Extrovert E-book Series

Design-Centered Introduction To Aerospace Engineering

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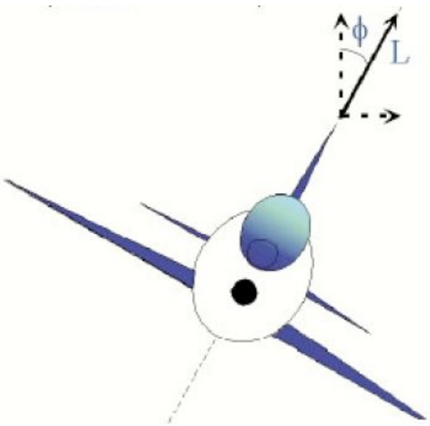

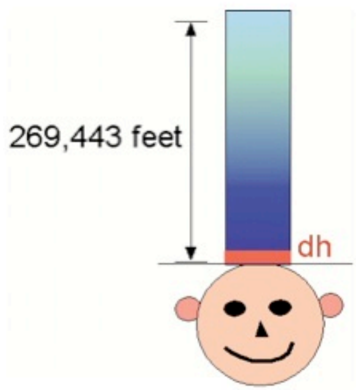

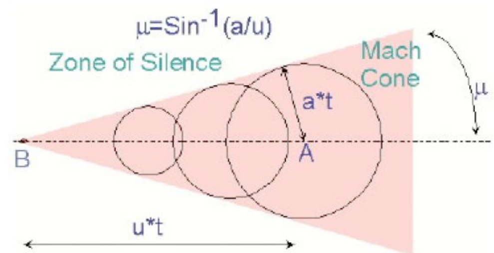

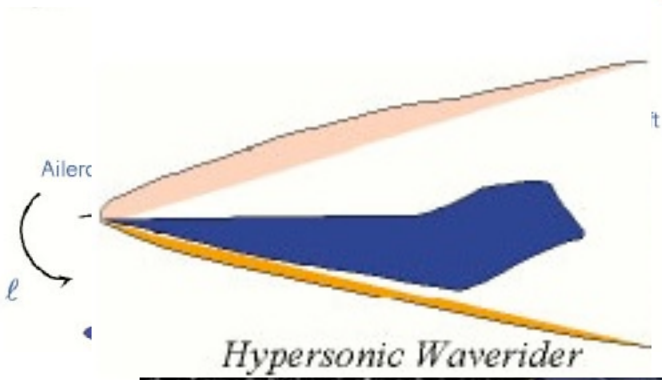

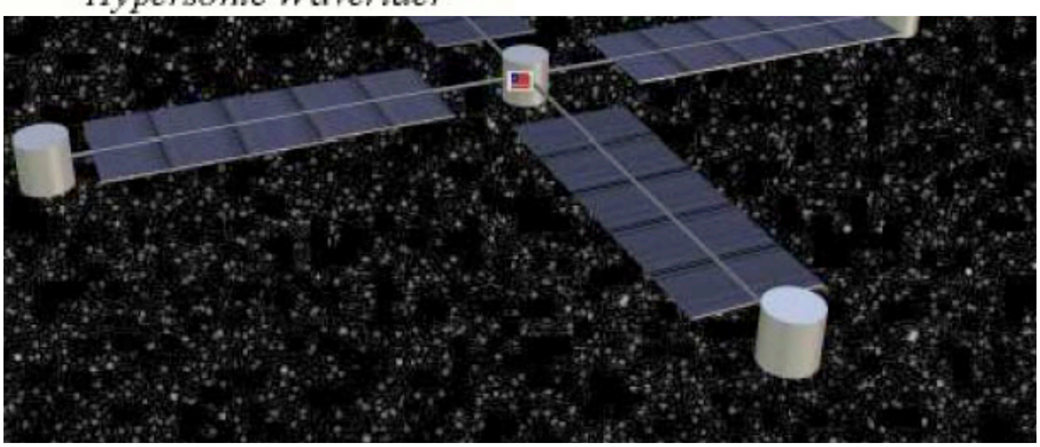

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Disclaimer

Design Centered Introduction to Aerospace Engineering

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Chapter 1

Welcome to Aerospace Engineering

Most pioneers learned the "disciplines" of engineering through the hard route of designing devices and systems. We try to capture this spirit by taking the learner through the process of conceptually designing a flight vehicle. In the process, we explore the gateways to the various disciplines of aerospace engineering, each leading to deep resources of knowledge and experience. Prepare for liftoff. We assume that you are a first year undergraduate, fresh out of high school. You will find yourself designing airliners and defining their flight envelopes, as you go through these chapters, follow the exercises and complete a few assignments.

This book is a condensed, refined version of a course that was first developed in the Summer of 1997, and first taught in Fall 1997 at Georgia Tech. In the 1990s, the aerospace industry was in recession, after the Cold War ended and the need for large numbers of combat aircraft disappeared. Many students who came to Georgia Tech to study aerospace engineering, found themselves studying with excellent students from other disciplines. There was no introduction to aerospace engineering in the first year of college, because it was assumed that students could not appreciate the issues in aerospace engineering until they had completed several courses in Mathematics, Physics, Chemistry, Statics, Dynamics and Strength of Materials. As they completed the second year, they found themselves having to decide between continuing alone in a school where they had never taken a course, or going with their study-group friends into other disciplines. The School of Aerospace Engineering decided to try introducing the excitement of aerospace engineering in the first year. Several different approaches were tried, but they were either too simplistic (avoiding Math and Physics) or far too complicated. The notion of using aircraft conceptual design as a vehicle to introduce students in their first week of college to aerospace engineering, was rather risky, but our students took to it with amazing enthusiasm. The course was taught several times in the subsequent years. It was condensed into a 1.5-day Workshop for engineering students, and then for new engineers from other disciplines.

There are several excellent textbooks set at the level of a college student learning about aerospace engineering. One is the textbook by Professor John D. Anderson [1]. Another book, set at a slightly higher level, but with excellent reference data, is the earlier book by Professor Shevell [2]. Dr. Tennekes, author of a famous and beautifully lucid textbook on the difficult science of Turbulence, has written a very simple book introducing flying vehicles [3]. This book is remarkable for the simple links between flying vehicles ranging from insects to jumbo jets, and showing, for example, that even a bird taking off for a flight across the Atlantic Ocean shares many design features with an airliner doing the same trip. Simmons [4] presents the aerodynamics of model airplanes. Alexander and Vogel [5] tell us about birds, insects and other flyers in nature, and Alexander [6] compares biomechanical flight to larger aircraft. Brandt [7] uses an aircraft design perspective to introduce engineering. Damon [8] introduces the science of space flight. Kemp [9] discusses the opportunities opened by the advent of sub-orbital space flights. The Civil Air Patrol has published a nice introduction to flight [10].

Aircraft design is, of course, a highly specialized subject. One widely-used resource for aircraft design is the series of textbooks by Professor Jan Roskam [11]. Aircraft designers introduce many variables to calculate the values of parameters to a high level of accuracy. We use a much simpler approach using the average values of these parameters. While less accurate, this approach gives useful results quickly, and illustrates the process of designing a flight vehicle.

1.1 Introduction to Aerospace Engineering: The Process of Designing a Flight Vehicle

Aerospace Engineering is about converting dreams to reality using science, engineering, imagination and determination. So we will first summarize today's dreams, and consider a route map of disciplines through which they can be converted to reality.

Today's Dreams

Although today's airplanes and spacecraft are amazing feats of technology, the field of aerospace engineering is still very young. There are so many immense challenges before us. Let us consider a few. As we consider each dream, we will have to wake up, and think a little about what exactly each entails. This is the first step in defining our requirements, putting numbers to the dreams.

Fly like a bird

Wouldn't it be nice if we could fly like birds? What exactly does that mean, though? We would have to fly in the speed range somewhere between 0 and about 160 kilometers (100 miles) per hour, We must be able to fly high enough and far enough to cross mountains and rivers, but we must also be able to land anywhere safely and at a very low landing speed. To avoid hitting the ground hard, the landing speed must be down in the range of 10 feet per second. The other feat that birds routinely accomplish is that of landing precisely on a branch and holding on to it as they spread and then fold their wings, hardly even shaking the branch. **That** is a lot tougher.

Commute by Air

This is a dream of millions of people who find themselves sitting in traffic for hours every day. We wish we could just fly from our home garage to the parking lot at our workplaces, and then back again later in the day, to our garages. What does this imply in technical terms? There are already some "flying cars", invented since the 1970s. However, if these are to become as affordable as today's cars, enough of them would have to be produced and sold. So we must imagine a traffic management system that can accomodate, say, 1 million cars over a big city. These cars may be moving at 200 kilometers per hour, and they must be able to operate safely in all types of weather, including thunderstorns, wind gusts and blizzards, and at night. The main problem is collision avoidance, and keeping the commuters from killing themselves and each other.

City to City, doorstep service

The next dream is to be able to fly from one city to another, across states, and again, be able to fly from our homes to our destimations. Such flight would have to be quite fast, perhaps at 600 kilometers per hour, so that we can visit friends and relatives who live, say, 1000 kilometers away.

Perhaps in this case the vehicle will not be a personal one such as the commuter vehicle, but a larger vehicle that might pick us up from near our homes, like a bus. It would have to take off and land vertically, so that no long runway is needed. While landing and taking off vertically, it must not make much noise, or cause a strong downward blast of air.

Cross the World In a Day

Anyone who has travelled on a 14-hour or 17-hour airliner ride, and done so in economy class, will appreciate the desire for something that travels about 2 to 3 times as fast. Such a vehicle might cruise at Mach 3, or 3 times the speed of sound. The speed of sound is roughly 600 miles of 960 kilometers per hour, so this would be around 1800 miles or 3000 kilometers per hour.

Note: Did you notice that we are being rather loose with our arithmetic? I didn't say that the speed of sound on a standard day in the Stratosphere above 33,000 feet altitude in the International Standard Atmosphere is precisely 583.47 Statute miles per hour etc. I have no idea if it is! That is quite all right for now. We need to develop the knack of making calculations in round numbers that we can work out while sitting on the bus to school or walking around the Student Center with headphones stuck on our ears. Once we get a grasp of the rough magnitudes of numbers, we can sit down with a calculator or computer and work things out in detailed and accurate mathematics. But simply being precise is no good if we don't have a grasp on the rough magnitudes.

Visit Low Earth Orbit

Many of us would like to experience the feeling of being in orbit around the Earth. The lowest altitudes where we can do this are about 300 to 400km above the surface. However, the issue in reaching orbit is not the height, but the speed. Going into orbit is like being a ball at the end of a string, being whirled around by a child. The string becomes taut as tangential speed of the ball increases, and the radial stress increases. If she lets go, the ball will zoom out along the tangent to the circle that she was describing in the air. One has to be moving at a tangential speed of about 24,000 kilometers per hour to balance Earth's gravity and be in orbit.

Our Orbital Joyrider (at least for me!) should be a reusable spaceliner, and it must take off, accelerate, re-enter the atmosphere, and land, routinely and in absolute safety and comfort. The "National Aerospace Plane" project announced by President Reagan was promptly dubbed the "Orient Express" by the media. As advertised, it was to be a hypersonic airliner that could take you from Los Angeles or New York to Tokyo at many times the speed of sound, zooming to the edge of Space.

Consider that in the days of the Space Shuttle, NASA invited the public to watch the spectacular liftoff from parkig lots located at a safe distance. Following the liftoff, people would rush to get in their cars and try to beat the rush out of the parking lot. Well.. before the average person could get their car out of the lot, they would be hearing the NASA commentator remark that the Shuttle was crossing Africa. So the prospect of a hypersonic airliner sounds very exciting indeed. But consider the acceleration and deceleration at the start and end of the journey. Accelerating at 3 or 4 times the acceleration due to gravity (3 Gs) or the vibration of a typical Space Shuttle liftoff are not very pleasant, even for highly trained astronauts. The airline would not have to feel bad about not serving any lunch on those expensive flights, as most passengers would not be able to eat any or keep it down for long. Not surprisingly, the National Aerospace Plane remains a dream, but surely some day some version of it will come about. You may be one of its designers who solves it's toughest problems.

Reaching orbit is like throwing a ball. Once the ball leaves your hand, it is in a ballistic trajectory (imposing-sounding word, but see the first 4 letters? Aerospace engineers used to be kids too, and the field is so exciting that it keeps us feeling young). The "throw" is the acceleration phase, when you are exerting thrust on the ball, and imparting an impulse or a packet of energy to the ball. If you think about what a space vehicle does, its engines "burn" for the first few minutes, and then it "coasts" for millions and millions of miles. We will see more on this when we study about spacecraft, but if you are burning fuel to produce this impulse, then, clearly, you want to burn up all the fuel as quickly as possible, because you don't want to spend the impulse on accelerating the fuel mass itself. So the ideal way to launch something is to put in all the impulse at the very start, as quickly as you can. This is why Space launch vehicles take off with such a spectacular display of immense power: they are burning the fuel as fast as the engine will allow, and accelerating the vehicle as much as its structure can take.

It still costs a huge amount of money to launch anything into Space. Very little of this is the actual cost of fuel, or even of the materials used to build the vehicle. Most of the cost is in the time and expertise it takes to design, fabricate, analyze, test, document, inspect, monitor and control the entire process. In other words, it is in the salaries of the people who work in the program. Without them, and without their full attention and expertise, space flight would not only be very difficult, it would be unacceptably dangerous. And no, this is not "government waste" and no, aerospace engineers do not get paid outrageous salaries or bonuses. But the net result is that today, the cost of sending anything to orbit, or to speeds similar to orbital speeds (as for that Saturday morning trip to Tokyo Mall) is immense. It costs something like \$30,000 per kilogram of mass, to go to orbit. Yes, that means that if you weigh 180 lbs on Earth, your mass is around 82 kilograms. It will cost at least \$2.46 million to send you to orbit - or to Tokyo Mall. For much less than that, you could go on the Internet from the comfort of your home, and in less time than it takes to say "10..9... Ignition!" you could charge that on your credit card and have the items shipped to you next day - usually on an airplane that aerospace engineers like you designed. This is the toughest problem of all in the Space program. The launch cost to orbit must be brought to about \$100 per kilogram or less, to make it really worthwhile to do many things in Space.

Visit nearby planets

Wouldn't it be cool to go and prance around in the 0.3G gravity field of Mars, or see the frozen seas of Titan, or zoom through the rings of Saturn? Some day, surely, people will. Right now, the prospect requires that we get up to speeds of over 100,000 kmph; months of endurance because the distance is just so immense, and, most difficult of all, we need some way of shielding ourselves from the radiation in Space, once we are outside the protection afforded by the lovely blue skies of our atmosphere. The Sun, from where most of our energy comes, is one huge nuclear reactor, with both nuclear fission reactions (large, massive atoms breaking into smaller ones) and nuclear fusion (tiny atoms like hydrogen joining to form more massive ones) occurring. These processes give us sunlight and warmth, but also release powerful ultraviolent rays, gamma rays and X-rays, all of which can break the cells in our bodies and kill us. In addition to these, in Space there are other wandering high-energy particles running away from awful events such as exploding stars. These are called Cosmic Radiation and they may be particles traveling at maybe 75 kilometers per second. They can go right through most things, which sounds harmless, except that they may hit the aluminum or steel shell of our craft and cause thousands of smaller particles and rays to emanate from the inside of these shells and kill us. And on top of these, let's not forget those tiny specks of whatever floating about in orbit. They may be as small as dust, but they can punch a hole a couple of millimeters in diameter in metal skins or circuit boards - or through pressure suits and people.

So look at those astronauts with a bit more respect. Yes, they ARE brave people, because for all the expertise of the Space Agencies looking after them, no one can really protect them against those last-named dangers.

If you really are determined to stop radiation and microscopic particles, there is one way: surround yourself with about 3 meters (roughly 6 feet) thickness of soil, either from the Earth or, say, from the Moon. That will stop most things. Unfortunately, if you calculate the mass of a vehicle that has such thick walls, and multiply that by $30,000 per kilogram, you decide again that it's not time to take that trip. Some day someone will solve this problem. That someone may be you.

Visit nearby star systems

The nearest star to the Solar System is , which is about 4.24 light-years, or 40 trillion kilometers away. Obviously, to traverse such a distance in any acceptable time, a vehicle must travel at a speed that is a substantial fraction of the speed of light. This is the relativistic speed domain, where strange things are expected to occur.

Travel to deep space

Many interesting places including stars that have Earth-like planets, are millions of light years away. How we might start traveling to such places, is a matter of speculation. Our ideas of travel may be completely changed as we consider methods to travel to such places. Maybe *they* will tell us?

Nanoprobes

At the other end of the size and speed spectrum, is the quest to go ever smaller, and explore phenomena at tiny size levels. Nanoprobes have dimensions on the order of one nanometer.

1.2 Issues in Designing a Flight Vehicle

In the table below, we summarize the process of flight vehicle design. The first column lists the steps, and the second says what each step does. The first step is th define the requirements, or decide what we want to do and why. That looks easy, but aircraft manufacturers will tell you that this is the most crucial step of all, and perhaps the one that is hardest to do as accurately as it must be done. This step requires the company to talk to hundreds, even thousands, of people, argue for hours, and eventually distill a clear idea of what the vehicle must be able to do. Why is this so? It is because the vehicle must eventually succeed in the marketplace, whether the customer is the military, a space agency, an airline, or a private owner. It must do better than predecessors have done, and yet not try to do too much.

The next step is, shockingly, to go and sneak a peek, or in fact spend a great deal of time and effort peeking, at what others have done before. This is called benchmarking. What has been shown to be possible? For instance, has anyone managed to design an aircraft that can carry 2000 people and their pets and bags and enough food and water for the trip, non-stop at 700 miles an hour average speed, for 10,000 miles? If not, what exactly have they achieved? Why did they do that, and what stopped them from doing any better? You see that this gets pretty deep, but at the beginning it is as simple as listing the specifications of flight vehicles that are in the same general class of size, speed, capacity to carry useful load, and range, as the one that you are contemplating designing.

In this process, we adopt two attitudes. First, we *respect* the engineers who designed the vehicles that flew in the past, or are flying today, and assume that they did about as well as anyone could have done, and that they were very smart and very hard-working people. In other words, a very hard act to follow. Secondly, we *boldly assume* that we will also learn what they learned, and be as smart and hard-working, and maybe, just maybe, a bit smarter or work a bit harder, since we have their experience to teach us. So we can do better, but not immensely better, than they did. This is the culture of aerospace engineering. Brash we may sound, but inside we have a healthy respect for our competition, because if they are anywhere near as smart and quick as we are, well, they must be pretty awesome too. But we can't afford to sit around being over-awed by them, because we *have* to do better.

The rest of the design process is pretty straightforward and logical. Please read the table, and as we get to each topic later, we will detail it. One observation to make: Note that the design process is *iterative*. We do not sit around until we can magically come out with a perfect design at the first attempt. Everything is linked to everything else, so where does one start? We break out of this dilemma by making a bold guess at the final answer, say how much the vehicle will weigh when it is all done. This is where it helps that we have the answers that our predecessors achieved. We will guess something a little better. Then figure out all the things that this implies, and suddenly we have a detailed specification of the vehicle. Maybe it won't work, and if that is the case, we will go back and refine our initial guess, and do everything over again, until everything fits perfectly.

In the old days this was painful, as one had to everything with a slide rule and pencil and eraser - but aerospace engineers still did it! Today it is so much easier to do with a spreadsheet on a computer. And take heart. My freshman students at Georgia Tech in 2010 complained bitterly at the immense amount of calculations that they had to do in SIX weeks, to get their first design done. So much calculation! So many hours! Oh, their poor brains! Their hands as they had to type and write! They had never had to work so hard in high school, the poor dears! So I asked them to revise the entire design, for a vehicle that was quite different - it would use liquid hydrogen as fuel instead of the usual petroleum jet fuel. I gave them one week to do it. They did it. Then I asked them to do the essential parts of those calculations for another vehicle, as one question out of 6 on a 3-hour final examination, with no computer available to help them. So they had all of 30 minutes to do it. They complained that that was hard - but the point is, they did it, and did it very well. THAT, dear reader, is the power of iteration, when there is an aerospace engineer learning from the experience. So yes, most people find this stuff difficult. But persist, pay attention, and you will excel at it.

Step	**Issues**
Define Requirements	What must the vehicle do? Why?
Survey past designs	What has been shown to be possible?
Weight estimation	How much will it weigh, approximately, going by past experience and our projection?
Aerodynamics	Wing size, speed, altitude, drag
Propulsion selection	How much thrust is needed? How many engines? How heavy? Fuel consumption?
Performance	Fuel weight, take off distance, speed/altitude/ maneuvering boundaries
Configuration	How should it look? Designer's decisions needed!
Stability & Control	Locate & size the tail, flaps, elevators, ailerons etc. Fuel distribution.
Structure	Strength of each part, material, weight reduction, life prediction.
Detailed engineering	Design each part, see how everything fits, and plan how to build and maintain the vehicle.
Life-cycle cost Life-cycle cost	Minimize cost of owning the vehicle over its entire lifetime.
Iteration Iteration	Are all the assumptions satisfied? Refine the weight and the design.
Flight Simulation Flight Simulation	Describe the vehicle using mathematics. Check the "flight envelope".
Testing Testing	Measure model characteristics, verify predictions. Build & test first prototype.
Refinement	Reduce cost and complexity, improve performance, safety and reliability.

1.3 Aerospace Design: A Route Map of Disciplines

Aerospace Engineering involves many "disciplines": each might warrant a separate division in a major company, with dedicated experts who spend decades specializing in it. Here we take a quick look at some of these disciplines which you will encounter in this course. To become an expert in each of these disciplines, one should pay careful attention to the basic courses in school which don't always seem at first sight to be very relevant to aerospace engineering.

Aspect	**Basic disciplines needed**
Mission Specification	Technology forecasting, market surveys, vehicle performance, economics, social sciences, political science
Weight Estimation	Statistics, technology forecasting
Aerodynamics	Physics, calculus, computer science
Propulsion	Physics, thermodynamics, chemistry, lasers, optics, environmental sciences, acoustics
Performance	Physics, Statics and Dynamics, calculus; flight mechanics
Structures	Materials, Statics, Dynamics, Strength of Materials.
Layout and detail design	Engg. graphics, psychology, economics, ergonomics
Stability	Statics, calculus
Controls	Laplace transforms, differential equations, electrical engg., computer science
Instrumentation & communications	Optics, electronics, signal processing, computing
Space propulsion	Electricity, magnetism, nuclear engg., chemistry, physics, dynamics, thermodynamics
Trajectories & space mission design	Dynamics, astronomy, modern physics
Spacecraft design	Heat transfer, materials, photoelectricity, thermodynamics, chemistry, physics, physiology, electrical circuits
Flight Simulation	Flight mechanics, image processing, engg. graphics, computer science, control theory.
Ground and flight testing and experimentation	Physics, chemistry, mechanical design, electronics, signal processing, image processing, computer science.
Lifecycle cost	Manufacturing, Systems Engg., Optimization, Economics, Political and Legal Issues.

1.4 Much Remains to be Learned About Flight

Today's designs can fly over 100 times as fast as the Wright Flyer, and go right out into space, circle the earth every 90 minutes or so, and return to precise touchdowns on earth.

In the 1920s, a whole 17 years since the Wright Brothers made their first flight, the newspapers claimed that airplanes had reached the limits of speed and altitude. So-called Experts "proved" that there was no "scope" in this field. After all, they argued, what more could be done, than what had already been demonstrated?

So as of 2013, humans have around 110 years of powered flight experience. Birds and insects have 1 million-plus years of evolution, i.e., iterative design improvements, built into them. We cannot match the control precision, landing versatility, payload fraction, engine weight fraction, fuel costs, maneuverability, reconfigurable geometry, or structure weight fraction of birds and insects. By comparison with birds and insects, today's aircraft are still fragile and clumsy.

They have stiff, nearly-rigid wings that can't flap, twist, fold or thrust to any significant degree. They need long runways and complex traffic control systems. You have to drive through 2 hours of downtown traffic and spend an hour and a half at the airport and another 30 minutes on the taxiway to make a flight of 200 miles. When we launch spacecraft, only 10% of the total launch mass ever reaches orbit: the rest is wasted.

1.5 Summarizing

You are already well into aerospace engineering. You saw how the dreams of far-out concepts are immediately translated into tangible requirements with numbers. You confidently put down numbers from calculations done in your head, without worrying that the arithmetic was not exact to 5 decimal places. You have seen how this process gets us thinking about how to meet those requirements, and eventially tells us what is needed. You have also seen some of the real obstacles in the way of achieving those dreams. Next, we will start the process of thinking through the requirements for something much closer to us: an airliner, or a "civilian transport aircraft".

Chapter 2

Estimating the TakeOff Weight of an Aircraft

Let us think about how an aircraft might be designed. How does one decide how big it should be, or its shape? How does one know whether it will travel as high and as fast and as far as we want it to do? All of its properties are related to one another. The designer needs a systematic process. If we knew how much the aircraft weighs, we could reason that it needs at least that much lift, and enough strength in the structure to support this weight, and so on. But until the aircraft is completely designed, how can one hope to know how much it weighs? Below we see how simple this process is. We start by guessing the most difficult answer of all: the Take Off Gross Weight of the aircraft. We try to reduce the uncertaintly of this guess as much as possible by looking at what other people have done, and how those aircraft turned out in practice. There is nothing wrong in taking such a guess. We will systematically calculate everything else based on this guess, and then at the end we will see if the weight really turns out to be close to the guess. Otherwise, we will know how to change the guess and "converge" on the right answer after a few iterations of the procedure.

Requirements Definition / Mission Specification

First, we have to decide what we want the contraption to do. Then we will think of a "typical mission profile". To do these, we must find out what our customers really want, what others have been able to achieve in the past, and what opportunities are opened by new developments. In industry, this is a crucial stage requiring massive effort and intense thinking, because this is going to be the basis to commit the company's future. It involves discussions, analyses and trade studies with the airlines, financiers, regulating agencies, airports, law-makers and advertising or marketing agencies. Engineers have to learn to excel in this environment.

Excercise: Mission Specification

Regional jet aircraft using hydrogen fuel
Range: 1600 km
Cruise Speed: 500 kmph minimum
80 passengers

Typical Mission Requirements:
Atlanta - Rayleigh-Durham NC
Albuquerque-San Antonio

Denver-Atlanta in summer
Short turn-around time. Fuel-efficient

Independent operation: No "jetway" needed. Reliable: All weather

Weight Estimation and Benchmarking

The mass to be carried is the "payload". This is the load which we get paid to carry. Once the payload is determined we ask, "haven't others tried to do something similar or close to this? How much did their aircraft weigh?"

This is known as "benchmarking", getting a rough idea of the weight fractions of the various systems involved. For example, fuel weight may be 50% of the takeoff weight of a long-range airliner.

How Takeoff Gross Weight (TOW) of an Aircraft is Broken Out

Component	**Fraction**
Payload Fraction: passengers+crew, baggage, food+water, cargo	$\frac{W_{payload}}{TOW}$
Propulsion Fraction: Engines, engine control systems, nacelles, fuel lines, fuel pumps, fuel tanks	$\frac{W_{engines}}{TOW}$
Structure and Controls Fraction: Everything else fixed to the aircraft: wings, fuselage, control surfaces, instruments, landing gear, hydraulic systems, air conditioning, lights, interior furnishings.	$\frac{W_{structure}}{TOW}$
Fuel Fraction	$\frac{W_{fuel}}{TOW}$
Total:	**1.0**

Note: However you break it out, you must make sure that everything is included somewhere, and only once.

Example: Computing TOW

Takeoff gross weight is simply the payload divided by the payload fraction. For example, if the payload is 30,000 lbs, and we find that *a reasonable payload fraction that we can achieve* is 0.15, then the TOW is 30,000/0.15 = 200,000 lbs.

This is an estimate. You just learned how to get across the most difficult "canyons" of technical uncertainty in engineering: you jump across it. You make a "reasonable guess", see where it leads, and then refine the guess as you learn more.

The rest of the design is to make sure we come in under this estimate, when we calculate everything else.

When we have a rough calculation of all the other things, we will go back and "iterate" many times to refine our estimates, so that the whole vehicle gets better. For this we will spare no technical effort, and it will take years.

Benchmarking

There is a wide range of answers to our question about the payload fraction. Some craft weigh only four times their payload; others way ten times the payload. There is some similarity between these payload fractions for aircraft which have similar missions and payloads. In our case, one way of classifying missions may be passenger-miles, the product of the number of passengers and number of miles of range. There are many other ways of doing the classification.

2.1 Example Aircraft with Statistics

Embraer 190 Commercial Regional Jet, Brazil

Wingspan 23.56 m
Length 36.15 m
Height 10.48 m

Cruise Speed: Mach 0.80

Boeing 787

Figure 2.1: Boeing 787. Image courtesy of http://inslee.house.gov/issue/aerospace

Length	57 m
Wingspan	60 m
Wing Area	325 m^2
Height	16.9 m
Max Takeoff Weight	228,000 kg
Range	14,000 - 15,000 km
Cruise Speed	Mach 0.85

F-35 Joint Strike Fighter

Figure 2.2: F35. Image courtesy of www.jsf.mil

Wingspan	35 feet
Wing Area	460 feet^2
Length	51 feet
Empty Weight	29,300 lb
Max Takeoff Weight	70,000 lb
Range	1,200 nmi

Sukhoi Su-30MKI

Figure 2.3: Sukhoi Su-30. Image Courtesy of http://www.usafe.af.mil/shared/media/photodb/web/080713-F-4964T-02.JPG

Wingspan	14.70 m
Length	21.94 m
Wing Area	62 m^2
Height	6.36 m
Empty Mass	17,700 kg
Max Takeoff Mass	34,000 kg
Range	3,000 km

2.2 Design Step 2

6. Determine payload. Mass per passenger = ( ) kg. Supplies per passenger = ( ) kg. Baggage per passenger = ( ) kg. Cargo per flight = ( ) kg. Total payload = ( ) kg.

7. Determine payload mass fraction.

8. Determine Takeoff Gross Weight (TOW).

Chapter 3

Force Balance

The wings (and horizontal tails to some extent) support the weight of the whole aircraft. The rest of the aircraft just hangs from these lifting surfaces. Of course the wings and tails themselves have weight. On most aircraft, the wings contain most of the fuel. We can use Newton's Laws of Motion to calculate the acceleration of an aircraft, and thus decide how the forces on the aircraft must be balanced to make it go in a desired direction.

3.1 Newton's Laws of Motion

Newton's First Law of Motion

The first law defines the concept of equilibrium. It says: An object continues to be in a state of rest or uniform motion unless there is a net force acting on it.

Newton's Second Law of Motion

states that the net force on an object is equal to the time rate of change of momentum.

$$F = \frac{d(mv)}{dt}$$

Using the chain rule:

$$F = (\frac{dm}{dt})v + m(\frac{dv}{dt})$$

If the mass remains unchanged,

$$F = m\frac{dv}{dt} = ma$$

Force and acceleration are vectors: they have magnitude and direction. If two vectors are equal, i.e., $\vec{A} = \vec{B}$, then,

$$A_x\vec{i} + A_y\vec{j} + A_z\vec{k} = B_x\vec{i} + B_y\vec{j} + B_z\vec{z}$$

or

$$A_x = B_x, A_y = B_y, A_z = B_z$$

Newton's Third Law of Motion

Every action has an equal and opposite reaction. For example, if the engine of an airplane produces thrust which pulls forward on the aircraft, then the aircraft pulls on the engine in the opposite direction.

Chapter 4

Equations to Describe Aircraft Motion

4.1 The Freestream and Aircraft Motion

Definition of Freestream Velocity

Since our interest is in the aircraft, and not so much on what happens to the air in a given place, we start by thinking of what we would see if we were attached to the aircraft. It appears that the aircraft is sitting still, and the whole atmosphere is rushing towards, past, and away from the craft. This is similar to the situation in wind tunnel where aircraft models are tested. The model is held in place in the test section of the wind tunnel, and the wind speed is adjusted for each test condition. This leads to the concept of a *freestream speed.*

The freestream is the air upstream of an aircraft. The freestream velocity is the velocity of this air relative to the aircraft. It is denoted as u_∞ *or* v_∞*, though this text more commonly uses the former. An aircraft can be said to be moving at a velocity* u_∞*, though in that context it refers to the velocity with the same magnitude but opposite direction of the freestream's direction.*

The definition of the freestream is very important, for the definitions of lift and drag depend on it. Lift is defined as the force perpendicular to the freestream velocity, and drag is defined as the force parallel to the freestream velocity.

Angle of Attack

The chord line of an airfoil or wing is the straight line from the center of curvature on the leading edge to the trailing edge. The angle of attack, denoted as α*, is the angle this line makes with the freestream, as shown below.*
It is important to note that the aircraft is still traveling in the direction of the freestream.

Angle with the Horizon

The angle between the freestream and the horizon is denoted by θ*.*

4.2 Coordinate System

We typically use a NED (north-east-down) coordinate system, with the x direction coming out of the the nose of the aircraft, the y direction exiting the starboard side of the aircraft. The z axis points *down to the earth, and is positive down.*

4.3 Equations Along Different Coordinate Axes

Weight acts towards the center of the earth (or whatever the closest massive heavenly body is). The two component equations are:

$$\text{Along } x\text{:} \quad -L\sin\theta - D\cos\theta + T\cos\theta = Ma_x$$

$$\text{Along } z\text{:} \quad L\cos\theta - D\sin\theta + T\sin\theta - W = Ma_z$$

The total acceleration vector of the aircraft is given by the components along the the x, y, and z directions.

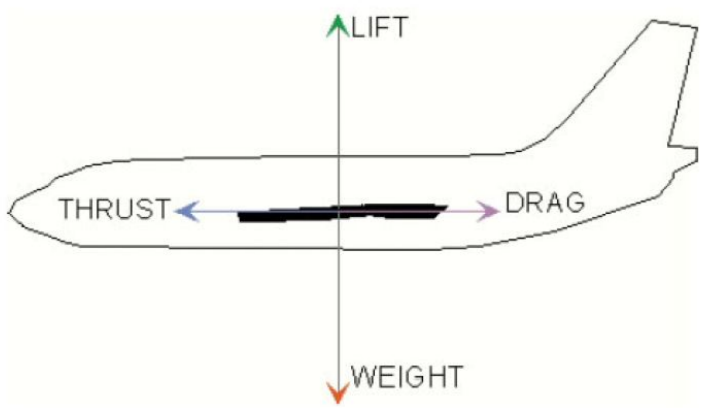
In the case above, there is no acceleration along the y-direction because there is no net force along that direction.

Case 1: Straight and Level Steady Flight

In straight and level steady flight, where all the accelerations are zero, lift is equal to weight, and thrust is equal to drag.

$$L = W$$

$$T = D$$



Case 2: Aircraft is almost flying level

So, if L > W, the vehicle accelerates upwards (note that upwards is -z). Also, if T>D, the vehicle accelerates forward. If L = W, the aircraft flies level, or rises/falls at constant speed. If T = D, the aircraft flies at constant speed.

Case 3:

Acceleration is zero, but $\theta \neq 0$.

From the X-momentum equation, $L\sin\theta = (T - D)\cos\theta$

From the Z-momentum equation, $L\cos\theta + (T - D)\sin\theta = W$

Rate of Climb

Dividing the X-momentum equation by the Z-momentum equation,

$$\tan\theta = (T - D)/L$$

$$w \sin U(T - D)/L$$

$$\tan\theta = \frac{w}{u}$$

If θ is small (as is usual under a routine climb condition where one is not in any desperate hurry), the value u is fairly close to the magnitude of the velocity vector, U. Then, approximately,

If w > 0, then the aircraft is climbing. From these we note:

1) If the lift is greater than the weight, then the aircraft will accelerate upwards.

2) If the thrust is greater than the drag, the aircraft can climb if the thrust acts at an angle to the flight direction. Thus there are different ways of achieving the same result.

Sideward Forces: Level Turn

By rolling an angle ϕ, the aircraft uses part of the lift force acting on the wings to execute turns, because lift acts perpendicular to the wings. As a result, a component of the lift, $L\sin\phi$, acts to make the aircraft turn. This force, called the centripetal force, is the force directed towards the center, and is equal to $\frac{MU^2}{R}$. The value $\frac{L\sin\phi}{M}$ is the radial acceleration.

(Image here)

Notes from the figure:

$$L\cos\phi = W$$

$$L\sin\phi = \frac{MU^2}{R}$$

To pull tighter turns (smaller radius R) at a smaller value of U, $L\sin\phi$ must be made larger.

If the aircraft is not to lose altitude during this maneuver, $L\cos\phi$ must be as large as the weight, W.

4.4 Roll, Pitch, Yaw

An airplane can use its control surfaces to pitch, roll, and yaw. Each of these is a rotation about the axes defined earlier.

Pitch

Pitch is typically controlled by the elevator, a horizontal control surface on the tail. Moving the elevator rotates the plane about the y-axis, which runs through the wings of the plane.

Roll

Rollis typically controlled by ailerons, horizontal control surfaces on the wings. Moving the ailerons rotates the airplane about the x-axis.

Yaw

Yaw is typically controlled by the rudder, the vertical control surface on the tail. Moving the rudder causes the plane to yaw, or rotate about the z-axis (vertical axis). Deflecting the rudder to the right causes the aircraft nose to yaw to the right.

Chapter 5

The Atmosphere

5.1 The Atmosphere

Earth's radius at the equator is about 6,378.137 km (3963 miles). The polar radius is about 6,356.750 km (3,950 miles). There are about 90 kilometers of gaseous atmosphere, or 270,000 feet of gaseous atmosphere. Outer space is over a hundred miles up, but there is very little air above 51 miles.

Because of gravity, the air above presses down on the air below. At sea level, air pressure is enough to support a column of mercury (Hg), 760 millimeters (mm) high: 101,325 N/m^2. For a given base area, this column of mercury weighs about the same as a column of air only 11 kilometers high at sea level air density. So most of the air is in the bottom layers of the atmosphere.

Hydrostatic equation

At a height h above the surface, let's say that pressure is p Newtons per square meter (N/m^2, or Pascals), and density is ρ kilograms per cubic meter (kg/m^3). The acceleration due to gravity is meters per second squared (m/s^2). If you go up a tiny distance dh, the pressure decreases by a tiny amount dp.

$$dp = -\rho g dh$$

This is because you no longer have to support the weight of the element dh of the air column that went below you.

Perfect Gas Law

The Perfect Gas Lawis a relation between pressure, density, temperature and compositio of a gas.

$$p = \rho RT$$

R depends only on the composition (i.e., the average molecular weight) of the gas, i.e., air. Knowing that air is generally composed of 20% diatomic oxygen (O_2; molecular weight MW = 32), 79% diatomic nitrogen (N_2; MW=28), and 1% argon (MW=44).

Average (or "mean") molecular weight of air is [(0.2)(32) + (0.79)(28) + (0.01)(44)]=28.96

The Universal Gas Constant is 8314 J/kmol-K in SI units.

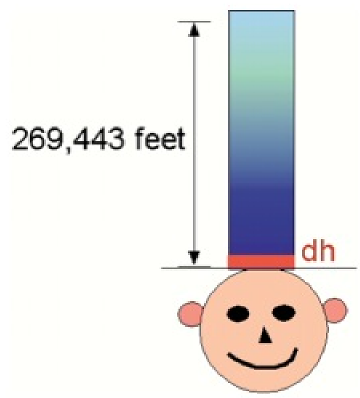


Figure 5.1:

Thus the gas constant for air is R = (8314 J/kmol-K) / (28.96 kg/kmol) = 287.04 J/kg-K
Differentiating the perfect gas law,

$$\frac{dp}{p} = \left(\frac{-g}{RT}\right) dh$$

$$\ln\left(\frac{P_2}{P_1}\right) = \left(\frac{-g}{RT}\right)(h_2 - h_1)$$

If T is constant,

$$\frac{P_2}{P_1} = \frac{\rho_2}{\rho_1} = e^{\left(\frac{-g}{RT}\right)(h_2 - h_1)}$$

This holds in the Stratosphere, the region between 11,000 meters and 25,000 meters.

In gradient regions, where T changes as altitude changes, we will assume that this variation is linear, i.e.,

$$T_2 = T_1 + a(h_2 - h_1)$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{-g}{aR}}$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\left(\frac{-g}{aR} - 1\right)}$$

This holds in the Troposphere, the region between sea level and 11,000 meters.

Troposphere

In the Troposphere (the region below 11,000 meters), the constant a is approximately -0.0065 Kelvin per meter. Thus, for a standard sea-level temperature of 288.12 Kelvin, the temperature in the troposphere is given by:

$$T = 288.12 - 0.0065h$$

where h is in meters. In this region, the pressure and density variations can be found as follows:

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{-g}{aR}}$$

density:

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{[\frac{-g}{aR}-1]}$$

Example:
What is the standard temperature at 5000 meters? $T = 288.12 - (5000)(0.0065) =$

Sea-Level Standard Conditions

Sea-Level Standard Conditions: Temperature = 288.12 K, Pressure = 101,325 N/m^2, Density = 1.225 kg/m^3.
We can express the pressure at a point on a given day as "so-many meters, , meaning: "If I were in a Standard Atmosphere, and measured this pressure, I would be at this altitude". Similarly, we can express and .

A major issue arises in tropical countries where there are high mountains, such as India for India: The density Altitude is often much higher than geometric altitude. For example, in the Himalayas, where long runways are anyway hard to come by, landing speed becomes too high for the available field length, on summer days. Helicopters find that they cannot clear ridges, and must fly through canyons and valleys instead, which is infinitely more dangerous.

Regions of the Atmosphere

Below 500 meters, we are in the Atmospheric Boundary Layer. The winds in this region get obstructed by hills, buildings, and by the friction of moving over the ground; hence they slow down and become turbulent. This is where we see most of the gusts, tornadoes, rain, snow, etc. Above this, and below 11,000 meters, is the Troposphere. Most of the "weather" occurs in this region, though some thunderstorms rise as high as 18,000 meters.
From about 11,000 meters to 25,000 meters is the Stratosphere, where the temperature is constant at a cold 216.7 Kelvins. Most of today's airliners cruise in this region.
From 25,000 meters to about 47,000 meters, the temperature rises again, linearly, reaching 270.65 K by 47,000 meters. Above that, the temperature is again assumed to remain quite constant. Composition starts changing approximately above 50,000 meters due to dissociation and ionization, caused by radiation and high-energy particles from space.

Some Sample Values

Altitude (m)	**Temp** (K)	**Density** (kg/m^3)	**Pressure** (N/m^2)	**Viscosity** (N$\cdot$ s/m^2)
0	288.15	1.225	101,327	0.00001789
11,000 (end of tropos.)	216.50	0.363925	22,633	0.00001421
25,000 (end of stratos.)	221.65	0.03946	2511.18	0.00001448
47,000 (end of linear temp increase)	270.648	0.00142	110.916	0.00001703
60,000	245.452	0.00028	20.3156	0.00001575
71,000	214.652	0.00006	3.95698	0.00001410

Note, in summary:

1. *It gets pretty cold and hard to breathe up there.*
2. *The "weather" is mostly below 11 km.*
3. *Most flight occurs below 20,000 meters today.*
4. *High-altitude winds can reach 200 mph.*
5. *The atmospheric boundary layer contains violent gusts and changes in conditions.*

5.2 Design Step 3

6. Determine conditions at cruise altitude. (Cruise altitude for short-range aircraft is lower than that for long-range)

Calculate the standard atmospheric conditions at a selected altitude for your aircraft.

On a day when sea-level temperature is 35 C and pressure is 101,000 Pascals, find the actual altitude if the aircraft altimeter indicates 12000 m.

1. Construct a spreadsheet calculation where you can specify the altitude and get all the standard conditions: temperature, pressure, density.

Homework:
2. Construct a spreadsheet where you can specify a pressure and temperature, and you can find the pressure altitude and density altitude.

Chapter 6

Aerodynamic Lift and Drag

6.1 Lift

Aerodynamic lift is the force perpendicular to the freestream. It is generated by deflecting the freestream air. According to Newton's 1st and 3rd laws, lift is the reaction to the rate of change of momentum of air, perpendicular to the freestream. Lift is related to freestream velocity by:

$$L = \frac{1}{2}\rho U_\infty^2 S C_L$$

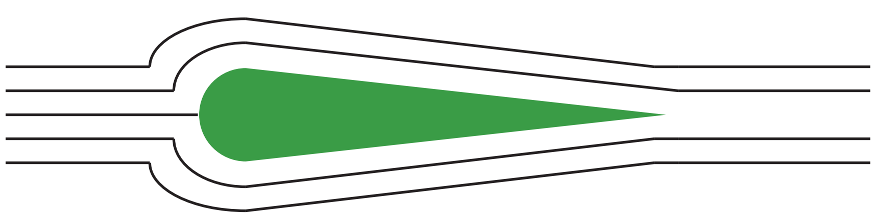
where U_∞ *is the freestream velocity,* C_L *is the wing lift coefficient, and* S *is the planform area of the wing.*

Lift Generation

In low-speed flows of air (<0.3 times the speed of sound, or Mach 0.3), there are three main ways to create aerodynamic lift. All involve directing the momentum perpendicular to the freestream. The angle of attack can be varied, camber can be added to the wing, or lift can be induced through vortices.

1. Varying the Angle of Attack

Imagine a wing with a symmetric airfoil as shown below. The airstream around it is as shown. No air is deflected down. Because the wing doesn't push any air down, there is no air pushing the wing up. This wing creates no lift.



Now imagine the same wing at some positive angle of attack. The airstream around it is deflected down. As the wing pushes air down, the air pushes the wing up, creating lift. This can be seen in Figure 6.1.

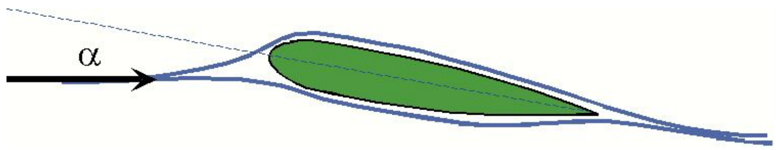
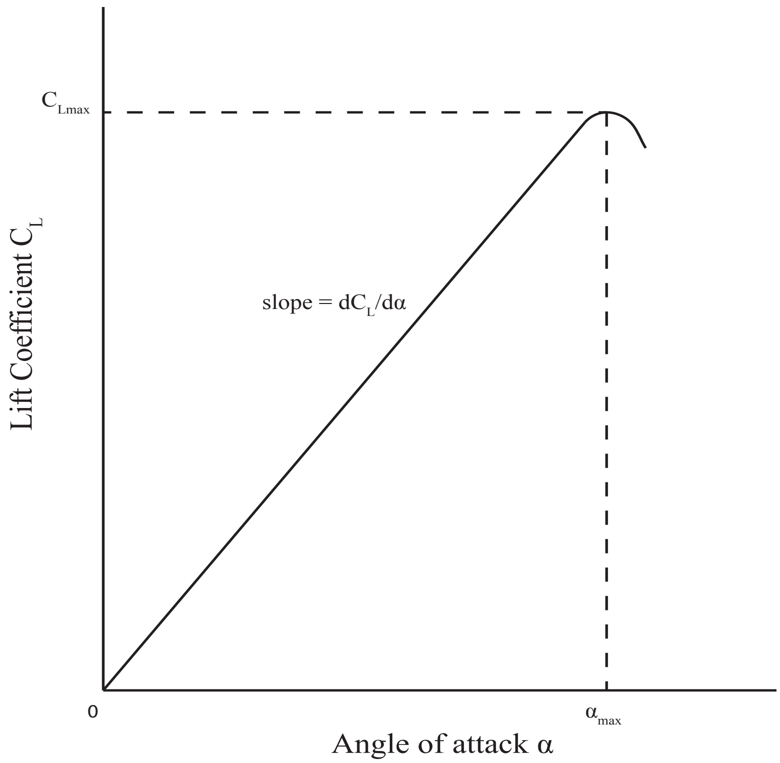


Figure 6.1: Symmetric wing at a positive angle of attack.

The lift coefficient of a wing increases linearly with the angle of attack. The slope of this curve, $\frac{dC_L}{d\alpha}$*, is called the lift curve slope.*



Note that the lift coefficient does not rise indefinitely with angle of attack. After some α_{stall}*, at which* C_L *reaches its max value, flow separation occurs along the wing. This is known as stall, and greatly reduces lift.*

2. Introducing Camber

In the C_L *vs* α *graph shown above, the lift coefficient is zero when the angle of attack is zero. We would say that the zero-lift angle of attack is 0 degrees. However, not all airfoils are symmetric. Some are cambered, which essentially means that they are not symmetric.*

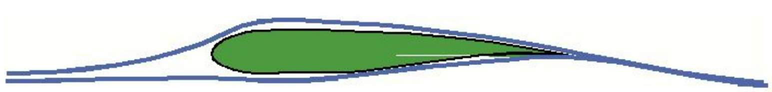


Figure 6.2:

3. Vortex-Induced Lift

The third way to generate lift in low-speed flight is vortex-induced lift. The vortex generated at the wing tip is generally bad news, because it means lift loss and drag rise. However, being a vortex, it has regions of high velocity and low pressure. If we can make the vortex go close to the upper surface of the wing, this low pressure can provide the suction we need to generate lift. This principle is used on aircraft which, for other reasons, must have wings with extremely low aspect ratio. In fact most aircraft designed for high-speed flight and high maneuverability have wings of small aspect ratio, with highly swept wing leading edges. The wing sweep is so high that we can think of the entire leading edge as the wing tip. Even at small angles of attack, a vortex forms along this edge (called, obviously, the Leading Edge Vortex), and this provides much of the lift of such wings when the aircraft is flying at low speed (even supersonic aircraft need to land, fairly slowly). When vortex lift is used, the wings can be very thin, and have sharp leading edges, which are good to minimize shocks and wave drag in high speed flight.

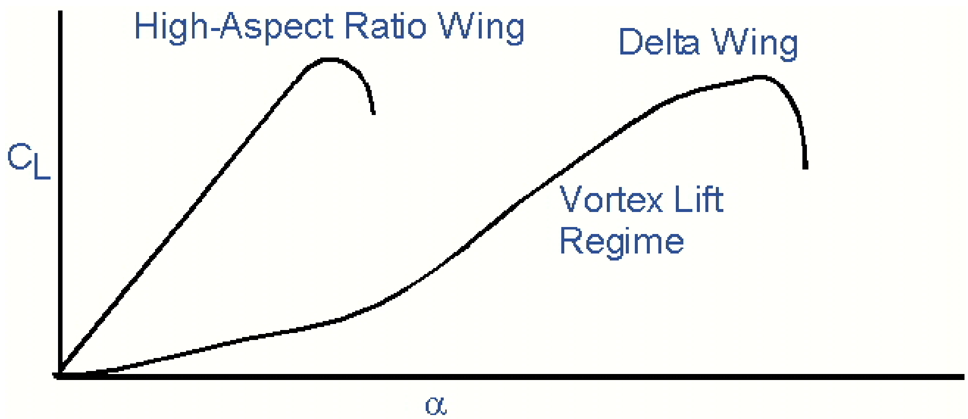


Figure 6.3:

The vortex lift-curve slope is very small compared to the ideal lift curve slope of 2π per radian. However, vortex lift can be obtained up to large angles of attack, sometimes up to 30 degrees angle of attack. So adequate lift can be obtained by going to high angles of attack during landing and low-speed flight. The North American XB-70 supersonic bomber, the British Aerospace - Aerospatiale Concorde (shown against the sun, below), and the Soviet Tupolev Tu-144 supersonic jetliners are examples of delta-winged aircraft. The delta wings are good for supersonic flight. When the aircraft comes in for a landing, it does so at a high angle of attack where the wings produce vortex lift.

Figure 6.4: (Left)North American XB-70. (Right)Concorde.

6.2 Drag

Drag is force along the freestream direction acting on the vehicle. It is due to irreversible loss of momentum. Drag is given by:

$$D = \frac{1}{2}\rho U_\infty^2 S C_D$$

The lift to drag ratio is:

$$\frac{L}{D} = \frac{C_L}{C_D}$$

We want our planes to have as high a L/D ratio as possible!

6.3 Pressure

Bernoulli's Principle

In each lift generation method presented in the "Lift Generation" section, the flow moves more rapidly at some places than at others. In these regions of high velocity, the pressure is lower. The relation between pressure and velocity in low-speed flow is given by the Bernoulli equation:

$$p_0 = p_1 + \frac{1}{2}\rho U_1^2$$

or

$$p_0 = p_1 + \frac{1}{2}\rho U_1^2 = p_2 + \frac{1}{2}\rho U_2^2$$

This equation is derived from Newton's Second Law of Motion, which expresses conservation of momentum.

p_0 is called the stagnation pressure, or total pressure, while p is called the static pressure. The term $\frac{1}{2}\rho U^2$ is called the dynamic pressure, also denoted as q.

Pressure Coefficient

The pressure coefficient is a way to express the pressure with respect to some reference pressure, as a "dimensionless" quantity.

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U_\infty^2} = \frac{p - p_\infty}{q_\infty} = 1 - (\frac{U}{U_\infty})^2$$

$C_p = 0$ *indicates the undisturbed freestream value of static pressure.*
$C_p = 1$ *indicates a stagnation point.*
$C_p < 0$ *indicates a suction region.*
Chordwise pressure distribution over an airfoil in low-speed flow.

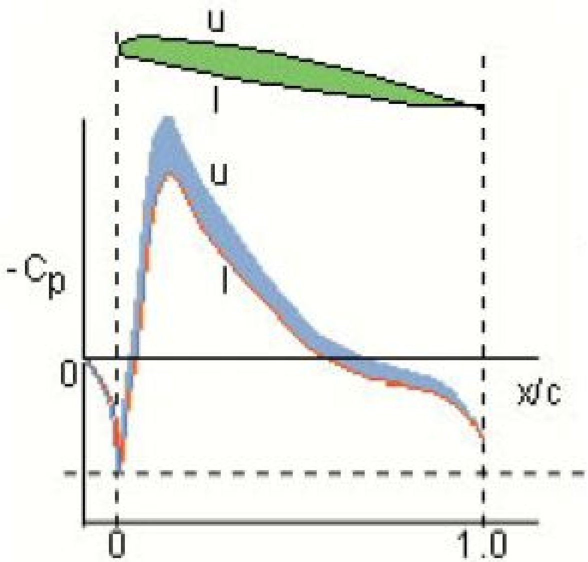


Figure 6.5: Chordwise pressure distribution over an airfoil in low-speed flow.

Excercise: Pressure Coefficient

What is the pressure coefficient at the stagnation point of an airfoil section?
What is the pressure coefficient on a flat surface aligned with the freestream?

C_p *at the suction peak of an airfoil is -1.2. What is the pressure there as a percentage of the freestream static pressure?*

What is the velocity at this point, as a percentage of the freestream velocity?

6.4 Airfoil (British: "aerofoil")

"Airfoil" means "shape of a section of a wing". It is a two-dimensional concept. Airfoils can't fly; wings fly. Airfoil properties are used to calculate and design wing properties.

The airfoil lift coefficient, also called the section lift coefficient, is denoted as c_l*. Note that this lowercase coefficient is not the same as the uppercase version,* C_L*, which refers to the wing lift coefficient. As with the wing lift coefficient, the airfoil lift coefficient also varies with angle of attack* α*.*

If the airfoil is cambered, the lift coefficient is positive even at zero angle of attack, and reaches zero only at some negative value of α*. This is called the "zero-lift angle of attack",* α_0*. As the camber increases,* α_0 *becomes more negative.*

Thus the airfoil coefficient is $c_l = \frac{dc_l}{d\alpha}(\alpha - \alpha_0)$

The lift curve slope is $\frac{dc_l}{d\alpha}$*.* $\frac{dc_l}{d\alpha} \leq 2\pi$*, where* α *is in radians.*

Excercise: Airfoil Lift Coefficient

The angle of attack of an airfoil is 12 degrees. The lift curve slope is 5.8 per radian. Zero-lift angle of attack is -2 degrees. Find the lift coefficient.

If the air density is 1/10 of sea-level standard, and the temperature is 20 deg. C higher than the standard sea-level, flight speed is 100 m/s, and the airfoil chord is 1.2m, find the lift per unit span of this airfoil section.

Chapter 7

Aerodynamics of Wings and Aircraft

7.1 Introduction

Unlike airfoils, studied in the last chapter, wings are 3-dimensional objects. They have a span, b, which is simply the distance from one wingtip to the other. They also have an aspect ratio, AR, defined by:

$$AR = \frac{b^2}{S}$$

where b is the span and S is the wing planform area. It is important to note that airfoils are considered to have an infinite span, and an aspect ratio of infinite.

Effects of Finite Aspect Ratio

At the ends of the wings, the pressure difference between the upper and lower sides is lost, as the flow rolls up into a vortex. This does not happen with airfoils, because, as previously stated, their spans are considered infinite and thus the flow never rolls up.

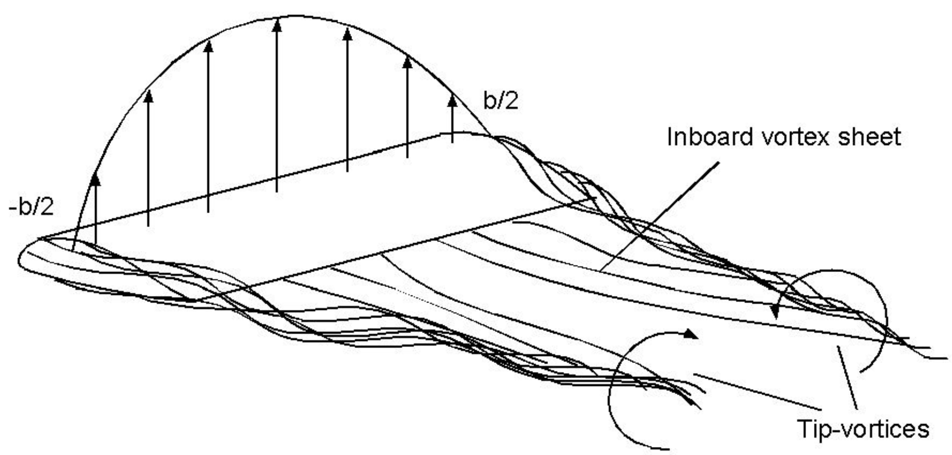


Figure 7.1: Wingtip vortices.

This effect causes overall lift to be reduced, relative to the airfoil lift value predicted for a section of an infinite wing.

The lift vector is tilted back, so that an induced drag is created.

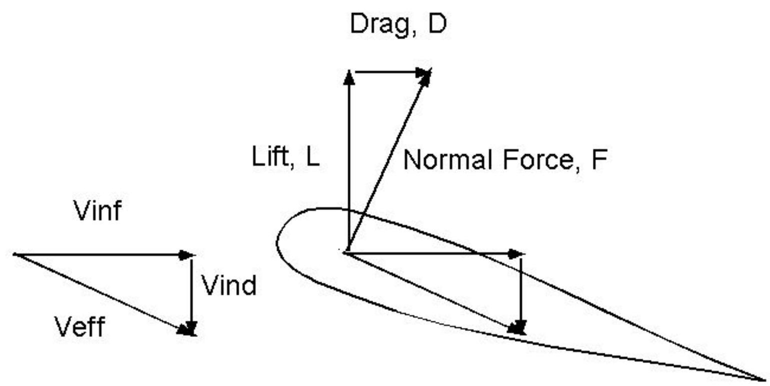


Figure 7.2:

Both of these (usually undesirable) effects are reduced by increasing the aspect ratio of the wing. As the aspect ratio approaches infinity, the wing resembles an airfoil, and will be less affected by these effects.

7.2 Lift Coefficient

Due to the effects of finite aspect ratio described above, the lift curve slope of a wing, $\frac{dC_L}{d\alpha}$*, will be smaller than the lift curve slope of the airfoil shape it uses. If we set the airfoil's lift curve slope to* $a_0 = \frac{dc_l}{d\alpha}$*, we can find the lift curve slope of a wing using this airfoil with the following equation:*

$$\frac{dC_L}{d\alpha} = \frac{a_0}{1 + \frac{a_0}{\pi ARe}}$$

The lift curve slope of a wing depends on the lift curve slope of its airfoil as well as its aspect ratio and spanwise efficiency factor. As the equation above shows, any wing with a finite aspect ratio will always have a smaller lift curve slope than its airfoil. As the aspect ratio of the wing approaches infinite, the lift curve slope of the wing approaches that of the airfoil. An aspect ratio of infinte would yield

$$\frac{dC_L}{d\alpha} = \frac{a_0}{1+0} = \frac{dc_l}{d\alpha}$$

In this chapter's introduction, it was stated that airfoils are considered to have an infinite aspect ratio. The results of this equation agree with this.

Although their lift curve slopes differ, wings do have the same zero-lift angle of attack α_0 *as their airfoils. Thus it is possible, given the* $\frac{dc_l}{d\alpha}$ *and* α_0 *of the airfoil, and the aspect ratio and spanwise efficiency of the wing, to find the lift coefficient of a wing at any angle of attack using the following equation:*

$$C_L = \frac{dC_L}{d\alpha}$$

7.3 Drag Coefficient

Drag is given by

$$D = \frac{1}{2}\rho U_\infty^2 S C_D$$

The drag coefficient in low-speed flow is composed of three parts:

$$C_D = C_{D0} + C_{D friction} + C_{Di}$$

where C_{D0} is the parasite drag coefficient, $C_{D friction}$ is the skin friction drag coefficient, and C_{Di} is the lift-induced drag. These three terms make up what the total drag coefficient, C_D, for low-speed flight.

Parasite/Profile Drag

The term C_{D0} in the drag coefficient equation above is the parasitic drag coefficient, also known as the profile drag coefficient. This term is independent of lift. It is usually due to the losses of stagnation pressure which occur when part of the flow separates somewhere along the wing or body surface. In high speed flight, the effects of shock and wave dragmust be added to this, and becomes the dominant source of drag. Most aircraft are designed to minimize C_{D0}.

The profile drag of an airfoil of chord 1 unit is about the same as that of a circular cylinder whose diameter is only 0.005 units.

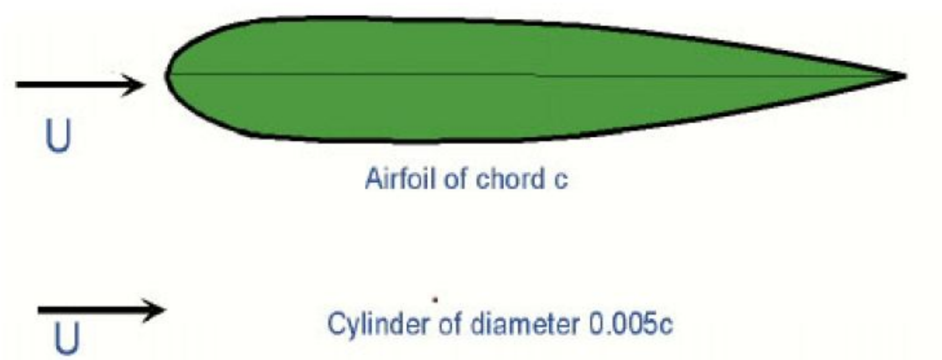


Figure 7.3: The profile drag of an airfoil is about the same as that of a circular cylinder whose diameter is 0.5% of the chord length.

Skin Friction Drag

$C_{D friction}$ is the skin friction drag coefficient, which is due to viscosity. This becomes important in two limits: one where the size of the wing, or the speed of the flow, is extremely small, as might be the case for an insect-sized aircraft. This is called the "low-Reynolds number" limit. We will see later what this Reynolds number is. The other limit is that of high-speed flight, where the skin friction can be severe enough to heat up the wing surface to melting point. If inviscid flow is assumed, this term can be neglected.

Induced Drag

C_{Di} is the lift-induced drag coefficient. In low-speed flight, this is the largest cause of drag, because you must have lift to fly, and this drag is caused by lift. The lift induced drag coefficient can be calculated with the following formula:

$$C_{Di} = \frac{C_L^2}{\pi ARe}$$

In the above equation, AR is the aspect ratio, C_L is the lift coefficient, and e is the spanwise efficiency factor. The spanwise efficiency factor answers the question: How does this wing rate compared to the ideal wing for this aspect ratio? Its value is usually close to 1, perhaps as high as 0.99. If unknown, it can usually be assumed to be anything from 0.9 to 1.

Two things should be noted from the equation for the induced drag coefficient:

1. $C_{Di} \propto C_L^2$ so that $C_{Di} \propto \alpha^2$. Induced drag increases with the square of the angle of attack. A smaller angle of attack reduces the induced drag.

2. C_{Di} approaches zero as aspect ratio approaches infinite. A larger aspect ratio reduces the induced drag.

To minimize induced drag, one should design wings with the largest possible aspect ratio, but also provide enough surface area so that you only need a small angle of attack to provide the necessary lift even at low speed.

Example: Calculating Drag

C_{D0} of a small airliner is 0.018. The wing aspect ratio is 6. Assume spanwise efficiency is 0.9. The lift coefficient is 0.5. Find the total drag coefficient. If the density is 1 kg/m^3, the span is 40 meters and the speed is 200 m/s, find the drag.

Speed for Minimum Drag

As mentioned before, total drag is composed of a part that depends on lift, and one that does not.

$$D = D_0 + D_i = (C_{D0} + C_{Di})(\frac{1}{2}\rho U_\infty^2 S)$$

$$D = (C_{D0} + \frac{C_L^2}{\pi(AR)e})(\frac{1}{2}\rho U_\infty^2 S)$$

Let us consider what it takes to keep lift equal to drag, L = W:

$$W = L = q_\infty S C_L where C_L = \frac{W}{q_\infty S}$$

So

$$D = q_\infty S C_{D0} + (\frac{W}{S})^2 \frac{1}{\pi(AR)e}(\frac{S}{q_\infty})$$

$$\frac{dD}{dq_\infty} = SC_{D0} - (\frac{W}{S})^2 \frac{1}{\pi(AR)e}(\frac{S}{q_\infty^2}) = 0$$

$$C_{D0} = (\frac{W}{Sq_\infty})^2 \frac{1}{\pi(AR)e}$$

$$C_{D0} = C_{Di}$$

Minimum Total Drag = twice zero-lift drag. This is a remarkable result. It means that aircraft, unlike other forms of transportation, have a definite speed for minimum drag.

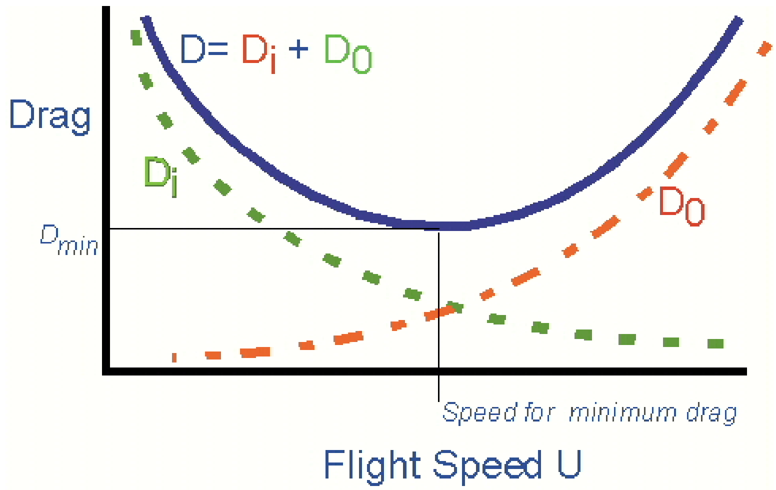


Figure 7.4: For aircraft, the speed for minimum drag is nonzero.

To fly an airplane of a given weight, straight and level, the condition for minimum drag (maximum lift-to-drag ratio) is that the profile drag coefficient is the same as the induced drag coefficient.

Example

An aircraft has a wing loading (W/S) of 130 pounds per square foot (6233 N/m^2), aspect ratio of 7.667, and wing span of 60.96 m. We'll assume that its spanwise efficiency factor will be 0.99. Let's assume that the profile drag coefficient is given by $C_{D0} = 0.015$.

Thus, for maximum Lift-to-Drag ratio (minimum drag, and lift is always equal to weight for straight and level flight),

$$C_{Di} = C_{D0} = 0.015$$

The corresponding C_L *is calculated as 0.598, and the dynamic pressure is 10423 N/m^2.*

At 11,000 meters in the Standard Atmosphere, density is 0.36 kg/m^3, so that the flight speed is 240.64 m/s.

Note: In practice, the C_{D0} *might change with flight Mach number, for high-speed flight. This is not taken into account in the above example.*

7.4 Aerodynamic Summary

Lift is the force perpendicular to the flow direction, due to pressure differences across surfaces. There are 3 ways of generating lift:

1. *angle of attack*
2. *camber*
3. *vortex-induced lift*

An infinite-span (2-dimensional) wing is entirely described by its airfoil section.

Finite wings have less lift than corresponding infinite wings at the same angle of attack, and also have lift-induced drag. The total drag acting on a wing in inviscid flow is composed of profile drag, which does not vary with lift, and induced drag, which rises with the square of the lift coefficient. The friction drag is ignored in the inviscid case.

To fly an airplane of a given weight, straight and level, the condition for minimum drag (maximum lift-to-drag ratio) is that the profile drag coefficient is the same as the induced drag coefficient.

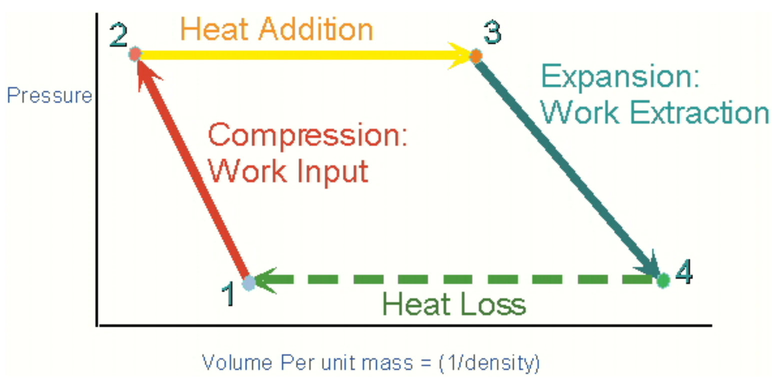
Chapter 8

Propulsion

As a rule of thumb, maximum takeoff thrust available must be around 30% of the gross weight of the aircraft. For airliners, takeoff is the most demanding thrust condition, so it drives the selection of the engines. Airliners must be able to take off with one engine out. Modern airliners use 2, 3 or 4 engines, trending towards using only two if possible.

8.1 How Jet Engines Work

Jet engines work using the Gas Turbine Cycle, a process which consists of the following four steps: compression, heat addition, expansion, and cooling.



1) Compression

Air enters the engine at P_1, and is compressed to a very high pressure P_2. This is done by doing work on the air using the rotating blades of a fan and a compressor. The temperature also rises as the air is compressed.

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}^{\frac{\gamma}{\gamma-1}} \quad \textit{where } \gamma = 1.4 \textit{ so that } \frac{\gamma}{\gamma - 1} = 3.5$$

2) Heat Addition

Heat is added to high-pressure air by burning fuel. Pressure remains constant, and temperature rises to its highest point in the engine. This temperature is called the turbine inlet temperature, as it will enter the turbine after this point. This quantity can be controlled by varying the amount of fuel added. In modern military engines, this temperature is greater than 2000 Kelvin.

3) Expansion and Work Extraction

Hot, high pressure air is blown out through a turbine, and then accelerated through a nozzle. The air moving through the turbine blades forces it to spin. The turbine is connected to the compressor with a shaft, and the turbine drives the compressor as air flows through it. This takes work out of the air, lowering its pressure and temperature. The air then blows out of the nozzle as a jet, with a velocity u_e *and a static pressure equal to ambient air pressure, or* $p_e = p_a$

4) Cooling:

To complete the cycle, we must consider the cooling of air (constant-temperature heat release) in the atmosphere before the next jet aircraft comes along and gulps it in. However, since we don't directly pay for this step inside the engine, we don't worry consider it much.

Thrust for Basic Jet Engine

Thrust is calculated using Newton's Second Law of Motion.

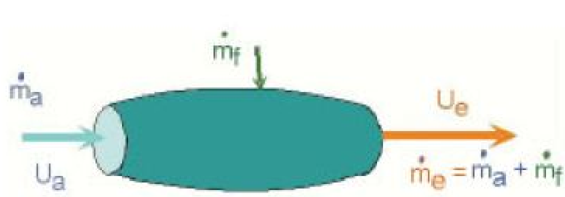


Figure 8.1: Basic jet engine.

Let's say that the speed of air entering the engine is u_a. *The engine gulps* $\dot{m}_a$ *kg/s of air, moving at* u_a *with respect to the engine. It adds* $\dot{m}_f$ *kg of fuel per second, which has about zero velocity with respect to the engine. All this mass then blows out of the exhaust at* u_e *m/s.*

$$(\dot{m}_a + \dot{m}_f)u_e - \dot{m}_a u_a - \dot{m}_f(0)$$

$$\textit{Defining fuel-to-air ratio as: } f \equiv \frac{\dot{m}_f}{m_a}$$

$$\textit{Rate of change of momentum} = \dot{m}_a[(1+f)u_e - u_a]$$

$$\textit{This must be equal to the thrust. } \tau = \dot{m}_a[(1+f)u_e - u_a]$$

The function of a jet engine is to increase the momentum of the fluid passing through it.

"Pressure Thrust"

The general thrust equation in the previoius section left out a pressure term. A more accurate equation of thrust would be:

$$\tau = \dot{m}_a[(1+f)u_e - u_a] + (p_e - p_a)A_e$$

where A_e *is the area of the engine's nozzle exit,* p_e *is the pressure of the gas exiting the engine, and* p_a *is the ambient pressure. The extra term could be called pressure thrust, which is the force due to the pressure difference between the exhaust plane and the outside, acting on the exhaust area.*

Ideally, a nozzle should expand the flow such that the exit pressure is equal to the ambient pressure, or $p_e = p_a$*. Most well-designed nozzles come pretty very to this, and in many cases this extra term can be dropped.*

8.2 Turbojets, Turboprops, Turbofans

Turbojets, turboprops, and turbofans are three different types of air-breathing jet engines that employ the use of a turbine (the "turbo" prefix in each of these words means a turbine is involved). However, some have additional turbines that extract energy out of the flow to do different things.

Turbojets

In turbojets, the turbine only takes energy out of the flow for the compressor and some auxiliary services, but little else. Most of the hot gas expanded during the burning phase is rushed out the nozzle as a fast jet of air.

Turbofans

In turbofans, there is another turbine in the engine that takes energy out of the flow to turn a fan at the front of an engine. This fan accelerates air outside the core of the engine, which adds to the thrust. The air accelerated by the fan is called bypass air. The ratio of the mass flow rate of air through the fan to the mass flow rate of air through the core of the engine is called the bypass ratio, denoted by β*.*

$$\tau = \beta \dot{m_{aH}}(u_{eC} - u_a) + \dot{m_{aH}}[(1+f)u_{eH} - u_a]$$

Turbofans are very efficient because the fan moves a lot of air at a slow velocity. For this reason, most commercial airliners use turbofans. The bypass ratio for most of these engines is usually between 4 to 8.

Turboprops

In turboprops, a turbine takes a lot of energy out of the flow, and is connected to a gearbox at the front of the engine, which is then connected to a propeller. This propeller is what gives the engine most of its thrust; only 15 - 20% comes from the jet. The gearbox is necessary to reduce the rotational speed of the enigne down to propeller speeds.

8.3 Ramjets

A ramjet is an air-breathing jet engine that has less moving parts than a turbojet. The air is compressed without a compressor, relying solely on the geometry of the engine inlet to compress the air. This relieves the need for a turbine. As a result, the combustor can heat the gas to higher temperatures. There are no turbine blades on which the flow can stagnate. This allows ramjets to expel the air-fuel mix at a much higher velocity. The ramjet contains less complicated machinery, which is good from a manufacturing standpoint.
The thrust equation for ramjets is the same as the general equation for thrust out of a jet engine:

$$\tau = \dot{m}_a[(1+f)u_e - u] + (p_e - p_a)A_e$$

Assuming an ideal nozzle, where $p_e = p_a$,

$$\tau = \dot{m}_a[(1+f)u_e - u]$$

A huge problem with ramjets is that they can't generate thrust when they are not moving. Because a ramjet has no compressor, it has no way of sucking air into the engine.

8.4 Rockets

Rockets do not take in any air, and must carry oxidizer as well as fuel. Rockets generate thrust by accelerating this fuel-oxidizer mix out the back of the engine. The thrust equation for a rocket is:

$$\tau = \dot{m}_e u_e + (p_e - p_a)A_e$$

The pressure term in the thrust for a rocket engine is more important than in the other engines examined. In most other cases, the engine can be designed so that the exit pressure is equal to the atmospheric pressure, thus eliminating the pressure term. However this is more difficult with rockets because of two reasons. First, rockets tend to work in low-pressure environments like high altitude, or zero-pressure environments like space. Thus it is harder for the nozzle to expand the flow to the correct conditions. Also, rockets often change altitude very quickly. This rapid change in the ambient pressure surrounding the rocket makes it difficult for a single nozzle design to expand the flow to ideal conditions at each altitude.

Rockets have a measure called the effective exhaust velocity, c_e. This term can be found by adding both types of thrust in the rocket thrust equation and dividing by the total mass flow rate of gas exiting the nozzle:

$$\tau = \dot{m}_e u_e + (p_e - p_a)A_e = \dot{m}_e c_e$$

$$c_e = u_e + \frac{(p_e - p_a)A_e}{\dot{m}_e}$$

8.5 Efficiency

Thermal Efficiency

shows that the efficiency in converting heat to work is highest when the heat is added at the highest possible pressure.

$$\eta_{thermal} = 1 - \frac{1}{P_r}^{(\gamma-1)/\gamma}$$

The compressor pressure ratio, denoted as P_r, is the ratio of pressures after and before the compressor. Modern turbine engines have high compressor pressure ratios, meaning they compress the incoming air to many times its original pressure. Note that even when P_r is 40, the thermal efficiency is only 0.65, or 65%. As with most efficiencies, thermal and propulsive efficiencies can only have values between 0 and 1.

Propulsive Efficiency

The propulsive efficiency measures the efficiency in converting the kinetic energy of the fluid (air) to thrust.

$$\eta_{propulsive} = \frac{2}{1 + \frac{u_e}{u}}$$

The exit velocity u_e will always be greater than the flight velocity u if thrust is created. Taking air in at a certain velocity, the engine must shoot it out at a greater velocity for there to be a change of momentum in the fluid. The thermal efficiency is maximized by driving the exhaust velocity as close as possible to the flight speed u. This is done by using high bypass fans, or turboprops. These devices move lots of air slowly, so that the exit velocity u_e is as close to u as possible.

Specific Thrust and Thrust Specific Fuel Consumption

Thrust Lapse Rate

As we go up in the atmosphere, the density decreases. So, for a given flight speed, thrust decreases as altitude increases. This rate of decrease is called the thrust lapse rate. In the absence of any data on this lapse, it is reasonable to assume that thrust is proportional to the density of air.

Empirical relations have been developed for large turbofan engines of the type employed on modern commercial transport aircraft:

$$\textit{Thrust at altitude} = \textit{(Sea-level static thrust)}$$

Below 5000 m, it can be assumed that thrust varies linearly, or scales with density.

The variation of thrust with altitude is milder than the variation with altitude. The expression is not monotonic (i.e., it does not just keep increasing, or keep decreasing, with increasing Mach number), so it is not attempted here.

Example: Thrust Calculation

Calculate the thrust and specific fuel consumption of the engine with the following conditions:

Hot air mass flow rate = 100 kg/s

Flight speed = 200 m/s

Fuel/air ratio = 0.015

Hot exhaust velocity = 800 m/s

Fan exhaust velocity = 250 m/s

Bypass ratio = 6

Solution:
Because there is a bypass ratio of 6, we know this is a turbofan engine. Using the equation provided in the turbofan section above,

$$\tau = \beta \dot{m_{aH}}(u_{eC} - u_a) + \dot{m_{aH}}[(1+f)u_{eH} - u_a]$$

$$\tau = (6)(100\ kg/s)(250\ m/s - 200\ m/s) + (100\ kg/s)[(1+0.015)800\ m/s - 200\ m/s]$$

$$\tau = 91200\ N$$

TSFC:

$$TSFC = \frac{\dot{m}_f}{\tau}$$

$$TSFC = \frac{(0.015)(100\ kg/s)}{91200\ N}$$

$$TSFC = 1.644\ x\ 10^{-5}\ kg/N\text{-}s$$

Some Implications:
Assume thrust to weight ratio of the engine is roughly 4.5, so that the weight of the engine is 20335 N. Assume that the engine is installed using the 30% of TOW guideline. TOW = 305333 N. Engine weight fraction = 20335/305333 = 6.65%.

If L/D = 12, thrust needed at flight altitude is only 25444 N. Can fly at altitude where density is about 0.28 of sea level.

8.6 Design Step 4

15. Select engines based on thrust and 1-engine-out criteria.
16. Find thrust-specific fuel consumption.
17. Estimate thrust variation with altitude.

Chapter 9

Performance

Aircraft performance is the study of how well an aircraft achieves specific goals. These goals depend on the specific mission of the aircraft in question. Transport aircraft need a lot of range, while patrol aircraft need a lot of endurance. Fighter aircraft need to have small turning radii so as to have the edge in a dogfight, but interceptor aircraft require good climb performance to quickly meet threats. This chapter explores some of the most common performance parameters, and shows how to evaluate them given some basic information about the aircraft.

9.1 Basic Performance Parameters

In this section, basic performance parameters will be discussed. These are either simple enough not to require an entire section, or necessary for understanding more complex parameters later in the chapter.

Wing Loading

The wing loading of an aircraft is given by the total weight of the aircraft divided by the planform area of the wing. The Boeing 747 has a wing loading of about 150 lb_f/ft^2*. In order to grasp the magnitude of this feat, try to visualize a square foot of metal. Then imagine putting 150 pounds of weight on it and trying to make it fly.*

Excess Thrust

For an aircraft to fly straight and level, its thrust must equal its drag. However, if an aircraft's engines are producing more thrust than is needed to fly, it is said to have excess thrust. Excess thrust is simply the difference between thrust and drag.

$$\textit{Excess Thrust} = T - D$$

Excess Power

The excess power is simply the excess thrust multiplied by the velocity at which the aircraft is moving.

$$\textit{Excess Power} = u_\infty (T - D)$$

9.2 Climb Performance

Steady, Unaccelerated Climb

Steady, unaccelerated climb refers to climb at a constant velocity u_∞ *at some nonzero angle* θ *to the horizontal. This angle* θ *is called the climb angle. Usually the most important parameter involved with climb performance is rate of climb, denoted* R_c*, which is the rate of change in altitude. It is the vertical speed of an aircraft, and is most commonly expressed in feet per minute, even in metric countries.*

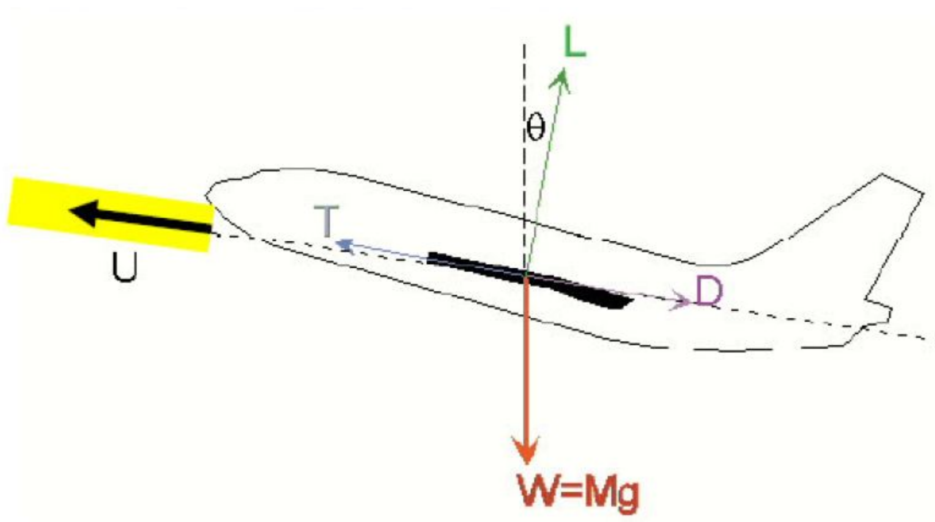


Figure 9.1: Aircraft in steady climb.

From Figure 9.1, we can derive the equation of motion along the freestream:

$$T - D - W\sin\theta = 0$$

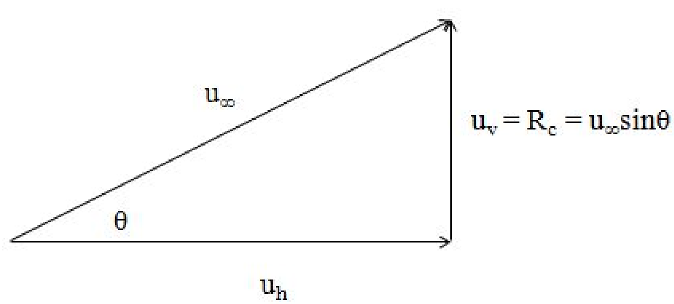
and the equation of motion perpendicular to the freestream:

$$L - W\cos\theta = 0$$

Solving for $\sin\theta$ *in the equation of motion along the freestream gives:*

$$\sin\theta = \frac{T - D}{W}$$

The aircraft is climbing with a speed u_∞ *at an angle* θ *to the horizontal. This can be broken into components, as shown below.*



Rate of climb. From Figure 9.2, we can see that the rate of climb, R_c, is equal to the vertical component of the velocity, u_v, which is equal to $u_\infty \sin\theta$.

$$R_c = u_\infty \sin\theta$$

Inserting the equation for $\sin\theta$ derived above,

$$R_c = u_\infty \frac{T - D}{W}$$

The term $u_\infty(T - D)$ is the term for excess power. Thus, the rate of climb can be expressed as excess power over weight.

$$R_c = \frac{Excess\ Power}{W}$$

Because of this relationship between excess power and rate of climb, it can be said that excess power determines the rate of climb. Similarly, excess thrust determines the angle of climb, θ. This can be seen if the expression for climb angle is slightly modified:

$$\sin\theta = \frac{T - D}{W} = \frac{Excess\ Thrust}{W}$$

.

Example: Steady Climb

An aircraft has a L/D of 15 and its thrust is 105% of the drag. Find the climb angle, assuming constant flight speed.

L/D = 15
T/D = 1.05

$$\tan\theta = (T - D)/L = (T/D - 1)/(L/D) = 0.05/15$$
$$= 0.0033$$
$$\sim 0.0033 * 180/\pi \ degrees$$
$$\sim 0.2 \ degrees$$

*If speed is 200 m/s, climb rate is 0.0033*200 = 0.66 m/s ∼ 120 feet per minute. Very low.*
Typical climb rate is ∼ 500 fpm.
Question: what is the thrust needed, as % of drag, for a 500 fpm climb?

Gliding Flight

gliding flight. Gliding flight, or unpowered flight, is flight with no thrust. Although there are some aircraft specifically built for gliding, any aircraft can become a glider if the engine fails or is shut off.

The glide angle is the angle between the horizontal and the flight path of the aircraft, and is denoted by θ. The equilibrium equations for the figure above are shown below:

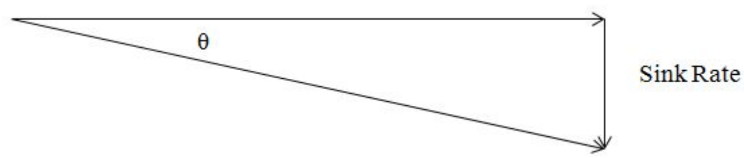
$$D = W \sin\theta$$

$$L = W \cos\theta$$

$$\frac{D}{L} = \tan\theta = \frac{1}{L/D}$$

The lift-to-drag ratio determines the glide angle θ. The higher the lift-to-drag ratio, the smaller the glide angle.

$$\tan\theta_{min} = \frac{1}{(L/D)_{max}}$$



sink rate. The sink rate, or rate of descent, is the same as the rate of climb but in the opposite direction. It is the vertical component of the aircraft's velocity.

$$Sink\ Rate = u_\infty \sin\theta$$

This can also be expressed using the drag and weight of the aircraft:

$$Sink\ Rate = u_\infty \frac{D}{W}$$

The minimum rate of descent occurs when the glide angle is smallest. Because the smallest glide angle is achieved with the highest lift-to-drag ratio, the minimum sink rate occurs when the L/D ratio is highest.

It is possible to find the glide velocity u_∞ for a given θ.

$$L = \frac{1}{2}\rho u_\infty^2 S C_L$$

$$\frac{1}{2}\rho u_\infty^2 S C_L = W \cos\theta$$

$$u_\infty = \sqrt{\frac{2\cos\theta}{\rho C_L}\frac{W}{S}}$$

Example: Sink Rate

An aircraft is coming in on final landing approach. The pilot has aligned the craft with the centerline of the runway, and trims for constant speed of 65 mph. If the aircraft weight is 2400 lb and drag is 220 lb, what should the thrust be, to achieve a constant sink rate of 500 ft/min?

Solution:

$$W = 2400\ lb$$

$$D = 220\ lb$$

$$u_\infty = 65\ \frac{mi}{hr} \cdot \frac{1\ hr}{60\ min} \cdot \frac{5280\ ft}{1\ mi} = 5720\ ft/min$$

$$Sink\ Rate = u_\infty \sin\theta$$

Unlike a problem with purely gliding flight, this one includes thrust:

$$W \sin\theta + T = D$$

$$\sin\theta = \frac{D - T}{W}$$

$$Sink\ Rate = u_\infty \frac{D - T}{W}$$

$$T = D - W \frac{Sink\ Rate}{u_\infty}$$

$$T = 220 - (2400)\left(\frac{500}{5720}\right)$$

The thrust must be 10 lb to achieve a constant sink rate of 500 ft/min.

Accelerated Climb

The sections on steady climb and gliding flight both focused on equilibrium situations, with no acceleration. Now we look at accelerated climb, but will use energy methods instead of dealing with free body diagrams and Newton's second law.

The first concept that must be understood is energy height. The total energy of an aircraft flying at some altitude h is given by the sum of its potential and kinetic energies.

$$Total\ aircraft\ energy = mgh + \frac{1}{2} m u_\infty^2$$

This is the total aircraft energy. This fixed amount of energy can be "spent" in any combination of potential energy or kinetic energy. The total aircraft energy could be turned into all potential energy, or all kinetic energy. An aircraft flying at an altitude h and some velocity u_∞ *could reduce its altitude to almost zero, which would convert all the potential energy into kinetic energy, increasing the speed of the aircraft. Alternatively, the aircraft could convert all of its kinetic energy into potential energy, which would increase the altitude of the aircraft (though it would have no velocity!). This last idea* *illustrates the idea of energy height, H_e. Energy height is simply the total aircraft energy divided by the weight of the aircraft:*

$$H_e = \frac{mgh + \frac{1}{2}mu_\infty^2}{mg}$$

$$H_e = h + \frac{u_\infty^2}{2g}$$

Energy height has units of height and represents the altitude an aircraft could reach if all of its kinetic energy were converted to potential energy.

9.3 Level Turn Performance

A level turn is a turn in which the altitude of the aircraft remains constant. In order to perform a level turn, an aircraft rolls an angle ϕ. This causes the lift vector to rotate by ϕ, as shown in Figure 9.3. The horizontal component of the rotated lift vector is what causes the aircraft to turn.

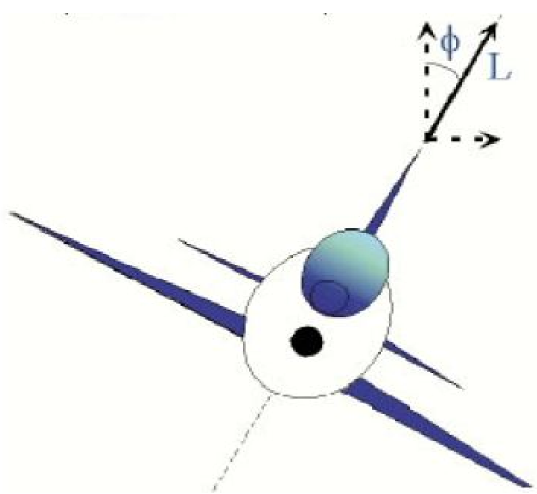


Figure 9.2: Plane performing a level turn.

Because the turn is level, and there is no change in altitude, there can be no net force in the vertical direction. This leads to the following relation between lift and weight:

$$L \cos \phi = W$$

In straight and level flight, the lift must equal the weight for the aircraft to remain in equilibrium. For any angle ϕ greater than 0° and less than 90°, the value $\cos\phi$ *will be less than one. Therefore lift must be greater than the weight if the aircraft is to remain at a level altitude during its turn.*

The centripetal force experience by the aircraft is equal to the horizontal component of the lift vector.

$$m\frac{u_\infty^2}{R_t} = L \sin \phi$$

In the equation above R_t is the turn radius. Thus the equation for turn radius is

$$R_t = \frac{m}{L}\frac{u_\infty^2}{\sin \phi}$$

Load Factor

The load factor n *is defined as the ratio of lift to weight. It is of great importance when looking at turn performance.*

$$n \equiv \frac{L}{W}$$

We have shown that for a level turn,

$$L \cos\phi = W$$

thus

$$n = \frac{L}{W} = \frac{1}{\cos\phi}$$

For a level turn, the load factor and the roll angle are directly related. The load factor is often expressed in terms of "g's". If an aircraft undergoes a load factor of 3, it is also said to be experiencing 3 g's.

Turn Radius and Load Factor

Given above, the turn radius for a level turn is

$$R_t = \frac{m}{L} \frac{u_\infty^2}{\sin\phi}$$

We wish to have load factor in the expression for R_t*. We begin by substituting* W/g *for* m *in the above equation.*

$$R_t = \frac{W}{L} \frac{u_\infty^2}{g \sin\phi} = \frac{u_\infty^2}{gn \sin\phi}$$

Above, we showed that

$$\cos\phi = \frac{1}{n}$$

Using the trig identity

$$\cos^2\phi + \sin^2\phi = 1$$

we can replace $\sin\phi$ *with a term that only has* n*:*

$$\frac{1}{n}^2 + \sin^2\phi = 1$$

$$\sin\phi = \sqrt{1 - \frac{1}{n^2}} = \frac{1}{n}\sqrt{n^2 - 1}$$

Substituting into the original equation for turn radius, the turn radius can be expressed with only u_∞ *and* n*:*

$$R_t = \frac{u_\infty^2}{g\sqrt{n^2 - 1}}$$

From the above equation, it can be seen that the smallest turn radius can be achieved by having the lowest velocity and the highest load factor. This relation makes the importance of load factor evident.

Example: Sideward Forces

Traveling at 250 m/s, an aircraft performs a level turn, with the radius limited by the need to keep the g-level (load factor) below 7.0. What is turn radius R_t, and what is the roll angle ϕ?

Solution:
We begin by with the two equations of motion:

$$L \sin\phi = \frac{mu_\infty^2}{R_t}$$

$$L \cos\phi = W$$

The second equation can be used to find the roll angle.

$$\cos\phi = \frac{W}{L} = \frac{1}{\frac{L}{W}} = \frac{1}{n}$$

$$\phi = \cos^{-1}\frac{1}{n}$$

$$\phi = \cos^{-1}\frac{1}{7} = 81.8^\circ$$

Dividing the centripetal equation of motion with the vertical equation of equilibrium yields:

$$\tan\phi = \frac{mu_\infty^2}{WR_t} = \frac{u_\infty^2}{gR_t}$$

$$R_t = \frac{u_\infty^2}{g\tan\phi} = \frac{(250)^2}{(9.8)(\tan 81.8^\circ)} = 920.5m$$

The roll angle is 81.8° and the turn radius is 920.5 m.

9.4 Range and Endurance

Breguet Range Equation

The range of an aircraft is the total distance it can travel on one load of fuel. The Breguet range equation is a good way to quickly estimate the range of an aircraft using some performance specifications of the aircraft. This equation assumes that the flight is steady and level.

The derivation begins by considering the definition of velocity:

$$u_\infty = \frac{ds}{dt}$$

Because we are solving for range, or total displacement, we arrange this so as to have ds alone on the left side:

$$ds = u_\infty dt$$

Using the definition of thrust specific fuel consumption for a jet-propelled airplane, dt in the above equation can be replaced with some of the aircraft's performance parameters:

$$c_t = -\frac{\dot{W}_{fuel}}{T} = -\frac{dW_{fuel}}{Tdt}$$

The negative sign is there because c_t is a positive value, but the weight of fuel is decreasing so the derivative is negative. Rearranging and solving for dt:

$$dt = -\frac{dW_{fuel}}{c_t T}$$

Substituting this into the displacement equation above:

$$ds = -\frac{u_\infty}{c_t}\frac{dW_{fuel}}{T}$$

We assumed steady, level flight. Thus, $T = D$ and we can replace T with D. We can also multiply the right hand side by L/D, as $L = W$; this is the same as multiplying by one. Also, it is reasonable to assume that $dW = dW_{fuel}$, or that the change in total aircraft weight is the same as the change in fuel weight. Applying these changes results in:

$$ds = -\frac{u_\infty}{c_t}\frac{L}{D}\frac{dW}{W}$$

Range is just the integration of ds. Thus we get:

$$R = \int_0^R ds = -\int_{W_0}^{W_1} \frac{u_\infty}{c_t}\frac{L}{D}\frac{dW}{W}$$

W_0 is the initial weight of the aircraft, when it is fully loaded with fuel, and W_1 is the final weight of the aircraft, when all the fuel has been used. Assuming that u_∞, c_t, and L/D are all constants:

$$R = \frac{u_\infty}{c_t}\frac{L}{D}\ln\frac{W_0}{W_1}$$

This equation is known as the Breguet range equation. This equation shows three independent methods to increase range. Firstly, specific fuel consumption (c_t) should be minimized. Also, the fraction $u_\infty(L/D)$ must be maximized. The speed at which a plane flies affects its lift and therefore its L/D ratio, which is why we join u_∞ and L/D in one term; they are not independent of each other. Lastly, the ratio of the initial weight to the final weight (W_0/W_1) should be maximized. Fuel should compose as much of the initial aircraft weight as possible.

Fuel Consumed During Climb

For the climb phase, we will assume the fuel consumption for cruise plus an increment depending on the cruise altitude. The table below is constructed from data on large commercial airliners (Shevell).

Altitude	% of takeoff weight as added fuel consumption
20,000 feet	0.75%
30,000 feet	1.25%
35,000 feet	1.60%

Endurance

While range is the distance an aircraft can travel on one load of fuel, endurance is the amount of time the aircraft can stay in the air on one load of fuel. It is a measure of time, and as such is measured in seconds, minutes, or hours. There are many applications in which endurance might be more important than range. Observation or patrol aircraft are prime examples. They might need to watch a small area for large amounts of time, and not necessarily cover a lot of ground.

The derivation of the endurance for jet-propelled aircraft is similar to the derivation of the Breguet range equation. Again we assume steady, level flight with constant specific fuel consumption and a constant L/D ratio. The derivation begins with the definition of thrust specific fuel consumption:

$$c_t = -\frac{dW_{fuel}}{Tdt}$$

This time we are interested in solving for total time, so we isolate dt on the left-hand side:

$$dt = -\frac{dW_{fuel}}{Tc_t}$$

As we did in the Breguet range derivation, we will replace thrust with drag, multiply by L/W, and replace dW_{fuel} with dW.

$$dt = -\frac{1}{c_t}\frac{L}{D}\frac{dW_{fuel}}{W}$$

Edurance can be found by integrating from $t = 0$, when the aircraft is fully loaded ($W = W_0$), to $t = E$, when the aircraft has no fuel ($W = W_1$):

$$E = \frac{1}{c_t}\frac{L}{D}\ln\frac{W_0}{W_1}$$

Similar to range, endurance can be increased by reducing the thrust specific fuel consumption c_t and by increasing the ratio of initial weight to final weight, W_0/W_1. Unlike range, endurance does not benefit from maximizing the product $u_\infty(L/D)$, as no u_∞ term appears in the endurance equation. Instead, endurace relies directly on the lift-to-drag ratio, L/D. Increasing the L/D ratio increases the endurance.

9.5 Minimum Velocity: Stall Speed

During steady and level flight, the lift of the airplane must equal the weight. These are both equal to the previously shown equation for lift:

$$W = L = \frac{1}{2}\rho u_\infty^2 SC_L$$

The right hand side of the above equation must equal the weight of the aircraft. The planform area S will probably remain constant (ignoring the possibility of some variable geometry wing), and so will the density, assuming level flight. To maintain a constant lift to support a (mostly) constant weight, u_∞ can decrease only if C_L increases. This is even more apparent if we rearrange:

$$u_\infty = \sqrt{\frac{2W}{\rho SC_L}}$$

As C_L increases, the speed required to stay in the air decreases. Thus, it is reasonable to say that the minimum speed at a given density and weight is:

$$u_{\infty min} = \sqrt{\frac{2W}{\rho S C_{Lmax}}}$$

9.6 Takeoff and Landing

Takeoff and Landing Distances

Net thrust is the thrust minus the ground roll friction, drag, etc. Assume net thrust is 0.2 W_{to}. Then $T - D = 0.2W_{to}$.

Kinetic Energy $= \frac{1}{2}mU^2 = \frac{1}{2}(\frac{W}{g})U^2 = 0.2WR$ *where R is the takeoff run distance.*

In distance R, we gained enough kinetic energy to be at takeoff speed, accelerating at the rate corresponding to net thrust of 0.2 W_{to}. Thus,

$$R = \frac{U^2}{0.4g}$$

The runway length should be twice this distance, in order to stop if the decision to abort takeoff is made at the takeoff speed.

Landing Procedure: Airliner

Descent to 5000 feet. Vectored to 12 miles downwind, make a 180-degree turn. Extend flaps and landing gear, and reduce speed to 150 mph. This leaves 5 minutes of final approach to do the flap deflections, landing gear deployment, lining up of the runway, etc.

Landing Procedure: Small Plane

Maintain constant angle of descent along a specified flight path (staying along the middle of the glide slope). Align with the runway centerline.

Arriving over the runway end, reduce power and increase angle of attack to level off with the landing gear a few feet off the ground. Use ground effect to continue flying level, as the speed reduces, and the aircraft slowly sinks to the ground.

Just before touchdown, flare (increase angle of attack, nearly to stall), and hold altitude. Rate of sink comes to very near zero,and as the speed comes below the stalling speed in ground effect, the main landing gear wheels touch the ground. Continue flying and bring nose slowly down until nose gear touches.

Use if any is available to slow down, otherwise roll with light braking, slow down to the speed for turning off the runway, and take the next taxiway turnoff as instructed by the tower.

Cross-Winds

Factors like cross-winds make life much more interesting. In the event of cross-winds, the airplane may be piloted down with the "controls crossed" (roll so that the lift vector is pointed into the wind, but use rudder in the opposite direction to keep from changing flight direction into the wind. As the airplane comes close to the runway, the wind will generally change quickly in speed and direction- and the pilot must compensate to keep the aircraft aligned with the runway. To land in a cross-wind, the airplane is held in a banked attitude, so that the landing gear on the side facing into the wind touches down first, then the other side, and the pilot holds the aircraft from swerving on the runway due to the greatly increased drag on one side.

Add gusts to the cross-wind, and the need for piloting skills becomes much more evident.

Sometimes, to increase the rate of descent down to the runway, the pilot may fly with the aircraft actually sideslipping, alternately one way and then the other, and again controls crossed to induce a side-slipping motion. The reasoning here is that the drag is much greater in sidelsip, so the forward speed is reduced and the angle of attack made a lot steeper.

Example

Gross weight = 200,000 N.
At altitude H, engine thrust is constant with speed, at 5000 N.
$W/S = 8{,}000\ N/m^2$
$C_{Lmax} = 1.4$

Lowest speed (stall): $L = W = qSC_{Lmax}$

$$u_{stall} = \sqrt{\frac{2W}{\rho S C_{Lmax}}}$$

Lowest speed (thrust available): $T_a = D = qSC_D$

$$C_D = C_{D0} + \frac{C_L^2}{\pi(AR)e}$$

Highest speed (thrust available): $T_a = D = qSC_D$

9.7 Design Step 5

18. *Find the fuel fraction needed for full mission range.*
19. *Determine propulsion and structure mass fractions.*
20. *Verify fuel weight and volume achievable-if not, iterate from # 14.*
21. *Develop steady flight envelop and plot. Determine extremes.*

22. *Verify ceiling, max speed and min speed.*
23. *Takeoff and landing field length; control surfaces/flaps needed.*

Chapter 10

Stability and Control

When you launch a paper airplane, the most frequent cause of an unsuccessful flight is that the airplane flips out of control. This is usually because the airplane is not statically stable. By carefully adjusting the weight distribution, or deflecting some control surfaces, the flight characteristics can be greatly improved.

10.1 Important Points

Center of Pressure

center of pressure. We have defined lift as the force acting on a wing perpendicular to the freestream, and drag as the force acting parallel to the freestream, but have not really visualized how these forces act on a wing. In reality, there are pressure and shear stress distributions acting over the surface. It is possible to find the net effect of these distributions, and represent them with a single concentrated force $\boldsymbol{R}$*, acting at a specific point. The specific point at which this resultant force* $\boldsymbol{R}$ *acts is the center of pressure, denoted by c.p. The force* $\boldsymbol{R}$ *can then be broken down into componenets perpendicular and parallel to the freestream, or lift and drag.*

Aerodynamic Center

The aerodynamic center of an airfoil is the point about which the pitching moment coefficient is independent of the angle of attack. Independent of the angle of attack means the pitching moment is independent of the lift coefficient. At the aerodynamic center, the pitching moment stays constant, no matter how the airfoil orients itself to the flow.

10.2 Static Stability

An aircraft is statically stable if it can recover from small disturbances by itself. It is statically unstable if the disturbance gets amplified and the aircraft does not recover.

Example of Static Instability

Imagine that the center of pressure on a wing is ahead of the center of gravity, as shown below. Suppose that α *increases due to some small, instantaneous gust. The coefficient of lift, and therefore lift, would increase because of the increase in* α*. The change in pitching moment is nose-up, which* *further increases α. Thus, a slight increase in α would continue to get larger; this is statically unstable.*

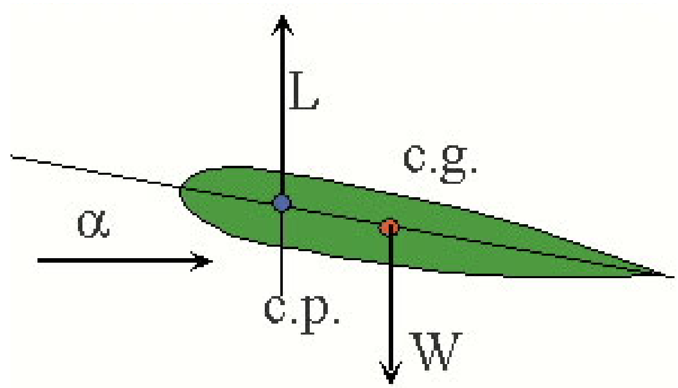


Figure 10.1: CP in front of CG.

Static Stability

If the center of pressure were located behind the center of gravity, lift would still increase if angle of attack were increased. However, this time the lift force is acting behind the center of gravity (at the c.p.), and causes a nose-down pitching moment. Thus the system acts to reduce α if α is suddenly increased, so this system is statically stable.

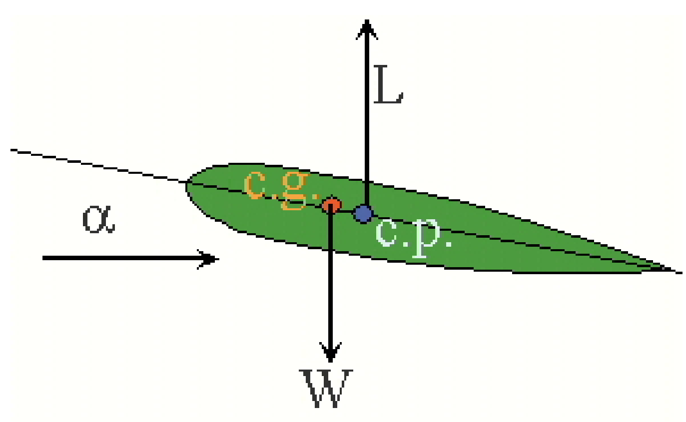


Figure 10.2: CP behind CG.

10.3 Aerodynamic Control Surfaces

In chapter 4, roll, pitch, and yaw were introduced as rotations about axes of the aircraft. In order to induce these rotations, aircraft need control surfaces.

Aileron: Roll Control

Ailerons are control surfaces located at the trailing edge of aircraft wings. Deflecting an aileron is like cambering the airfoil section of the wing. It changes the lift at the same angle of attack. If the *aileron is deflected down, the wing produces more lift; an aileron that is deflected up causes the wing to produce less lift (or even negative lift). Because of this force imbalance, the aircraft rolls about the roll axis.*

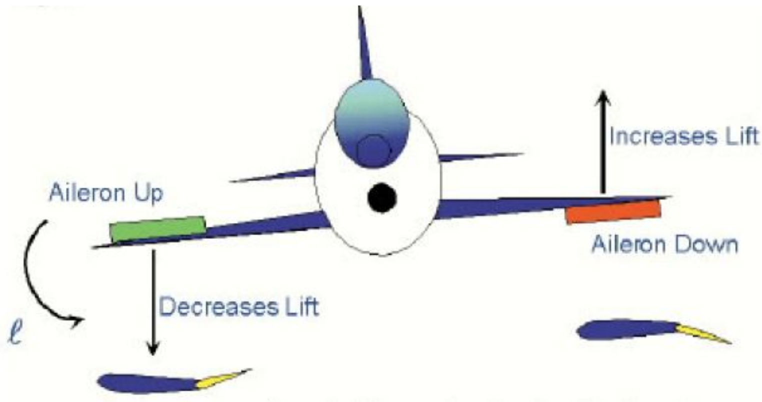


Figure 10.3:

Elevator: Pitch Control

Elevators are flight control surfaces found on the horizontal part of the tail structure of most aircraft.

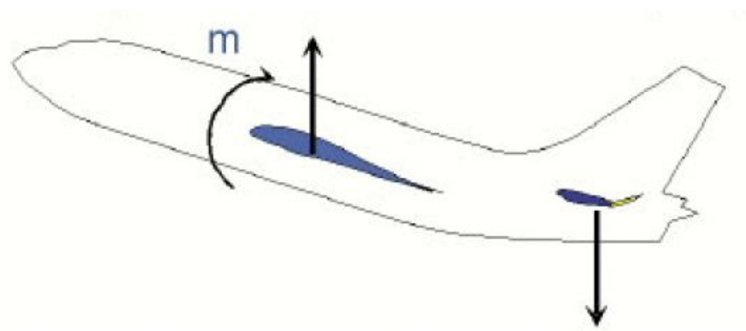


Figure 10.4: Deflecting the elevator up causes the aircraft nose to pitch up.

Rudder: Yaw Control

The rudder is the vertical control surface found on the tail structure of most aircraft. If the lift (side force in this case) on the vertical tail is changed, the aircraft tends to yaw. The aircraft must then roll to avoid sideslipping. In aircraft, yaw is not used to turn aircraft. As was shown in the performance chapter, aircraft roll in order to turn. The main use of the rudder is to make minor changes and corrections as the aircraft goes through a turn.

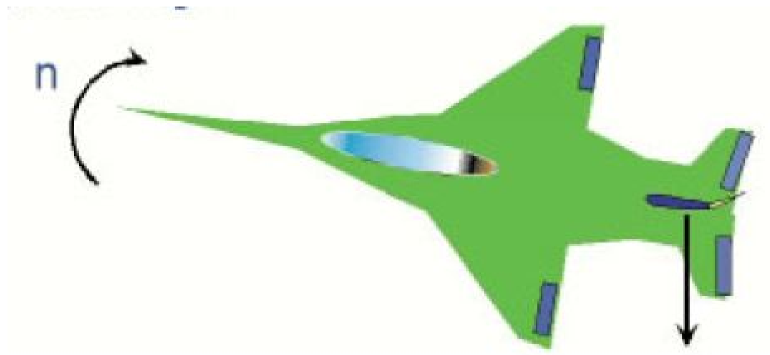


Figure 10.5: Deflecting the rudder to the right causes the aircraft nose to yaw to the right.

Chapter 11

Structures

The study of structures is a critical part of aerospace engineering. Whereas buildings can take advantage of several tons of concrete and steel for their construction, aerospace vehicles are typically weight limited and must be incredibly efficient with their use of supports. Furthermore, the large forces encountered during flight present new structural difficulties not present in the building of static structures. As a result, the aerospace industry is constantly making use of the latest structural advances. This chapter is intended to make the student aware of some basic issues in aircraft structures, as well as some of these advances and their implications. This chapter does not include a detailed explanation on the theory of structures, as this is expected to be covered in structures-specific courses later in the student's career.

11.1 Aeroelasticity

Aeroelasticity refers to the study of structures under aerodynamic loading. Not only do aircraft encounter large aerodynamic loads when flying at high speeds, they have structures that are specifically designed to create yet more forces (like wings that create lift). Therefore it is very important to see how structures and aerodynamic forces interact.

Flutter

Flutter is a phenomenon in the field of dynamic aeroelasticity, and explains a rapid up and down vibration of the wings, and typically causes failure. It occurs when aerodynamic forces mix with elastic and inertial forces to create a dynamic instability. Although it can happen to bridges and chimneys, it is a huge design consideration when designing wings and other lift-creating surfaces. The vibration is usually caused by the aerodynamic force on the wings. If the vibration increases the aerodynamic force, the effect is self-exciting oscillation, and the wing undergoes rapid periodic motion before failing and breaking.

Aeroelastic flutter is what caused the famous collapse of the Tacoma Narrows Bridge in 1940. Caused by 40 mph winds, the aerodynamic forces on the bridge induced an oscillation in the main span that eventually caused the bridge to collapse. Now aerodynamic forces are taken into consideration when designing bridges and other stationary structures. Even chimneys are designed with this in mind so that they do not flutter in high winds. The forces experienced by an aircraft in high-speed flight are much more severe, and for this reason aeroelastic effects are heavily considered when designing an aircraft.

Figure 11.1: This image shows some of the kind of vibration experienced by the Tacoma Narrows Bridge due to aeroelastic flutter.

Active Aeroelastic Wing

Until this point, aeroelastic phenomena have mostly been shown in a negative light. However, this does not have to be the case, and aeroelasticity's effects can be used to increase performance and control. This idea dates back to the Wright brothers, who, lacking the sophisticated control surfaces we take for granted today, tried to control their Wright Flyer by twisting and warping the wings. However, control surfaces such as ailerons quickly became the preferred method of aircraft control.

Although the Wright brothers did not use aeroelastic effects to twist their wings, the NASA Active Aeroelastic Wing program is currently studying the feasibility of using aeroelastic effects to cause wing-warping in order to increase control and performance. The X-53 used to test the idea is a modified F-18 with multiple "aerodynamic tabs" meant to induce aeroelastic effects. The hope is that control of the aircraft can be achieved with smaller control surfaces that turn the entire wing into a control surface, rather than relying on a single large aileron. The larger ailerons add a lot of weight to the wing, and have minimum size requirements, due to the fact that they must be able to control the aircraft even at low speeds. This causes a lot of strain at higher speeds, and also reduces the aspect ratio of the wing. By using the smaller control surfaces to induce twist in the wing, lighter, more efficient wings with higher aspect ratios can be used instead.

Figure 11.2: The X-53 attempts to use small control surfaces to induce wing-warping and control the aircraft.

11.2 Weight

Aerospace vehicles must be as light as possible so they can perform as efficiently as possible. The most structural integrity is gleaned from the least amount of material possible. This requires innovative structural designs not necessary in other fields.

Rocketry

The implications of structural mass are perhaps most evident when considering rockets. The ideal rocket equation presented in the spaceflight chapter gives some insight into this. Achieving low earth orbit requires a velocity incrememnt of over 9,000 m/s. If we use some of the best fuel available, with an I_{sp} *of about 350 s, and a final to initial mass fraction of 0.1, the ideal rocket equation yeilds a velociy increment of only about 7,900 m/s. The structure and payload of the rocket only represent 10% of the initial rocket mass; the other 90% is the fuel! Although multistage rockets are used to circumvent this problem, this example nonetheless makes very clear the need for highly efficient structures, capable of supporting many times their weight. The Atlas rocket would collapse under its own weight if the fuel tanks were empty. A structure so thin that it requires the pressure of fuel in the fuel tanks to keep it supported is precisely the kind of structural shenanigans that must be used in order to have successful vehicles.*

Figure 11.3: The Atlas family of rockets would collapse under their own weight if the fuel tanks were empty.

Carbon Fiber

The aircraft industry has primarily created airframes out of aluminum for many years due to its light weight and high strength. However, carbon fiber is a material that is both stronger and lighter. Carbon fiber is made up of thin strands of carbon. These strands are thinner than human hair, and can be woven together like yarn. When coated with some sort of resin, these strands can be molded into particular shapes. This is similar to paper mache, which involves coating newspaper strips with glue so that they mold to some shape.

Being both lighter and stronger than aluminum, carbon fiber has attracted the attention of commercial aircraft designers. The Boeing 787 is 50% carbon fiber by weight and 80% carbon fiber by volume. This represents a major advance in commercial aviation. The aircraft is estimated to be 20% more fuel efficient than a similarly sized aircraft that is built with the more traditional aluminum.

Figure 11.4: The Boeing 787 is the first commercial airliner made mostly of carbon fiber.

Although there are many great advantages to using carbon fiber, there are also many hurdles impeding its use. While fractures and imperfections in aluminum are easy to test for (in many cases being visible to the naked eye), it is much more difficult to test and perform maintenance on carbon fiber. Carbon fiber is also more expensive than aluminum, but is becoming cheaper with time.

Chapter 12

High Speed Flight

12.1 Introduction

Low-speed flows can be considered incompressible. This means that density is assumed constant. However, this assumption cannot be made for high-speed flows. This makes calculations for high-speed flows more difficult.

12.2 Mach Number and Mach Angle

Mach Number and the Speed of Sound

Small disturbances in air propagate at the speed of sound. This means that the speed of sound is the speed through at which information propagates through the fluid. The speed of sound, a, can be calculated with the formula below:

$$a = \sqrt{\gamma R T}$$

where γ is 1.4 for air, T is an absolute temperature (Kelvin or Rankine), and R is the gas constant of air.

The Mach number M is defined as:

$$M \equiv \frac{u}{a}$$

The Mach Cone Angle

The Mach Cone defines the boundaries of where the weakest disturbances created by an object moving faster than sound, reach, around and behind the object. As shown in Figure 12.1 the Mach cone is the result of the disturbance spreading outwards in an expanding sphere radiating out from the source, as the source moves faster than the speed of sound. Note that the picture shown in the figure is an instantaneous picture. At the next instant, the object will have moved, and the cone will have grown in size in the window that we are viewing.

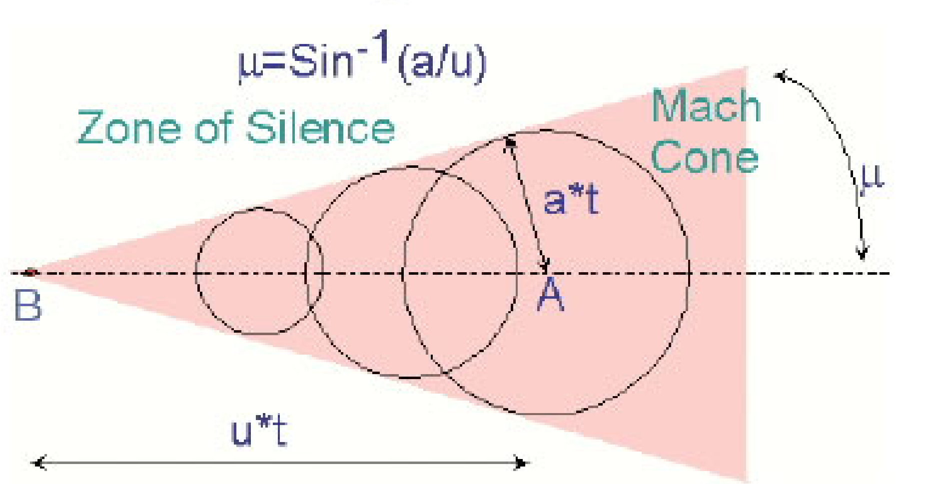


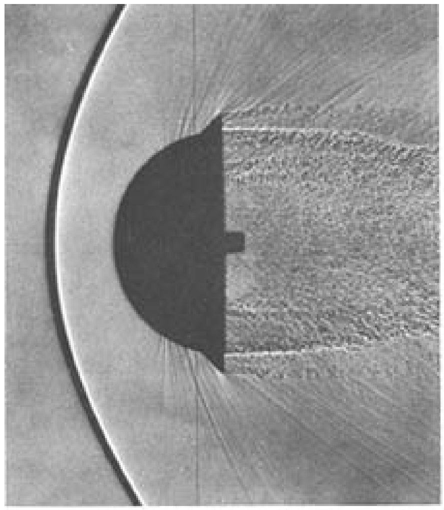
Figure 12.1:

$$\sin\mu = \frac{a}{u} = \frac{1}{M}$$
$$\mu = \sin^{-1}\frac{1}{M}$$

12.3 Shocks

In aeronautics, the term "shocks" refers to shock waves. These waves develop in front of bodies travelling faster than the speed of sound. Because the speed of sound is the speed at which information propagates through a fluid, air in front of an object travelling faster than the speed of sound is unaware of the impending disturbance. The shock develops to slow down the flow before it strikes the object. Across a shock wave, the Mach number of the fluid decreases, and huge increases in pressure, density, temperature, and entropy occur.

If a body is blunt, the shock wave forms in front of the body, as shown in the left side of Figure 12.2. If the body is sharp, an oblique shock wave forms.



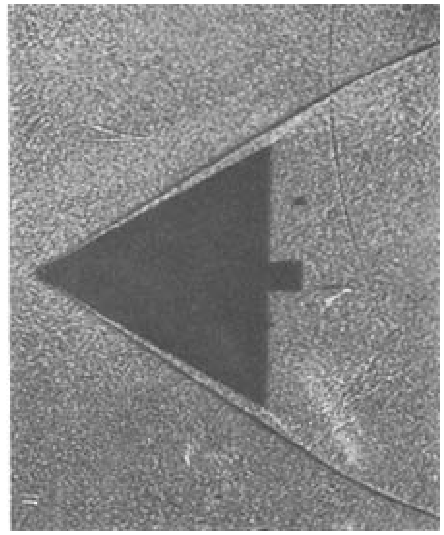


Figure 12.2: (Left)A bow shock wave. (Right)An oblique shock wave.

Oblique shocks turn the flow before it hits the object. The oblique shock angle β is greater than the Mach angle μ.

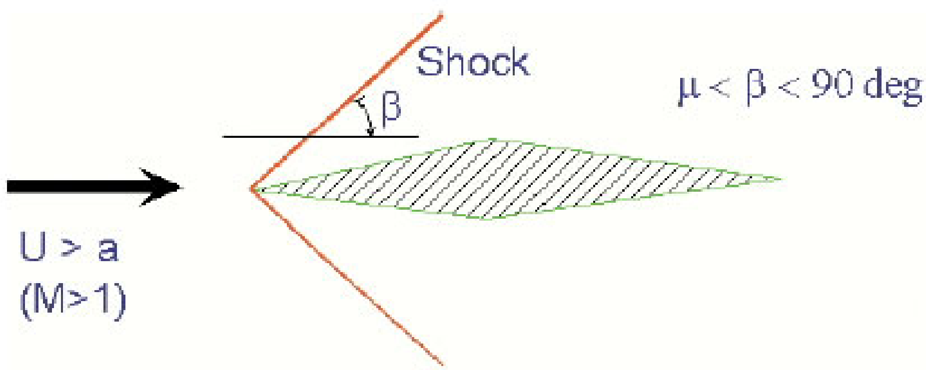


Figure 12.3: Oblique shock angle β.

Hypersonic Regime

The hypersonic regime exists above Mach 4 or 5. Across a shock there is a large pressure increase, as well as a large temperature increase. The Mach angle is extremely small. It is possible to generate lift with a large pressure rise below the lower surface of a hypersonic aircraft.

Figure 12.4:

The Power of Shocks

If an asteroid is so big that it still remains as a solid object when it nears the ground (i.e., comes down to a few thousand feet), the shock waves can hit the ground with terrible strength. This can flatten trees, forrests, and buildings (happened in Siberia just around the time the Wright Brothers were getting off the ground). When the object finally reaches the ground, the shock in front blasts a huge crater in the earth's surface. The crater is huge, even though the object that actually reaches the ground may be quite small. The Meteor Crater in Arizona is one mile in diameter, and hundreds of feet deep. It is widely believed that an immense object hit the Gulf of Mexico during the age of the dinosaurs, and the havoc was such that it caused the extinction of the dinosaurs, worldwide.

12.4 Flight at High Subsonic and Transonic Speeds

Above a Mach number of about 0.3, air can no longer be considered incompressible. This makes calculations more difficult, as density can no longer be considered a constant.

Subsonic Flight: Prandtl-Glauert Compressibility Correction

Information about an airfoil's lift and drag coefficients in incompressible flow is not valid in the compressible regime. However, the incompressible lift and drag coefficients can be converted into lift and drag coefficients in the compressible regime using the Prandtl-Glauert compressibility correction. The lift and drag coefficients of an airfoil at a given angle of attack increase with flight Mach number, according to the Prandtl-Glauert Compressibility Correction:

$$c_l = \frac{c_{lo}}{\sqrt{1 - M_\infty^2}}$$

$$c_d = \frac{c_{do}}{\sqrt{1 - M_\infty^2}}$$

where c_{lo} *and* c_{do} *are the lift and drag coefficients before the correction. To get the same lift coefficient at a higher Mach number, one requires a smaller angle of attack. This is good, because the drag* *coefficient also increases like this. This expression is valid for Mach numbers which are lower than the critical Mach number.*

Critical Mach Number

As a wing approaches Mach 1, the flow at some point along the wing will reach Mach 1 even if the freestream Mach number is lower. Once any flow along the airfoil reaches Mach 1, huge increases in drag occur, as the graph below shows. For this reason, commercial aircraft fly just below their critical Mach number, so a huge drag increase can be avoided.

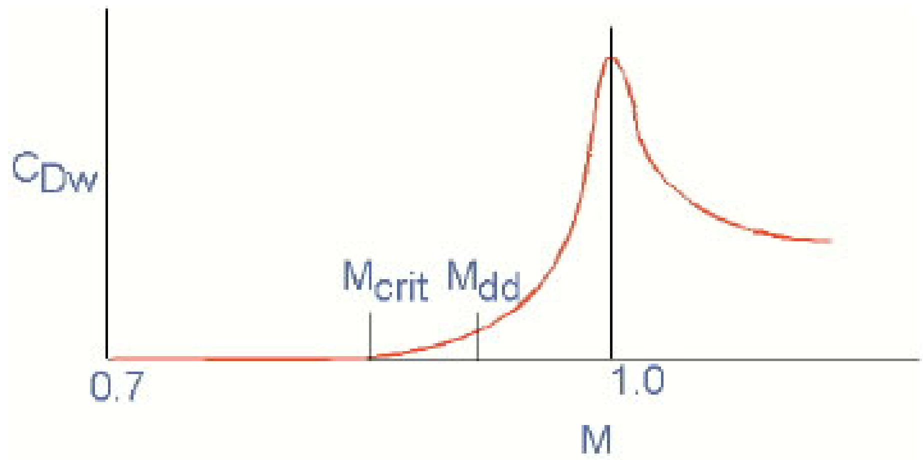


Figure 12.5: Drag increases greatly near Mach 1.

It is possible to "trick" the flow with swept wings and achieve higher critical Mach numbers. The motivation to do so should be apparent; an increase in the critical Mach number is effectively an increase in the cruise speed. The most notable method is the use of wing sweep. If a wing is swept back an angle Λ, the wing only experiences the component of the freestream normal to it. This component is $u_\infty \cos\Lambda$. Because this normal component is smaller than the freestream velocity, the aircraft can travel at a higher freestream velocity before the flow over the wing at any point reaches Mach 1. Thus, the critical Mach number is higher with wing sweep.

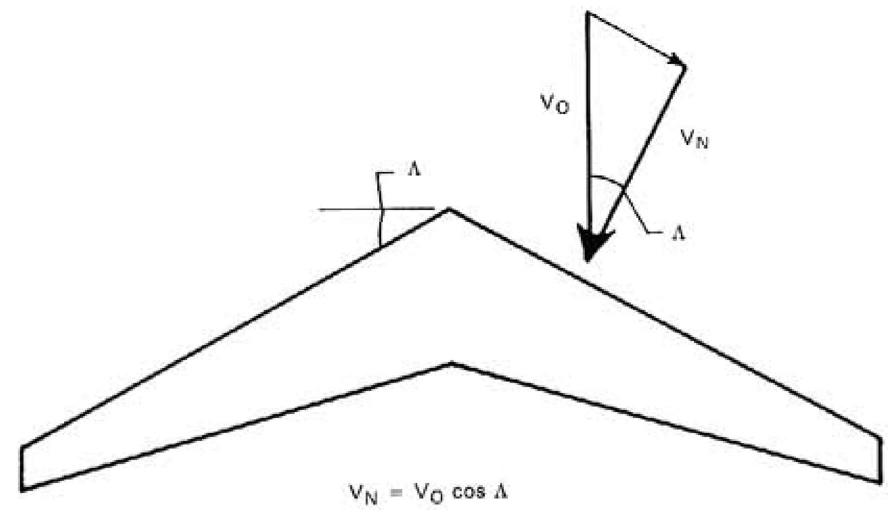


Figure 12.6: Wing sweep increases the critical Mach number by decreasing the normal component of velocity experienced by the wing.

The positive effects of wing sweep can be seen in commercial aircraft. The table below demonstrates the relationship between the sweep angles of various airliners and their cruise Mach numbers.

Aircraft	**Sweep Angle (°)**	**Typical Cruise Mach**	**Max Cruise Mach**
B737-800	25.0	0.785	0.82
B757-200	25.0	0.80	0.82
B767-300	31.5	0.80	0.86
B777-200	31.64	0.84	0.89
B747-400	37.5	0.85	0.92

Suggested Method to Calculate Transonic Drag Peak

The part up to Mach 1 can be described as a Lorentz function.

$$C_D = \frac{1}{\pi\gamma[1 + (\frac{M - M_{cr}}{\gamma})^2]}$$

Where the peak

$$C_{Dmax} = \frac{1}{\pi\gamma}$$

Chapter 13

Space Flight

Lunar Excursion Module. From the Boeing Web Page, Gallery, History Section. See www.boeing.com

Figure 13.1:

13.1 Rockets Revisited

Velocity Increment ΔV

Consider a rocket with effective exhaust velocity c_e. *The effective exhaust velocity is a way of expressing the thrust in a simple manner by adding up the momentum thrust and the pressure thrust and dividing this sum by the mass flow rate. As propellant is blasted from the exit nozzle, the mass of the vehicle decreases. This is substantial in the case of the rocket as compared to air-breathing engines, because all the propellant comes from inside the vehicle. From Newton's Second Law,*

$$Thrust = M\frac{dV}{dt} = -c_e\frac{dM}{dt}$$

$$dV = -c_e \frac{dM}{M}$$

$$\Delta V = -c_e \int_{M_1}^{M_2} \frac{dM}{M}$$

$$\Delta V = c_e \ln \frac{M_1}{M_2}$$

where M_1 is the initial mass, which includes the propellant, and M_2 is the mass after the propellant has been used up to achieve the velocity increment ΔV.

Specific Impulse and Mass Ratio

Define the specific impulse of the propellant as:

$$I_{sp} \equiv \frac{c_e}{g}$$

where g is the standard value of acceleration due to gravity at sea-level (9.8 m/s^2). The unit of specific impulse is seconds. Using this definition,

$$\Delta V = g I_{sp} \ln \frac{M_1}{M_2}$$

Mass ratio of a rocket is

$$\frac{M_1}{M_2} = e^{\frac{\Delta V}{g I_{sp}}}$$

Note: Some space agency websites express specific impulse without the g. Thus their "I_{sp}" is simply c_e.

13.2 The Laws of Newton and Kepler

Newton's Law of Gravitation

To find the velocity increment required for various missions, we must calculate trajectories and orbits. This is done using Newton's Law of Gravitation:

$$F_r = -\frac{GMm}{r^2}$$

Here the left hand side is the "radial force" of attraction due to gravitation, between two bodies; the larger is of mass M, and the smaller is of mass m.

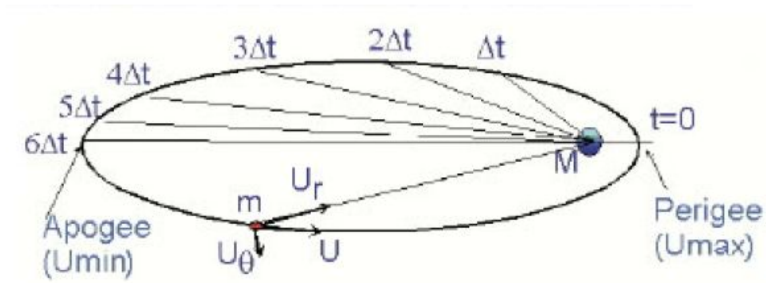
The universal gravitational constant is:

$$G = 6.670 \ x \ 10^{-11} \ Nm^2/kg^2$$

Kepler's Laws

These can be applied to a satellite of mass m, orbiting a much larger object of mass M ($m \ll M$).

1. The satellite travels in an elliptical path around its center of attraction, which is located at one focii of the eclipse. The orbit must lie in a plane containing the center of attraction.



2. The radius vector from the center of attraction sweeps equal areas of the orbit per unit time. As the satellite moves away, its speed decreases. As it nears the center of attraction, its speed increases.

3. The ratio of the squares of the orbital periods of any two satellites about the same body equals the ratio of the cubes of the semi-major axes of the respective orbits.

$$\frac{t_1^2}{t_2^2} = \frac{a_1^3}{a_2^3}$$

Calculating Velocities Along an Elliptical Orbit

The speed at any point in an elliptical orbit is:

$$v = \sqrt{g_0 r_0^2 (\frac{2}{r} - \frac{1}{a})}$$

where g_0 is the acceleration due to gravity at radius r_0.
The minimum velocity is obtained at the apogee, or highest point of the orbit:

$$v_{apogee} = \sqrt{g_0 r_0^2 \frac{(1-\varepsilon)}{a(1+\varepsilon)}} = v_{min}$$

where eccentricity is given by

$$\varepsilon = 1 - \frac{r_{min}}{a}$$

The max velocity is obtained at the perigree, or lowest point of the orbit:

$$v_{perigree} = \sqrt{g_0 r_0^2 \frac{(1+\varepsilon)}{a(1-\varepsilon)}} = v_{max}$$

The time for one orbit is:

$$\tau = \frac{2\pi\sqrt{a^3}}{r_0\sqrt{g_0}}$$

Circular Orbit and Escape Velocity at Radius r

$$v_{circ} = \sqrt{g_0 \frac{r_0^2}{r}}$$

$$v_{escape} = v_{circ}\sqrt{2}$$

$$v_{escape} = \sqrt{2g_0 \frac{r_0^2}{r}}$$

Example: Speed for Circular Orbit and Speed for Escape

Earth's volumetric radius ≈ 6370.998685023 km (≈ 3958.755 mi; ≈3440.064 nmi). However, note that in calculating the standard value of g_0, a slightly different radius is used:

$$g = G\frac{m1}{r^2} = (6.5742 \times 10^{-11})\frac{5.9736 \times 10^{24}}{(6.37101 \times 10^6)^2} = 9.822\frac{m}{s^2}$$

Consider a circular orbit at r = 7000 km.

$$v_{circ} = \sqrt{g_0 \frac{r_0^2}{r}}$$

Escape speed at radius r.

$$v_{escape} = v_{circ}\sqrt{2}$$

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