Fluid Mechanics

3rd Year Mechanical Engineering

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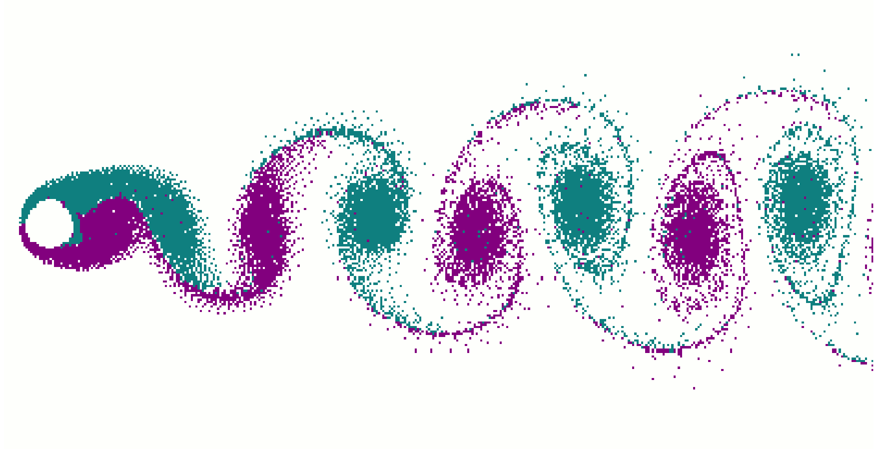
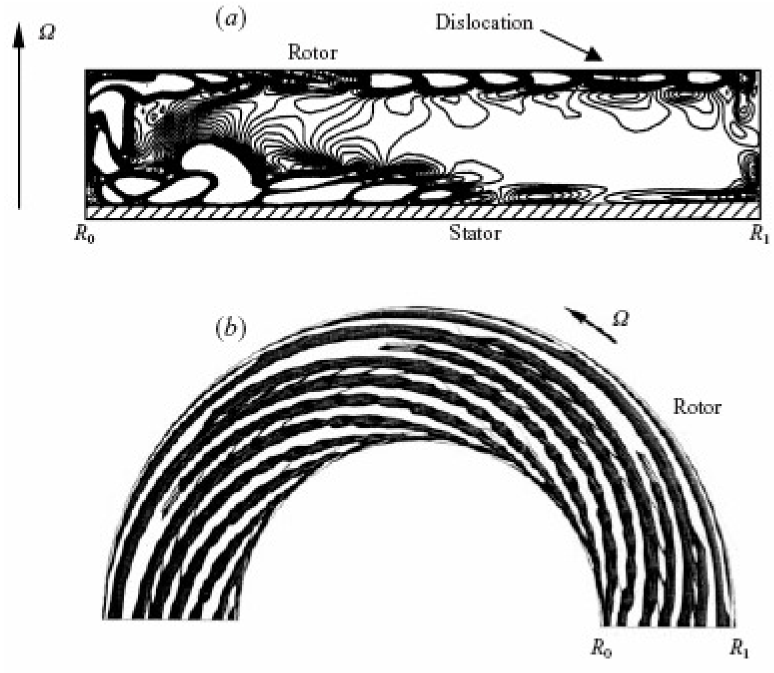
Lecture 9: **Instability & Transition**

Flow instability

- In practice it is found that steady laminar flow only persists at “low” flow rates.
- When some critical level is reached the flow may in some circumstances develop an *unsteady laminar motion.*
- Examples: the Karman vortex street behind a cylinder for Reynolds numbers greater than 40 or the Ekman spirals that form near a spinning disc.

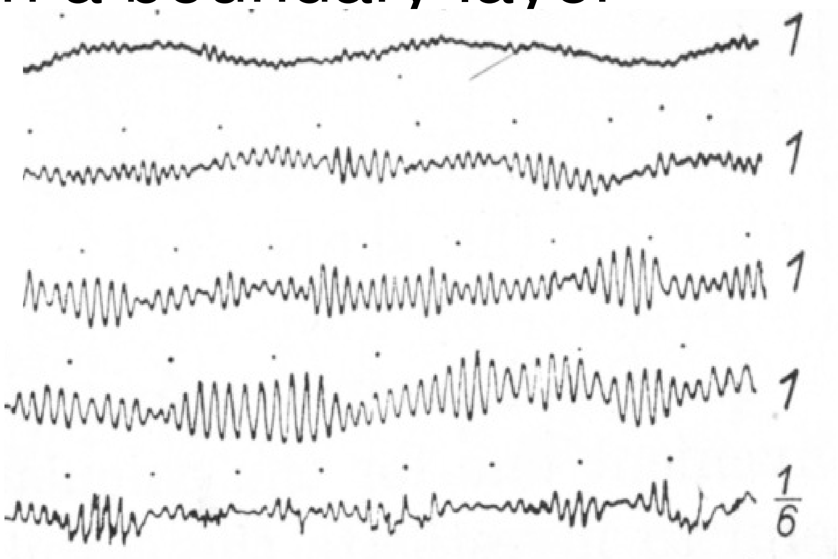




Chaotic instability

- When the damping effects of viscosity are not sufficiently large, however, a regular unsteady laminar motion is not possible and the flow instability develops a chaotic growth pattern.
- The following traces show the development of an instability in a boundary layer

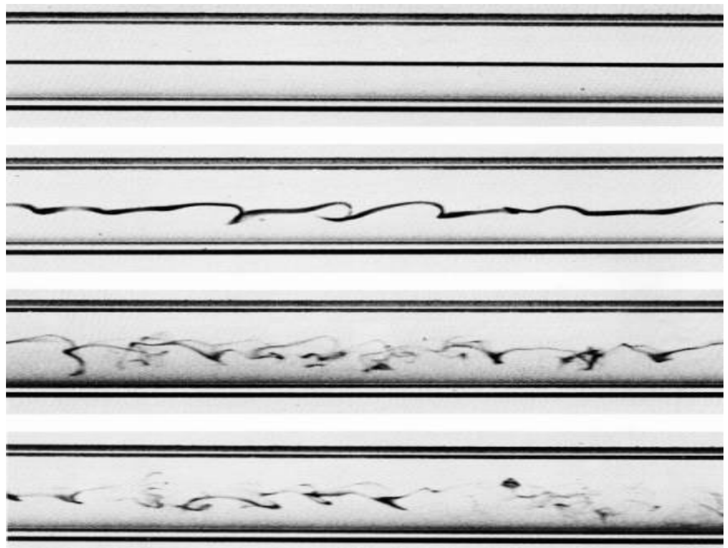
Down-stream



- Note change in scale at most downstream position

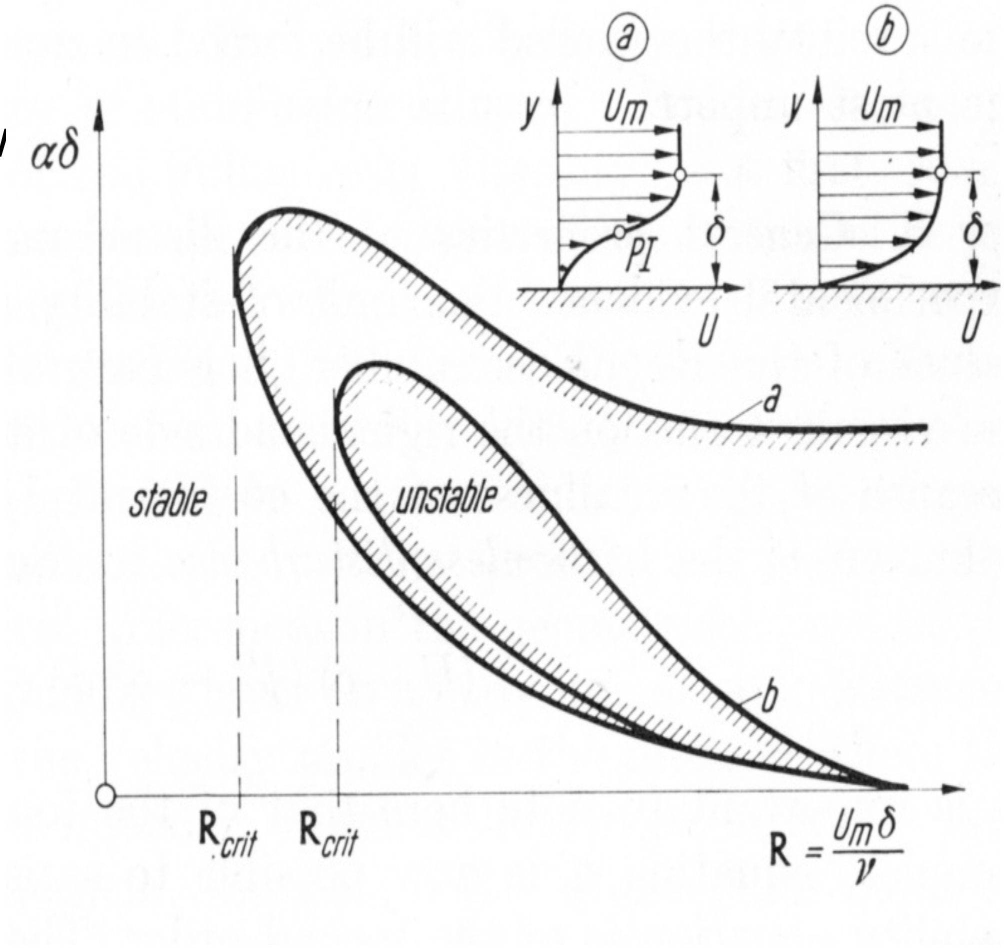
First mapping of such chaotic behaviour

- Osborne Reynolds (first professor of engineering in England) showed that the ***flow through a pipe*** underwent such a transitional behaviour when the dimensionless group UD/ν exceeded a certain critical value.
- He found the most consistent agreement if he identified transition as the value of this group **below** which turbulent flow could not occur.



Classical stability theory

- Explores conditions under which perturbations from a steady state behaviour will grow (unstable) or decay (stable).
- **Orr-Sommerfeld** (O-S) equation used as the basis of such studies.
- Derived from N-S equation by supposing small 2D perturbations to the velocities and discarding quadratic terms.
- Numerical solutions identify the critical wave number and Reynolds number for stability.
- The most important question is: *below what Reynolds numbers is the boundary layer stable at ALL wave numbers?*
- Note the strong dependence of stability on the shape of the mean velocity profile



What is the Orr-Sommerfeld Equation?

- Formed by examining a **2D perturbation** of the Navier-Stokes equations

$$\rho \frac{\partial u'}{\partial t} + \rho U \frac{\partial u'}{\partial x} + \rho v' \frac{dU}{dy} = -\frac{\partial p'}{\partial x} + \mu \left[\frac{\partial^2 u'}{\partial x^2} + \frac{\partial^2 u'}{\partial y^2} \right]$$

- Note: all quadratic products of fluctuating velocities are discarded as small.
- A stream-function ψ is used to characterize the velocity fluctuations: $\psi(x, y, t) = \phi(y) e^{i(\alpha x - \beta t)}$
- Substitution for velocities leads to the 4th order O-S equation in ϕ given in most advanced texts on boundary layers.
- No further knowledge of the eq'n is expected in this course.

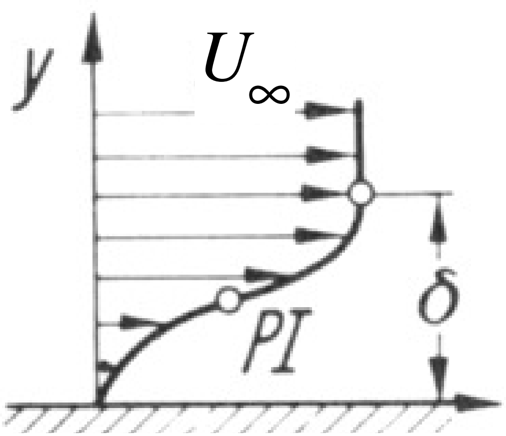
Factors that affect stability

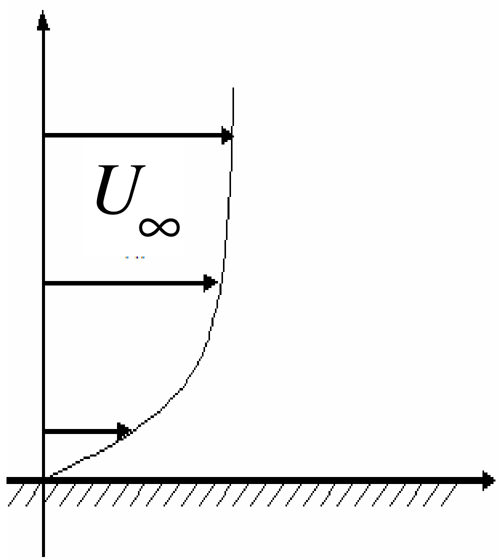
- The most important feature affecting stability is the **shape of the velocity profile**.
- Velocity profiles with a point of inflexion (where .$\partial^2 U / \partial y^2$ changes sign) are especially unstable.
- Hence free shear flows tend to be less stable than boundary layers.
- Streamwise pressure gradients (which greatly affect the velocity profile shape), strongly modify stability – see Slide 8.
- Curvature of the surface and mass transfer through it also have strong effects. These effects discussed further under “flow management” lectures

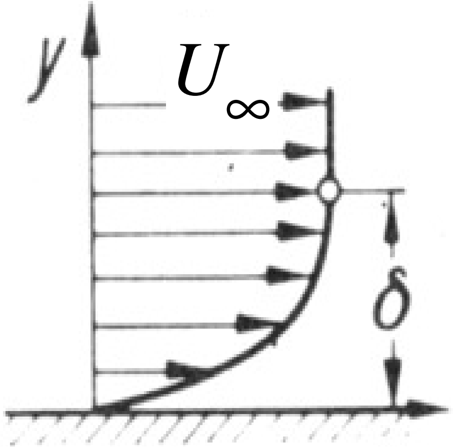
Effect of Velocity Profile Shape on Stability

- **Second derivative of velocity profile at wall gives a good indication**

$$\frac{\partial^2 U}{\partial y^2}\Big|_{y=0} > 0 \qquad \frac{\partial^2 U}{\partial y^2}\Big|_{y=0} = 0 \qquad \frac{\partial^2 U}{\partial y^2}\Big|_{y=0} < 0$$



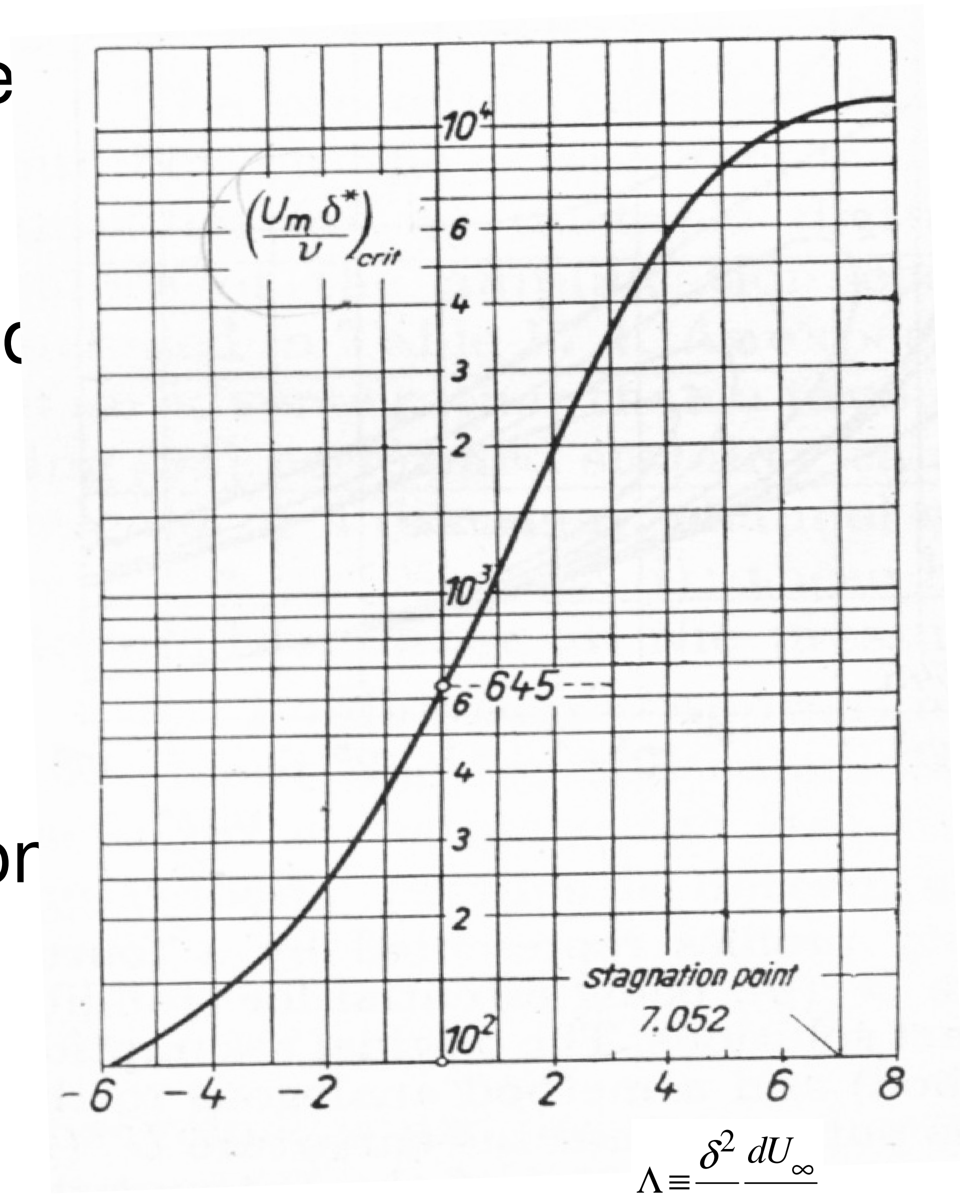




- **Unstable** **Stable**

Stability curve for b.l. in pressure grad.

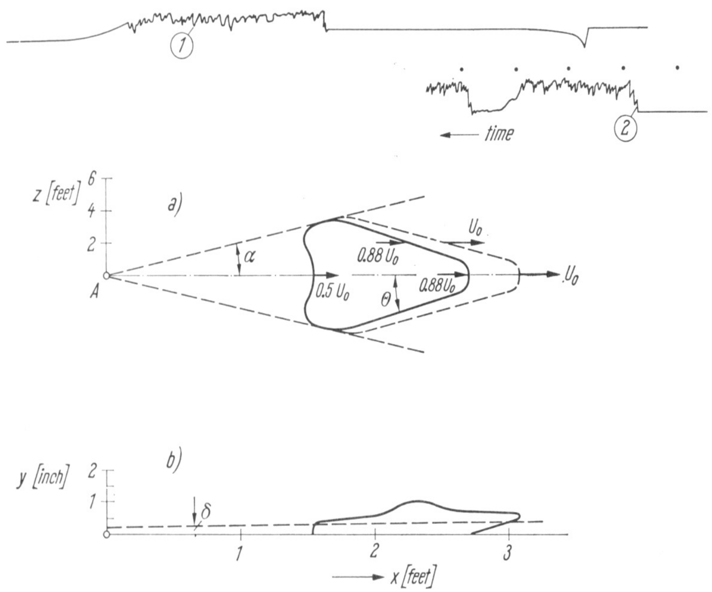
- In a boundary flow, at the wall: $dP/dx = \mu \partial^2 U/\partial y^2$
- In an accelerating flow (dP/dx –ve) which will tend to stabilize the flow.
- In a +ve pressure gradient the reverse occurs.
- In flow over a turbine blade it is sometimes assumed that transition on the suction surface will occur at the minimum pressure point



$$\Lambda \equiv \frac{\delta^2}{\nu}\frac{dU_\infty}{dx}$$

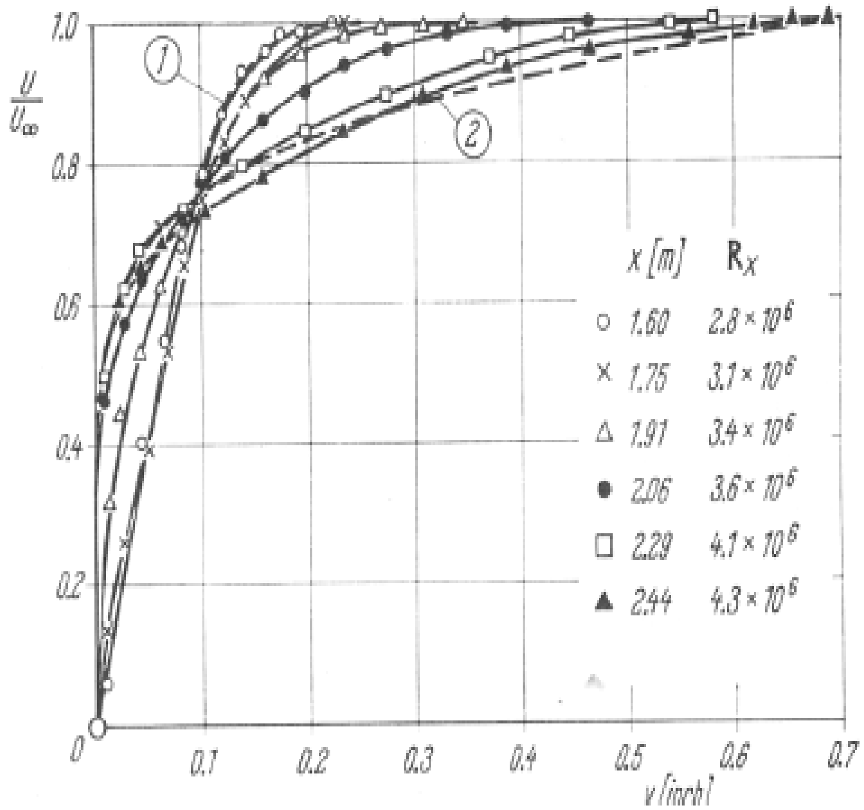
Effect of Free-Stream Turbulence

- Just because a shear flow is un-stable, it doesn't mean that instability and transition will immediately ensue. It depends greatly on the level of **free-stream turbulence (FST)** or wall vibrations.
- With **very low levels of FST** lam-inar flow may persist to Reynolds numbers several times higher than those at which the flow is mathematically 'unstable'.
- When transition occurs at **low FST** it will be "spotty" -see diagram and **very hard to predict**
- Transition at FST higher than about 2% can usually be handled by "turbulence models" (see later).



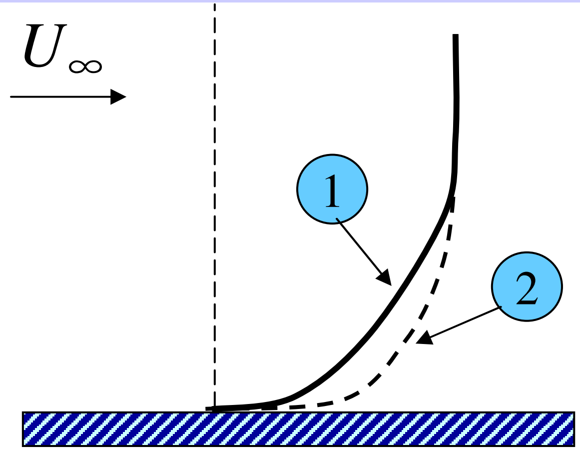
Changes in velocity profile in transition

- As the b.l undergoes transition the velocity profile undergoes a transformation becoming very steep near the wall and much flatter over the remainder of the layer.
- This change in shape occurs because, as the wall is approached, the turbulence becomes less effective at transferring momentum.
- At the wall the velocity fluctuations must VANISH (no slip at the wall) so there *all* the momentum transfer (or 'shear stress') is by viscous action
- Next to the wall there has to be a **viscous sublayer**

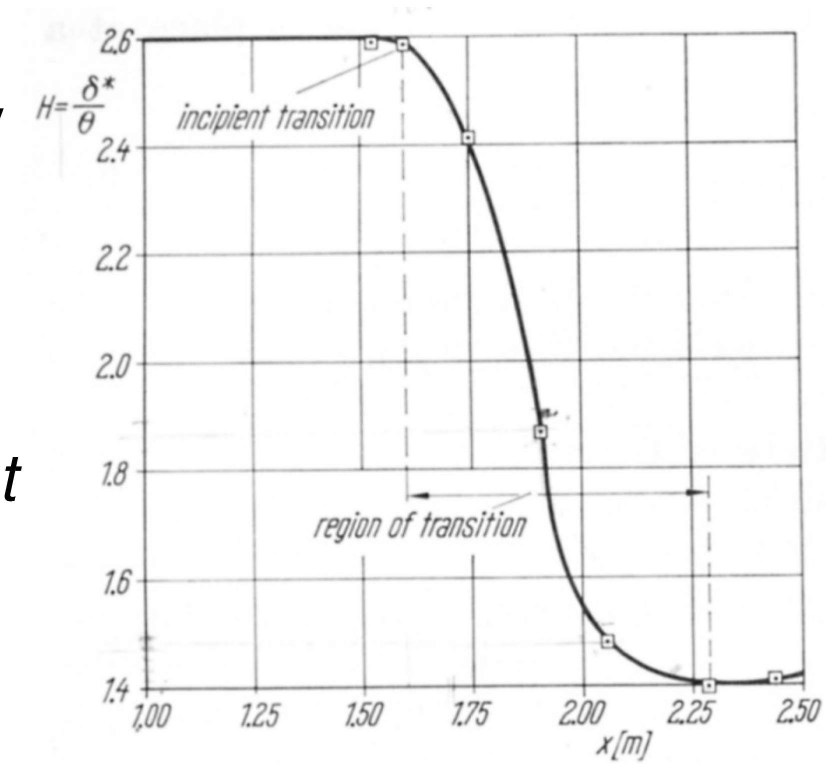


Revisiting: “The shape factor”

- The definition: $H = \delta^*/\theta$
- A change in shape factor indicates a change in the velocity profile.

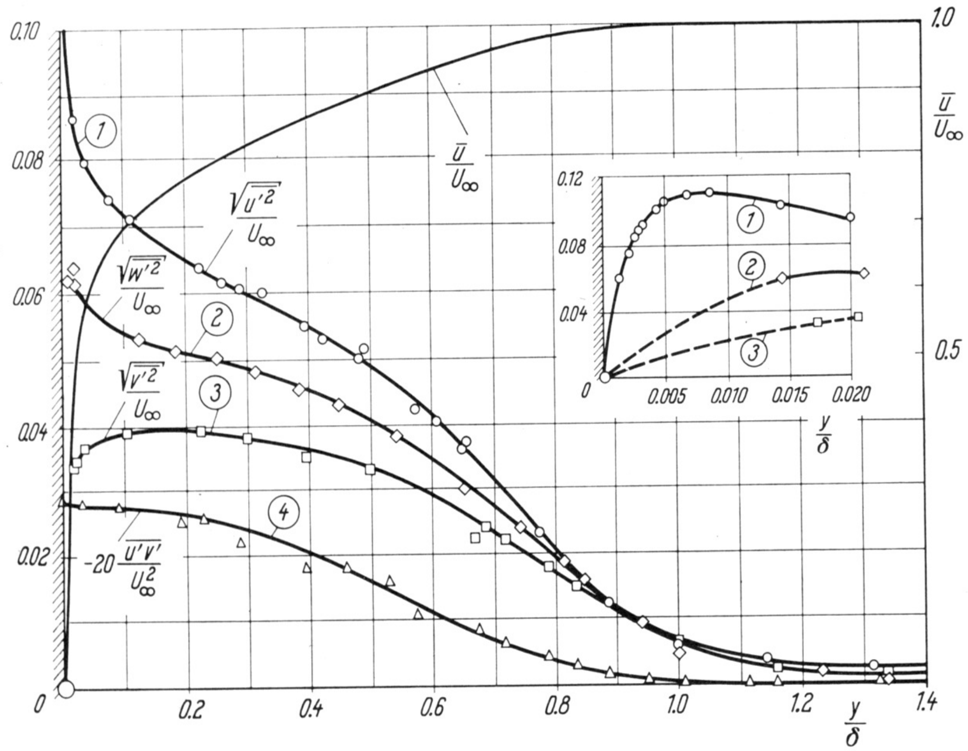


- Shape factor has distinctly different values for boundary layers in different states
  - Fõr a flat-plate *laminar* boundary layer: H 2.6
  - For a typical flat plate *turbulent* boundary layer: H=1.3 -1.5



Turbulent fluctuations near a wall

- As noted above, turbulent velocity fluctuations vanish at the wall to comply with the 'no-slip' condition.
- But (paradoxically) the **maximum** turbulence intensity also occurs very close to the wall, at the edge of the viscous sublayer.
- An explanation of why this is so must await a formal framework for the analysis of turbulent flows (next lecture)



$$\sqrt{\overline{w'^2}} \equiv rms..velocity..fluctuations..in..z-direction..etc.$$

What has been learned- 1

- In most situations steady laminar flow becomes unstable above a certain Reynolds number (or some equivalent dimensionless group)
- The flow stability depends very sensitively on the shape of the velocity profile: the presence of a point of inflexion leads to instabilities.
- Under some conditions an unstable flow may continue as laminar but with a regular, periodic, unsteady character, e.g., the vortex street behind a cylinder or Ekman spirals.
- When the Reynolds number is high enough there is a breakdown of the wave-like fluctuations into a **chaotic turbulent motion.**

What has been learned - 2

- The rate at which a shear flow changes from laminar to turbulent depends on the level of background fluctuations present (whether in the fluid or by transmission through rigid bounding surfaces).
- The very great increase in $\partial U/\partial y$ as the wall is approached reflects the fact that the momentum transfer by turbulent mixing is reduced, eventually to zero at the wall by virtue of the no-slip condition.
- The intensity of turbulent velocity fluctuations **increases** as the wall is approached but then rapidly diminishes across the viscous layer to vanish at the wall.