or mass polar moment of inertia. We may write:

$$\frac{N_1}{N_2} = \frac{\sqrt{\frac{H_1}{J_1}}}{\sqrt{\frac{H_2}{J_2}}}$$

As it is not usually possible to change the entire shaft line up, the variation in critical speed is less than this; which means that for any considerable change in the critical speed the alterations in the shaft arrangement may need to be of considerable magnitude.

It is generally considered better practice to raise a critical speed above the operating range, if it is possible to do so, rather than to lower it. If the critical is lowered, the engine must pass through it every time it is started or stopped, and even if this is only a momentary operation the stresses arising may be dangerously high.

The critical speeds may sometimes be adjusted by the introduction of a flexible coupling having the proper amount of torsional elasticity. There are a number of types of flexible couplings manufactured. The majority of them, however, being designed as misalinement couplings, are quite stiff in torsion and therefore not suitable if any considerable change must be made. More torsional flexibility can usually be introduced by the use of a length of high grade alloy steel of reduced diameter than in any other way. Considerable change may sometimes be made by alterations in the dimensions or position of the flywheel.

Damping

67. The best known damping device is the Lanchester damper. This consists of a flywheel at the forward end of the engine, connected to the shaft only through friction surfaces. The flywheel tends to run at a constant speed, and, when the shaft vibrates, exerts a retarding influence by means of its friction surfaces. The amplitude of the vibration can thus be decreased. The device has had considerable application in engines of small power, such as motor cars. For heavy Diesels it is more satisfactory to alter the criticals rather than to damp them.

Mr. F. Fox (21) describes a damper of this type applied to an 800-horsepower engine. Gümbel (27) describes several types of friction and hydraulic dampers.

It is not generally possible to alter an installation sufficiently for the second method of damping to be applied. We have seen that certain types of installation have an inherent damping quality of this kind.

68. The third general method of eliminating critical speeds consists in alterations in the exciting cause, the variable torque. It is impossible to alter the torque variation of a single cylinder, but the cylinders may be so arranged that the harmonic components delivered by each tend to counteract each other. We have already seen how the minor criticals may be reduced in magnitude by a proper choice of crank arrangement and firing order. If, instead of the cylinders firing in even succession, a properly chosen set of crank angles be used, any one particular critical speed, major or minor, can be eliminated. Wydler (28) proposed to accomplish this by dividing the engine into two halves and offsetting one, in angle, with respect to the other. This would not completely eliminate the critical unless all the cylinders vibrate with equal amplitude. We readily see that

the general condition for the complete elimination of any given critical is that the vector sum $\Sigma\bar{\beta}$ shall be zero. If the engine is of more than three cylinders, there are an infinite number of crank arrangements that make this possible.

There are two outstanding difficulties in way of this method of elimination: (1) while one critical was being eliminated others, which were before of small magnitude, would be increased; (2) the balance of the engine would be disturbed. We do not know of any actual engine in which this scheme has been carried out, and mention it largely because of its theoretic interest.

VI. SPEED REGULATION—ACTION OF THE FLYWHEEL

69. While the subject of the regulating action of the flywheel is not directly concerned with critical speeds, it will be of interest to treat it here because of the accuracy with which it may be handled by the use of the harmonic coefficients. The degree of regulation is defined by the equation

$$\delta = \frac{\text{maximum speed} - \text{minimum speed}}{\text{mean speed}}$$

It is ordinarily calculated on the assumption that the shafting is infinitely rigid; also that only the engine and its flywheel enter into the result. In a great many, perhaps in the majority, of cases, these assumptions will lead to results that are very far from the truth. The degree of regulation is not constant throughout the shaft but varies from point to point. If at certain points of the shaft there are nodes, the degree of regulation of those points will be zero. A flywheel located at or near a node would have no effect whatever in regulating engine speed, no matter how heavy it might be made. It is quite possible for a flywheel to be so located in an actual installation.

Let α be the maximum amplitude of vibration at any point and δ the degree of regulation.

$$\frac{d\Theta}{dt} = \alpha p \cos pt \qquad \frac{d\Theta}{dt}\text{ max.} = \alpha p$$

$$\delta = \frac{v\text{ max.} - v\text{ min.}}{v\text{ mean}} = \frac{2\,\alpha p}{2\,\pi N^1} = \frac{4\,\alpha\pi N}{2\,\pi N^1}$$

N = vibrations per second; N^1 = revolutions per second.

or

$$\delta = 2\alpha\lambda \tag{34}$$

where λ is the order of vibration under consideration.

70. The amplitude α at any point of an installation may be found by the method of articles 35 to 37. The example given in article 35 shows how it may be calculated for an installation of the "short connected" type. As an example of another type we find the amplitude at 85 revolutions per minute of the *Seekonk*. We take the data from article 60. At 85 revolutions per minute p for the third order torque variation will be

$$p = 2\pi N = 2\pi \times \frac{85 \times 3}{60} = 26.7 \qquad p^2 = 715$$

If we consider the engine and flywheel as being disconnected from the line shaft, the amplitude will be given by equation (3)

$$\Theta = \frac{Mg}{Jp^2}$$

$$\alpha = \frac{2630000 \times 384}{179500000 \times 715} = .00786$$

and the degree of regulation $= 6 \times .00786 = .04716$.

If full account is to be taken of line shaft and propeller, the method of article 37 is used. The work is given in the tabulation below:

Mass.	$\frac{Jp^2}{g}$	Θ	$\frac{Jp^2\Theta}{g}$	$\sum\frac{Jp^2\Theta}{g}$	C	$\sum\frac{Jp^2\Theta}{g\,c}$
Prop.	46,600,000	ϵ	46,600,000 ϵ	46,600,000 ϵ	22,700,000	2.05 ϵ
Eng.	334,000,000	-1.05ϵ	$-$351,000,000 ϵ	$-$304,400,000 ϵ + 2,630,000		

$$\text{Propeller amplitude} \quad \epsilon = \frac{2630000}{304400000} = .00863 \text{ radians.}$$

$$\text{Engine amplitude} \quad \alpha = -.00863 \times 1.05 = -.00907 \text{ radians.}$$

$$\text{Shaft torque} \quad = 46600000\,\epsilon = 402{,}000 \text{ in.-lbs.}$$

Note that the engine amplitude is actually greater than when the engine is at its critical speed of 62 revolutions per minute. The shaft torque, which is the better measure of vibration, is of course less. We should in theory take account in this manner of all the harmonic components acting upon the engine. It will generally be sufficient, however, to consider only the order equal to the number of firing impulses per revolution, unless we are near the critical speed of some other order. We should not lose sight of the fact that it is not usually essential to fix the degree of regulation with any great degree of accuracy, and that in many cases the size and position of the flywheel are fixed by considerations quite other than those of the regulation desired.

Bibliography

71. *The Torsional Vibration of Reciproacting Engines and related topics:*

(1) Bauer: "Untersuchungen über die periodischen Schwankungen in der Umdrehungsgeschwindigkeit der Wellen von Schiffsmaschinen." Jahrbuch der Schiffbautechn. Ges. 1900, 1.B.

(2) Frith, J., and E. H. Lamb: "The Breaking of Shafts in Direct Coupled Units, Due to Oscillations Set up at Critical Speeds." Journal Inst. of Elect. Engs., Vol. 31, p. 646, 1901.

(3) Gümbel: "Ueber Torsionsschwingungen von Wellen." Schiffbau, 1901–1902, S. 580.

Translation: "Torsional Vibrations of Shafts." Jl. Am. Soc. Naval Engs., May, 1902, p. 432. Inst. Naval Architects, 1902.

(4) Frahm: "Neuere Untersuchungen über die dynamischen Vorgänge in den Wellenleitungen von Schiffsmaschinen mit besonderer Berücksichtigung der Resonanzschwin-

gungen." Zeit., d. Verein Deutsches Ing. 1902, S. 797. Translation in Jl. Am. Soc. Naval Engs., August, 1902, p. 721.

(5) LORENZ: "Dynamik der Kurbelgetriebe." Leipzig, 1901.

(6) MELVILLE, G. W.: "The Vibration of Steam Ships." Engineering, Vol. lxxv, 1903, p. 1.

(7) ROTH: "Schwingungen von Kurbelwellen." Z. d. V. D. I., 1904, S. 564.

(8) HOLZER: "Torsionsschwingungen von Wellen mit beliebig vielen Massen." Schiffbau, 1907, S. 823.

(9) LORRAIN: "Étude sur les vibrations de torsion." Bulletin de l'Ass. Tech., May, 1909.

(10) HUMPHREY, H. A.: "Cyclic Irregularity and Angular Deviation." Proc. Inst. Elect. Engs., 1909, Vol. 43.

(11) DIXON, J.: "Torsional Vibrations of Massive Loaded Shafts." Univ. of Durham, Phil. Soc., 1909–1910, p. 287.

(12) GÜMBEL: "Verdrehungsschwingungen eines Stabes mit fester Drehachse und beliebig zur Drehachse symmetrischer Massenverteilung unter dem Einfluss beliebiger harmonischer Kräfte." Z. d. V. D. I., 1912, S. 1025.

(13) GÜMBEL: "Torsional Vibrations of Elastic Shafts of any Cross Section and Mass Distribution and Their Application to the Vibration of Ships." Inst. of Naval Architects, 1912.

(14) ROWETT: "Elastic Hysteresis in Steel." Proc. Royal Soc. of London, Ser. A, 89, 1913–1914, p. 528.

(15) GEIGER: "Ueber Verdrehungsschwingungen von Wellen, insbesondere von mehrkurbligen Schiffsmaschinenwellen." Dissertation, Augsburg, 1914.

(16) GEIGER: "Der Torsiograph, ein neues Instrument zur Untersuchung von Wellen." Z. d. V. D. I., 1916, S. 811.

(17) JANSON, E. N., and J. O. RICHARDSON: "Investigation of Shafting Failures and Engine Vibration on Vessels of the Louisiana Class." Jl. Am. Soc. Naval Engs., Feb., 1917.

(18) FRAHM: "Ein neuer Torsionindikator mit Lichtbildaufzeichnung und seine Ergebnisse." Z. d. V. D. I., 1918, S. 177.

(19) DREVES: "Neues graphisches Verfahren auf statischer Grundlage zur Untersuchung beliebiger Wellenmassensysteme auf freie Drehschwingungen." Z. d. V. D. I., 1918, S. 588.

(20) LEWIS, F. M.: "Torsional Vibrations of Irregular Shafts." Jl. Am. Soc. of Naval Engs., Nov., 1919, p. 857. "Critical Speeds of Torsional Vibration." Jl. Soc. Automotive Engs., Nov., 1920, p. 418

(21) FOX, J F.: "Torsional Vibration and Critical Speeds in Crankshafts." Jl. Soc. Automotive Engs., Nov., 1920, p. 413.

(22) SESINI, O.: "Le Oscillazioni Torzionali degli Alberi di Trasmissione, con massa propria e con masse Concentrate in Punti Intermedi." Reale; Accademia della Scienze di Torino. Atti. 55, 1919–1920.

(23) FÖPPL, O.: "Dreh- und geradlinige Schwingungen von Wellen und Massen." Zeit. für Ang. Math. und Mech., 1921, S. 367.

(24) SASS: "Beiträge zur Berechnung kritischer Torsions-Drehzahlen." Z. d. V. D. I., 1921, S. 67.

(25) GEIGER: "Zur Berechnung der Verdrehungsschwingungen von Wellenleitungen." Z. d. V. D. I., 1921, S. 1241.

(26) HOLZER: "Die Berechnung der Drehschwingungen." Verlag von Julius Springer 1921.

(27) GÜMBEL: "Verdrehungsschwingungen und ihre Dämpfung." Z. d. V. D. I., 1922, S., 252 and 281.

(28) WYDLER, H.: "Drehschwingungen in Kolbenmaschinenanlagen und das Gesetz ihres Ausgleichs." Verlag von Julius Springer, 1922.

(29) TIMOSHENKO, S.: "Torsion of Crankshafts." Am. Soc. of Mech. Engs., Vol. 44, 1922. "The Bending and Torsion of Multi-Throw Cranks." Am. Soc. of Mech. Engs., 1923.

(30) *Motorship:* Falk Supplement, May, 1923.

(31) STEINEN, VON DEN: "Berechnung der Eigenfrequenzen bei Verdrehungsschwingungen von Schiffswellenleitungen." Werft Reederei Hafen, June 7, 1923.

(32) BLONDEL, A.: "Danger of Resonance in Crankshafts of Internal-Combustion Engines Driving Electric Generators." Comptes Rendus, Dec. 31, 1923 March 31, 1924, and April 28, 1924.

(33) PLATIRER, C.: "Torsional Resonance of Shafts." Comptes Rendus. March 31, 1924.

(34) REISSNER: "Ueber die Kraftschlüssigkeit von Zahnradgetrieben." Zeit. für Ang. Math. und Mech., Feb., 1924.

(35) MÜLLER: "Die Doppelschrauben Fracht und Passagier Motorschiffe." Monte Sarmiento und Monte Olivia. Werft Reederei Hafen, Nov. 22, 1924.

(36) FRAHM: "Gear Drives for Turbine and Motorships." Jahrbuch der Schiffbautechn. Ges. 1924, also Werft Reederei Hafen, Dec. 7, 1924.

(37) LACK, A., and C. B. JAHNKE: "Torsional Vibration and Critical Speeds of Crankshafts." Am. Soc of Mech. Engs., 1925.

(38) DREVES: "Gear Drives for Motorships." Schiffbau, June, 1925. Trans. Jl. Am. Soc. Nav. Engs., Aug., 1925.

DISCUSSION

THE PRESIDENT:—Gentlemen, the paper is now before you for discussion.

MR. JOHN L. BOGERT, *Member:*—I think the thanks of the Society are certainly due to Mr. Lewis. He has given something that every Diesel engineer, every man interested in reciprocating engines of any kind, will appreciate. That is a paper which I want to study not for one day but for weeks. That is my opinion about it. (Laughter and applause.)

MR. JAMES C. SHAW, *Member:*—Mr. Lewis has made a very valuable contribution to this interesting and important subject and is deserving of the thanks of the Society.

His calculations in reference to the *Seekonk* are of a particularly personal interest. The conclusions to be drawn are that the only critical speed to be concerned with in this vessel is that of the third order which is below the operating speed. The reason that this critical speed has not been observed is probably due to the rapidly passing over it when stopping and starting and its small magnitude for reasons stated.

Experience shows with vessels of this class that torsional troubles are infrequently met with. Mr. Lewis' calculations, however, would indicate it possibly necessary to investigate the criticals when the machinery is placed aft, with short length of shafting, as done in tankers.

In reference to the factor C for shafting, it is noted that no account has been taken in the formula for variation in shaft diameter. I would like to know if Mr. Lewis could include this as an addenda.

As an example where preliminary calculations would have been of much value might be cited the experience of the Cramp Company some ten years ago with some direct-drive twin-screw turbine destroyers in which small reciprocating compound steam engines were coupled in ahead at cruising speed. After they were in service it was found that they had a tendency to break their shaft connecting engines with turbines when running at approximately 250 revolutions per minute, the shafts becoming hot enough to be discolored before the break occurred. The calculation was fairly simple, due to the turbine rotor being relatively large, which could be considered infinite in comparison with the engine with cranks at 180 degrees. This showed the natural frequency of the connecting shaft so loaded by engine parts and clutch was exactly double the frequency of the steam impulses on the pistons. The remedy suggested was one of the methods in Mr. Lewis' paper, in substituting for the solid shaft a hollow shaft of larger diameter which raised the polar moment of inertia of the shaft.

MR. ROBERT HAIG, *Council Member:*—I have little to say. I agree that Mr. Lewis has given us enough here to set up technical indigestion (Laughter), but we have gone through some of the troubles that he has told us about today. We were the builders of some dredges for the War Department. The War Department found that they had to go to their sister department to help them out with their troubles, which they did very effectually. We were in no way responsible, because we neither built the engines nor designed them, nor did we have anything to do with the generators or anything else that was sent with them. They were purchased by the War Department. I am explaining that because in this paper you see diagrams of the *Dan C. Kingman* shaft, which Mr. Lewis has used to illustrate his paper, and that is one of the dredges we built.

These dredges ran their trial trips excellently, with no trouble whatever, and one of them, which was sent down south after completing trials, was worked at lower revolutions and has run all right. But in three boats working up north, the revolutions per minute came into the critical zone, and the crank-shafts came to grief.

Their troubles were similar to troubles that had occurred elsewhere. As you probably know, the government departments knew abundantly well that when serious shaft failures started to happen in Diesel-engine jobs some years ago, they found that the only people who then knew much about torsional vibration in Diesel engines were the Germans. They knew far more than anyone else knew. They had it down to a very fine science and were able to clear up a great many difficulties, clear them up very effectually, for the simple reason that they had gone through them.

In 1920 I happened to be over on the other side, and while there, a twin-screw job came back to the builders. The trouble was similar to what happened here, and to eliminate that trouble they had to increase some of the shafts 2 inches. In those dredges I mentioned, I think they increased the shafts an inch and a quarter, or something like that (that is, the shafts aft of the crank-shaft), and, of course, renewed the broken section of the crankshaft. They also changed the location of the flywheel and by that means brought the shafting up to a condition where the critical zone was higher up, and the boat could operate at the designed speed without encroaching on the dangerous zone.

I think, in this subject of the development of Diesel engines, the torsional vibration that we have to look out for will require our very close attention. When we built steam engines and boilers, as all of us know, little thought was given to vibrations unless we had very high power and a light ship. The ship went away. It vibrated, and people said it was the fault of the builder, or the loading, but the ship went out. Today, when you install Diesel engines and set up vibrations in the hull, you must find out whether you are the transgressor, whether your engine is not wrongly designed, whether you have or have not provided for the conditions that you get.

All kinds of nostrums are put up to us: "If you hang this or that on the ship, it will stop the vibrations," and so on, but those who have the time to investigate find that very few of these cures do apply and do produce the results expected. It is a condition that can be very clearly met, but it must be gone into.

Some time ago, when one of the outlying government departments asked for large Diesel engines for power purposes, people commenced to realize what they were going to meet with when they ran up into high speed with large units. That had not been considered, and it brought back to me what a very eminent builder once stated—they found that when they attempted to build small Diesel engines that went into higher revolutions per minute, they had to forget many things they had learned about large slower moving Diesel engines.

The man who has come up from the small Diesel engines, such as our automobile engine, knows that. We have gone into this very thoroughly and are familiar with their experience and the results they have achieved. They got into vibrations that would not allow their cars to run with comfort, and which simply meant a very short life for the machinery, and Diesel engineers will find, unless this subject has been very carefully dealt with, that their engines will not run, and that their engines will set up conditions that are, to the man who stands outside, almost impossible to diagnose as to what is the trouble. But when you get right down to it, it is just one of the real fundamentals of engineering, and nothing more.

I think Mr. Lewis has given us an enormously rich paper. The thing that is valuable about the meetings of this Society and kindred societies is the meat contained in papers such as this that men can sit down to at night and read and learn. I think, sir, we owe a debt of gratitude to Mr. Lewis for his very excellent paper. (Applause.)

MR. H. SCHRECK, *Visitor:*—Professor Lewis has contributed a very valuable paper on torsional vibration of shafts. I have studied this paper and would like to make a few suggestions on various points contained therein.

Professor Lewis has adapted for his calculations a method of converting crank-shaft, propeller shaft, etc., into a system of equal elasticity by introducing a constant "C" for the various parts of the shaft, thereby leaving the length of the shaft the same as the actual length. This method is, as far as I know, different from all other methods of figuring critical speeds used in this country and abroad. I personally think that this method complicates matters unnecessarily against the older method on which we take the main diameter of shafting as standard and reduce all other parts of the system in length to this "standard shaft."

It is surprising to me to note the difference in stiffness of shafts; that is, calculated against actual shaft. However, it is valuable to note that the stiffness of shaft as calculated by the formula of Geiger agrees with the value determined on the actual shaft when bearing clearance is small; that is, similar to the condition on the Diesel engines in operation.

The author states that the stiffness is dependent, to a certain extent, on the amount of restraint at the bearings, and that some of the formulae take account of this. May I ask which authority does this and where such information can be found in the literature?

In paragraph 27 the author tells how to figure the elasticity of flanged couplings. I may state in this connection that I have always used the suggestion of Holzer to consider a flange coupling equivalent to a piece of shafting equal in length to the thickness of the coupling flange, and a diameter equal to the shaft diameter of which the coupling is a part. I have followed the suggestions of Frahm and Gümbel to omit the length of shafting, which forms the seat of a flywheel, motor, propeller, etc.

Calculations which I made on critical speed of two different engines, both 6-cylinder, 4-cycle engines, using above stated rules, agreed very well with the actual conditions on these engines. The two engines were of widely different size and speed. I used in my calculations the above-mentioned data, and the same might, therefore, be of interest.

I would like to ask Mr. Lewis about the effect of the external torque of the cylinders. In my calculations of critical speed, I did not consider the torque which is set up by the gas pressure and by the inertia forces. In other words, in figuring critical speed, I have only considered the masses. Results obtained in that way showed that calculated critical speed and actual critical speed were so close that the difference could not be detected without the use of a torsiograph.

In this connection, I would like to refer to paragraph 38, where Mr. Lewis states that "the constant harmonic torque of the cylinders becomes smaller, and the forced vibration becomes larger and larger until at the critical speed it would be infinite." Therefore, I think we are justified in neglecting, for the purpose of figuring the critical speed (not for figuring the amplitude, but for figuring the critical speed), the gas pressure and forces set up by the acceleration.

As to the historical development of the problem of torsional vibration, and the statement that Dr. Bauer in 1900 was probably the first to show the danger of it, I wish to state, in remembrance of and respect to my former superior—Dr. Frahm, director of the Blohm & Voss Shipyard in Hamburg—that he was, to my knowledge, the first engineer engaged in research in connection with torsional vibration of shafts. As a matter of fact, Dr. Frahm states in his paper (compare Bibliography 3, Translation) that in 1899 he was given by Messrs. Blohm & Voss the problem of investigating the breaking of propeller shafts due to torsional vibration, and the statement referred to by Dr. Bauer was made in the year 1900.

I think this is a small matter, but the greatest acknowledgment that we have to make, among German engineers, is to Dr. Frahm, and I therefore wished to make this statement. (Applause.)

Captain Emory S. Land (CC) U. S. N., *Member:*—This is an excellent paper.

Having been "shipmates" on one of the ships that was mentioned here as having this torsional vibration (the U. S. S. *Connecticut*), and remembering very distinctly that it was decided at that time by the seagoing element that all the trouble came from the structure—due to the tail weaving and various other things—it is interesting now to see how science has developed enough to determine that it is inherent in the engine itself, and probably had little or nothing to do with the structure. This was proven later, but it was rather a prevalent opinion that some of the difficulty in those days was structural.

We also had difficulties along the same line with our submarines, as you gentlemen well know, which brings me to the point that while the Germans may have been the only people that knew anything about it, I was under the impression that the Swiss had a pretty intimate knowledge of these difficulties in Diesel engines. Maybe they were of German extraction. (Laughter.) Anyway, they knew something about it and had arrived at somewhat the same conclusion that Mr. Lewis has arrived at as to the solution; that is, it must be taken care of on the drawing-board and in the design. The engine must be so designed that this critical speed, if it is bound to occur, will occur preferably beyond the revolutions which you are actually going to use in operating.

They also suggested, in 1920, the flywheel cure.

My main purpose in getting up here is to get a little information on something that may have a bearing in aviation engines. We do not have the Diesel engine in heavier-than-air. We are on the border line in lighter-than-air and we see possibilities in the future for Diesel engines in heavier-than-air.

Are we to expect these difficulties in the short shafts that we have there? If so, perhaps Mr. Lewis can tell us how to avoid this torsional vibration in the short shafts which we have in aviation engines. And is there an analogy to be expected between the Diesel engine and the gasoline engine?

We do have today certain difficulties with our aviation engines, in which there is a critical speed, and I am wondering if the answer is to go back to the drawing-board and find out where this torsional vibration occurs, or if it does occur. I would be very glad if Mr. Lewis would give us some information on that point or show us some line of development which we can follow up, or if he has made any investigation of the aviation engines. (Applause.)

MR. JOHN F. FOX, *Visitor:*—I wish to say that we at the New York Navy Yard have been in very close touch with Professor Lewis and his work. In fact, I think we can claim the credit for having started him on his investigations back in 1920.

We have worked out a large number of submarine and other installations, among them the dredges *Dan C. Kingman* and class, using Professor Lewis's method of calculation. In every case we have been able to hit the critical speeds within one or two revolutions.

The main things to be considered in this subject are:

First, the location of the critical speeds, or, in other words, the natural period of the shaft.

Second, the orders of the critical speeds to be expected.

Third, the relative violence of the critical speeds, and in fact the absolute violence.

Fourth, the range of speeds over which each critical speed will extend. This is considered of particular importance. It has been found that some of the critical speeds extend over a very broad range and are very violent. Other critical speeds, equally violent, extend over a very short range.

The paper covers thoroughly all the factors involved except the question of range. We are all hoping that Professor Lewis will continue his work and give a little more information as to the question of the ranges over which each critical speed will extend. (Applause.)

Mr. Lewis:—With regard first to the question of Mr. Shaw regarding the calculation of "C" for shafts of varying diameter, I note that this is given by formula (6). Each element of the summation refers to an element of shaft length of constant diameter. The formula of Geiger for crank-shaft stiffness was given because of its simplicity. Formulas of possibly greater accuracy and taking account of bearing restraint will be found in Bibliography (26) and (29).

Mr. Schreck evidently misunderstands my statement regarding the effect of the torque due to the gas pressure. This only enters into the calculations for amplitude; no account is taken of it in calculations for the natural frequency.

I confess to having no first-hand knowledge of the early history of the subject. The statement made, crediting the initial investigations to Dr. Bauer, is quoted from Dr. Gümbel's paper of 1901, Bibliography (3).

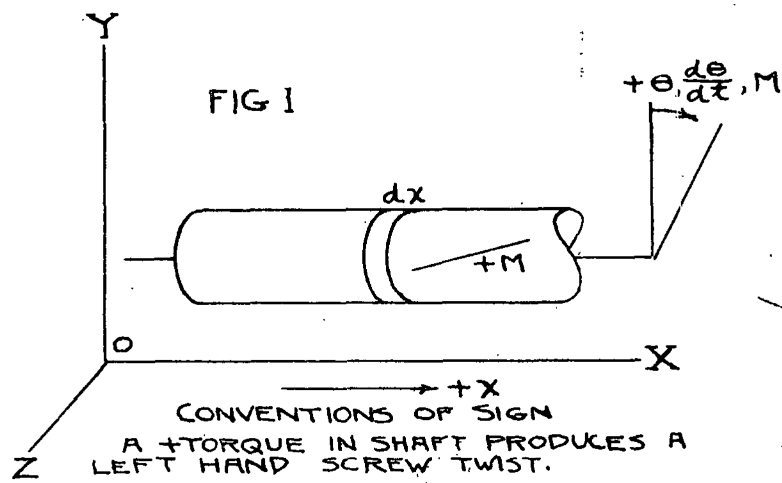
In reply to the question of Captain Land I would state definitely that these same troubles from torsional vibration are to be expected in aircraft, and that the answer to the problem is just the one that he has suggested, namely, go back to the drawing-board, make calculations to see what may be expected, and then remedy the difficulties, all before the engine is built.

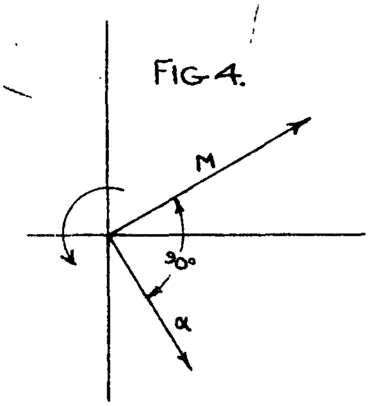
The President:—Gentlemen, our thanks are due to Mr. Lewis for his very illuminating paper upon a difficult but very live subject.

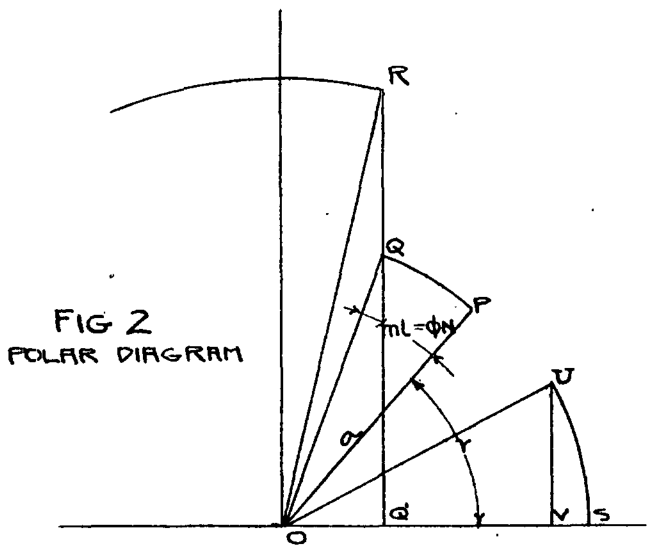
The next on our program is Paper No. 10, "The Launch of the Airplane Carrier U. S. S. *Saratoga*," by Mr. Ernest H. Rigg, Council Member.

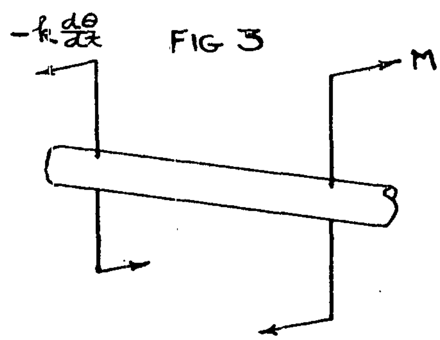
Mr. Rigg presented this paper in abstract.

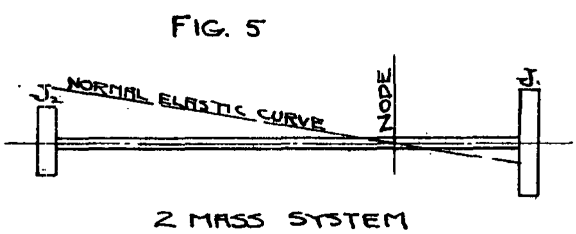
To illustrate paper on "Torsional Vibration in the Diesel Engine,"
by Frank M. Lewis, Esq., Member.

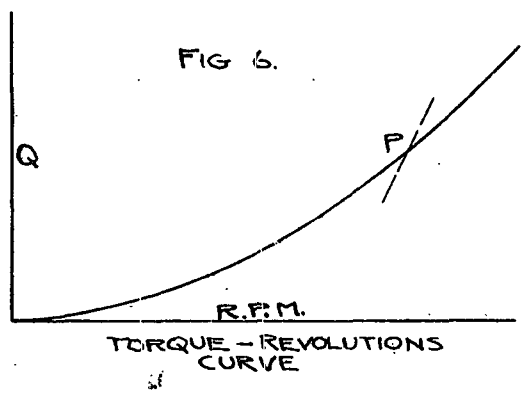








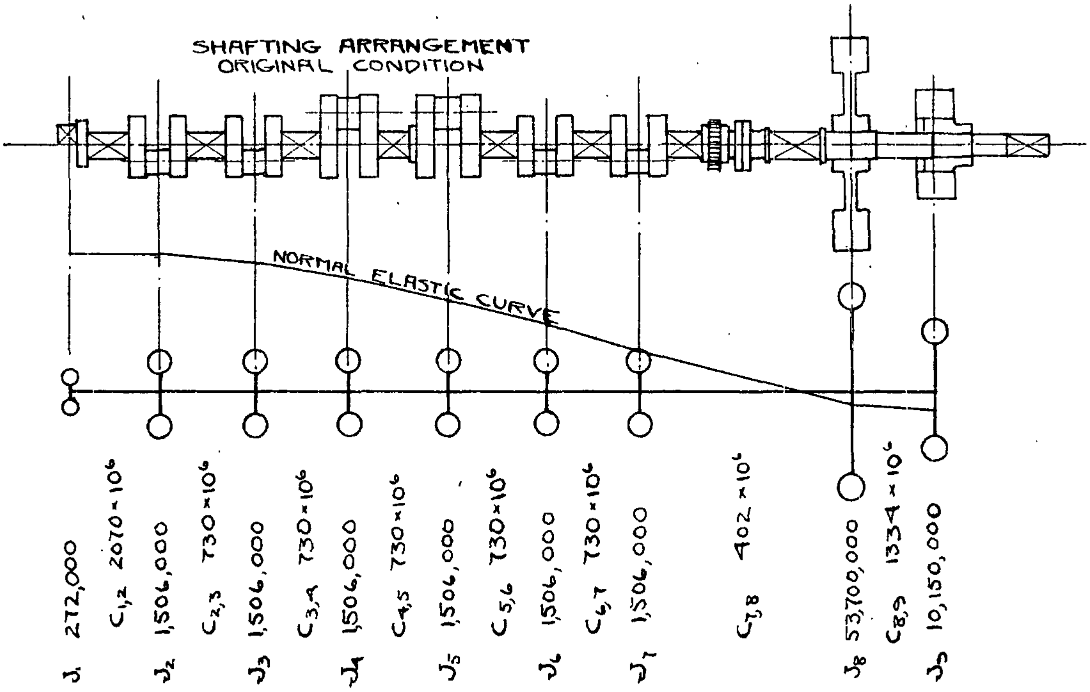




To illustrate paper on "Torsional Vibration in the Diesel Engine," by Frank M. Lewis, Esq., Member.

DREDGES DAN C. KINGMAN ETC.

CALCULATION OF NATURAL FREQUENCY CONCENTRATED MASS METHOD



	1	2	3	4	5	6	7
MASS NO.	J/g	$\frac{Jp^2}{g}$	θ	$\frac{Jp^2\theta}{g}$	$\sum\frac{Jp^2\theta}{g}$	C	$\frac{\sum\frac{Jp^2\theta}{g}}{C}$

N = 15.3 p = 96.2 p^2 = 9250

1	708	6.55×10^6	1	6.55×10^6	6.55×10^6	2070×10^6	.0031
2	3920	36.25×10^6	.9969	36.1×10^6	42.65×10^6	730×10^6	.0585
3	3920	36.25×10^6	.9383	34.0×10^6	76.65×10^6	730×10^6	.105
4	3920	36.25×10^6	.8333	30.2×10^6	106.85×10^6	730×10^6	.1462
5	3920	36.25×10^6	.6871	24.9×10^6	131.75×10^6	730×10^6	.1803
6	3920	36.25×10^6	.5068	18.38×10^6	150.1×10^6	730×10^6	.206
7	3920	36.25×10^6	.3008	10.90×10^6	161.0×10^6	402×10^6	.401
8	139800	1293×10^6	−.1002	-130.0×10^6	31.0×10^6	1334×10^6	.0233
9	26400	244×10^6	−.1235	-30.2×10^6	+ 800,000		

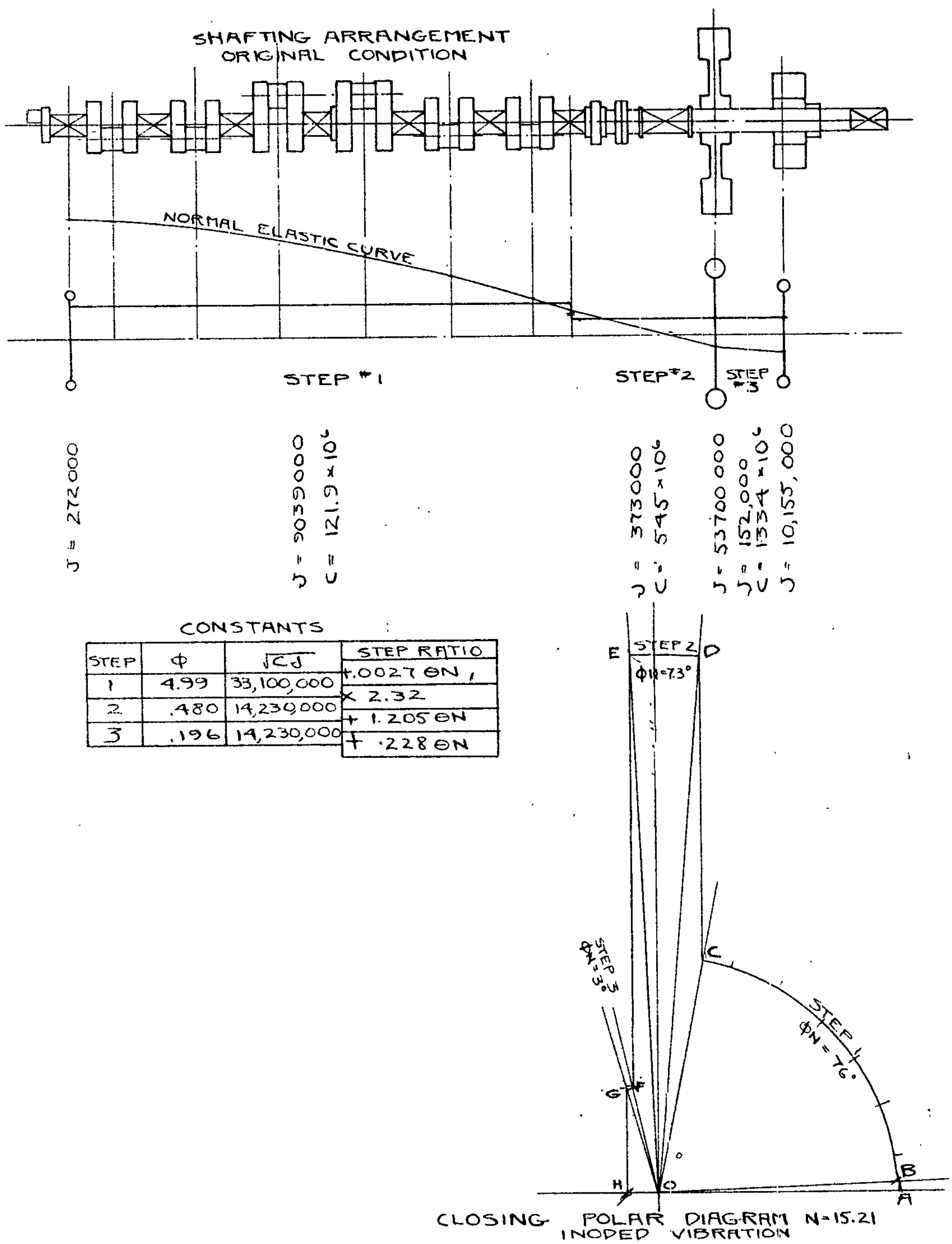
N = 15.4 p = 96.8 p^2 = 9380

1	708	6.65×10^6	1	6.65×10^6	6.65×10^6	2070×10^6	.0032
2	3920	36.8×10^6	.9968	36.7×10^6	43.35×10^6	730×10^6	.0594
3	3920	36.8×10^6	.9374	34.5×10^6	77.85×10^6	730×10^6	.1069
4	3920	36.8×10^6	.8305	30.6×10^6	108.45×10^6	730×10^6	.1487
5	3920	36.8×10^6	.6818	25.1×10^6	133.55×10^6	730×10^6	.1830
6	3920	36.8×10^6	.4988	18.36×10^6	151.91×10^6	730×10^6	.208
7	3920	36.8×10^6	.2908	10.7×10^6	162.61×10^6	402×10^6	.404
8	139800	1312×10^6	−.1132	-148.5×10^6	14.11×10^6	1334×10^6	.0106
9	26400	248×10^6	−.1238	-30.7×10^6	-16.59×10^6		

To illustrate paper on "Torsional Vibration in the Diesel Engine," by Frank M. Lewis, Esq., Member.

DREDGES DAN C. KINGMAN ETC.

CALCULATION OF NATURAL FREQUENCY
POLAR DIAGRAM METHOD

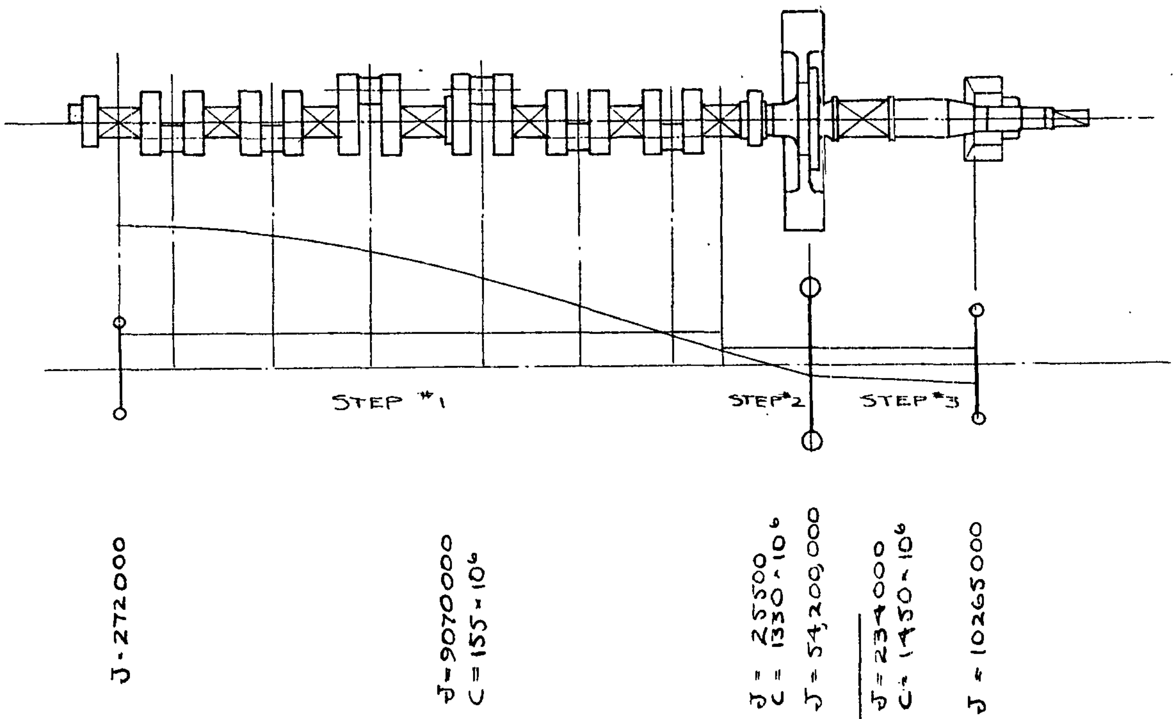


CONSTANTS

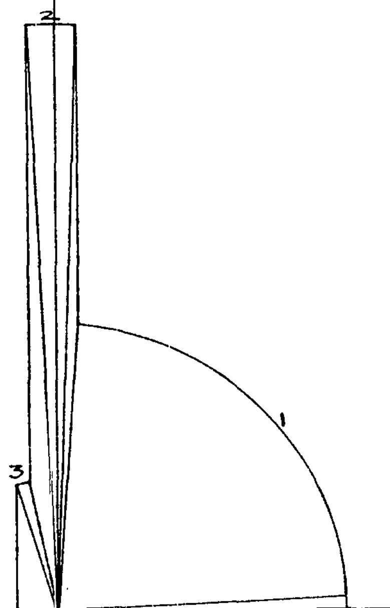
STEP	Φ	√CJ	STEP RATIO
1	4.99	33,100,000	÷ .0027 ΘN,
2	.480	14,230,000	× 2.32
3	.196	14,230,000	+ 1.205 ΘN
			+ .228 ΘN

To illustrate paper on "Torsional Vibration in the Diesel Engine," by Frank M. Lewis, Esq., Member.

DREDGES DAN C. KINGMAN ETC.
REVISED ARRANGEMENT OF SHAFTING.



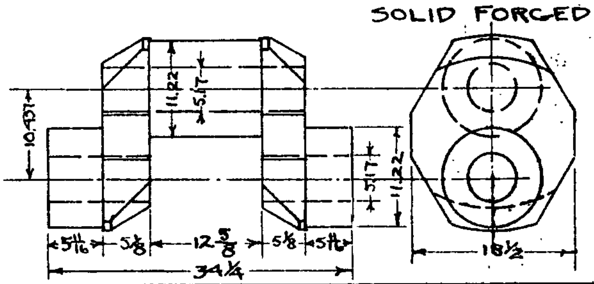
STEP	Φ	$\sqrt{CJ}$	STEP RATIO
1	4.43	37400000	+ .0023 θN
			× 2.04
2	.254	18380000	+ .943 θN
3	.233	18380000	+ .1787 θN



CLOSING POLAR DIAGRAM N = 1865
1 NODED VIBRATION

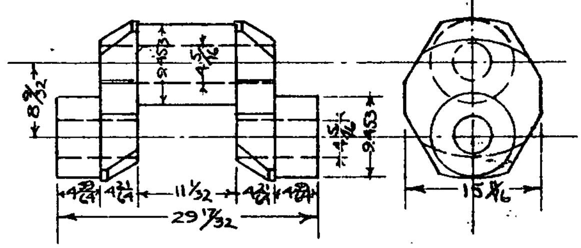
To illustrate paper on "Torsional Vibration in the Diesel Engine," by Frank M. Lewis, Esq., Member.

SOLID FORGED SHAFT



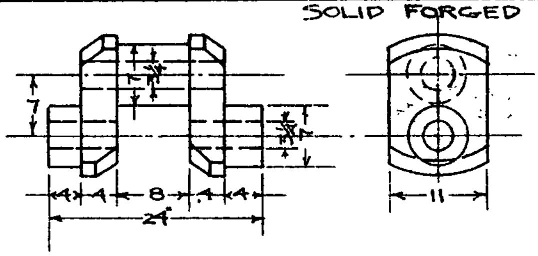
MEAN BEARING CLEAR.	STIFFNESS RATIO
.005	1.02
.007	.985
.011	.967
FREE SHAFT	.943
BY GEIGERS FORM.	1.05

SOLID FORGED SHAFT



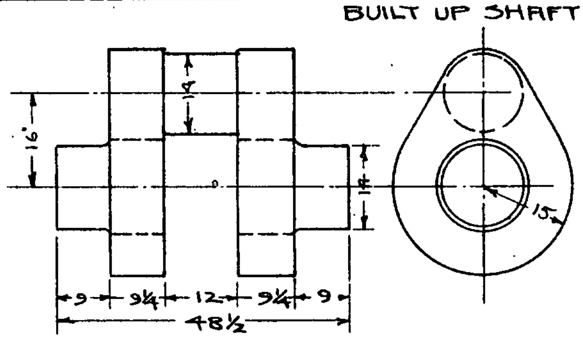
MEAN BEARING CLEAR.	STIFFNESS RATIO
.004	1.02
.006	1.002
FREE SHAFT	.940
BY GEIGERS FORM.	1.05

SOLID FORGED SHAFT

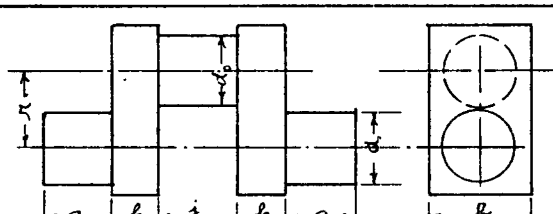


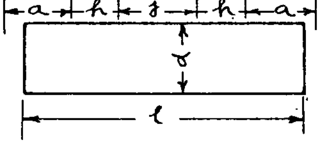
SHAFT TESTED IN BEARINGS—CLEARANCE UNKNOWN—STIFFNESS RATIO 1.17

BUILT UP SHAFT CAST WEBS AND CRANKPIN



SHAFT TESTED FREE
STIFFNESS RATIO 1.12





GEIGERS FORMULA FOR STIFFNESS OF CRANKSHAFT

H_W = MOMENT OF INERTIA OF CRANKSHAFT
H_K = MOMENT OF INERTIA OF CRANKPIN

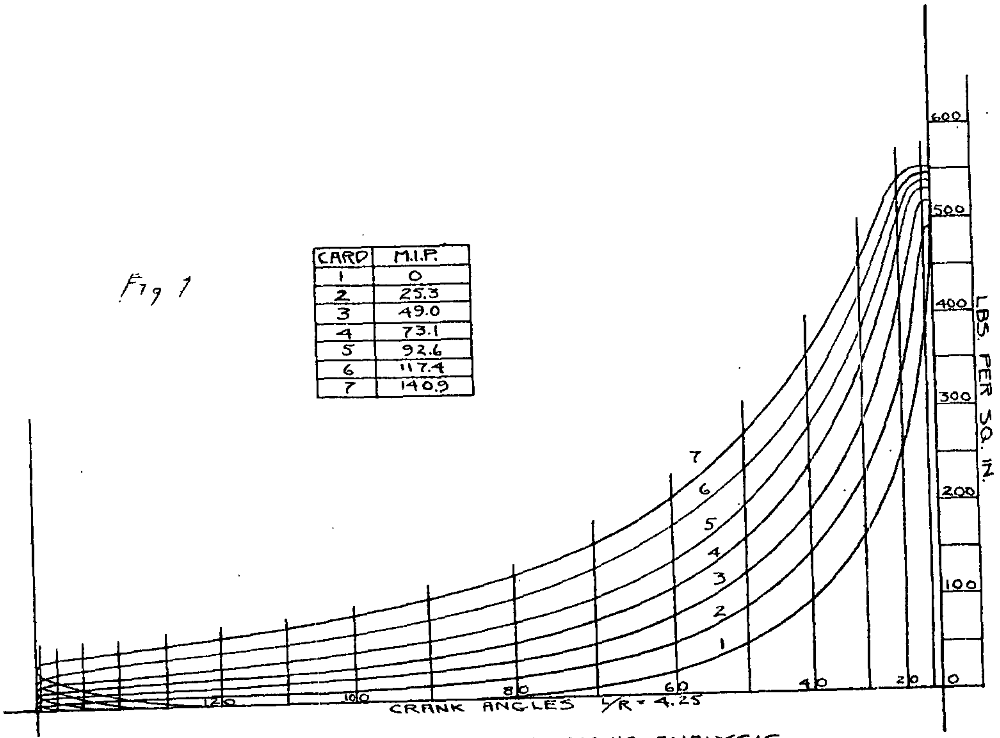
$\gamma = 0$ FOR $b/d = 1.6$ AND $r/d = 1.2 - .92$
$\gamma = .4$ FOR $b/d = 1.49$ AND $r/d = .84$
$H_A = \frac{h b^3}{12}$

EQUIVALENT LENGTH – SHAFT OF SAME ELASTICITY AND DIA $l = 2a + 4h + (j + .4h)\frac{H_W}{H_K} + .113(r - \gamma d)\frac{H_W}{H_A}$

$$\text{STIFFNESS RATIO} = \frac{M/\theta \text{ OF ACTUAL SHAFT}}{M/\theta \text{ OF SHAFT WITHOUT CRANKS OF THE SAME LENGTH AND DIA.}}$$

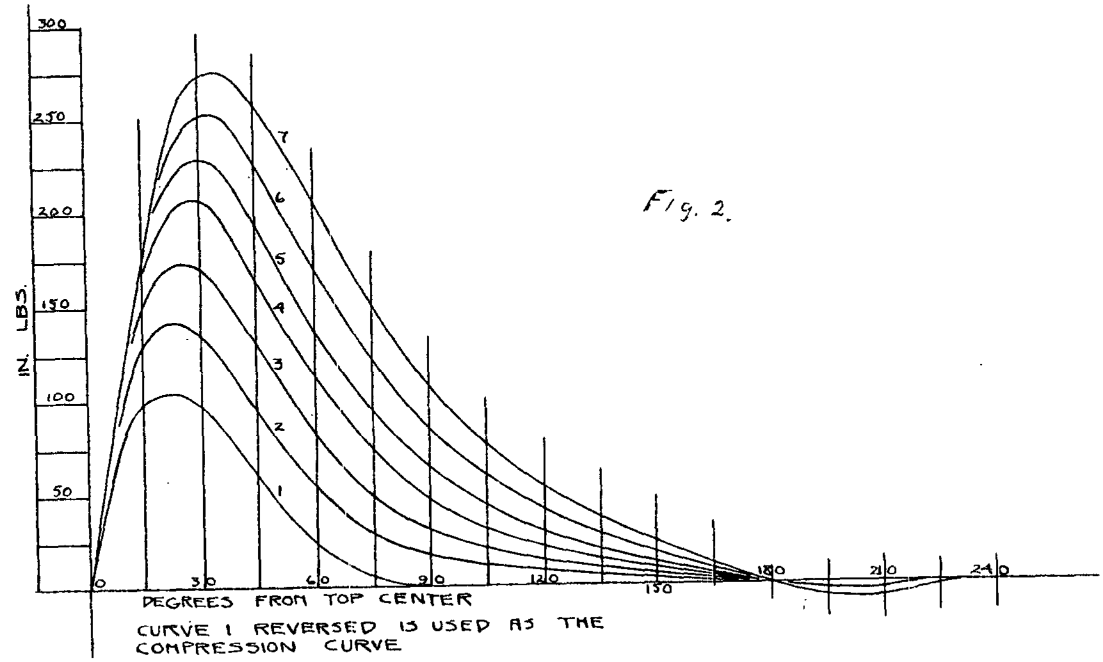
To illustrate paper on "Torsional Vibration in the Diesel Engine," by Frank M. Lewis, Esq., Member.

Fig 1



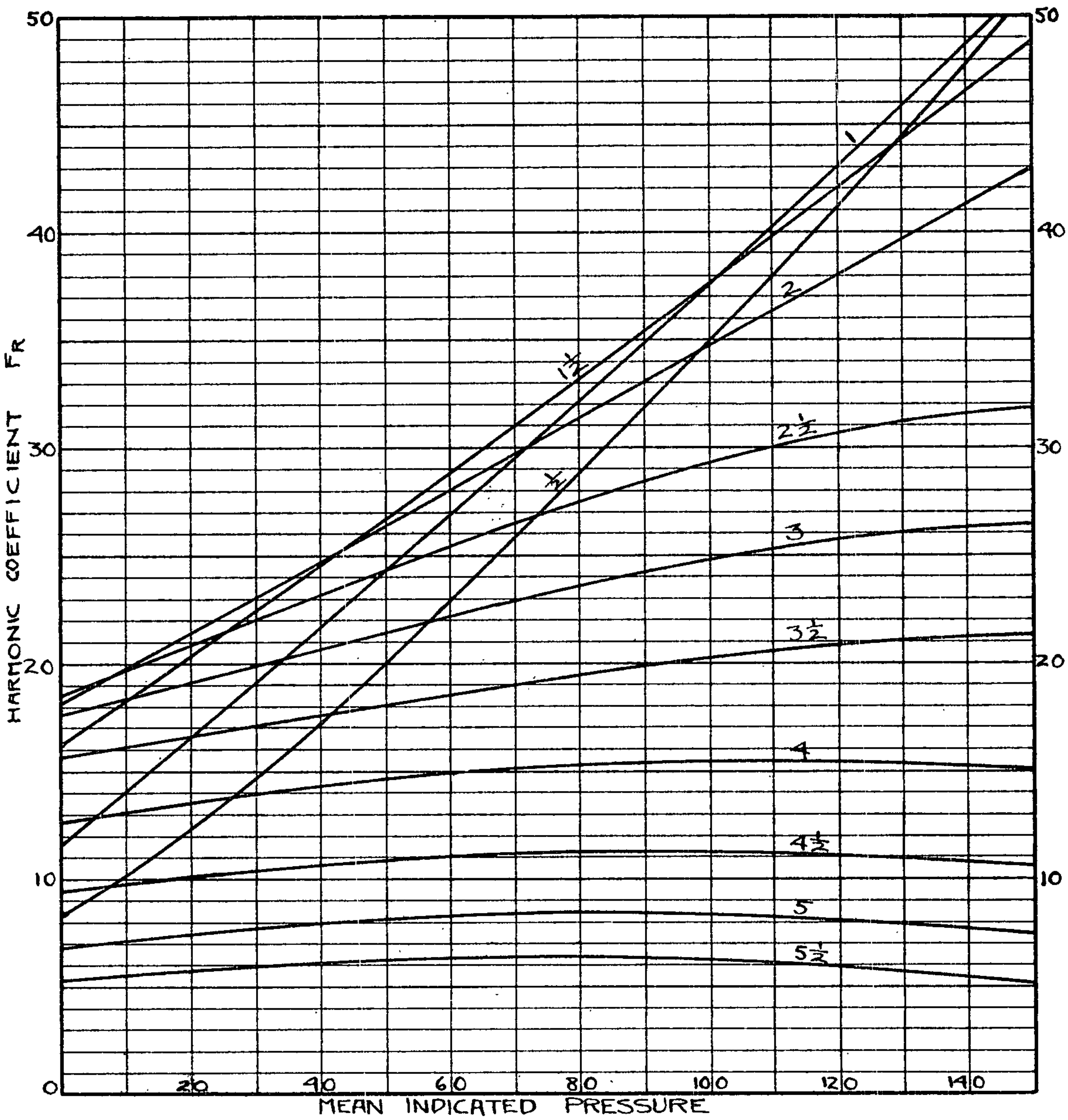
DIESEL CARDS FOR HARMONIC ANALYSIS

Fig. 2.



TURNING EFFORT CURVES FOR DIESEL CARDS

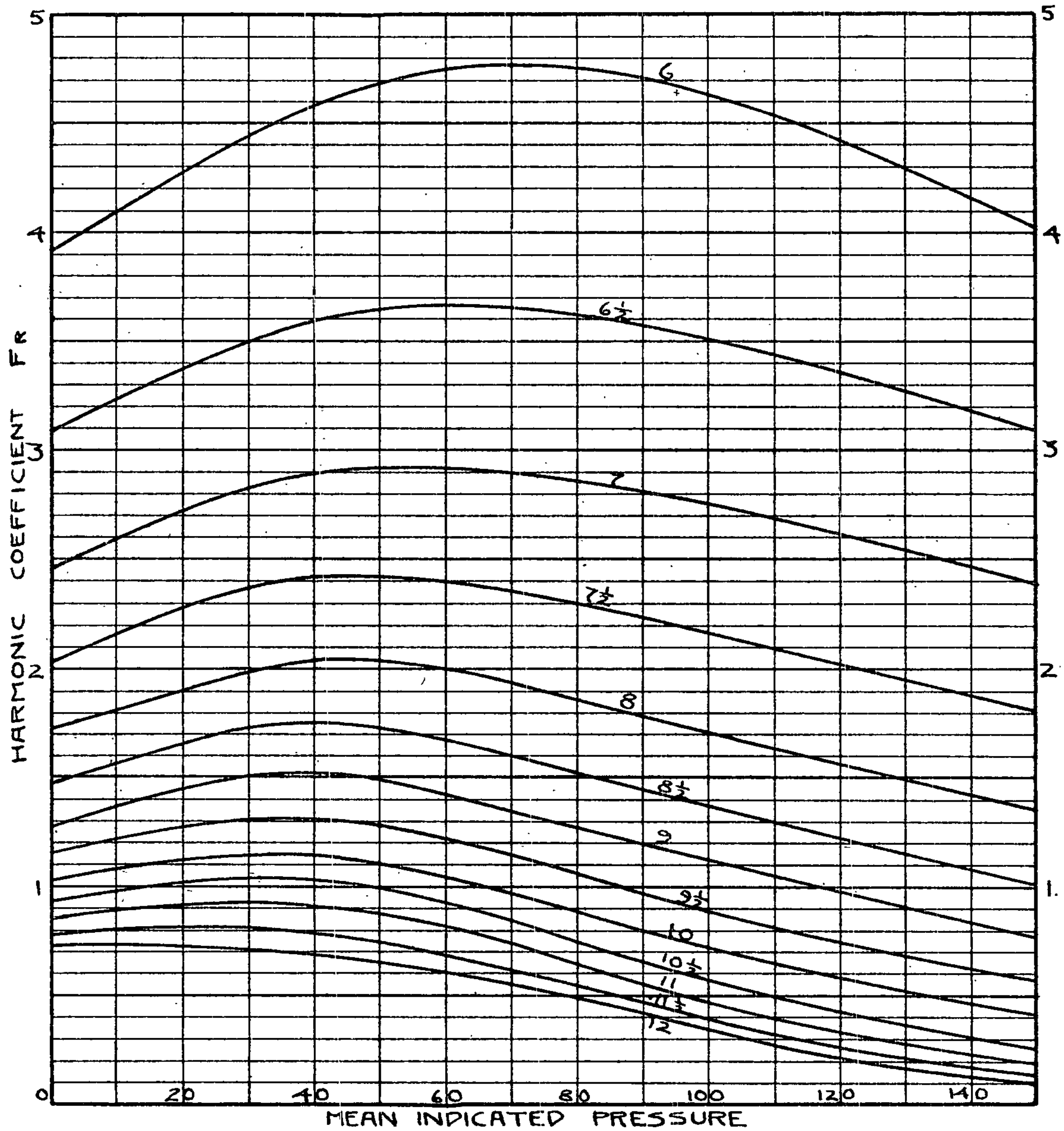
To illustrate paper on "Torsional Vibration in the Diesel Engine," by Frank M. Lewis, Esq., Member.



RESULTANT HARMONIC COEFFICIENTS ORDERS ½ TO 5½
4 CYCLE DIESEL ENGINE

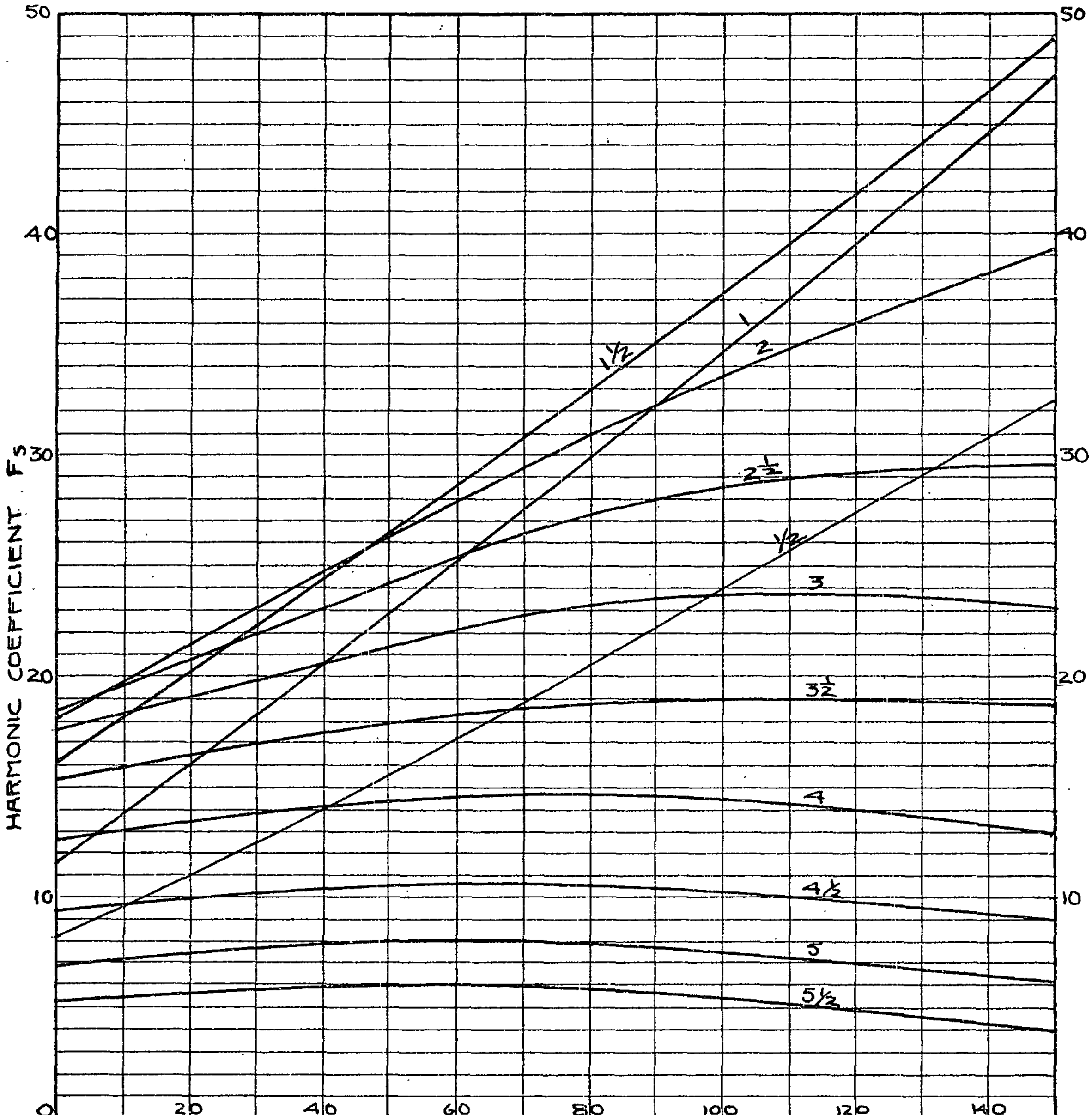
HARMONIC TORQUE = F_R × CYLINDER AREA × CRANK RADIUS
INCH POUND UNITS $F_R = \sqrt{F_S^2 + F_C^2}$

To illustrate paper on "Torsional Vibration in the Diesel Engine," by Frank M. Lewis, Esq., Member.



RESULTANT HARMONIC COEFFICIENTS ORDERS 6 TO 12

To illustrate paper on "Torsional Vibration in the Diesel Engine," by Frank M. Lewis, Esq., Member.



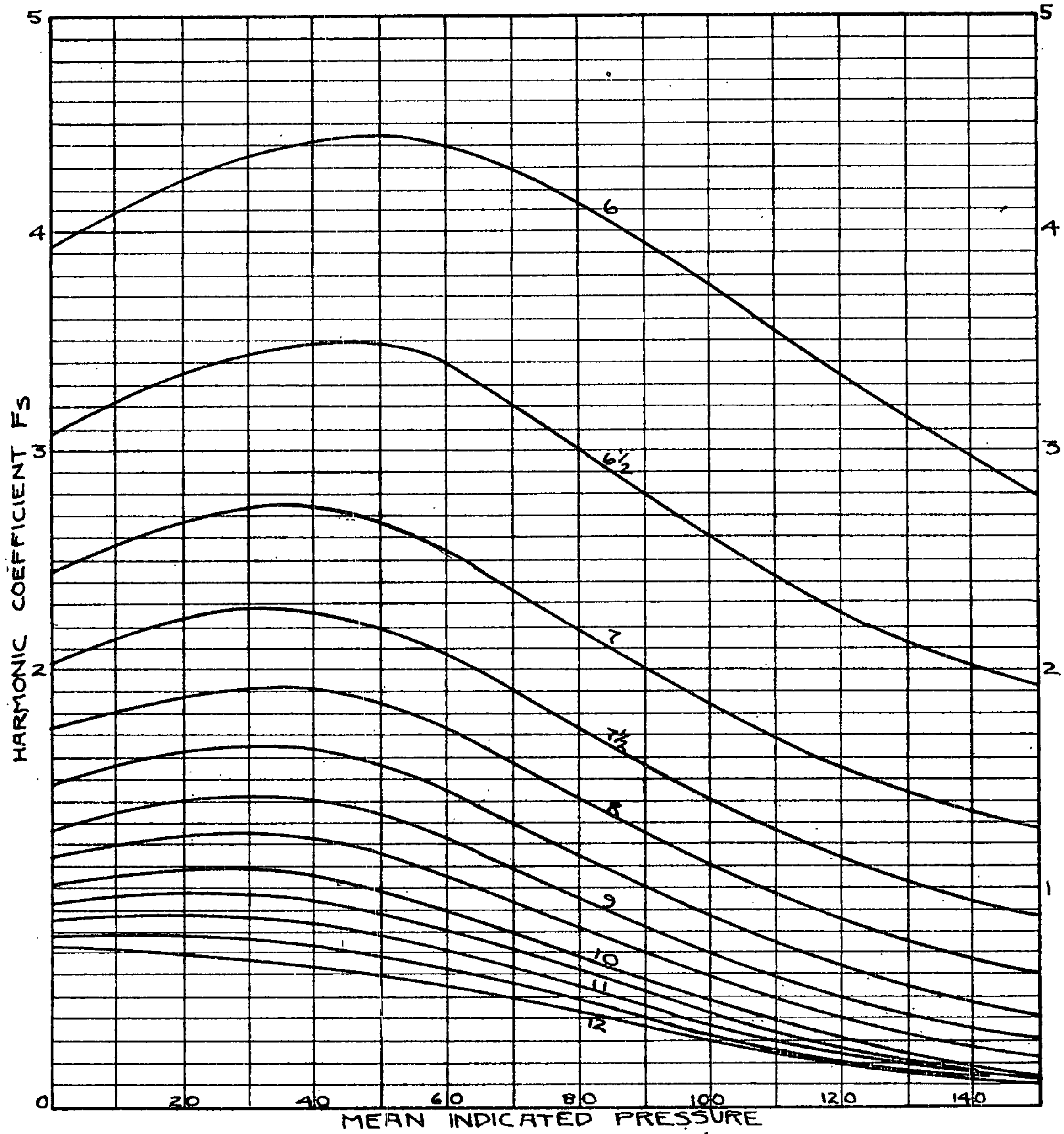
MEAN INDICATED PRESSURE

HARMONIC COEFFICIENTS – SINE TERMS

ORDERS 1/2 TO 5 1/2

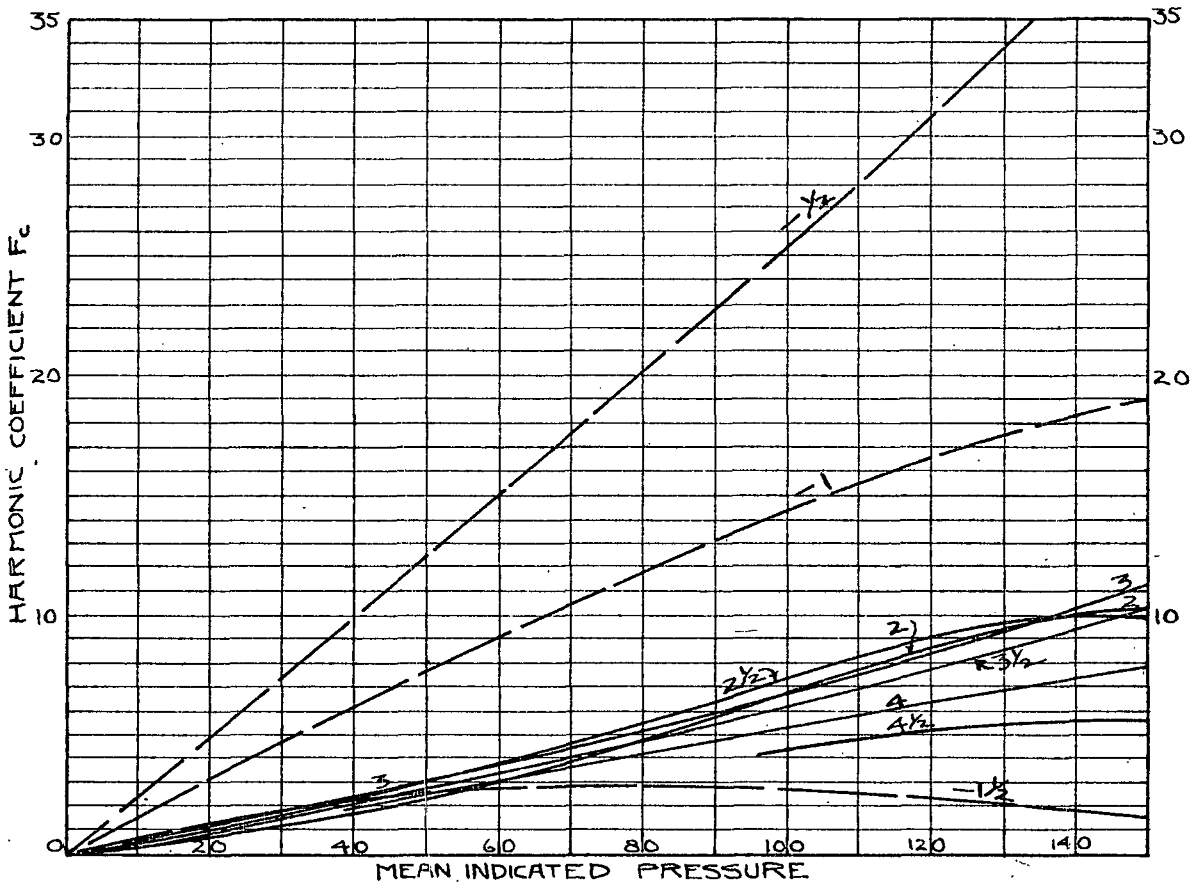
REFERENCE POINT – TOP DEAD CENTER BEFORE FIRING STROKE

To illustrate paper on "Torsional Vibration in the Diesel Engine,"
by Frank M. Lewis, Esq., Member.



MEAN INDICATED PRESSURE
HARMONIC COEFFICIENTS - SINE TERMS
ORDERS 6 TO 12

To illustrate paper on "Torsional Vibration in the Diesel Engine," by Frank M. Lewis, Esq., Member.



HARMONIC COEFFICIENTS COSINE TERMS

ORDERS ½ TO 4½

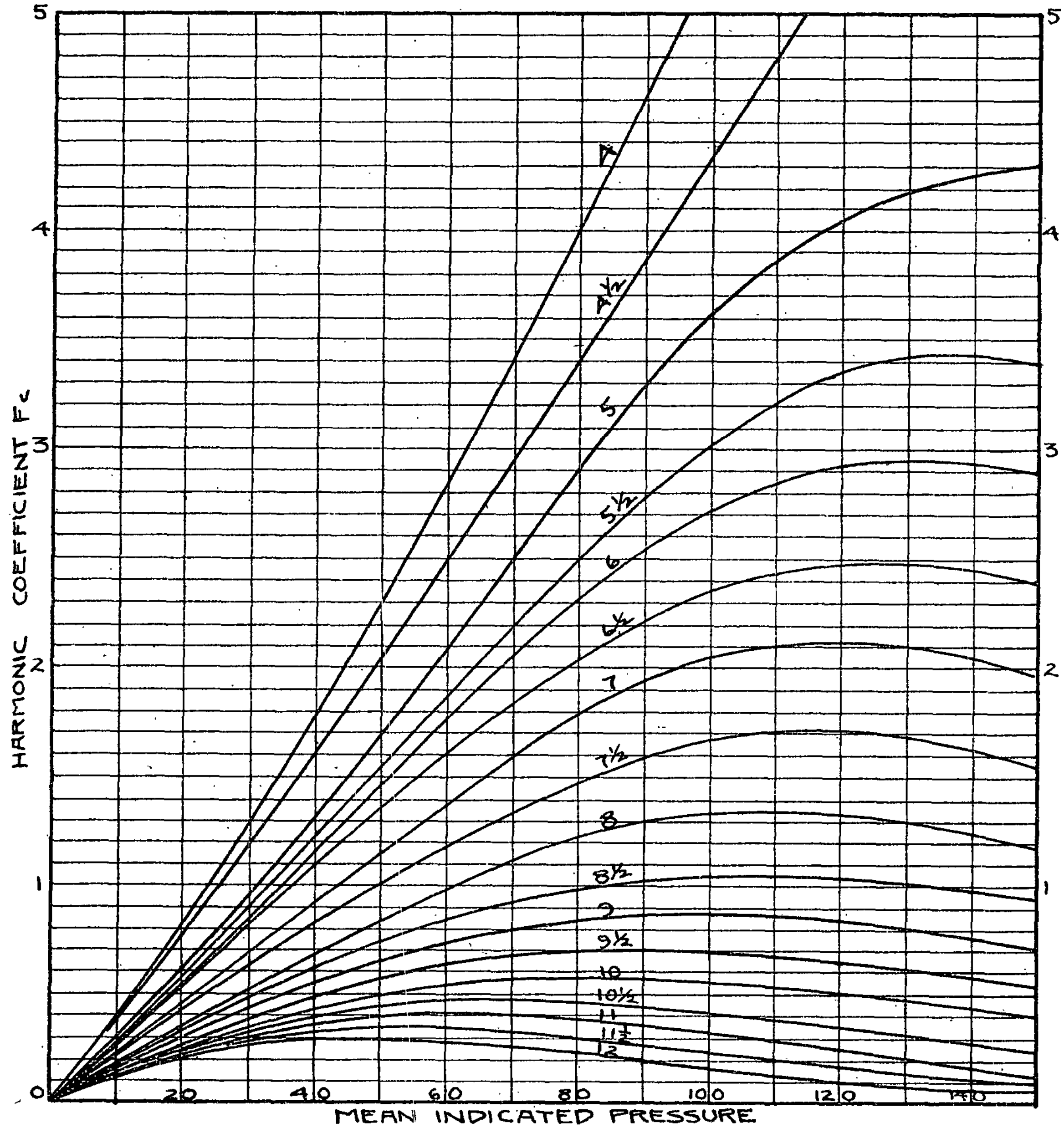
ORDERS ½, 1, 1½ ARE NEGATIVE

HARMONIC COMPONENTS OF TORQUE CURVE DUE TO INERTIA OF RECIPROCATING PARTS

HARMONIC COMPONENT (SINE ONLY) = $F \times \frac{W}{g} \times$ CRANK RADIUS$^2 \times$ ANGULAR VELOCITY^2

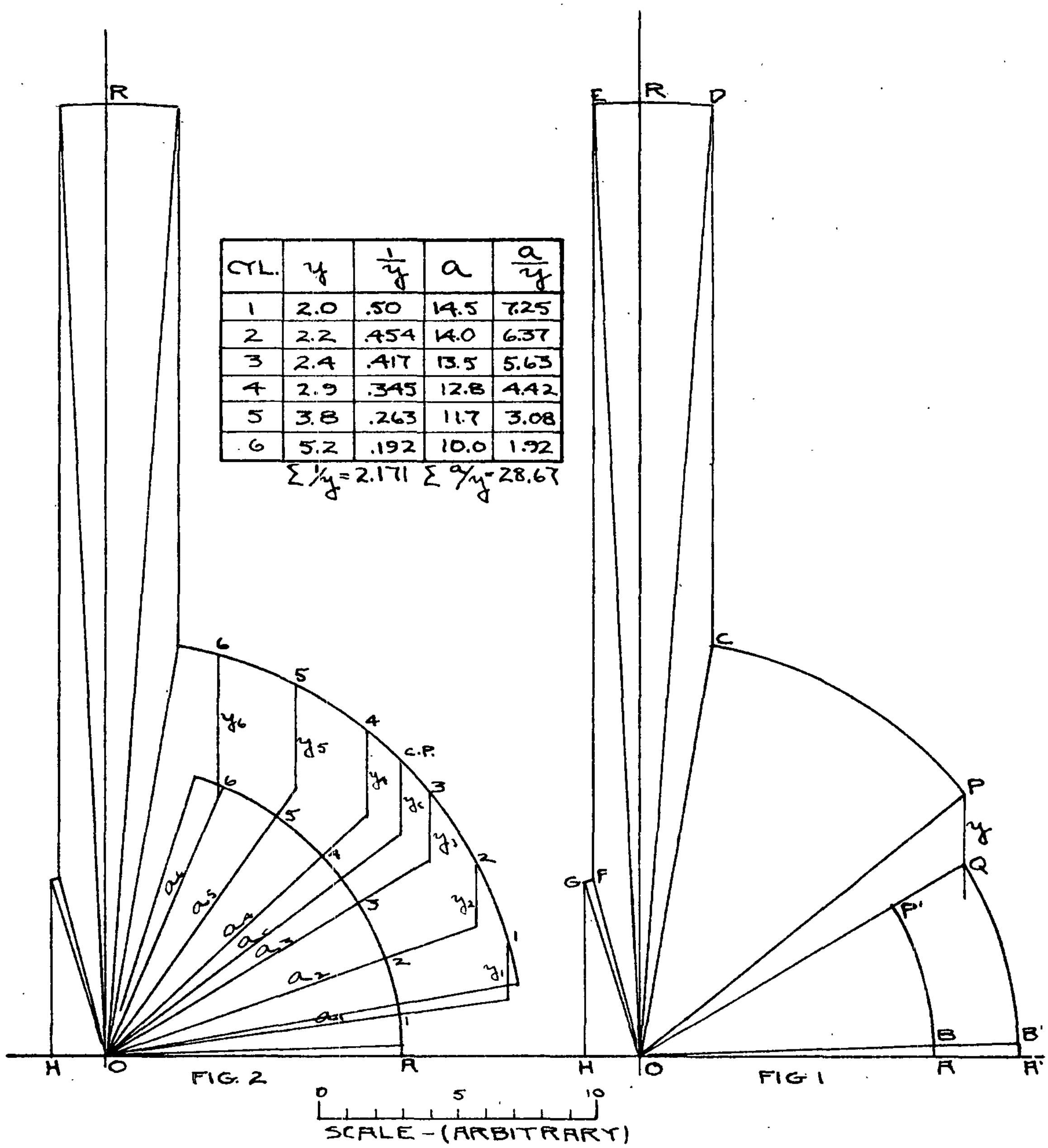
		F FOR CRANK CONNECTING ROD RATIO GIVEN								
ORDER	SIGN	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8
1	+	.06351	.06192	.06038	.05894	.05757	.05626	.05500	.05380	.05266
2	−	.50012	.50011	.50010	.50009	.50008	.50008	.50007	.50006	.50006
3	−	.19217	.18718	.13246	.17804	.17385	.16973	.16601	.16234	.15886
4	−	.01613	.01532	.01458	.01389	.01325	.01265	.01200	.01159	.01110
5	+	.00258	.00239	.00222	.00206	.00193	.00180	.00167	.00157	.00148
6	+	.00014	.00013	.00012	.00010	.00009	.00009	.00008	.00007	.00007
7	−	.00004	.00004	.00003	.00003	.00002	.00002	.00002	.00002	.00002
8	−	.00001	.00001	.00001	.00001	.00000	.0	.0	.0	.0

To illustrate paper on "Torsional Vibration in the Diesel Engine,"
by Frank M. Lewis, Esq., Member.



HARMONIC COEFFICIENTS – COSINE TERMS
ORDERS 4 TO 12

To illustrate paper on "Torsional Vibration in the Diesel Engine,"
by Frank M. Lewis, Esq., Member.

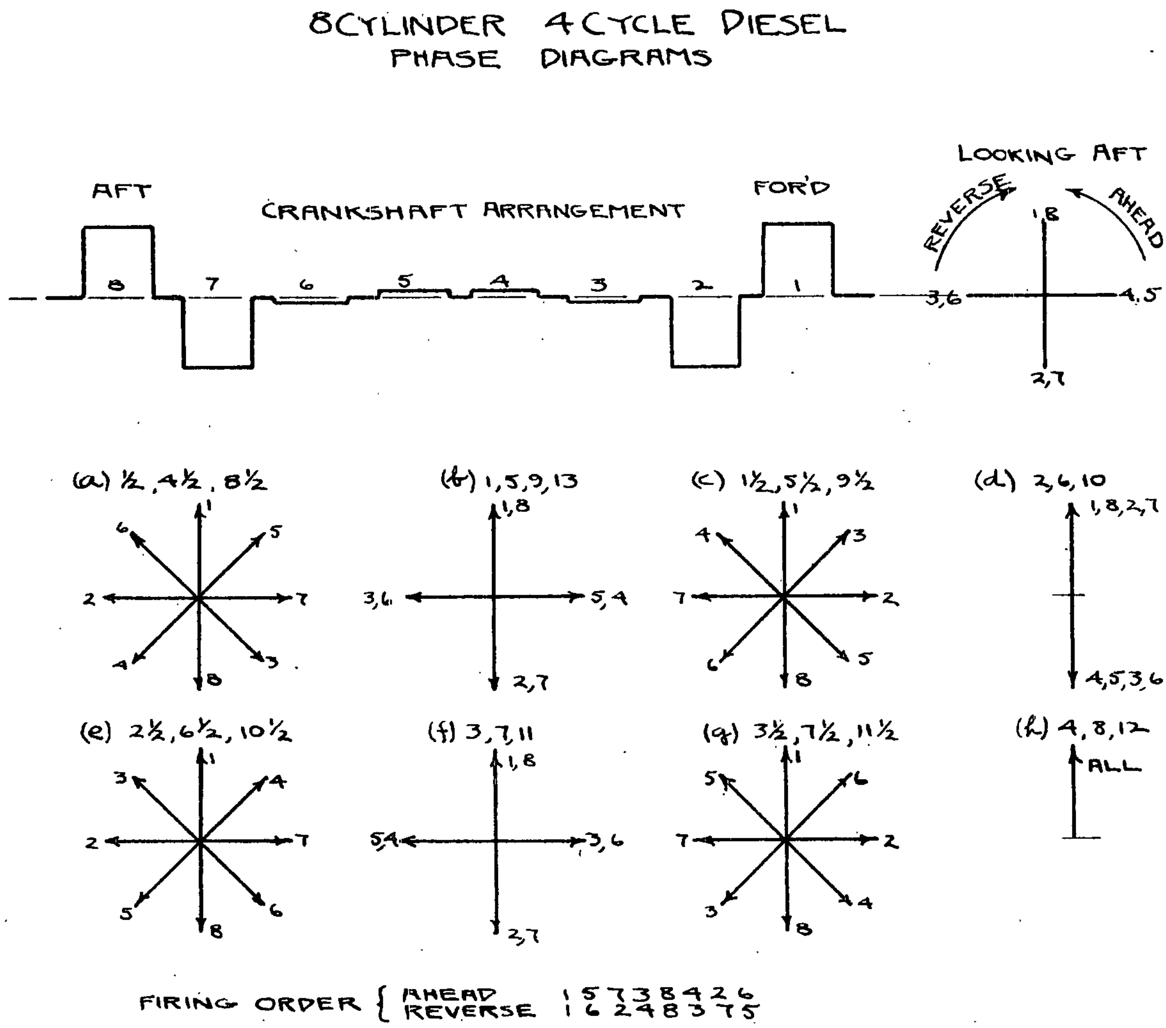


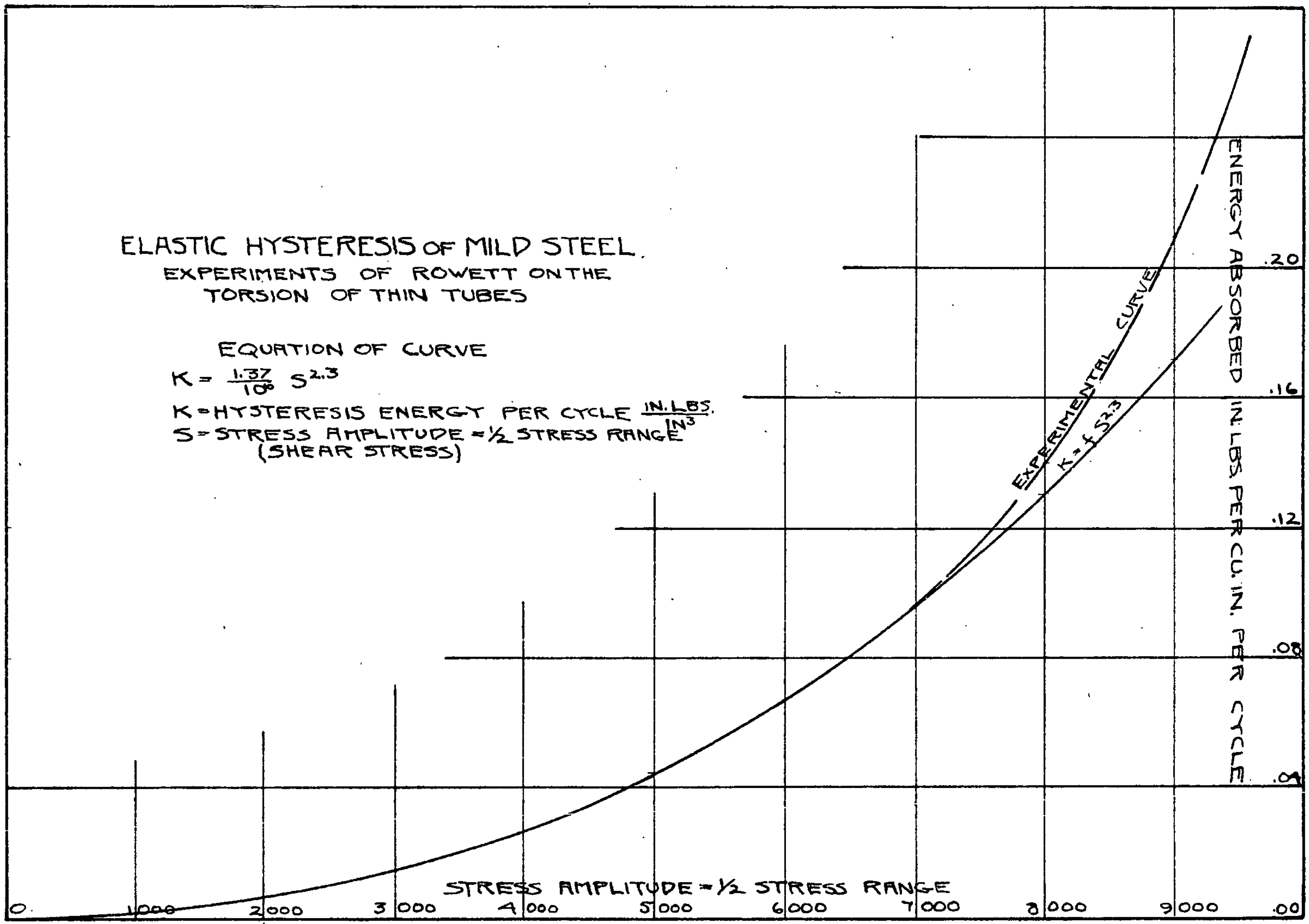
CYL.	y	$\frac{1}{y}$	a	$\frac{a}{y}$
1	2.0	.50	14.5	7.25
2	2.2	.454	14.0	6.37
3	2.4	.417	13.5	5.63
4	2.9	.345	12.8	4.42
5	3.8	.263	11.7	3.08
6	5.2	.192	10.0	1.92

$\Sigma \, 1/y = 2.171$ $\Sigma \, a/y = 28.67$

FORCED VIBRATION

To illustrate paper on "Torsional Vibration in the Diesel Engine," by Frank M. Lewis, Esq., Member.





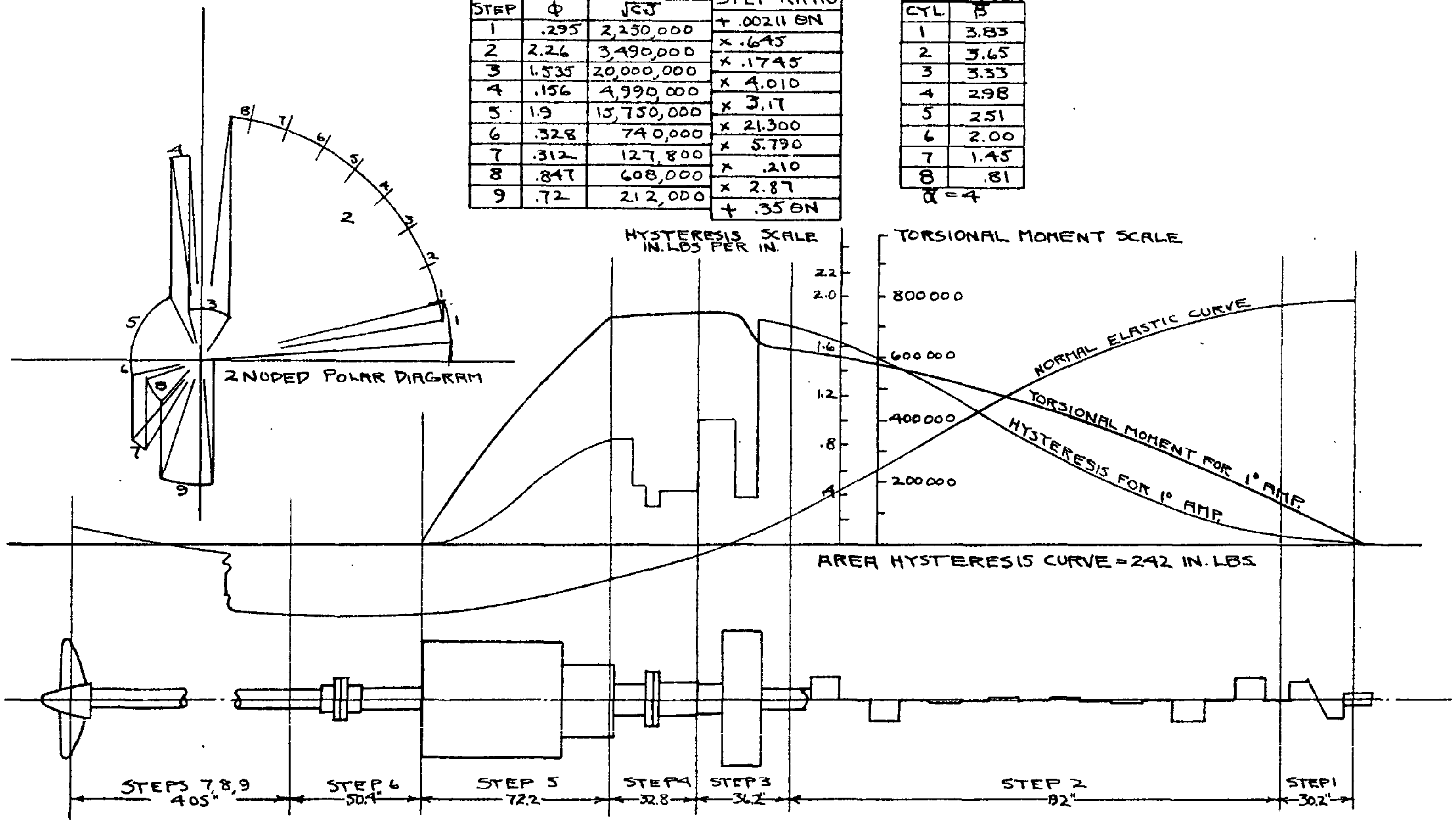
Transactions Society Naval Architects and Marine Engineers, Vol. 33, 1925. *Plate 98*

To illustrate paper on "Torsional Vibration in the Diesel Engine," by Frank M. Lewis, Esq., Member.

STEP	Φ	$\sqrt{CJ}$	STEP RATIO
1	.295	2,250,000	+ .00211 ON
2	2.26	3,490,000	× .645
3	1.535	20,000,000	× .1745
4	.156	4,990,000	× 4.010
5	1.9	15,750,000	× 3.17
6	.328	740,000	× 21.300
7	.312	127,800	× 5.790
8	.847	608,000	× .210
9	.72	212,000	× 2.87
			+ .35 ON

CYL.	β
1	3.83
2	3.65
3	3.53
4	2.98
5	2.51
6	2.00
7	1.45
8	.81

$\alpha = 4$



SUBMARINE ENGINE — AMPLITUDE OF VIBRATION

Transactions Society Naval Architects and Marine Engineers, Vol. 33, 1925.

To illustrate paper on "Torsional Vibration in the Diesel Engine," by Frank M. Lewis, Esq., Member.

Plate 99

Transactions Society Naval Architects and Marine Engineers, Vol. 33, 1925. Plate 100

To illustrate paper on "Torsional Vibration in the Diesel Engine," by Frank M. Lewis, Esq., Member.

SUBMARINE ENGINE – AMPLITUDE OF VIBRATION

1	2	3	4	5	6	7	8	9	10	11	12
ORDER OF CRITICAL	R.P.M	Q	MEAN ENG. TORQUE	TORQUE PER CYL.	M.I.P.	F_R	HARMONIC TORQUE	$\Sigma\bar{\beta}$	K_2	α CALCULATED AMPLITUDE	α OBSERVED BY TORSIOGRAPH
4½	433	129500	168200	21020	123	11.0	11810	2.36	384	1.42	A
5	390	105000	143700	17900	105	8.3	8920	.488	60	.34	B
5½	355	87100	125800	15700	92	6.4	6870	5.13	485	1.71	1.30
6	325	73000	111700	13960	82	4.7	5150	1.08	77	.41	B
6½	300	62000	100700	12590	74	3.6	3870	5.13	273	1.09	.70
7	280	54100	92800	11600	68	2.9	3110	.488	21	.15	B
7½	260	46600	85300	10660	63	2.4	2580	2.36	84	.44	B
8	244	41100	79800	9975	59	2.0	2150	20.56	610	2.03	1.40
8½	230	36500	75200	9400	55	1.7	1820	2.36	59	.34	B
9	217	32400	71100	8880	53	1.5	1610	.488	11	.09	B
9½	205	28900	67600	8450	50	1.3	1400	5.13	99	.50	B

NOTE 8 CYL. 4 CYCLE NELSECO ENG.
600 B.H.P AT 380 R.P.M. 15" STROKE 13.5" BORE
NATURAL FREQUENCY 1950 VIB. PER MIN. (2 NODES)

$$K_2 = \frac{M\pi^2\alpha}{180\,\bar{\alpha}}\Sigma\bar{\beta} = .01376\, M\Sigma\bar{\beta}$$

HYSTERESIS ENERGY 242 IN. LBS.

$$\text{COLUMN } 11 = \sqrt[1.3]{\frac{K_2}{242}}$$

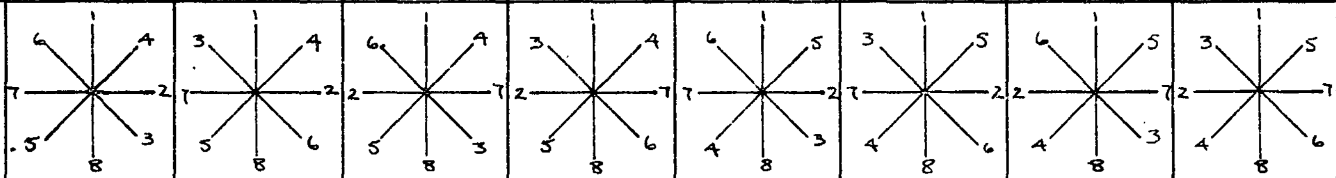
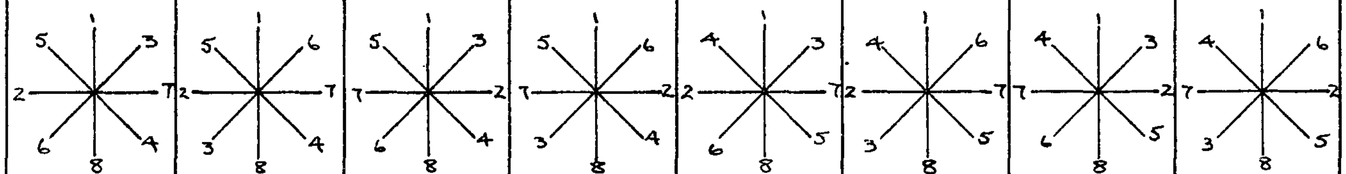
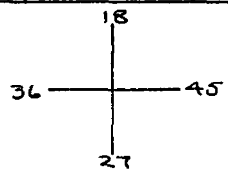
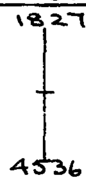
A – ENGINE NOT OPERATED ABOVE 405 R.P.M.

B – TORSIOGRAPH AMPLITUDE TOO SMALL TO BE OBSERVED OR CONFUSED BY OTHER VIBRATIONS

Transactions Society Naval Architects and Marine Engineers, Vol. 33, 1925.

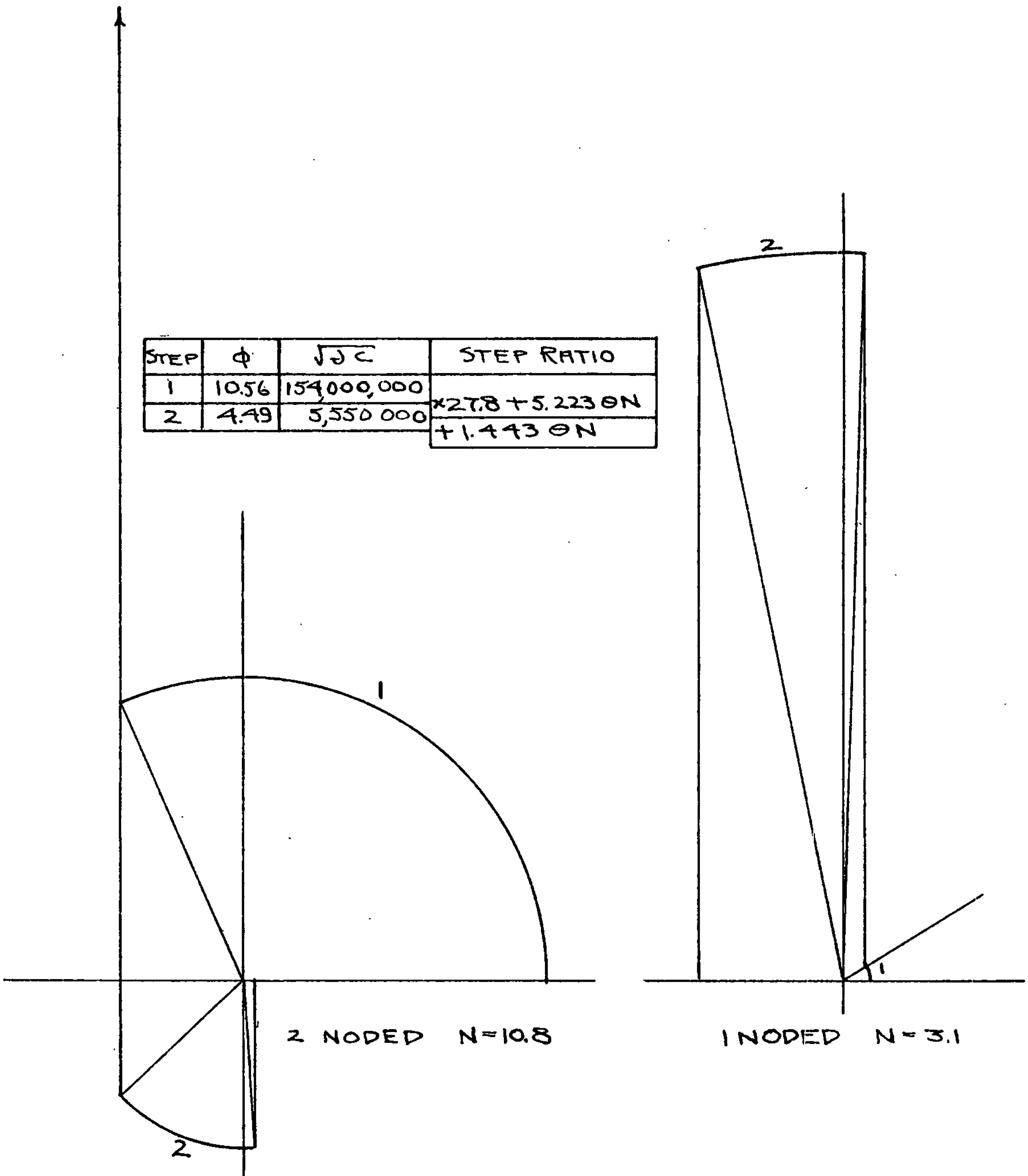
To illustrate paper on "Torsional Vibration in the Diesel Engine,"
by Frank M. Lewis, Esq., Member.

8 CYLINDER 4 CYCLE DIESEL—Σβ FOR THE 8 FIRING ORDERS

ARRGT NO	1	2	3	4	5	6	7	8
AHEAD	14238576	14268573	14738526	14768523	15238476	15268473	15738426	15768423
REVERSE	16758324	13758624	16258374	13258674	16748325	13748625	16248375	13248675
MINORS ½, 4½, 8½ AND 3½, 7½, 11½  Σβ =	4.17	4.49	2.52	5.13	3.31	3.75	2.36	5.02
MINORS 1½, 5½, 9½ AND 2½, 6½, 10½  Σβ =	3.75	3.31	5.02	2.36	4.49	4.17	5.13	2.52
MINORS 1, 5, 9, 13 AND 3, 7, 11  Σβ =	.488							
MINORS 2, 6, 10  Σβ =	1.08							
MAJORS 4, 8, 12  Σβ =	20.56							

To illustrate paper on "Torsional Vibration in the Diesel Engine," by Frank M. Lewis, Esq., Member.

STEP	ϕ	$\sqrt{JC}$	STEP RATIO
1	10.56	154,000,000	×27.8 + 5.223 θN
2	4.49	5,550 000	+1.443 θN



"SEEKONK" POLAR DIAGRAMS