Algebra Term Test 3 Sample Questions

Problem 1.

Give the multiplication table for the group of units of Z_{12}. What group is it isomorphic to?

Problem 2.

Describe all solutions to the congruence

$$36x \equiv 15 \ (mod) \ 24$$

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Problem 3.

Use Euler's theorem to calculate

$$7^{98} \ (mod) \ 24$$

Problem 4.

a) Show in a commutative ring with identity that every maximal ideal is prime.

b) Show that every prime ideal is maximal in Z_6.

c) Find all the maximal ideals of Z_6.

Problem 5.

a) Show that $x^3 + x^2 + 1$ is irreducible over Z_2.

b) Show how to construct a field of eight elements.

Problem 6.

Mark the following T (true) or F (false).

1) If F is a field, the units of $F[x]$ are preciseley the units of F.

2) The product of two non units in Z_n may be a unit.

3) $x^2 + 3$ is irreducible in Z_7.

4) If F is a field, then every ideal in $F[x]$ is a principal ideal.

5) $Q[x]/ < x^2 - 5x + 6 >$ is a field.