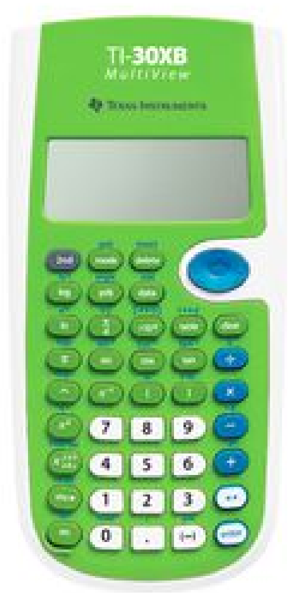
Revision :
Fluid mechanics

FAQ 1

- Can we take other calculators into the exam?

- No, sorry that you have to use the “green one” (TI-30XB)



FAQ 2

- If your answer to a later part of a question is wrong because of a numerical slip-up in an earlier part, will you lose marks?

- No. If you show your working, you will still get the credit for the later part of the question.

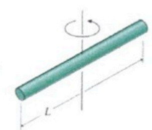
FAQ 3

- Will you lose marks for not quoting correct numbers of significant figures?

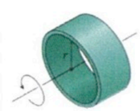
- Only in extreme cases. If you quote something similar to the data given in the question, you will not lose marks.

Formula sheet

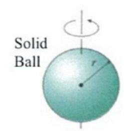
LINEAR MECHANICS		ROTATIONAL MECHANICS	
$v = v_0 + at$	$x - x_0 = \frac{1}{2}(v_0 + v)t$	$\omega = \omega_0 + \alpha t$	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$v^2 = v_0^2 + 2a(x - x_0)$	$x - x_0 = v_0 t + \frac{1}{2}at^2$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$\vec{F}_{net} = m\vec{a} = \frac{d\vec{p}}{dt}$	$\vec{w} = m\vec{g}$	$\vec{\tau}_{net} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$	$s = r\theta \quad \omega = \frac{d\theta}{dt} \quad \alpha = \frac{d\omega}{dt}$
$W = \vec{F} \cdot \Delta\vec{r} = F\Delta r \cos\theta$	$f_s \le \mu_s n \quad f_k = \mu_k n$	$\lvert\vec{\tau}\rvert = \lvert\vec{r}\times\vec{F}\rvert = rF\sin\theta$	$\vec{F}_r = m\vec{a}_r = \frac{mv^2}{r}$
$W = \int_{x_1}^{x_2} F\,dx$	$F_s = -kx$ $\Delta U_s = \frac{1}{2}k(x_f^2 - x_i^2)$	$I = \sum_i m_i r_i^2$	$v = r\omega \qquad a_t = r\alpha$ $\vec{a}_{net} = \vec{a}_r + \vec{a}_t$
$W_{net} = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$	$W_c = -\Delta U \quad U_g = mgy$	$K_R = \frac{1}{2}I\omega^2$	$K_{roll} = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv_{cm}^2$
$\Delta K + \Delta U = W_{nc} = -F_{fric}d$	$P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$	$P_R = \frac{dW}{dt} = \vec{\tau}\cdot\vec{\omega}$	$x_{cm} = \frac{\sum_i m_i x_i}{\sum_i m_i}$
$\vec{p} = m\vec{v}$ $\vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_{1,f} + \vec{p}_{2,f}$	$\vec{J} = \int_{t_1}^{t_2} \vec{F}\,dt = \Delta\vec{p} = \vec{F}\Delta t$	$\vec{L} = I\vec{\omega}$ $\vec{L}_{1,i} + \vec{L}_{2,i} = \vec{L}_{1,f} + \vec{L}_{2,f}$	$\lvert\vec{L}\rvert = \lvert\vec{r}\times\vec{p}\rvert = mvr\sin\theta$



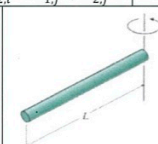
$I = \frac{1}{[illegible]}ML^2$



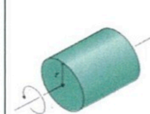
$I = MR^2$



$I = \frac{2}{[illegible]}MR^2$



$I = \frac{1}{[illegible]}ML^2$



$I = \frac{1}{[illegible]}MR^2$

FLUID MECHANICS			
$p = \frac{F}{A} \quad F_B \propto \rho V g$	$p = p_0 + \rho g h \quad \rho = \frac{m}{V}$	$p + \frac{1}{2}\rho v^2 + \rho g y = const$	$A_1 v_1 = A_2 v_2 = const$

THERMODYNAMICS			
$\frac{\Delta L}{L} = \alpha\Delta T \quad \frac{\Delta V}{V} = \beta\Delta T$	$pV = nRT = Nk_BT$	$\frac{1}{2}m\,\overline{v^2} = \frac{3}{2}k_BT$	$n = \frac{N}{N_A} = \frac{m}{M}$
$Q = mc\Delta T \quad Q = m\,L$	$PV = \frac{1}{3}m\,\overline{v^2}$	$H = \frac{Q}{\Delta t} = -kA\frac{dT}{dx}$	$P_{net} = \sigma Ae(T^4 - T_{amb}^4)$

ELECTRICITY			
$F = k_e\frac{q_1 q_2}{r^2}$	$E = k_e\frac{q}{r^2} = \frac{F_e}{q}$	$i = \frac{\Delta q}{\Delta t}\ ,\ i = \frac{V}{R}$	$P = Vi = i^2R = \frac{V^2}{R}$
$V_b - V_a = \frac{1}{q}(U_b - U_a) = \frac{-W_{ba}}{q}$	$E = -\frac{V_b - V_a}{d}$	$q = CV$	$v = \sqrt{\frac{F}{\mu}} \qquad f_n = \frac{n}{2L}v$
$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$ parallel	$R_{eff} = R_1 + R_2 + R_3 + \cdots$ series	$C_{eff} = C_1 + C_2 + C_3 + \cdots$ parallel	$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$ parallel

Formula sheet

FLUID MECHANICS			
$p = \frac{F}{A}$ $F_B \propto \rho V g$	$p = p_0 + \rho g h$ $\rho = \frac{m}{V}$	$p + \frac{1}{2}\rho v^2 + \rho g y = const$	$A_1 v_1 = A_2 v_2 = const$

Formula sheet

Acceleration due to gravity at the earth's surface	g	9.80 m/s^2
Avogadro's constant	N_A	6.02 x 10^{23} mol^{-1}
Boltzmann's constant	k_B	1.38 x 10^{-23} J/K
Ideal gas constant	R	8.31 J/mol K
Stefan constant	σ	5.67 x 10^{-8} W/m^2 K^4
Atomic Mass Unit	u	1.66 x 10^{-27} kg
Density of water		1.00 x 10^3 kg/m^3
Density of helium		0.18 kg/m^3
Density of concrete		2200 kg/m^3
Density of Styrofoam		160 kg/m^3
Co-efficient of linear expansion of steel		12 x 10^{-6}
Specific heat of aluminium		900 J/kg °C
Specific heat of ice		2050 J/kg °C
Specific heat of iron		447 J/kg °C
Specific heat of Styrofoam		1300 J/kg °C
Specific heat of water		4186 J/kg °C
Specific heat of wood		1400 J/kg °C
Latent heat of fusion of ice		3.33 x 10^5 J/kg
Latent heat of vaporisation of water		2.26 x 10^6 J/kg
Thermal Conductivity of iron		80.4 W/m °C
Thermal Conductivity of water		0.61 W/m °C
Thermal Conductivity of Styrofoam		0.029 W/m °C
Thermal Conductivity of wood		0.11 W/m °C
Atomic mass of argon, Ar		40 u
Molecular mass of hydrogen, H_2		2.0 u
Molecular mass of nitrogen, N_2		28.0 u
Molecular mass of oxygen, O_2		32.0 u

Conversion factors

1 atm ≡ 1.013 x 10^5 Pa

K = °C + 273

1 litre = 10^{-3} m^3

1 revolution per minute = 2 π radians per 60 seconds

Fluid Mechanics key facts (1/5)

- Basic definitions:

- Density $\rho = \frac{Mass}{Volume}$ [units: $kg\ m^{-3}$]

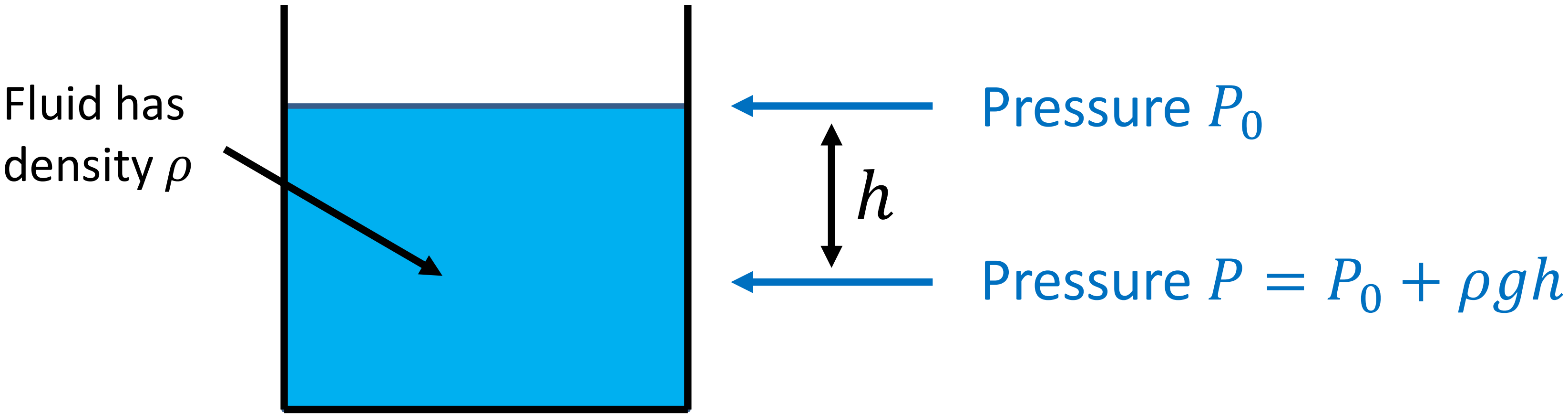
- Pressure $P = \frac{Force}{Area}$ [units: $N\ m^{-2}$ or Pa]

- Pressure can also be measured in atmospheres:
  $1\ atm = 1.013\ \times 10^5\ Pa$ [on formula sheet]

- Gauge pressure means the extra pressure above the atmospheric pressure

Fluid Mechanics key facts (2/5)

- A fluid at rest obeys hydrostatic equilibrium - where its pressure increases with depth to balance its weight : $P = P_0 + \rho g h$



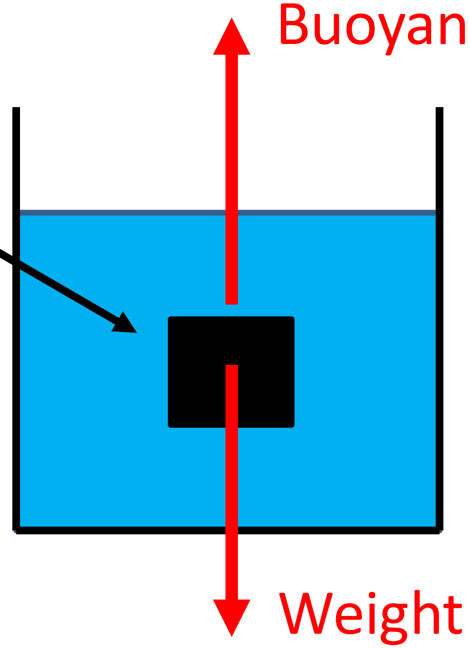
- Points at the same depth below the surface are all at the same pressure, regardless of the shape

Fluid Mechanics key facts (3/5)

- An object immersed in a fluid will feel a buoyancy force equal to the weight of the fluid displaced

Buoyancy force $= m_{fluid}\ g = \rho_{fluid}\ V\ g$

Object has volume V

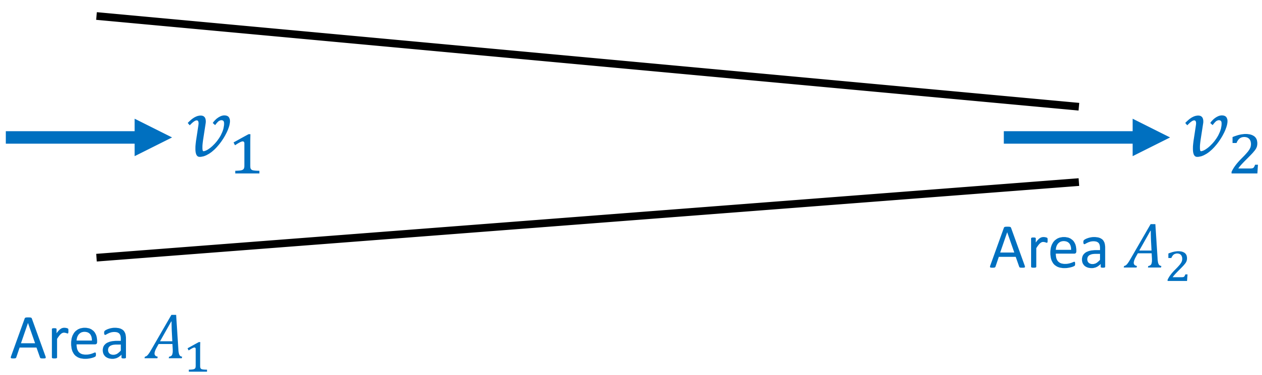


Buoyancy > Weight if $\rho_{object} < \rho_{fluid}$

Weight force $= m_{object}\ g = \rho_{object}\ V\ g$

Fluid Mechanics key facts (4/5)

- Flow of an incompressible fluid obeys the continuity equation : $v\,A = constant$ where $v =$ fluid velocity and $A =$ pipe area



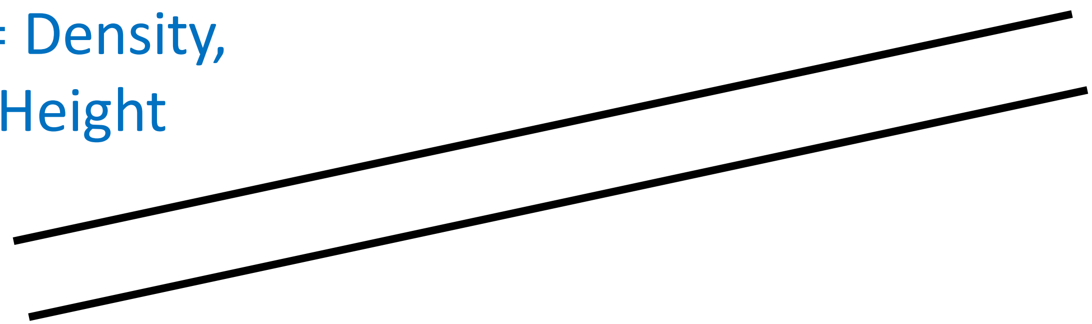
$$v_1\,A_1 = v_2\,A_2$$

- This equation results from conservation of mass. It's the same as saying $volume\ flow\ rate = constant$

Fluid Mechanics key facts (5/5)

- The pressure in a flowing fluid obeys Bernoulli's equation : $P + \frac{1}{2}\rho v^2 + \rho g h = constant$

P = Pressure, ρ = Density,
v = Velocity, h = Height



- This equation results from the conservation of energy

- For a horizontal pipe, $P + \frac{1}{2}\rho v^2 = constant$

Practice exam questions: Section A

2. A toy wooden boat floating in a container of water displaces 0.35 kg of water. If the boat is now placed in a container filled with a fluid whose density is 20% larger than that of water, how much fluid is displaced?

A. 0.35 kg
B. 0.42 kg
C. 0.28 kg
D. 0.70 kg

Weight of fluid displaced = weight of boat
Same weight is displaced before and after [A]

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3. A fluid flows with a velocity of 2 m/s in a horizontal tube of cross-sectional area of 10 cm^2. The cross-sectional area is then reduced to 5 cm^2. At this constriction:

A. the velocity **increases** and the pressure in the fluid **increases.**
B. the velocity **decreases** and the pressure in the fluid **increases**
C. the volume flow rate **decreases** and the pressure in the fluid **decreases**
D. the volume flow rate remains **constant** and the pressure in the fluid **decreases**

Continuity equation $v\,A = constant$: velocity increases / constant flow rate
Bernoulli's equation $P + \frac{1}{2}\rho v^2 = constant$: pressure decreases [D]

Practice exam questions: Section A

A10. You are designing a rectangular swimming pool, with length L, width W, and depth D. Which of these quantities will affect the pressure at the bottom of the pool when it is filled with water?

A. only D

B. only L and W

C. all three of them; L, W, and D

D. Not enough information to determine

Hydrostatic equilibrium only depends on depth : A

A11. An iron anchor is thrown into a lake. As the anchor descends to the bottom of the lake, how do the pressure on the anchor and the buoyant force on the anchor vary?

A. the pressure and buoyant force both increase

B. the pressure and buoyant force both decrease

C. the pressure increases, the buoyant force decreases

D. the pressure increases, the buoyant force is constant

Lake is in hydrostatic equilibrium : pressure increases with depth

Buoyancy force = weight of water displaced = constant : D

Practice exam questions: Section A

A12. The continuity equation in fluids shows that the quantity Av is conserved, where A is area and v is flow speed. The units of this quantity show that Av measures

A. the energy of the fluid flow

B. the volume flow rate

C. the pressure of fluid

D. the mass flow rate

Units of Av are $m^2 \times ms^{-1} = m^3s^{-1}$

This is volume/second, or volume flow rate : B

A13. A fluid flowing in a horizontal tube encounters a narrowing in the tube. In this narrow section

A. the velocity **increases** and the pressure in the fluid **increases.**

B. the velocity **decreases** and the pressure in the fluid **increases.**

C. the velocity **decreases** and the pressure in the fluid **decreases.**

D. the velocity **increases** and the pressure in the fluid **decreases.**

Same question as earlier: velocity increases (continuity equation) and pressure decreases (Bernoulli's equation) : D

Practice exam questions: Section B

B8. You unbend a paper clip made from 1.5 mm diameter wire and push the end against the wall. Calculate what force you must apply to give a pressure against the wall of 12×10^6 Pa

$$Pressure = \frac{Force}{Area}$$

$$Area = \pi r^2 = \pi \left(\frac{1.5 \times 10^{-3}}{2}\right)^2 = 1.77 \times 10^{-6}\, m^2$$

$$Force = Pressure \times Area = 12 \times 10^6 \times 1.77 \times 10^{-6} = 21\, N$$

2

Practice exam questions: Section B

B9. A vertical tube open at the top contains 5.0 cm of oil with density 820 kg/m^3, floating on 5.0 cm of water. Calculate the gauge pressure at the bottom of the tube.

Gauge pressure is the additional pressure above the atmospheric pressure

Hydrostatic equilibrium: $P = P_0 + \rho g h$

Gauge pressure $= \rho_{oil}\, g\, h_{oil} + \rho_{water}\, g\, h_{water} =$
$820 \times 9.8 \times 0.05 + 1000 \times 9.8 \times 0.05 = 890\ Pa$

2

Practice exam questions: Section B

B10. A steel drum has volume 0.23 m^3 and mass 16 kg. Determine whether the drum will float in water when it is filled with gasoline. *(ignore the volume of the steel walls of the drum)*

Weight of drum $= mg = 16 \times 9.8 = 157\ N$

Weight of gasoline $= \rho_{gasoline} V g =$
$860 \times 0.23 \times 9.8 = 1940\ N$

Total weight of drum + gasoline = 2097 N

Buoyancy force = Weight of water displaced $=$
$\rho_{water} V g = 1000 \times 0.23 \times 9.8 = 2250\ N$

Buoyancy force > Total weight, so drum floats

2

Practice exam questions: Section C

4. (a) Water discharges from a **horizontal** pipe at the rate of 5.00×10^{-3} m^3/s. At point A in the pipe where the cross-sectional area is 2.00×10^{-3} m^2, the absolute pressure is 1.60×10^5 Pa.

(i) What is the velocity of the water at point A?

Volume flow rate $= v\,A = 5 \times 10^{-3}\ m^3\ s^{-1}$

$$v = \frac{5 \times 10^{-3}}{2 \times 10^{-3}} = 2.5\ m\ s^{-1}$$

1

Practice exam questions: Section C

(ii) What is the velocity of the water at point B in the pipe where the pressure is reduced to 1.20×10^5 Pa?

Bernoulli's equation : $P + \frac{1}{2}\rho v^2 = constant$

$$P_A + \frac{1}{2}\rho {v_A}^2 = P_B + \frac{1}{2}\rho {v_B}^2$$

$P_A = 1.6 \times 10^5\ Pa,\qquad P_B = 1.2 \times 10^5\ Pa,$

$v_A = 2.5\ ms^{-1},\qquad \rho = 1000\ kg\ m^{-3}$

$$\rightarrow v_B = 9.3\ m\ s^{-1}$$

2

(iii) What is the cross-sectional area of the pipe at point B?

Continuity equation : $v\ A = constant$

$$v_A\ A_A = v_B\ A_B$$

$$A_B = \frac{v_A\ A_A}{v_B} = \frac{2.5 \times 2 \times 10^{-3}}{9.3} = 5.4 \times 10^{-4}\ m^2$$

1

Practice exam questions: Section C

(b) A 3.0 kg block of silver is completely immersed in water.

Calculate

(i) the volume of the silver block

$$Density = \frac{Mass}{Volume} \rightarrow V = \frac{3.0}{10.5 \times 10^3} = 2.9 \times 10^{-4}\ m^3$$

1

(ii) the mass of water displaced by the silver block.

$$m_{water} = \rho_{water} V = 1000 \times 2.9 \times 10^{-4} = 0.29\ kg$$

1

(iii) the buoyant force on the silver block.

$$\text{Buoyant force} = m_{water}\ g = 0.29 \times 9.8 = 2.8\ N$$

1

Practice exam questions: Section C

C3. A glass beaker measures 14 cm high by 5.0 cm in diameter. Empty, it floats in water with one-third of its height submerged. Calculate how many 12 gram rocks can be placed in the beaker before it sinks.

Volume of beaker $= V_{beaker} = \pi r^2 h = \pi(0.025)^2(0.14) = 2.7 \times 10^{-4}\ m^3$

Buoyancy force when empty = $\frac{1}{3} V_{beaker}\, \rho_{water}\, g = \frac{1}{3} \times 2.7 \times 10^{-4} \times 1000 \times 9.8 = 0.90\ N$ [= weight of beaker]

Buoyancy force when full = $V_{beaker}\, \rho_{water}\, g = 2.69\ N$

Extra buoyancy $= 2.69 - 0.90 = 1.79 = N_{rock}\, m_{rock}\, g$

$\rightarrow N_{rock} = 15$

Next steps

- Make sure you are comfortable with unit conversions (especially Pressure in Pa or atm)
- Review the fluid mechanics key facts
- Familiarize yourself with the fluid mechanics section of the formula sheet
- Try questions from the sample exam papers on Blackboard and/or the textbook