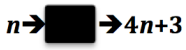
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

Title: SHIFTING TIMES TABLES (Grades 7 to 9)

Count in 4's to give the 4 times table: 4, 8, 12, 16, 20, … 80, 84, 88, …

Now shift the 4 times table up 3 to give 7, 11, 15, 19, 23, … 83, 87, 91, …



How does this mapping diagram represent shifting the 4 times table up by 3?

Which tables were shifted to give the following sequences? By how much? Explain how you know.

(a) 7, 12, 17, 22, 27, … 82, 87, 92, …

(b) 9, 11, 13, 15, 17, … 89, 91, 93, …

(c) 13, 20, 27, 34, 41, … 83, 90, 97, …

(d) 4, 7, 10, 13, 16, … 79, 82, 85, …

(e) 5, 11, 17, 23,, 29, … 71, 77, 83, …

Match each of the following rules for mapping *n = 1, 2, 3 …* to numbers in the sequences above. Explain how you do this.

(1) $n \rightarrow 7n + 6$

(2) $n \rightarrow 5n + 2$

(3) $n \rightarrow 3n + 1$

(4) $n \rightarrow 6n - 1$

(5) $n \rightarrow 2n + 7$

Now make up your own sequence and rule.

Solution

$n \rightarrow an+b$ where a is the multiplication table and b is the shift so $n = 1$ gives the first term of the sequence.

Rule	Sequence
(1) $n \rightarrow 7n+6$	(c) 13, 20, 27, 34, 41, … 83, 90, 97, …
(2) $n \rightarrow 5n+2$	(a) 7, 12, 17, 22, 27, … 82, 87, 92, …
(3) $n \rightarrow 3n+1$	(d) 4, 7, 10, 13, 16, … 79, 82, 85, …
(4) $n \rightarrow 6n - 1$	(e) 5, 11, 17, 23, 29, … 71, 77, 83, …
(5) $n \rightarrow 2n+7$	(b) 9, 11, 13, 15, 17, … 89, 91, 93, …

Notes for teacher

Why do this activity?

This activity builds on what learners already know about multiplication tables and counting in 2s, 3s, 4s etc, and what they can easily recognise as number patterns and sequences, and takes them forward to the concept of a mapping or function. By thinking of shifting the whole multiplication table they are naturally led to think about many pairs of numbers simultaneously, for example: (1, 7), (2, 11), (3, 15), (4, 19), (5, 23) … In this activity the learners concentrate on shifting different multiplication tables and working out the corresponding mappings. Learning can then progress with the activity Steps https://aiminghigh.aimssec.ac.za/grades-7-to-9-steps/ from shifting multiplication tables to using the pairs of numbers as coordinates of points and then joining the points to form a straight line. The beauty of this idea is that counting in 2's leads to a line with gradient 2 (1 across, 2 up), counting in 3's leads to a line with gradient 3 (1 across, 3 up) etc. The Shifting Tables activity focuses on the rules for functions that map the natural numbers to the terms of arithmetic sequences and so the activity can be used later as an introduction to a lesson on arithmetic sequences.

Possible approach

Start with the whole class counting in 4s aloud and challenge them to count up to 100. Tell them to join in the counting when they can. Ask them how they work out the next number each time. Then write the second sequence 7, 11, 15, 19, 23, … 83, 87, 91, … on the board and ask the learners how they worked out the next number after 23 where there is a gap for this sequence. Write 4, 8, 12, 16, 20, … 83, 87, 91, … under this sequence and ask the learners what they notice. Ask what is the same and what is different about the two sequences. Your purpose should be to lead learners to suggest that the 4 times table has shifted up by 3 without you telling them. Then the class should chant the 4 times table. (You could use the term 'multiples' but rather talk about multiplication tables which all the learners should understand.)

Show the class the function box for the rule or mapping $n \rightarrow 4n+3$ and ask them what the output will be for the input n=1. Then for n=2, then n=3 … What do they notice?

Either give out copies of the worksheet (see below) or write the question on the board or write the 5 sequences (a), (b), (c), (d) and (e) above on the board and ask the learners to work out what the multiplication table was in each case and how it has been shifted. It is best to encourage your learners to read questions for themselves and decide for themselves what to do. You could decide that the learners should work alone for 10 minutes and then compare their findings with a partner before sharing their findings in a class discussion.

For the learners who finish before the rest of the class, tell them to make up some sequences based on shifting multiplication tables and to decide on the corresponding rules mapping the counting numbers 1, 2, 3, 4, … to their sequences. These learners can then exchange their sequences with other learners and each try to find the mapping for these newly created sequences.

When most of the learners have done all they can and checked their answers, then ask some learners to explain their answers to the whole class. Summarise what has been learned and the connections between the multiplication tables, the sequences and the rules for the functions.

Key questions
What is the same and what is different about those 2 sequences?
What is the next term in that sequence? And the one after that? How did you find those terms?
What do you add on each time to get the next number in the sequence?
What multiplication table gives you a sequence going up like that?
By what number has the multiplication table been shifted?

Possible extension
Learners could go on to do Steps https://aiminghigh.aimssec.ac.za/grades-7-to-9-steps/

Possible support
If learners have serious difficulties with this activity it may be that they do not know their multiplication tables. They could write out the lists of multiples of 2, 3, 4, … up to multiples of 12 and use these lists to help them do the activity.

SHIFTING TIMES TABLES WORKSHEET

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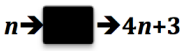

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