FUNDAMENTALS OF FLUID MECHANICS

Chapter 12 Pumps and Turbines

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MAIN TOPICS

- Introduction
- Basic Energy Considerations
- Basic Angular Momentum Considerations
- The Centrifugal Pump
- Dimensionless Parameters and Similarity Laws
- Axial-Flow and Mixed-Flow Pumps
- Fans
- Turbines
- Compressible Flow Turbomachines

Pumps and Turbines

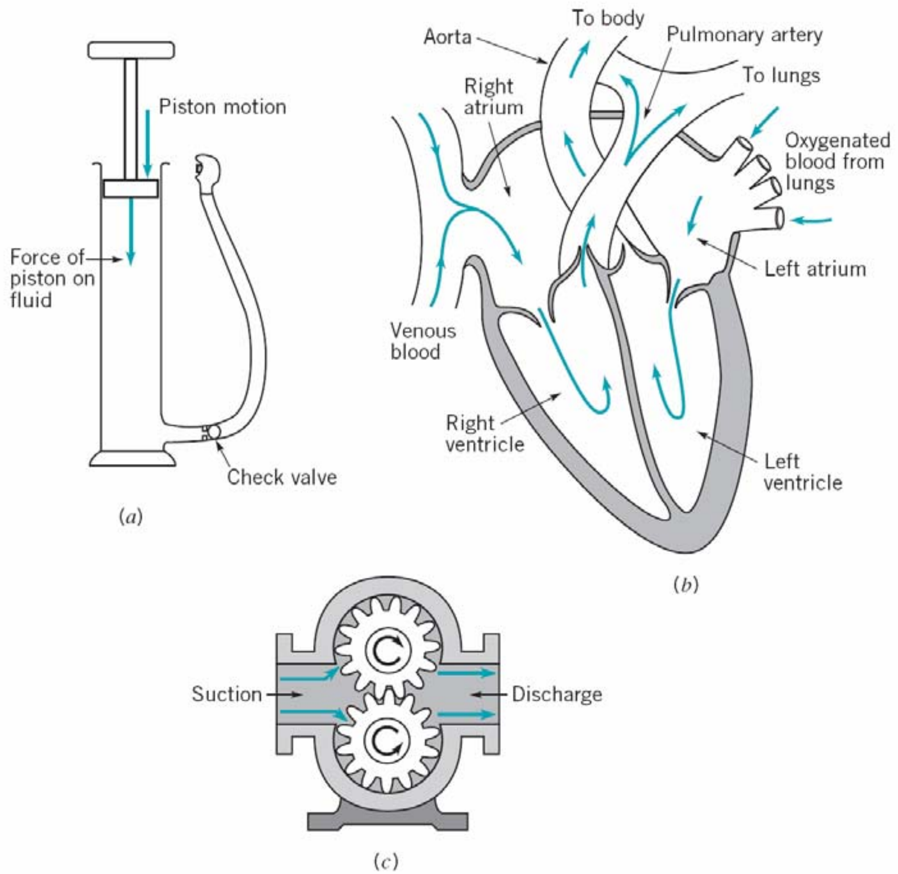
- Pumps and turbines: Fluid machines.
- Pumps: Add energy to the fluid – they do work on the fluid.
- Turbines: Extract energy from the fluid – the fluid does work on them.

Fluid Machines

- ❖ Positive displacement machines (denoted as the static type)
- ❖ Turbomachines (denoted as the dynamic type).

Positive Displacement Machines

- ❖ Force fluid into or out of a chamber by changing the volume of the chamber.
- ❖ The pressure developed and the work done are a result of essentially static forces rather than dynamic effects.



Typical positive displacement pumps: (*a*) tire pump, (*b*) human heart, (*c*) gear pump.

Turbomachines

- Turbomachines involve a collection of blades, buckets, flow channels, or passages arranged around an axis of rotation to form a rotor.
- Turbomachines are mechanical devices that either extract energy from a fluid (turbine) or add energy to a fluid (pump) as a result of dynamic interactions between the device and the fluid.
- The fluid used can be either a gas or a liquid.

Operating Principles of Turbomachines

- The basic operating principles are the same whether the fluid is a liquid or a gas.
- Cavitation may be an important design consideration when liquids are involved if the pressure at any point within the flow is reduced to vapor pressure.
- Compressibility effects may be important when gases are involved if the Mach number becomes large enough.

Structure of Turbomachines

- Many turbomachines contain some type of housing or casting that surrounds the rotating blades or rotor, thus forming a n internal flow passageway through which the fluid flows.
- Some turbomachines include stationary blades or vanes in addition to rotor blades. These stationary vanes can be arranged to accelerate the flow and thus serve as an nozzles.
- These vanes can be set to diffuse the flow and act as diffusers.

Classification of Turbomachines

- Axial-flow machines: The fluid maintains a significant axial-flow direction component from the inlet to outlet of the rotor.
- Mixed-flow machines: There may be significant radial- and axial-flow velocity components for the flow through the rotor row.
- Radial-flow mahcines: The flow across the blads involves a substantial radial-flow component at the rotor inlet, exit, or both.

Basic Energy Considerations

By considering the basic operation of

Household fan (pump).

Windmill (turbine).

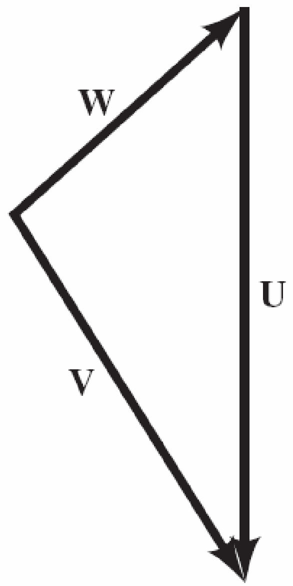
Household Fan 1/2

- Consider a fan blade driven at constant angular velocity by the motor.
- Absolute velocity is the vector sums of relative and blade velocities.

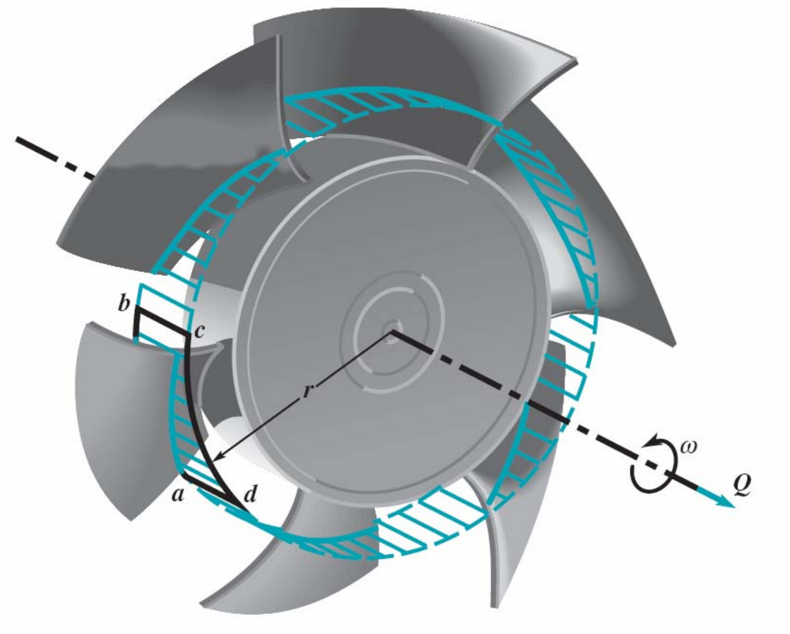
The blade velocity $\vec{U} = \omega r$

The absolute velocity **V** seen by a person sitting stationary at the table on which the fan rests.
The relative velocity seen by a person riding on the fan blade **W**

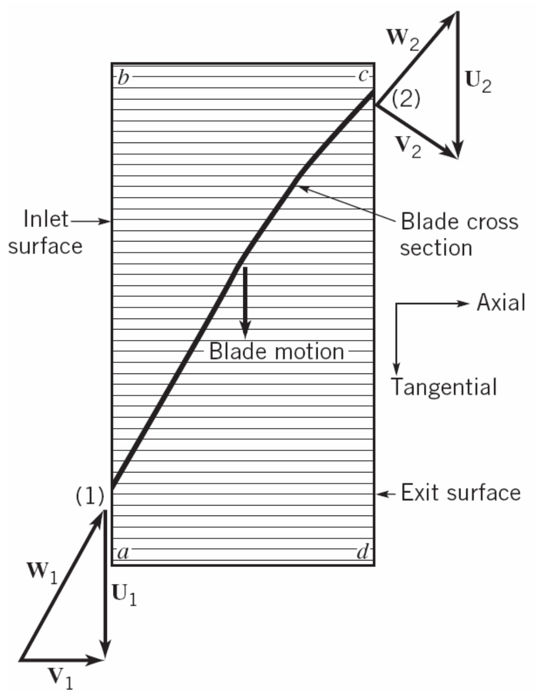
$$\vec{V} = \vec{W} + \vec{U}$$



Household Fan 2/2



(a)

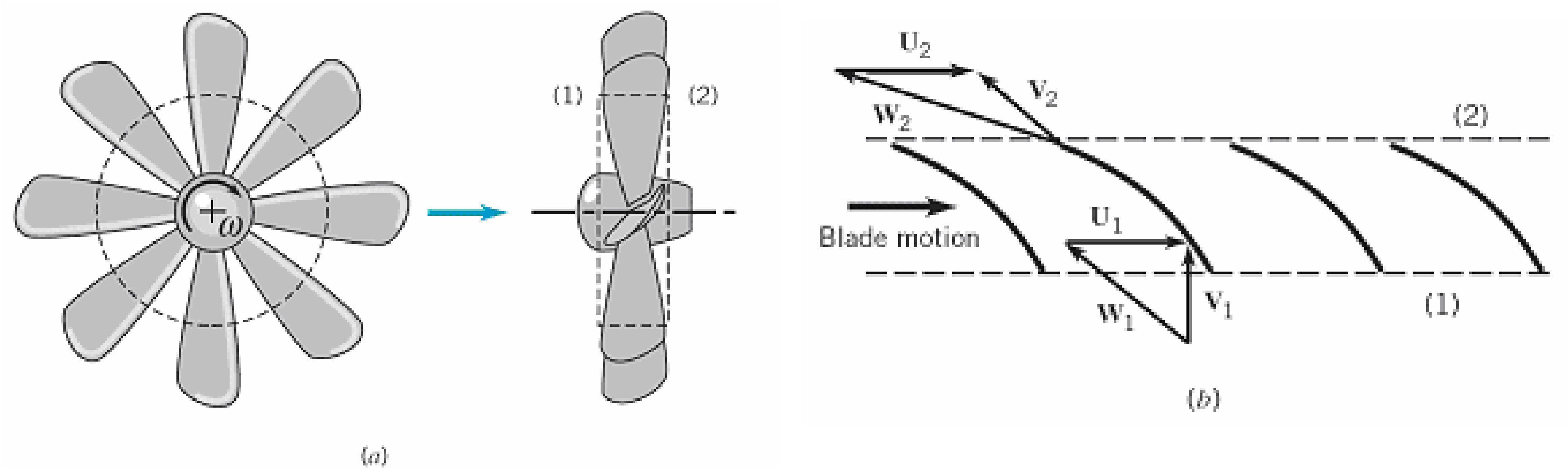


(b)

Idealized flow through a fan: (a) fan blade geometry: (b) absolute velocity, V; relative velocity, W, and blade velocity, U at the inlet and exit of the fan blade section.

Windmill

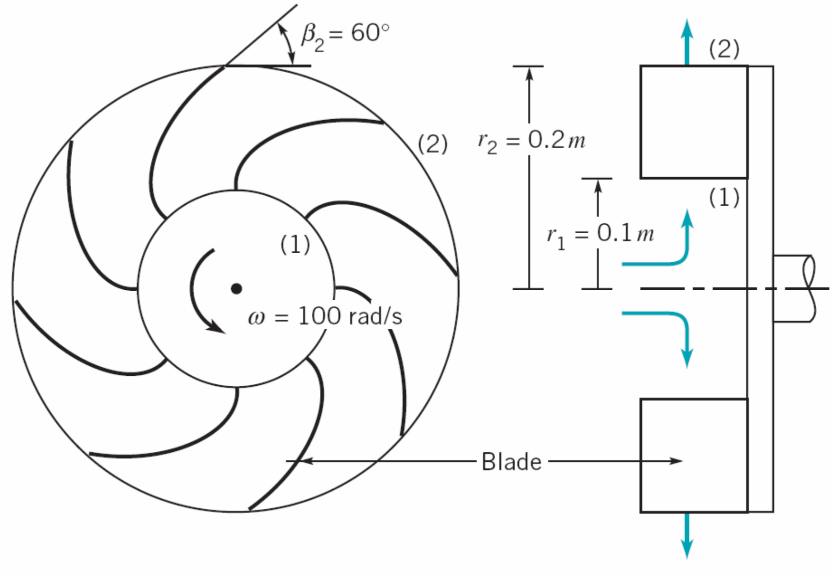
❖ Consider the windmill. Rather than the rotor being driven by a motor, it is rotated in the opposite direction by the wind blowing through the rotor.



Idealized flow through a windmill: (a) windmill blade geometry; (b) absolute velocity, V; relative velocity, W, and blade velocity, U at the inlet and exit of the windmill blade section.

Example 12.1 Drag from Pressure and Shear Stress Distributions

- The rotor shown in Fig. E12.1a rotates at a constant angular velocity of ω= 100 rad/s. Although the fluid initially approaches the rotor in an axial direction, the flow across the blades is primarily radial. Measurements indicate that the absolute velocity at the inlet and outlet are V_1 = 12 m/s and V_2 = 15 m/s, respectively. Is this device a pump or a turbine?



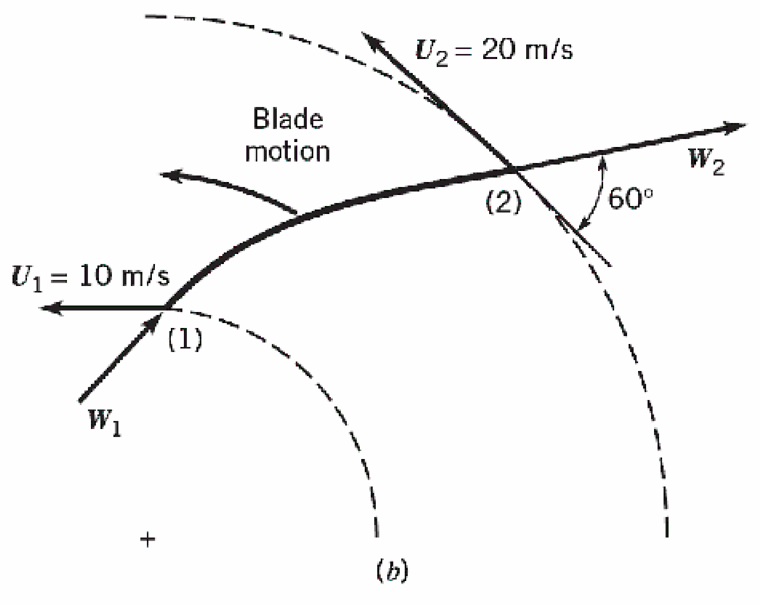
(a)

Example 12.1 Solution[1/2]

If the tangential component f the force of the blade on the fluid is in the direction of the blade motion (a pump) or opposite to it (a turbine).

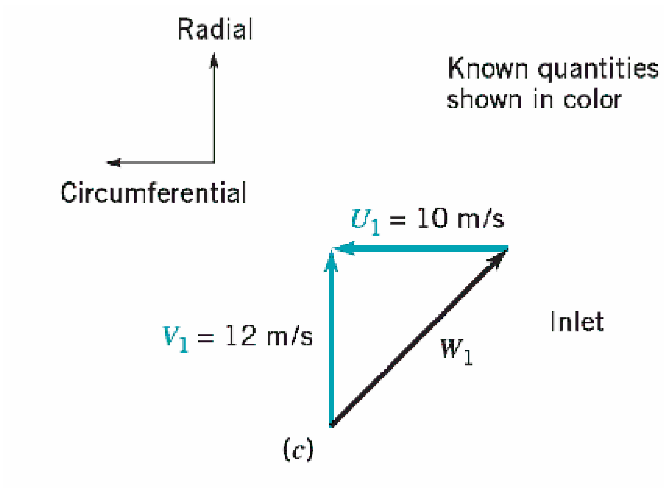
The inlet and outlet blade

$U_1 = \omega r_1 = 10 m/s \qquad U_2 = \omega r_2 = 10 m/s$

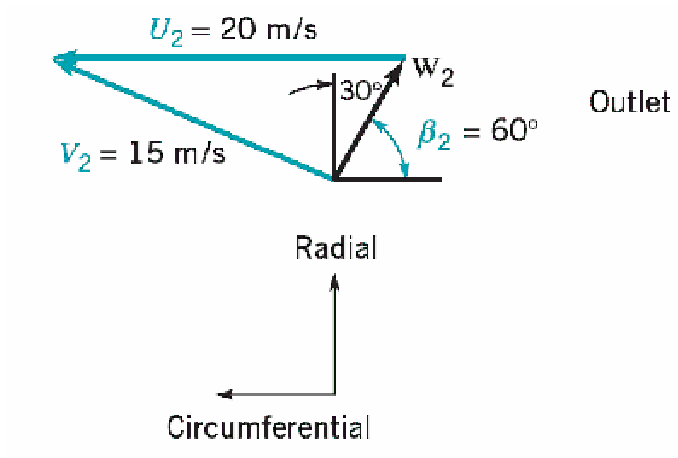


Example 12.1 Solution$^{2/2}$

The inlet velocity triangle



The outlet velocity triangle



At the inlet there is no component of absolute velocity in the direction of rotation; at the outlet this component is not zero. That is, the blade pushes and turns the fluid in the direction of the blade motion, thereby doing work on the fluid. → **This device is a pump.**

Basic Angular Momentum Considerations

Angular Momentum Considerations 1/6

- Work transferred to or from a fluid flowing through a pump or a turbine occurs by interaction between moving rotor blades and the fluid.
  - Pump: The shaft toque (the torque that the shaft applies to the rotor) and the rotation of the rotor are in the same direction, energy is transferred from the shaft to the rotor and from the rotor to the fluid.
  - Turbine: The torque exerted by the shaft on the rotor is opposite to the direction of rotation, the energy transfer is from the fluid to the rotor.

Angular Momentum Considerations 2/6

- All of the turbomachines involve the rotation of an impeller or a rotor about a central axis, it is appropriate to discuss their performance in terms of **torque and angular momentum**.

Angular Momentum Considerations 3/6

- In a turbomachine a series of particles (a continuum) passes through the rotor.
- For steady flow, the moment of momentum equation applied to a control volume

$$\underbrace{\sum (\vec{r} \times \vec{F})}_{\text{Sum of the external torques}} = \underbrace{\int_{CS} (\vec{r} \times \vec{V})\rho \vec{V} \cdot \vec{n} dA}_{\text{Net rate of flow of moment-of-momentum (angular momentum) through the control volume}}$$

Sum of the external torques

Net rate of flow of moment-of-momentum (angular momentum) through the control volume

Angular Momentum Considerations 4/6

Applied to the one-dimensional simplification of flow through a turbomachine rotor, the axial component

$$T_{shaft} = \underline{-\dot{m}_1(r_1 V_{\theta 1}) + \dot{m}_2(r_2 V_{\theta 2})} \qquad (2)$$

Shaft work applied to the contents of the control volume

"+" : in the same direction as rotation

"-" : in the opposite direction as rotation

Euler turbomachine equation

Euler turbomachine equation : the shaft torque is directly proportional to the mass flowrate. The torque also depends on the tangential component of the absolute velocity, V_θ.

Angular Momentum Considerations 5/6

(2) $\Rightarrow$ $$\dot{W}_{shaft} = T_{shaft}\,\omega \qquad (3)$$

$$\dot{W}_{shaft} = -\dot{m}_1 (U_1 V_{\theta 1}) + \dot{m}_2 (U_2 V_{\theta 2}) \qquad (4)$$

$$w_{shaft} = \frac{\dot{W}_{shaft}}{\dot{m}} = -(U_1 V_{\theta 1}) + (U_2 V_{\theta 2}) \qquad (5)$$

$$\dot{m} = \dot{m}_1 = \dot{m}_2$$

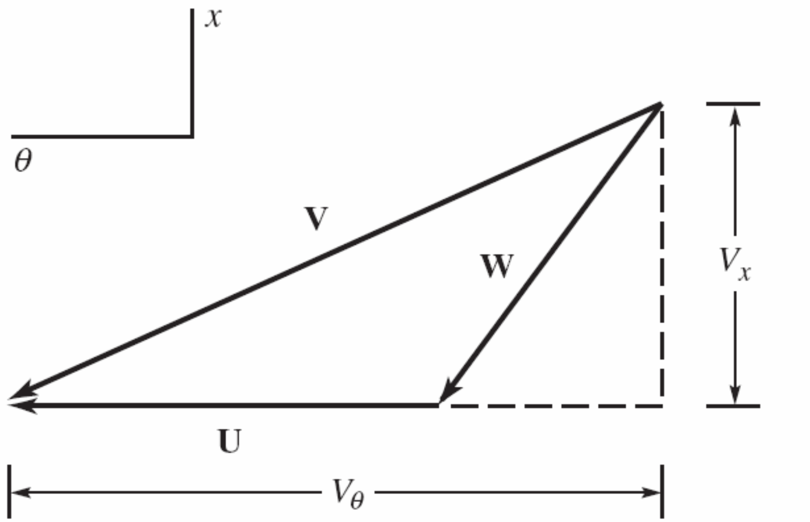
(3) (4) (5) :**The basic governing equations for pumps or turbines whether the machines are radial-, mixed, or axial-flow devices and for compressible and incompressible flows.**

Angular Momentum Considerations 6/6

Another useful but more laborious form.
Based on the velocity triangles at the entrance or exit.

$$V^2 = V_\theta^2 + V_x^2 \qquad V_x^2 = V^2 - V_\theta^2 \qquad (6)$$

$$V_x^2 + (V_\theta - U)^2 = W^2 \qquad (7)$$



(6)+(7) $\Rightarrow$ $$V_\theta U = \frac{V^2 + U^2 - W^2}{2}$$

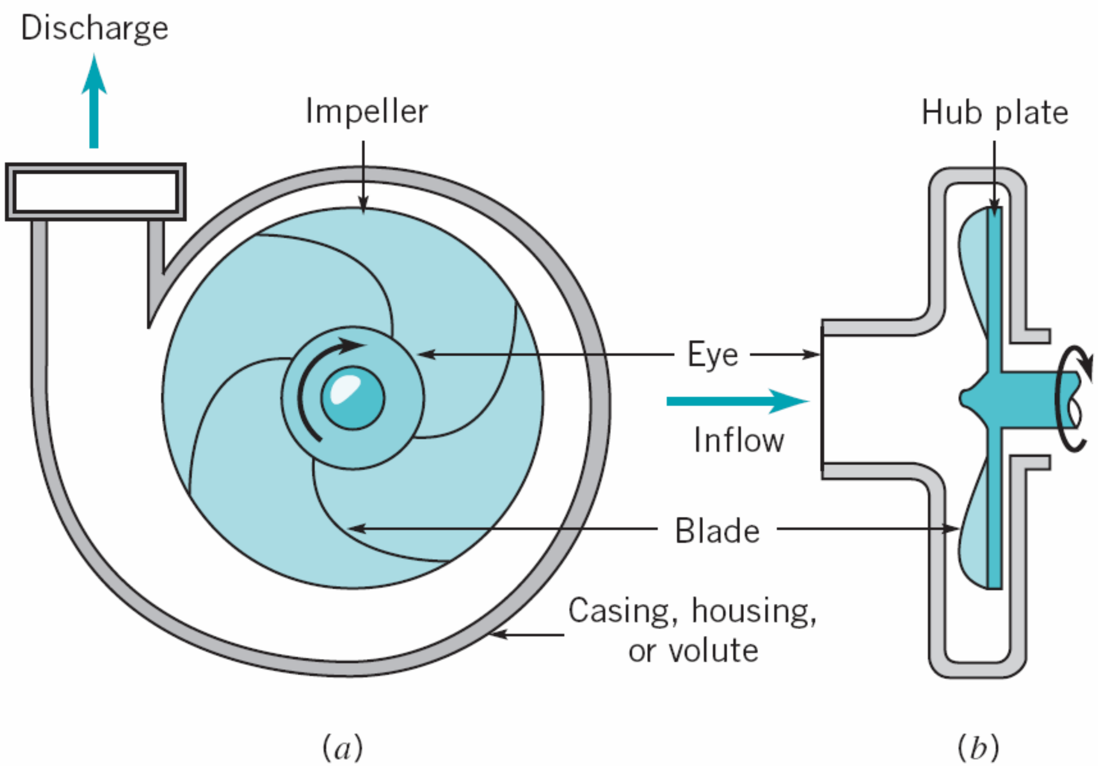
(5) $\Rightarrow$ $$w_{shaft} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 - (W_2^2 - W_1^2)}{2} \qquad (8)$$

Turbomachine work is related to changes in absolute, relative, and blade velocities.

The Centrifugal Pump

Structure of the Centrifugal Pump 1/3

❖ Centrifugal pump has two main components: an impeller and a stationary casing, housing, or volute.



Structure of the Centrifugal Pump 2/3

❖ An impeller attached to the rotating shaft. The impeller consists of a number of blades, also sometimes called vanes, arranged in a regular pattern around the shaft.

(a) Type of impeller *(b)*

(*a*) Open impeller, (*b*) enclosed or shrouded impeller

Structure of the Centrifugal Pump 3/3

- A stationary casing, housing, or volute enclosing the impeller.
  - The casing shape is designed to reduce the velocity as the fluid leaves the impeller, and this decrease in kinetic energy is converted into an increase in pressure.
  - The volute-shaped casing, with its increase area in the direction of flow, is used to produce an essentially uniform velocity distribution as the fluid moves around the casing into the discharge opening.

Operation of the Centrifugal Pump

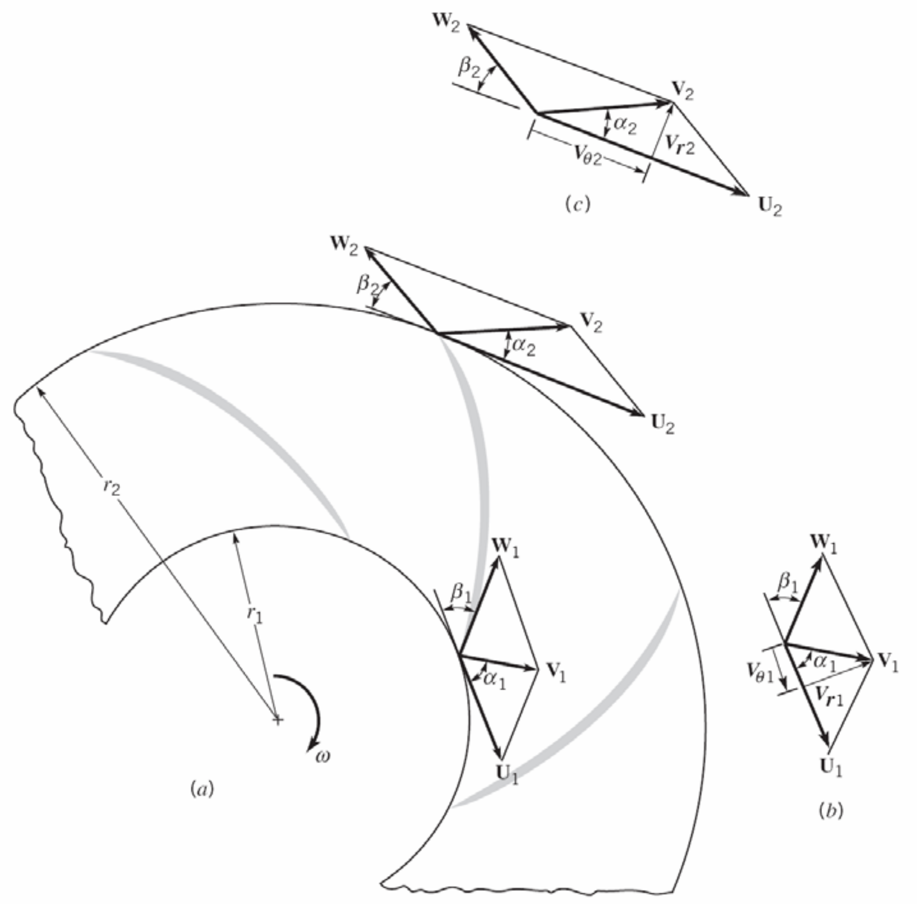
- As the impeller rotates, fluid is sucked in through the eye of the casing and flows radially outward.
- Energy is added to the fluid by the rotating blades, and both pressure and absolute velocity are increased as the fluid lows from the eye to the periphery of the blades.

Stages of the Centrifugal Pump

- Simple stage pump: Only one impeller is mounted on the shaft.
- Multistage pump: Several impellers are mounted on the same shaft.
  - The flowrate is the same through all stages.
  - Each stage develops an additional pressure rise.
  - For a very large discharge pressure.

Theoretical Considerations 1/5

❖ The basic theory of operation of a centrifugal pump can be developed by considering the average one-dimensional flow of the fluid as it passes between the inlet and the outlet sections of the impeller as the blades rotate.



Velocity diagrams at the inlet and exit of a centrifugal pump impeller.

Theoretical Considerations 2/5

- The moment of momentum equation indicates that the shaft torque required to rotate the pump impeller is

$$T_{shaft} = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1}) = \rho Q(r_2 V_{\theta 2} - r_1 V_{\theta 1}) \quad (9)\ (10)$$

$$\dot{m} = \dot{m}_1 = \dot{m}_2$$

The tangential components of the absolute velocity

$$\dot{W}_{shaft} = T_{shaft}\,\omega = \rho Q \omega (r_2 V_{\theta 2} - r_1 V_{\theta 1}) = \rho Q(U_2 V_{\theta 2} - U_1 V_{\theta 1}) \quad (11)$$

$$w_{shaft} = \frac{\dot{W}_{shaft}}{\dot{m}} = U_2 V_{\theta 2} - U_1 V_{\theta 1} \quad (12)$$

Theoretical Considerations 3/5

- The head that a pump adds to the fluid is an important parameter. The ideal or maximum head rise possible, h_i

$$\dot{W}_{shaft} = \rho g Q h_i$$

+(12) ⇒ $$h_i = \frac{1}{g}(U_2 V_{\theta 2} - U_1 V_{\theta 1}) \quad (13)$$

(8)+(12) ⇒ $$h_i = \frac{(V_2^2 - V_1^2) + (U_2^2 - U_1^2) + (W_1^2 - W_2^2)}{2g} \quad (14)$$

Theoretical Considerations 4/5

❖ An appropriate relationship between the flowrate and the pump ideal head rise:

$\alpha_1 = 90^o$ +(12) ⇒ $$h_i = \frac{U_2 V_{\theta 2}}{g} \quad (15)$$

$$\cot\beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

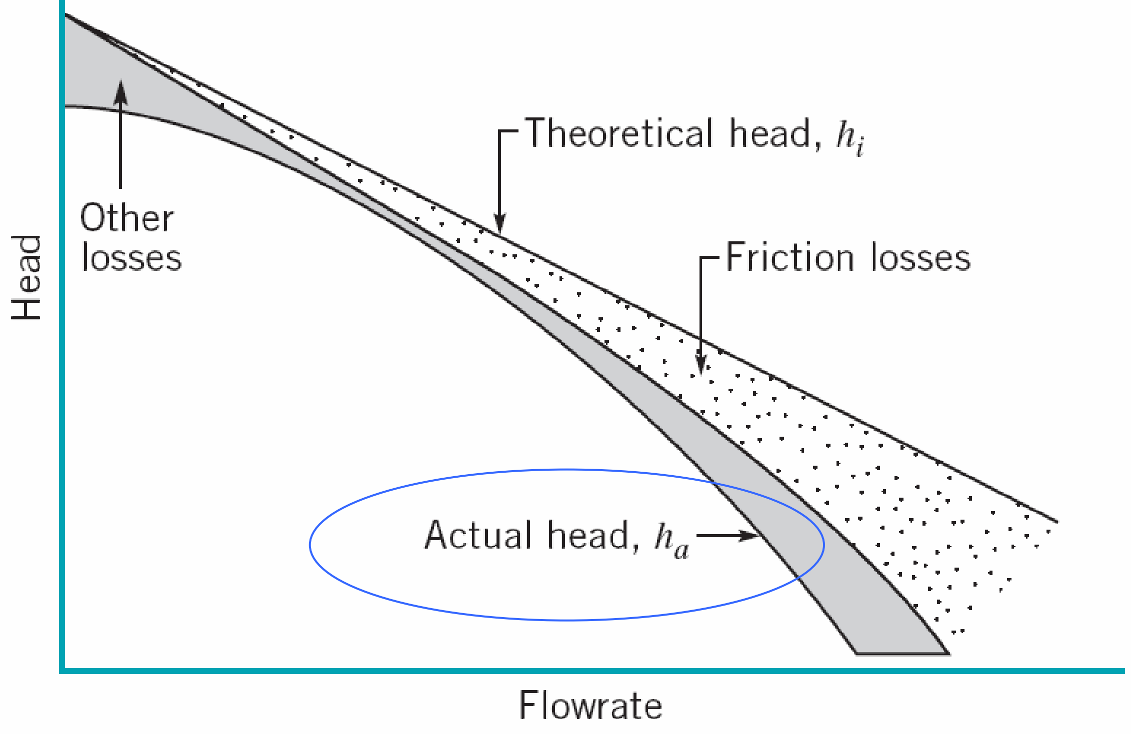
(15) ⇒ $$h_i = \frac{U_2^2}{g} - \frac{U_2 V_{r2} \cot\beta_2}{g} \quad (16)$$ $$Q = 2\pi r_2 b_2 V_{r2} \quad (17)$$

(16)+(17) ⇒ $$h_i = \frac{U_2^2}{g} - \frac{U_2 \cot\beta_2}{2\pi r_2 b_2 g} Q \quad (18)$$

Theoretical Considerations 5/5

$$h_i = \frac{U_2^2}{g} - \frac{U_2 \cot\beta_2}{2\pi r_2 b_2 g} Q$$

For a centrifugal pump with backward curved vanes ($\beta_2 < 90^o$)



Example 12.2 Centrifugal Pump Performance Based on Inlet/Outlet Velocities

- Water is pumped at the rate of 1400 gpm through a centrifugal pump operating at a speed of 1750 rpm. The impeller has a uniform blade length, b, of 2 in. with $r_1 = 1.9$ in. and $r_2 = 7.0$ in., and the exit blade angle is $\beta = 23^\circ$. Assume ideal flow conditions and that the tangential velocity component, $V_{\theta 1}$, of the water entering the blade is zero ($\alpha_1 = 90^\circ$). Determine (a) the tangential velocity component, $V_{\theta 2}$, at the exit, (b) the ideal head rise, h_a, and (c) the power, $\dot{W}_{shaft}$, transferred to the fluid. Discuss the difference between ideal and actual head rise. Is the power, $\dot{W}_{shaft}$, ideal or actual? Explain.

Example 12.2 Solution[1/2]

The tip velocity of the impeller

$$U_2 = \omega r_2 = (7/12\text{ft})(2\pi\text{rad}/\text{rev})(1750\text{rpm}/60\text{s}/\text{min}) = 107\text{ft}/\text{s}$$

Since the flowrate is given

$$Q = 2\pi r_2 b_2 V_{r2} \qquad V_{r2} = \frac{Q}{2\pi r_2 b_2} = 5.11\text{ft}/\text{s}$$

$$\Rightarrow \cot\beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}} \Rightarrow V_{\theta 2} = U2 - V_{r2}\cot\beta_2 = 95.0\text{ft}/\text{s}$$

(15) $$\Rightarrow h_i = \frac{U_2 V_{\theta 2}}{g} = 316\text{ft}$$

Example 12.2 Solution[2/2]

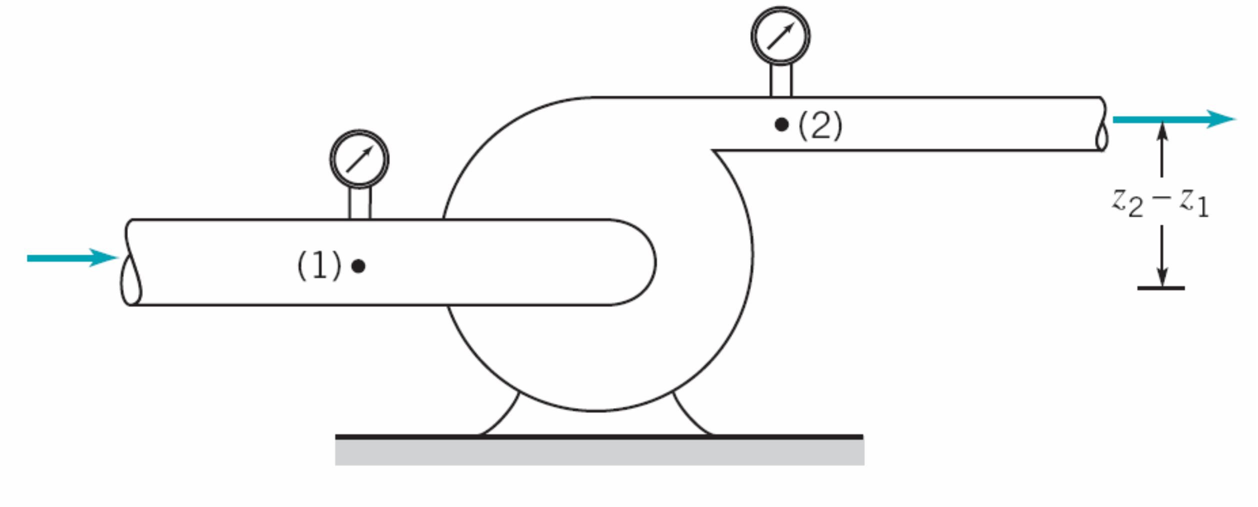
The power transferred to the fluid

$$\dot{W}_{shaft} = \rho Q U_2 V_{\theta 2} = \ldots = 112 hp$$

Pump Performance Characteristics 1/8

- Typical experimental arrangement for determining the head rise, h_a, gained by a fluid flowing through a pump.
- Using the energy equation with $h_a = h_s - h_L$

$$h_a = \frac{p_2 - p_1}{\gamma} + z_2 - z_1 + \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad (19)$$



Pump Performance Characteristics 2/8

The differences in elevations and velocities are small $\Rightarrow$ $$h_a \approx \frac{p_2 - p_1}{\gamma} \qquad (20)$$

The power gained by the fluid

$$P_f = \gamma Q h_a \qquad (21)$$

$$\Rightarrow \quad P_f = \text{water horsepower} = \frac{\gamma Q h_a}{550} \qquad (22)$$

Overall efficient

$$\eta = \frac{\text{power gained by the fluid}}{\text{shaft power driving the pump}} = \frac{P_f}{\dot{W}_{shaft}} = \frac{\gamma Q h_a / 550}{\text{bhp}} \qquad (23)$$

Pump Performance Characteristics 3/8

- The overall pump efficiency is affected by the ***hydraulic losses*** in the pump, and in addition, by the ***mechanical losses*** in the bearings and seals.
- There may also be some power loss due to leakage of the fluid between the back surface of the impeller hub plate and the casing, or through other pump components.
- This leakage contribution to the overall efficiency is called the volumetric loss.

Pump Performance Characteristics 4/8

- The overall efficiency arises from three source, the hydraulic efficiency, η_{h},, the mechanical efficiency, η_{m},, and the volumetric efficiency, η_{v}

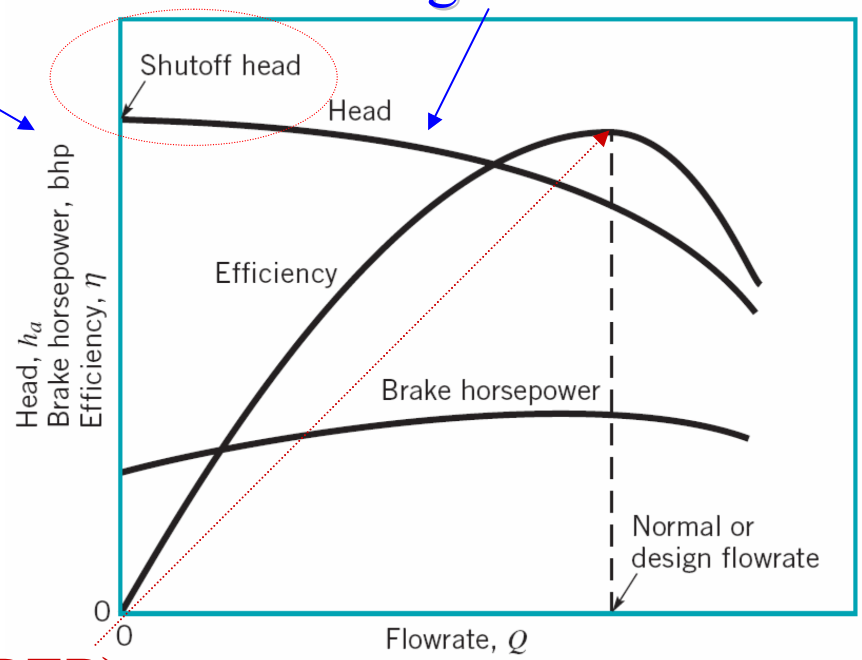
  ⇨ $\eta = \eta_{h}\eta_{m}\eta_{v}$

Pump Performance Characteristics 5/8

Performance characteristics for a given pump geometry and operating speed are usually given in the plots of h_a, η , and bhp versus Q.

Rising head curve

Typical performance characteristics for a centrifugal pump of a given size operating at a constant impeller speed.



Best efficiency points (BEP)

capacity

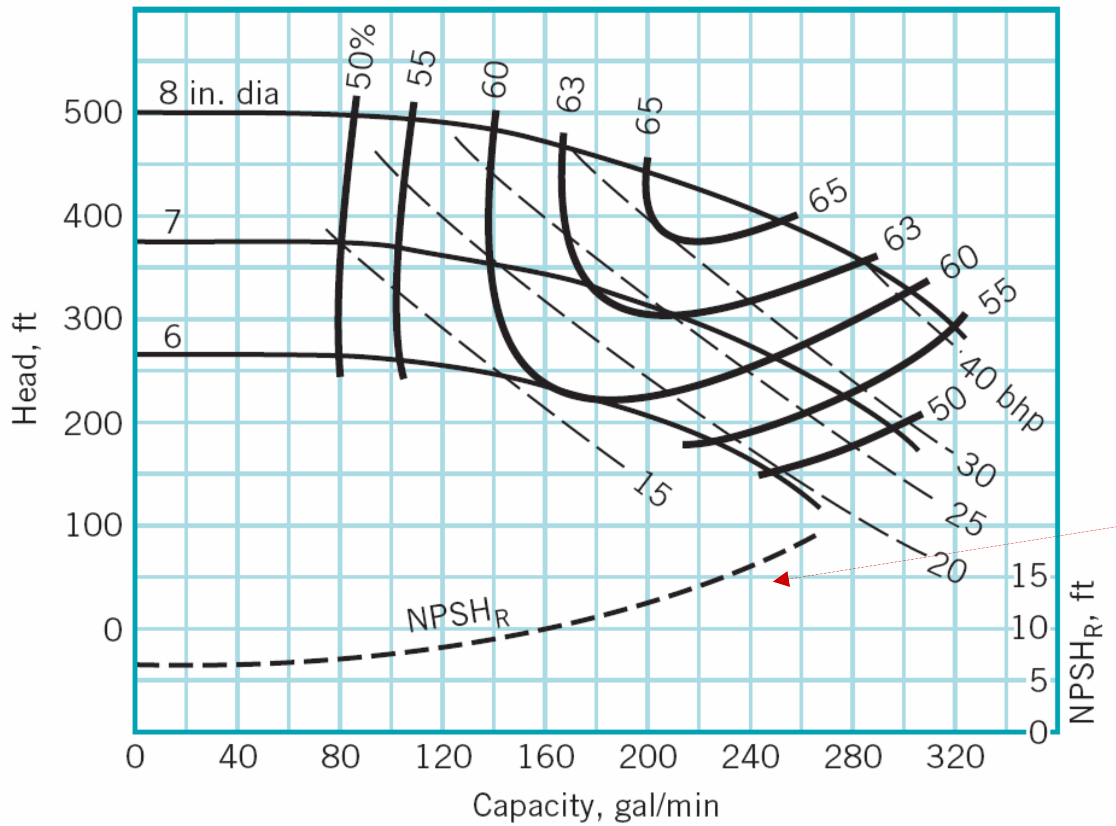
Pump Performance Characteristics 6/8

- **Rise head curve:** the head curve continuously rises as the flowrate decreases.
- **Falling head curve**: ha-Q curves initially rise as Q is decreased from the design value and then fall with a continued decrease in Q.
- **Shutoff head**: the head developed by the pump at zero discharge. It represents the rise in pressure head across the pump with the discharge valve closed.
- **Best efficiency points (BEP):** the points on the various curves corresponding to the maximum efficiency.

Pump Performance Characteristics 7/8

- As the discharge is increased from zero the brake horsepower increases, with a subsequent fall as the maximum discharge is approached.
- The efficiency is a function of the flowrate and reaches a maximum value at some particular value of the flowrate, commonly referred to as the normal or design flowrate or capacity for the pump.
- The performance curves are very important to the engineer responsible for the selection of pumps for a particular flow system.

Pump Performance Characteristics 8/8



NPSH$_R$
Required net positive suction head

Related to conditions on the suction side of the pump

Performance curves for a two-stage centrifugal pump operating at 3500 rpm. Data given for three different impeller diameters.

Net Positive Suction Head 1/2

- On the suction side of a pump, low pressures are commonly encountered, with the concomitant possibility of cavitation occurring within the pump.
- Cavitation occurs when the liquid pressure at a given location is reduced to the vapor pressure of the liquid. When this occurs, vapor bubbles form; this phenomenon can cause a loss in efficiency as well as structural damage to the pump.
- How to characterize the potential for cavitation…

Net Positive Suction Head 2/2

❖ To characterize the potential for cavitation, define the net positive suction head (NPSH) as

$$\mathrm{NPSH} = \underbrace{\frac{p_s}{\gamma} + \frac{V_s^2}{2g}}_{\text{The total head on the suction side near the pump impeller inlet}} - \frac{p_v}{\gamma} \qquad (24)$$

The liquid vapor pressure head

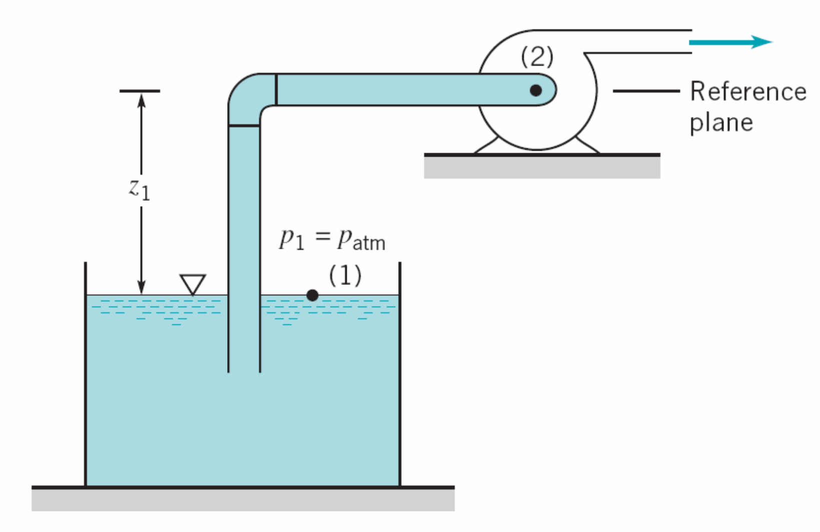
There are actually two values of NPSH of interest.

$NPSH_R$ and $NPSH_A$ [1/3]

- **Required NPSH**, denoted $NPSH_R$, that must be maintained, or exceeded, so that cavitation will not occur. Since pressure lower than those in the suction pipe will develop in the impeller eye, it is usually necessary to determine experimentally, for a given pump, the required $NPSH_R$.
- **Available NPSH**, denoted $NPSH_A$, represents the head that actually occurs for the particular flow system. This value can be determined experimentally, or calculated if the system parameters are known.

NPSH_R and NPSH_A 2/3

For a typical flow system



The energy equation applied between the free liquid surface and a point on the suction side of the pump near the impeller inlet

$$\frac{p_{atm}}{\gamma} - z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} + \sum h_L$$

Head losses between the free surface and the pump impeller inlet.

NPSH$_R$ and NPSH$_A$ 3/3

$$\underbrace{\frac{p_s}{\gamma} + \frac{V_s^2}{2g}}_{\text{The head available at the pump impeller inlet}} = \frac{p_{atm}}{\gamma} - z_1 - \sum h_L$$

$$\text{NPSH}_A = \frac{p_{atm}}{\gamma} - z_1 - \sum h_L - \frac{p_v}{\gamma} \qquad (25)$$

For proper pump operation

$$\text{NPSH}_A \geq \text{NPSH}_R$$

Example 12.3 Net Pressure Suction Head

- A centrifugal pump is to be placed above a large, open water tank, as shown in Fig. 12.13, and is to pump water at a rate of $0.5 ft^3/s$. At this flowrate the required net positive suction head, $NPSH_R$, is 15 ft, as specified by the pump manufacturer. If the water temperature is 80ºF and atmospheric pressure is 14.7 psi, determine the maximum height, z_1, that the pump can be located above the water surface without cavitation. Assume that the major loss between the tank and the pump inlet is due to filter at the pipe inlet having a minor loss coefficient $k_L = 20$. Other losses can be neglected. The pipe on the suction side of the pump has a diameter of 4 in. If you were required to place a valve in the flow path would you place it upstream or downstream of the pump? Why?

Example 12.3 Solution

(25) $$\text{NPSH}_A = \frac{p_{atm}}{\gamma} - z_1 - \sum h_L - \frac{p_v}{\gamma}$$

and the maximum value for z_1 will occur when $\text{ZPSH}_A = \text{NPSH}_R$

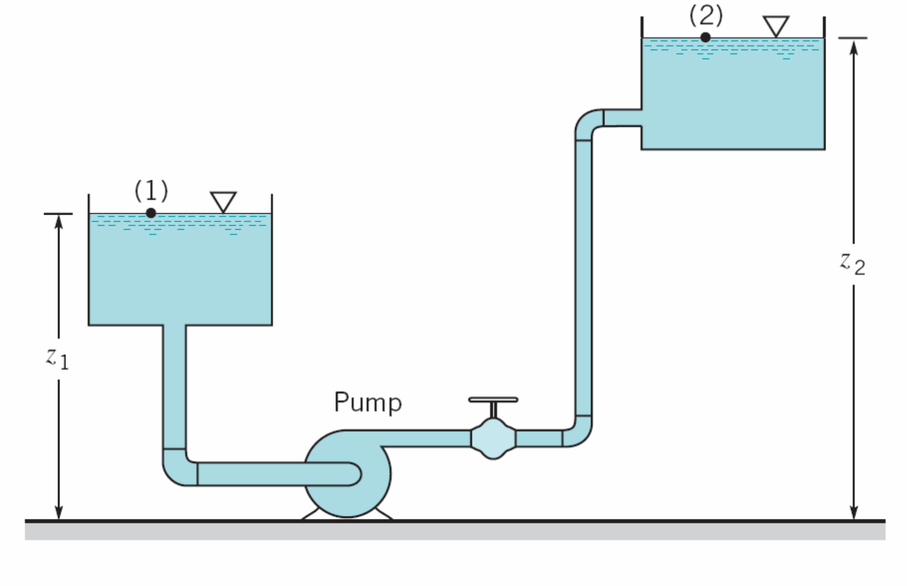
$$(z_1)_{max} = \frac{p_{atm}}{\gamma} - \sum h_L - \frac{p_v}{\gamma} - \text{NPSH}_R$$

$$V = \frac{Q}{A} = 5.73 \text{ft/s} \quad \sum h_L = K_L \frac{V^2}{2g} = \ldots = 10.2 \text{ft}$$

$$(z_1)_{max} = \frac{p_{atm}}{\gamma} - \sum h_L - \frac{p_v}{\gamma} - \text{NPSH}_R = \ldots = 7.65 \text{ft}$$

System Characteristics and Pump Selection 1/4

For a typical flow system in which a pump is used



The energy equation applied between points (1) and (2)

$$h_p = z_2 - z_1 + \sum h_L \qquad (26)$$

The actual head gained by the fluid from the pump.

All friction losses and minor losses

System Characteristics and Pump Selection 2/4

(26) $\Rightarrow$ $h_p = z_2 - z_1 + KQ^2$ (27)

$h_L \propto KQ^2$

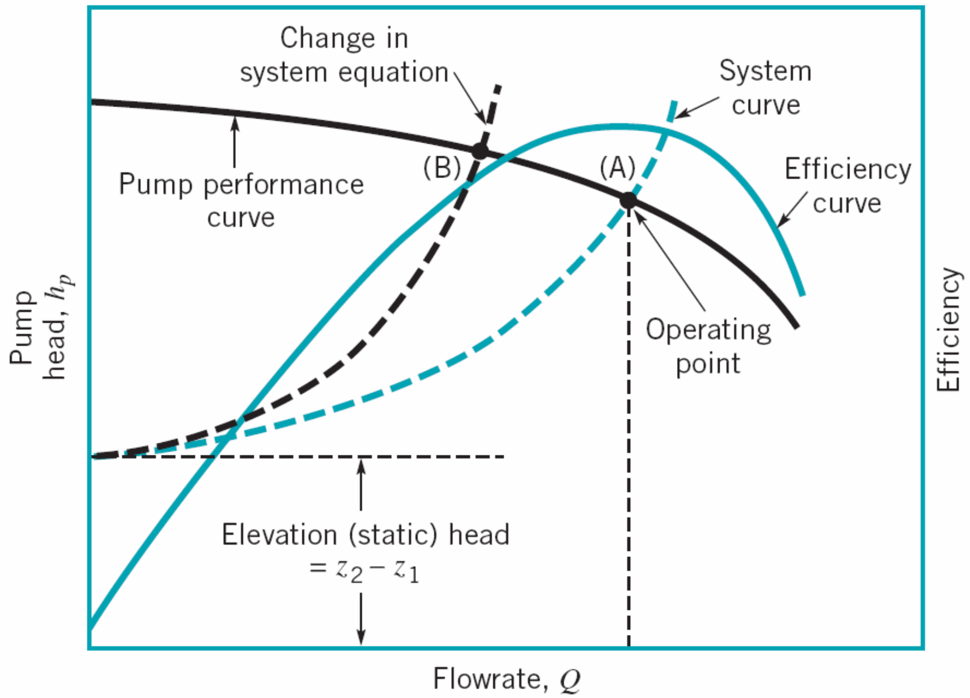
K depends on the pipe size and lengths, friction factors, and minor loss coefficients.

(27) is the system equation which shows how the actual head gained by the fluid from the pump is related to the system parameters.

System Characteristics and Pump Selection 3/4

❖ There is also a unique relationship between the actual pump head gained by the fluid and flowrate, which is governed by the pump design.

Pipe friction increase due to wall fouling.
(A)→ (B) flowrate ↓
efficiency ↓

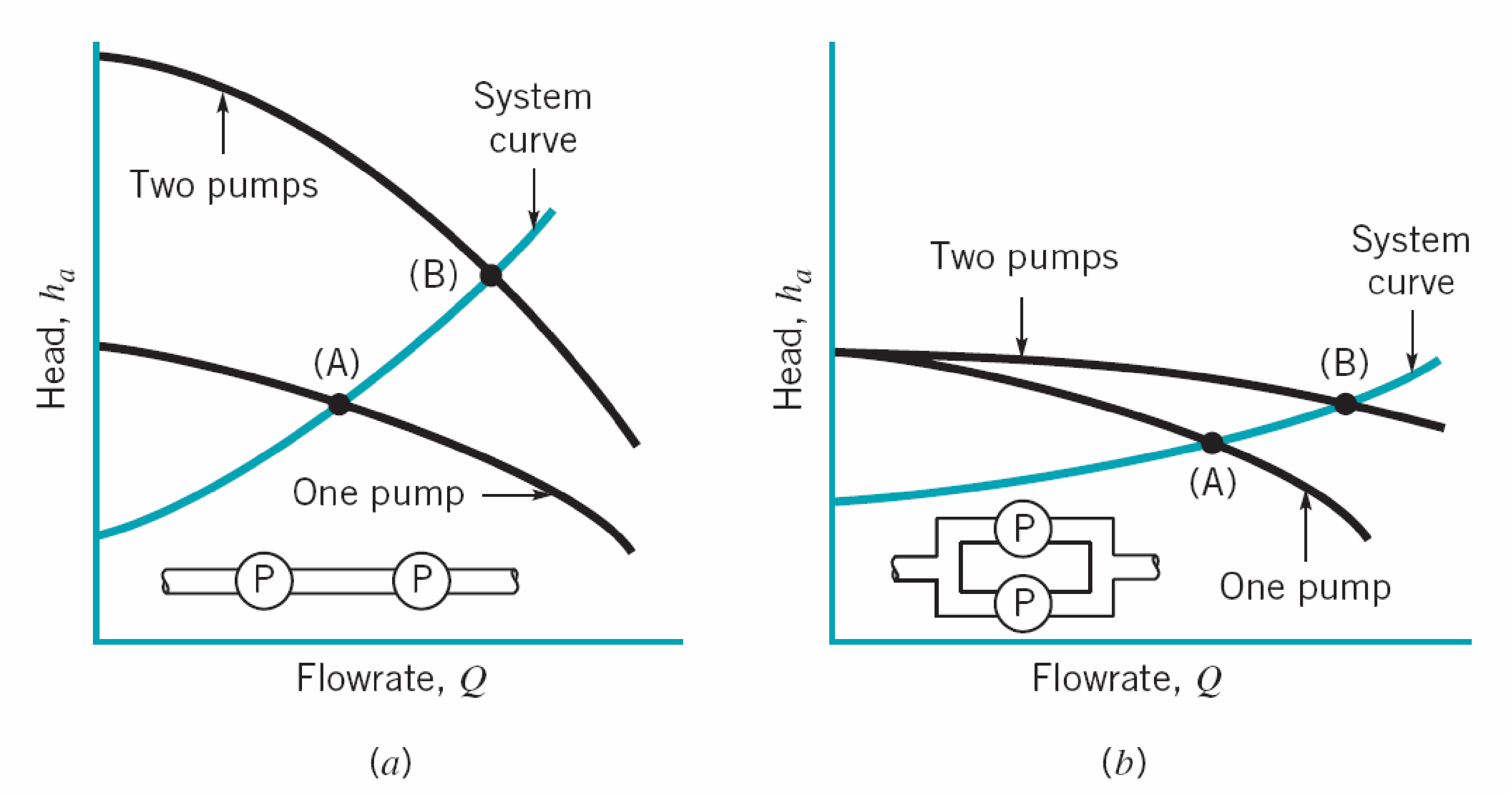


Utilization of the system curve and the pump performance curve to obtain the operating point for the system.

System Characteristics and Pump Selection 4/4

- To select a pump for a particular application, it is necessary to utilize both the system curve, determined by the system equation, and the pump performance curve.
- The intersection of both curves represents the operating point for the system.
  - ⇨The operating point wanted to be near the best efficiency point (BEP).

Pumps in Series or Parallel 1/3



Effect of operating pumps in (*a*) series and (*b*) in parallel.

Pumps in Series or Parallel 2/3

- When two pumps are placed in series
  - The resulting pump performance curve is obtained by adding heads at the same flowrate.
  - Both the actual head and the flowrate are increased but neither will be doubled.
  - The operating point is moved from (A) to (B).

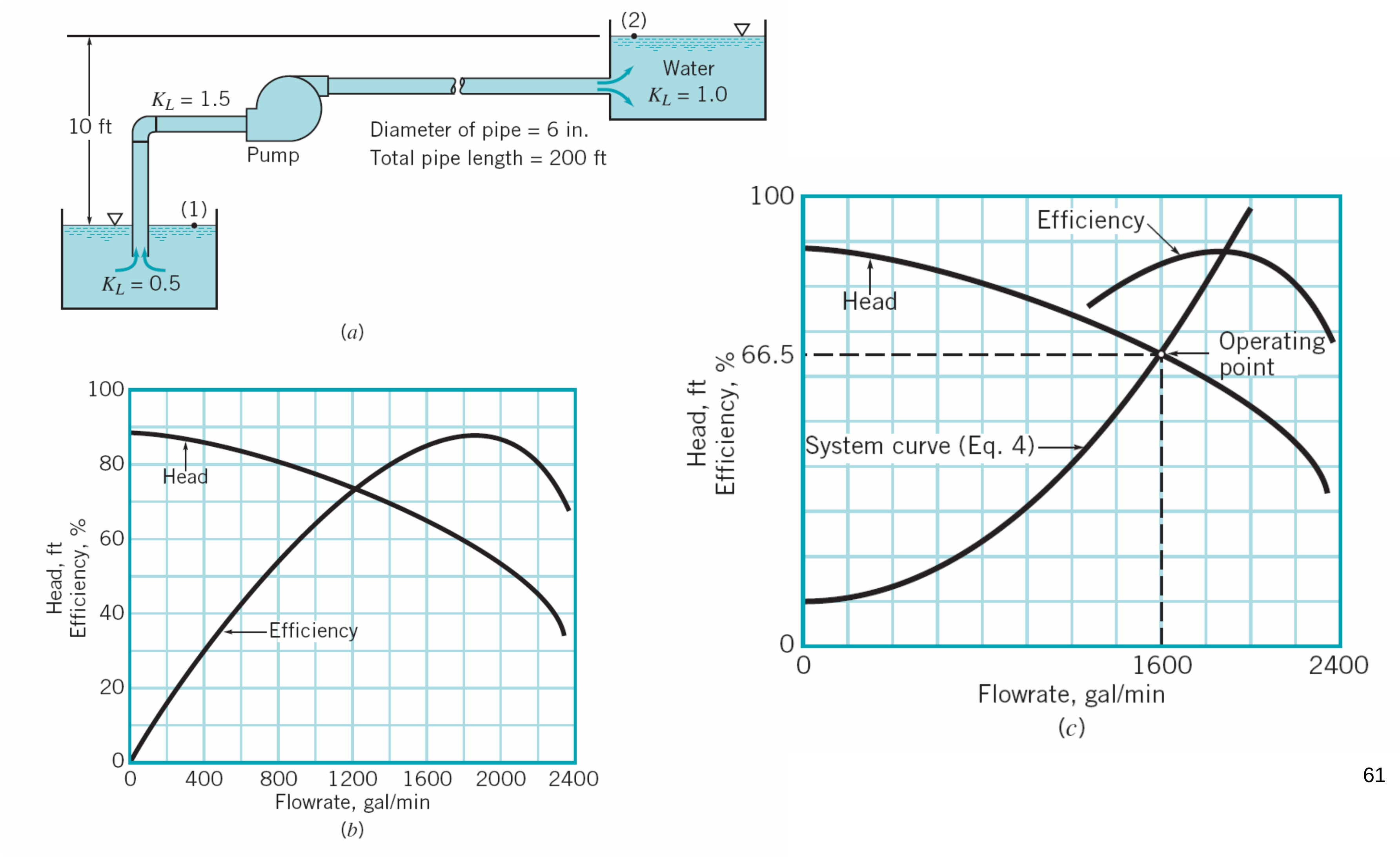
Pumps in Series or Parallel 3/3

- When two pumps are placed in parallel
  - ⇨ The combined performance curve is obtained by adding flowrate at the same head.
  - ⇨ The flowrate is increased significantly, but not be doubled.
  - ⇨ The operating point is moved from (A) to (B).

Example 12.4 Use of Pump Performance Curves 1/2

- Water is to be pumped from one large, open tank to a second large, open tank as shown in Fig. E12.4a. The pipe diameter throughout is 6 in. and the total length of the pipe between the pipe entrance and exit is 200 ft. Minor loss coefficients for the entrance, exit, and the elbow are shown on the figure, and the friction factor for the pipe can be assumed constant and equal to 0.02. A certain centrifugal pump having the performance characteristics shown in Fig. E12.4b is suggested as a good pump for this flow system. With this pump, what would be the flowrate between the tanks? Do you think this pump would be a good choice?

Example 12.4 Use of Pump Performance Curves 2/2



Example 12.4 Solution[1/2]

Application of the energy equation between the two free surfaces, points (1) and (2) as indicated, given

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}+\sum K_L\frac{V^2}{2g}$$

With $p_1=p_2=0$, $V_1=V_2=0$, $z_2-z_1=10\text{ft}$, $f=0.02$, $D=6/12\text{ft}$, and $\ell=200\text{ft}$

$$h_p=10+\left[0.02\frac{(200\text{ft})}{(6/12\text{ft})}+(0.5+1.5+1.0)\right]\frac{V^2}{2(32.2\text{ft}/\text{s}^2)}$$

$$V=\frac{Q}{A}=.. \qquad h_p=10+4.43Q^2 \qquad Q\ \text{is in ft}^3/\text{s}$$

Example 12.4 Solution$^{2/2}$

With Q in ft^3/s

$$h_p = 10 + 4.43Q^2 \quad \text{Eq. (3)}$$

With Q in gal/min

$$h_p = 10 + 2.20 \times 10^{-5} Q^2 \quad \text{Eq. (4)}$$

System equation for this particular flow system and reveals how much actual head the fluid will need to gain from the pump to maintain a certain flowrate.

With intersection occurring at Q=1600 gal/min

With the corresponding actual head gained equal to 66.5ft

Dimensionless Parameters and Similarity Laws

Dimensionless Parameters 1/4

- The principal, dependent pump variables
  - Actual head rise h_a
  - Shaft power $\dot{W}_{shaft}$ Efficiency η
- The important variables
  - Characteristic diameter D Pertinent lengths ℓ_i
  - Surface roughness ε Flowrate Q
  - Pump shaft rotational speed ω
  - Fluid viscosity μ
  - Fluid density ρ

Dimensionless Parameters 2/4

$$\text{dependent variables}: h_a, \dot{W}_{shaft}, \eta$$

$$= f(D, \ell_i, \varepsilon, Q, \omega, \mu, \rho)$$

$$\Rightarrow \text{dependent pi term} = \phi\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho\omega D^2}{\mu}\right)$$

Dimensionless Parameters 3/4

The dependent pi term may be

Head rise coefficient $$C_H = \frac{gh_a}{\omega^2 D^2} = \phi_1\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho\omega D^2}{\mu}\right)$$

Power coefficient $$C_p = \frac{\dot{W}_{shaft}}{\rho\omega^3 D^5} = \phi_2\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho\omega D^2}{\mu}\right)$$

Efficiency $$\eta = \frac{\rho g Q h_a}{\dot{W}_{shaft}} = \phi_3\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho\omega D^2}{\mu}\right)$$

Dimensionless Parameters 4/4

- For simplicity, ε /D can be neglected in pumps since the highly irregular shape of the pump chamber is usually the dominant geometry factor rather than the surface roughness.
- With these simplicity and for geometrically similar pumps, the dependent pi terms are function of only $Q/\omega D^3$.

$$\frac{gh_a}{\omega^2 D^2} = \phi_1\left(\frac{Q}{\omega D^3}\right) \quad (29)$$

$$\frac{\dot{W}_{shaft}}{\rho\omega^3 D^5} = \phi_2\left(\frac{Q}{\omega D^3}\right) \quad (30)$$

Flow coefficient C_Q

$$\eta = \phi_3\left(\frac{Q}{\omega D^3}\right) \quad (31)$$

Similarity Laws 1/3

Above three equations provide the desired similarity relationship among a family of geometrically similar pumps

If $$\left(\frac{Q}{\omega D^3}\right)_1 = \left(\frac{Q}{\omega D^3}\right)_2 \quad (32)$$

Then $$\left(\frac{gh_a}{\omega^2 D^2}\right)_1 = \left(\frac{gh_a}{\omega^2 D^2}\right)_2 \quad (33) \qquad \left(\frac{\dot{W}_{shaft}}{\rho\omega^3 D^5}\right)_1 = \left(\frac{\dot{W}_{shaft}}{\rho\omega^3 D^5}\right)_2 \quad (34)$$

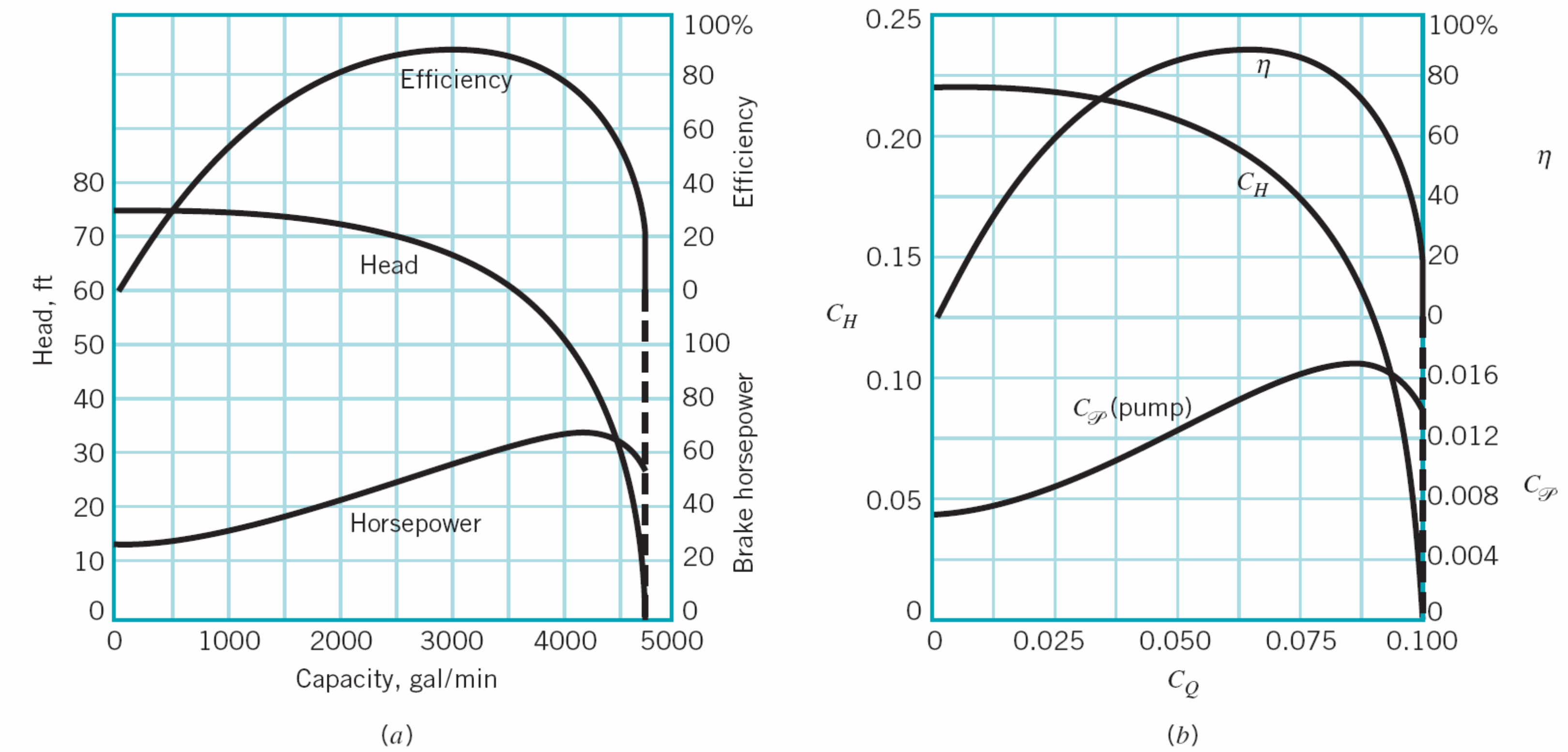
$$\eta_1 = \eta_2 \quad (35)$$

Where the subscripts 1 and 2 refer to any two pumps from the family of geometrically similar pumps.

Similarity Laws 2/3

- With these so-called pump scaling laws, it is possible to experimentally determine the performance characteristics of one pump in laboratory and then use these data to predict the corresponding characteristics for other pumps within the family under different operating conditions.
  - ⇨ From these curves of the performance of different-sized, geometrically similar pumps can be predicted.

Similarity Laws 3/3



Typical performance data for a centrifugal pump:
(*a*) characteristic curves for a 12-in. centrifugal pump operating at 1000 rpm, (*b*) dimensionless characteristic curves.

Example 12.5 Use of Pump Scaling Laws

- An 8-in.-diameter centrifugal pump operating at 1200 rpm is geometrically similar to the 12-in.-diameter pump having the performance characteristics of Fig. 12.17a and 12.17b while operating at 1000 rpm. For peak efficiency, predict the discharge, actual head rise, and shaft horsepower for this smaller pump. The working fluid is water at 60 ºF.

Example 12.5 Solution[1/2]

(31) For a given efficiency the flow coefficient has the same value for a given family of pumps.

Fig. 12.17(b) At peak efficiency C_Q=0.0625

$$Q = C_Q \omega D^3 = 2.33 \mathrm{ft}^3/\mathrm{s}$$

$$= (2.33 \mathrm{ft}^3/\mathrm{s})(7.48 \mathrm{gal}/\mathrm{ft}^3)(60 \mathrm{s}/\mathrm{min}) = 1046 \mathrm{gpm}$$

The actual head rise and the shaft horsepower can be determined in a similar manner since at peak efficiency C_H=0.019 and C_p=0.014

Example 12.5 Solution$^{2/2}$

$$h_a = \frac{C_H \omega^2 D^2}{g} = \ldots = 41.6\text{ft}$$

$$\dot{W}_{shaft} = C_p \rho \omega^3 D^5 = \ldots = 7150\text{ft} \cdot \text{lb/s} = 13.0\text{hp}$$

Special Pump Scaling Laws 1/6

- How a change in the operating speed, ω, for a given pump, affects pump characteristics.

(32) ⇒
$$\frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2} \quad (36)$$

For the same flow coefficient with $D_1 = D_2$

Where the subscripts 1 and 2 refer to the same pump operating at two different speeds at the same flow coefficient

(33) ⇒
$$\frac{h_{a1}}{h_{a2}} = \frac{\omega_1^2}{\omega_2^2} \quad (37)$$

(34) ⇒
$$\frac{\dot{W}_{shaft1}}{\dot{W}_{shaft2}} = \frac{\omega_1^3}{\omega_2^3} \quad (38)$$

Special Pump Scaling Laws 2/6

These scaling laws are useful in estimating the effect of changing pump speed when some data are available from a pump test obtained by operating the pump at a particular speed.

Special Pump Scaling Laws 3/6

❖ How a change in the impeller diameter, D, of a geometrically similar family of pumps, operating at a given speed, affects pump characteristics.

(32) $\xRightarrow{\omega_1=\omega_2}$ $$\frac{Q_1}{Q_2}=\frac{D_1^3}{D_2^3} \quad (39)$$

(33) $\Rightarrow$ $$\frac{h_{a1}}{h_{a2}}=\frac{D_1^2}{D_2^2} \quad (40)$$

(34) $\Rightarrow$ $$\frac{\dot{W}_{shaft1}}{\dot{W}_{shaft2}}=\frac{D_1^5}{D_2^5} \quad (41)$$

Special Pump Scaling Laws 4/6

- With these scaling laws are based on the condition that, as the impeller diameter is changed, all other important geometric variables are properly scaled to maintain geometric similarity.
- Geometric scaling is not always possible
- (39)~(41) will not, in general, be valid.

Special Pump Scaling Laws 5/6

- However, experience has shown that if the impeller diameter change is not too large, less than about 20%, these scaling relationships can still be used to estimate the effect of a change in the impeller diameter.
- (36)~(41) are sometimes referred to as the **pump affinity law**.

Special Pump Scaling Laws 6/6

- It has been found that as the pump size decreases these effects more significantly influence efficiency because of smaller clearance and blade size.
- An approximate, empirical relationship to estimate the influence of diminishing size on efficiency is

$$\frac{1-\eta_1}{1-\eta_2} = \left(\frac{D_1}{D_2}\right)^{1/5} \qquad (42)$$

Specific Speed 1/5

❖ Specific speed is a useful pi term obtained by eliminating diameter D between the low coefficient and the head rise coefficient

$$N_s = \frac{(Q/\omega D^3)^{1/2}}{(gh_a/\omega^2 D^2)^{3/4}} = \frac{\omega\sqrt{Q}}{(gh_a)^{3/4}} \quad (43)$$

Specific speed varies with flow coefficient just as the other coefficients and efficiency.

Specific Speed 2/5

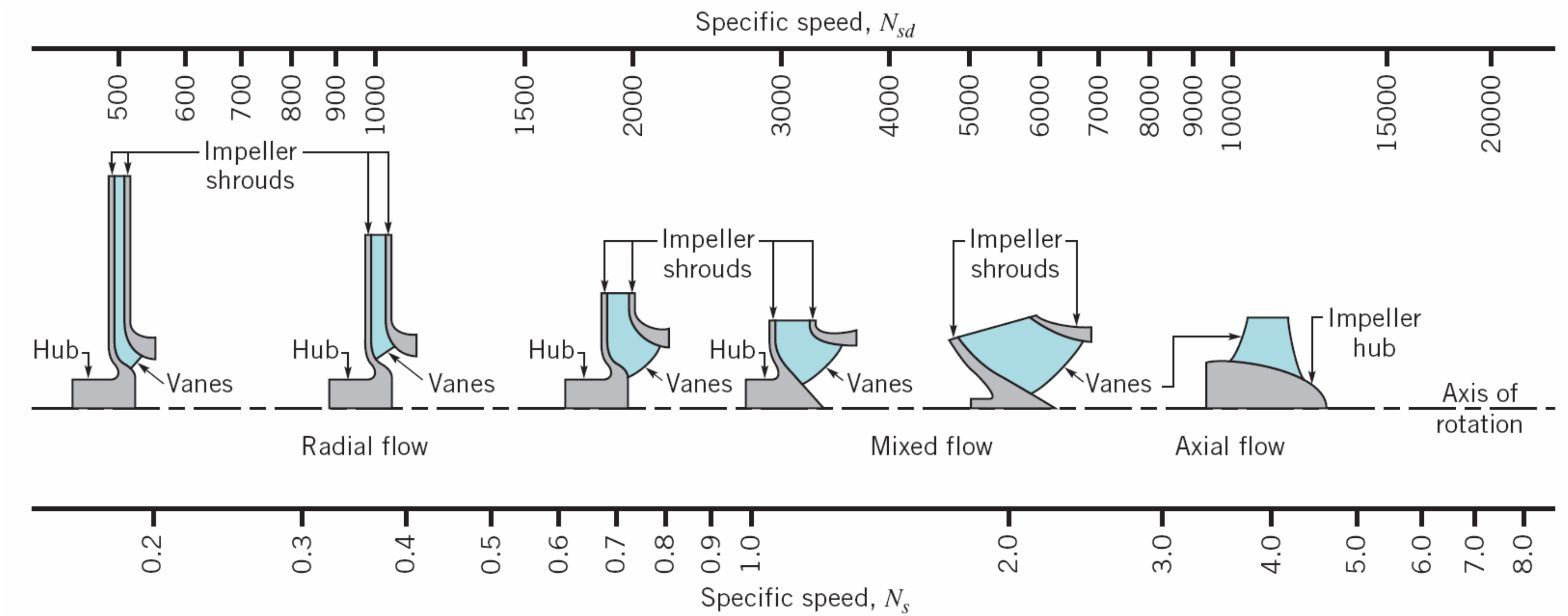
- For nay pump it is customary to specify a value of specific speed at the flow coefficient corresponding to peak efficiency only.
- In the United States a modified, dimensional form of specific speed, N_{sd}

$$N_{sd} = \frac{\omega(\text{rpm})\sqrt{Q(\text{gpm})}}{[h_a(\text{ft})]^{3/4}} \qquad (44)$$

Specific Speed 3/5

- N_{sd} is expressed in U.S. customary units.
- Typical value of N_{sd} are in the range 500 to 4000 for centrifugal pumps.
- Both N_s and N_{sd} have the same physical meaning, but their magnitudes will differ by a constant conversion factor N_{sd}=2733N_s when ω is expressed in rad/s.

Specific Speed 4/5



Variation in specific speed with type of pump.

Specific Speed 5/5

- Each family or class of pumps has a particular range of values of specific speed associated with it.
- Pumps that have low-capacity, high-head characteristics will have specific speed that are smaller than that have high-capacity, low-head characteristics.
- As N_{sd} increases beyond about 2000 the peak efficiency of the purely radial-flow centrifugal pump starts to fall off, and other type of more efficient pump design are preferred.

Suction Specific Speed 1/2

- Suction specific speed is defined

$$S_s = \frac{\omega\sqrt{Q}}{[g(NPSH_R)]^{3/4}} \qquad (45)$$

- In the United States a modified, dimensional form of suction specific speed, S_{sd}

$$S_{sd} = \frac{\omega(rpm)\sqrt{Q(gpm)}}{[NPSH_R(ft)]^{3/4}} \qquad (44)$$

Suction Specific Speed 2/2

- Typical values for S_{sd} fall in the range 7000 to 12000.
- Note that S_{sd}=2733S_s ,with ω expressed in rad/s.

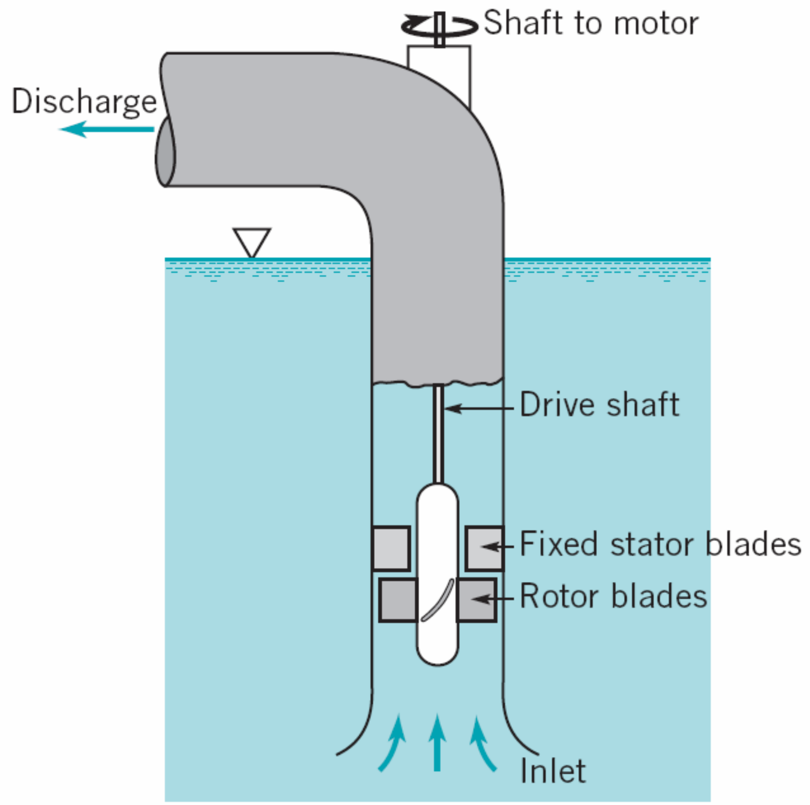
Axial-Flow and Mixed-Flow Pumps

Axial-Flow and Mixed-Flow Pumps 1/2

- Centrifugal pumps are radial-flow machines that operate most efficiently for applications requiring high heads at relatively low flowrate.
- For many applications, such as those associate with drainage and irrigation, high flowrate at low head are required.
  - ⇨ Centrifugal pumps are not suitable.
  - ⇨ Axial-flow pumps are commonly used.
- Axial-flow pump, consists of a propeller confined within a cylindrical casing, is often called propeller pump.

Axial-Flow and Mixed-Flow Pumps 2/2

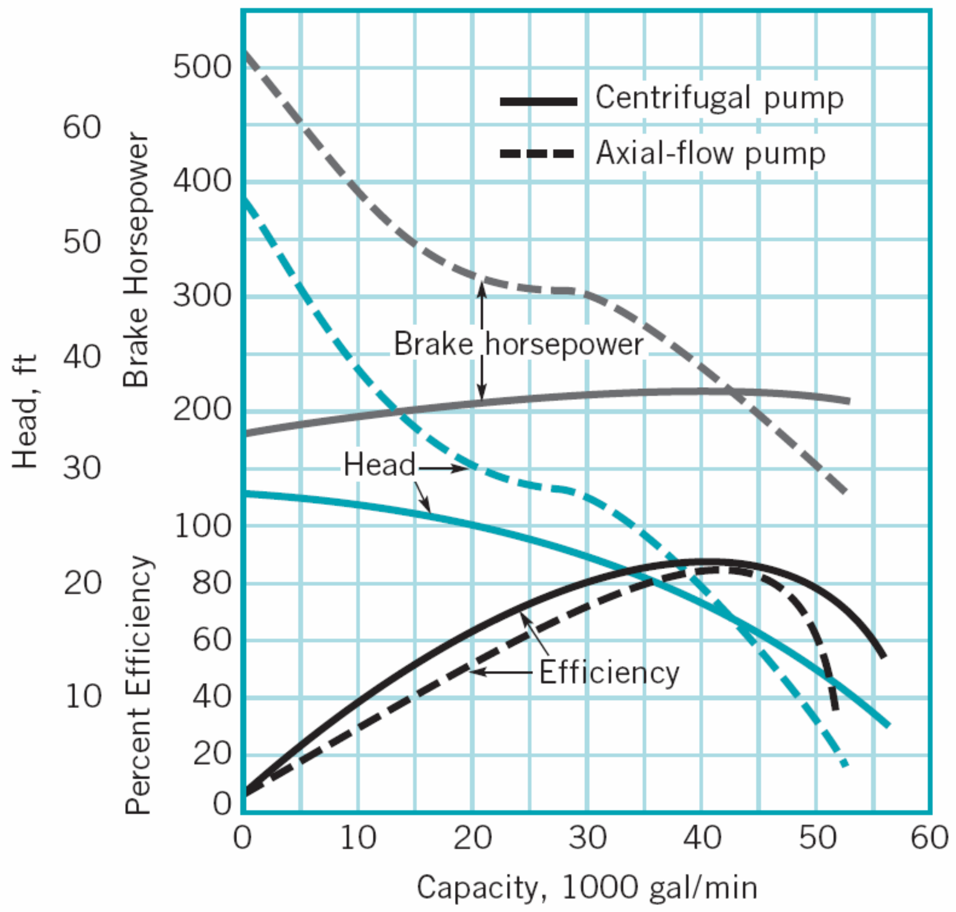
❖ Schematic diagram of an axial-flow pump arranged for vertical operation.



⇨ A rotor is connected to a motor through a shaft.

⇨ As the rotor rotates the fluid is sucked in through the inlet.

Centrifugal Pump vs. Axial-Flow Pump 1/2

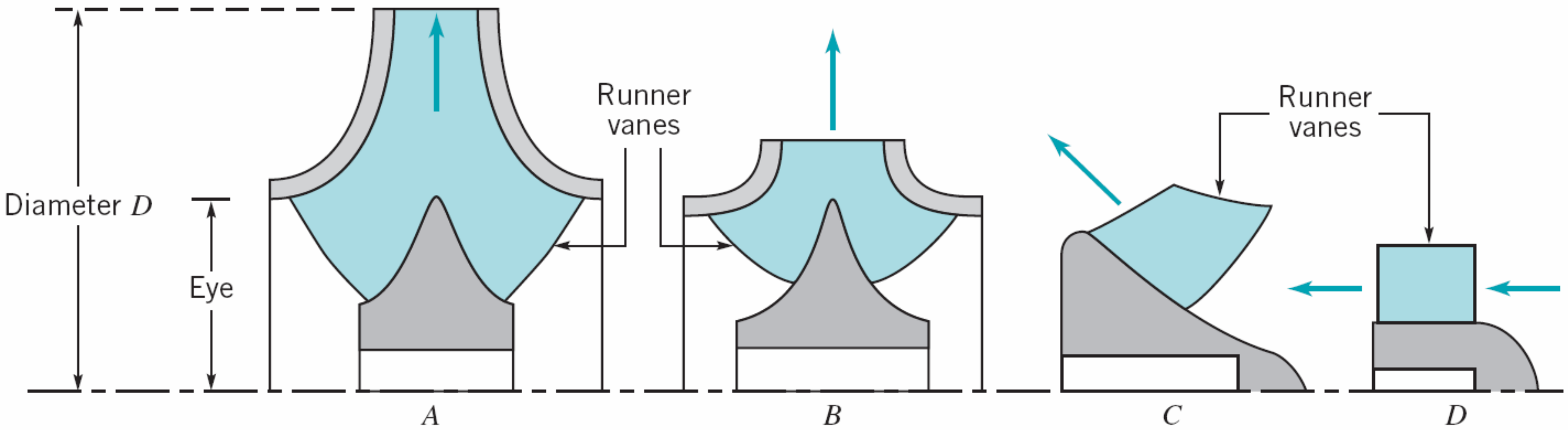


- ❖ At design capacity (maximum efficiency) the head and brake horsepower are the same for the two pumps.
- ❖ As the flowrate decreases, the power input to the centrifugal pump falls to 180 hp at shutoff.

Centrifugal Pump vs. Axial-Flow Pump 2/2

- Whereas for the axial-flow pump the power input increases to 520 hp at shutoff.
- The axial-flow pump can cause overloading of the drive motor if the flowrate is reduced significantly from the design capacity.
- The head curve for the axial-flow pump is much steeper than that fir the centrifugal pump.
- Except at design capacity, the efficiency of the axial-flow pump is lower than that o the centrifugal pump.

Comparison of Different Types of Impellers



Type	Centrifugal	Centrifugal	Mixed flow	Axial flow
N_{sd}	1,250	2,200	6,200	13,500
Gal/min	2,400	2,400	2,400	2,400
Head, ft	70	48	33	20
Rpm	870	1,160	1,750	2,600
D, in.	19	12	10	7
D_{eye}/D	0.5	0.7	0.9	1.0

Fans

Fans 1/3

- Fans: used to move the fluid.
- Types of fans varying from small fan used for cooling desktop computers to large fans used in many industrial applications.
- Fan are also called blowers, boosters, and exhausters depending on the location within the system.
- As in the case for pumps, fans designs include centrifugal (radial-flow) fans, mixed-flow and axial-flow (propeller) fans.

Fans [2/3]

- Analysis of fans performance closely follows that previously described for pumps.
- Scaling relationships for fans are the same as those developed for pumps, that is, equations (32)~(35) apply to fans and pumps.
- For Fans

(33)

$$\left(\frac{p_a}{\rho\omega^2 D^2}\right)_1 = \left(\frac{p_a}{\rho\omega^2 D^2}\right)_2 \qquad (47)$$

Replace the head, h_a, with pressure head $p_a/\rho g$

Fans 3/3

- Equations (47), (32) and (34) are called the fan laws and can be used to scale performance characteristics between members of a family of geometrically similar fans.

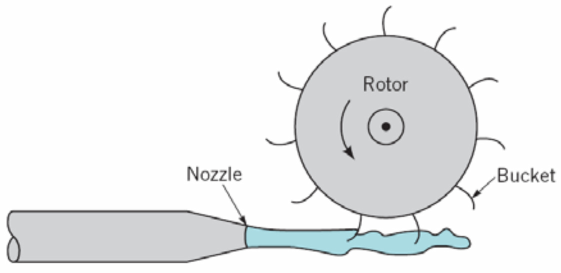
Turbines

Turbines 1/6

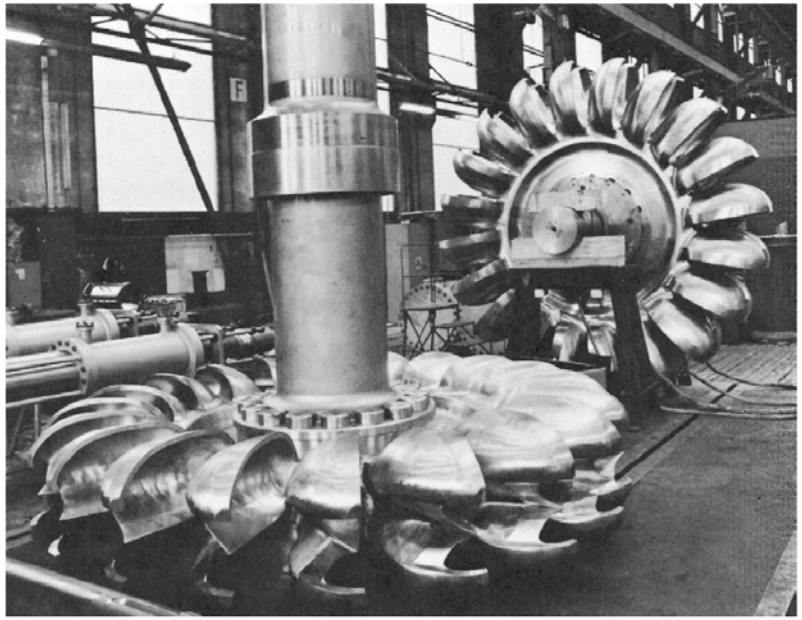
- Turbines are devices that extract energy from a flowing fluid.
- The geometry of turbines is such that the fluid exerts a torque on the rotor in the direction of its rotation.
- The shaft power generated is available to derive generators or other devices.
- The two basic types of hydraulic turbines are **impulse and reaction turbines.**

Turbines 2/6

- For hydraulic impulse turbines, the pressure drop across the rotor is zero; all of the pressure drop across the turbine stages occurs in the nozzle row.
- The Pelton whell is a classical example of an impulse turbines.

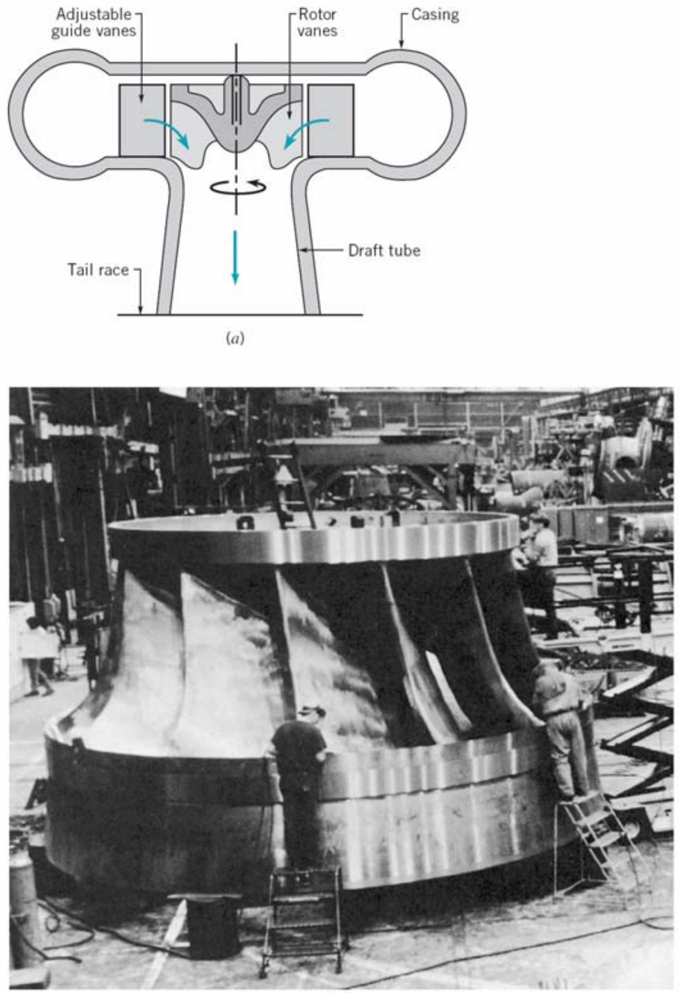


(a)


(b)

Turbines 3/6

❖ Diagram shows a reaction turbine.



(b)

Turbines 4/6

- For impulse turbines
  - ⇨ The total head of the incoming fluid is converted into a large velocity head at the exit of the supply nozzle.
  - ⇨ Both the pressure drop across the bucket (blade) and the change in relative speed of the fluid across the bucket are negligible.
  - ⇨ The space surrounding the rotor is not completely filled with fluid.
  - ⇨ The individual jets of fluid striking the buckets that generates the torque.

Turbines 5/6

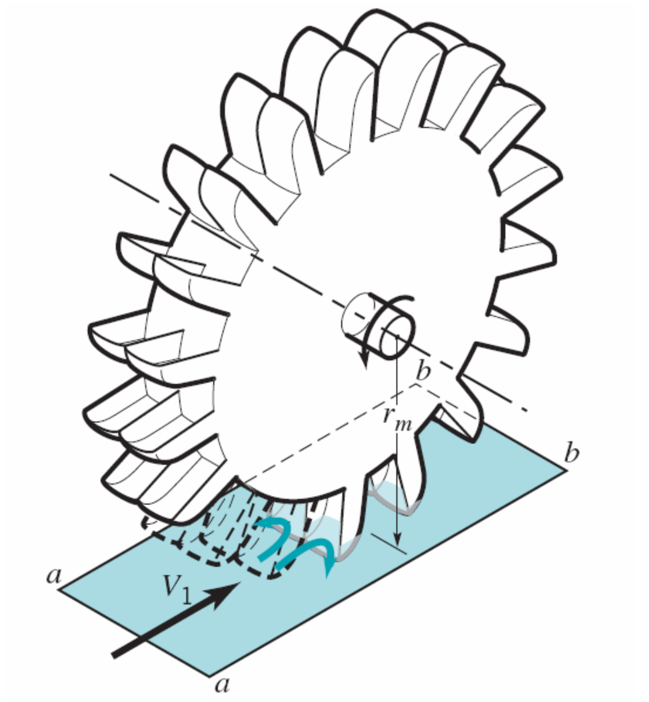
- For reaction turbines
  - There is both a pressure drop and a fluid relative speed change across the rotor.
  - Guide vanes act as nozzle to accelerate the flow and turn it in the appropriate direction as the fluid enters the rotor.
  - Part of the pressure drop occurs across the guide vanes and part occurs across the rotor,

Turbines 6/6

- **Summary**
  - **Impulse turbines: High-head, low flowrate devices.**
  - **Reaction turbines: Low-head, high-flowrate devices.**

Impulse Turbines 1/6

- The easiest type of impulse turbines design is the Pelton wheel.
- Lester Pelton (1829~1908), an American mining engineer during the California gold-mining days, is responsible for many of still-used features of this type of turbine.

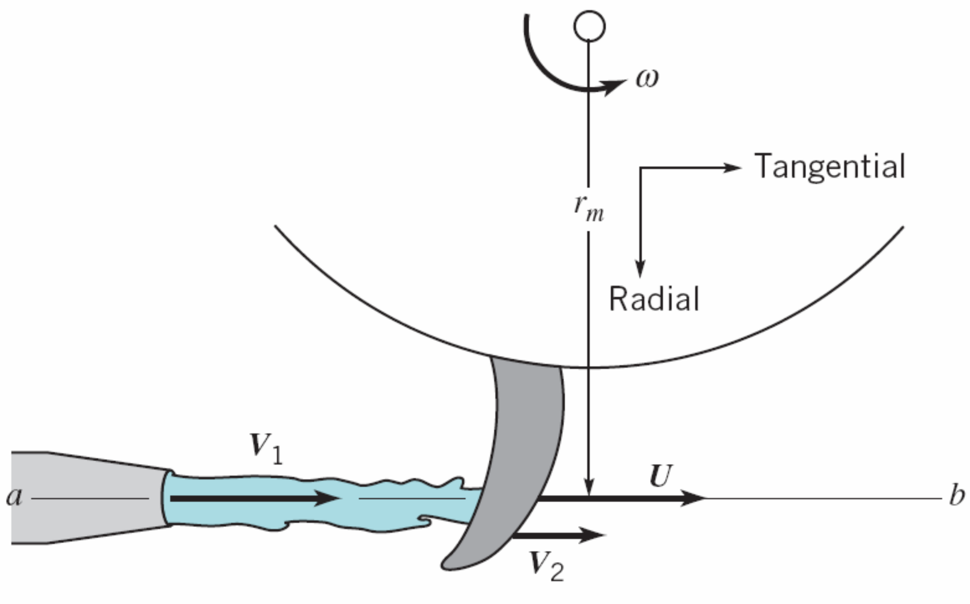


Impulse Turbines 2/6

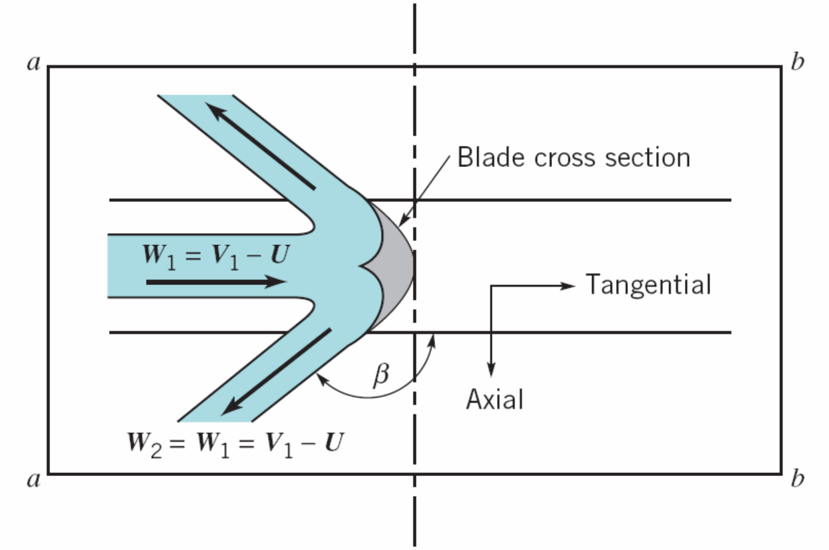
- A high-speed jet of water strikes the Pelton wheel buckets and is deflected.
- The water enters and leaves the control volume surrounding the wheel as free jet.
- A person riding on the bucket would note that the speed of the water doest not change as it slides across the buckets. That is, the magnitude of the relative velocity does not change, but its direction does.

Impulse Turbines 3/6

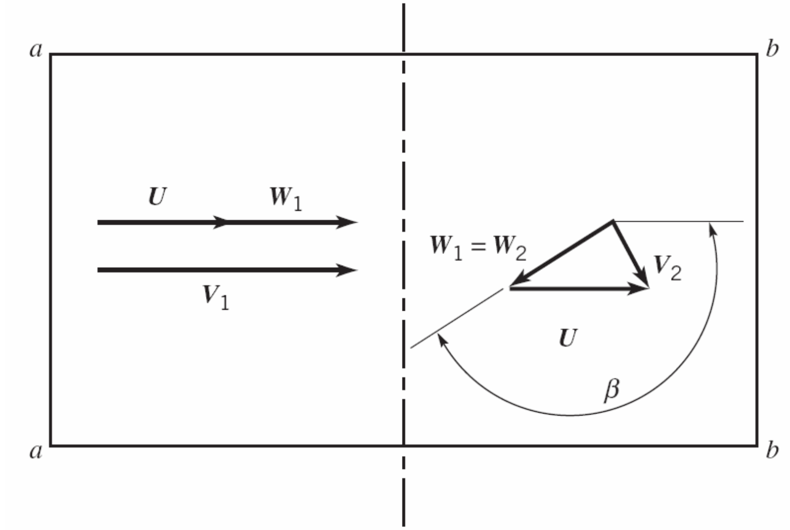
- Ideally, the fluid enters and leaves the control volume with no radial component of velocity.
- The buckets would ideally turn the relative velocity through a 180° turn, but physical constraints dictate that β, the angle of the exit edge of the blade, is less than 180 °



Impulse Turbines 4/6



Flow as viewed by an observer riding on the Pelton wheel – relative velocities



Inlet and exit velocity triangles for a Pelton wheel turbine.

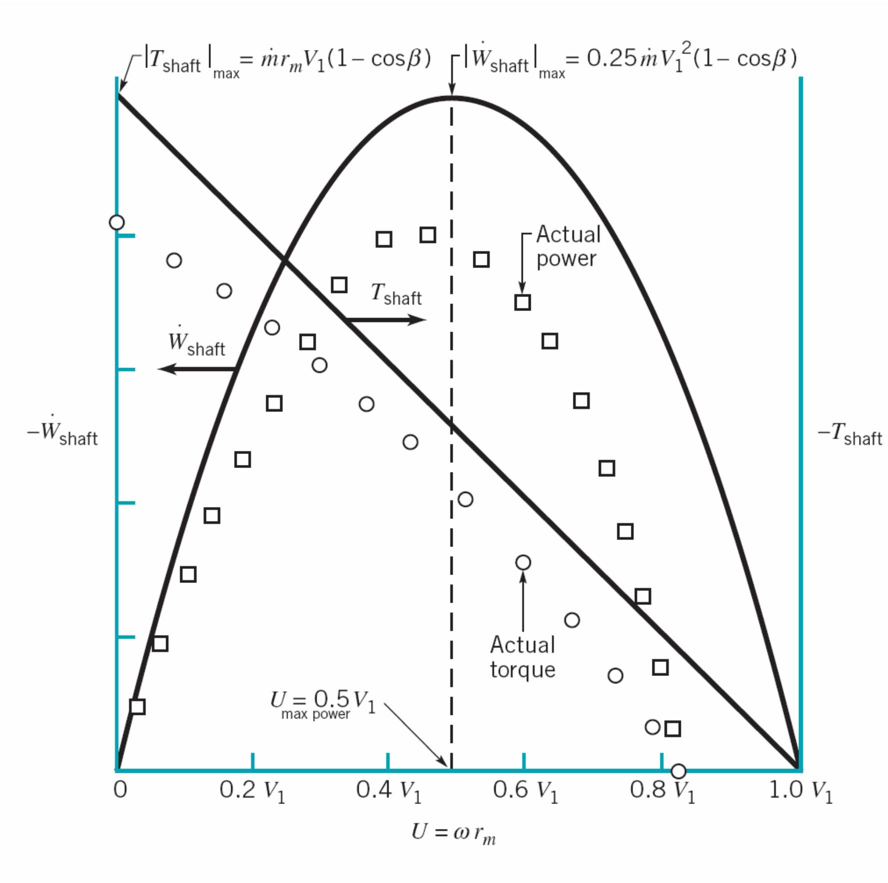
$$V_{\theta 1} = V_1 = W_1 + U \quad (48)$$

$$V_{\theta 2} = W_2 \cos\beta + U \quad (49)$$

With $W_1 = W_2$ (48)+(49) ⇨ $V_{\theta 2} - V_{\theta 1} = (U - V_1)(1 - \cos\beta)$ (50)

Impulse Turbines 5/6

(50)+(2)+(4) ⇒ $T_{shaft} = \dot{m} r_m (U - V_1)(1 - \cos\beta)$



$$\dot{W}_{shaft} = T_{shaft}\,\omega$$
$$= \dot{m} U (U - V_1)(1 - \cos\beta) \quad (51)$$

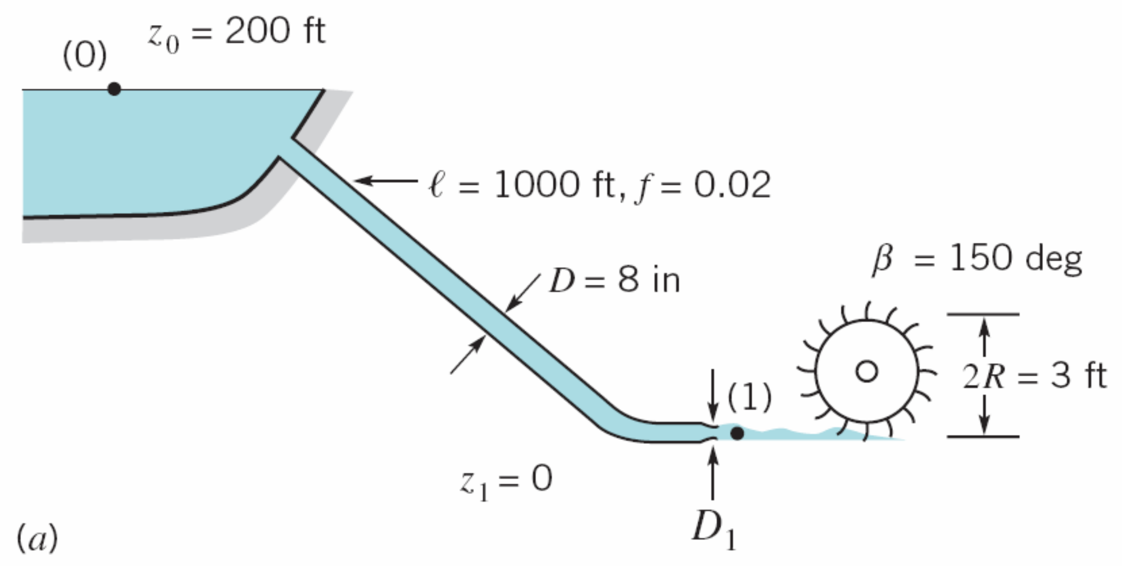
Typical theoretical and experimental power and torque for a Pelton wheel turbine as a function of bucket speed.

Impulse Turbines 6/6

- From above results:
  - The power is a function of β. A typical value of β=165º results in a relatively small reduction in power since 1-cos165º=1.966.
  - Although torque is maximum when the wheel is stopped (U=0), there is no power under this condition – to extract power one needs force and motion.
  - The power output is a maximum when U=V/2. (52)
  - The maximum speed occurs when T_{shaft}=0.

Example 12.6 Pelton Wheel Turbine Characteristics

- Water to drive a Pelton wheel is supplied through a pipe from a lake as indicated in Fig. E12.6a. Determine the nozzle diameter, D_1, that will give the maximum power output. Include the head loss due to friction in the pipe, but neglect minor losses. Also determine this maximum power and the angular velocity of the rotor at this condition.



Example 12.6 Solution[1/3]

(51) $$\dot{W}_{shaft} = \rho Q U (U - V_1)(1 - \cos\beta)$$

The nozzle exit speed, V_1, can be obtained by applying the energy equation between a point on the lake surface (where $V_0=p_0=0$) and the nozzle outlet (where $z_1=p_1=0$) to give

$$z_0 = \frac{V_1^2}{2g} + h_L \qquad h_L = f\frac{\ell}{D}\frac{V^2}{2g}$$

$$z_0 = \left[1 + f\frac{\ell}{D}\left(\frac{D_1}{D}\right)^4\right]\frac{V_1^2}{2g} = \frac{113.5}{\sqrt{1+152D_1^4}}$$

$$Q = \pi D_1^2 V_1 / 4$$

Example 12.6 Solution 2/3

$$\dot{W}_{shaft} = \frac{323UD_1^2}{\sqrt{1+152D_1^4}}\left[U - \frac{113.5}{\sqrt{1+152D_1^4}}\right]$$

The maximum power occurs when $U=V_1/2$

$$\dot{W}_{shaft} = \frac{1.04\times10^6 D_1^2}{(1+152D_1^4)^{3/2}}$$

The maximum power possible occurs when $d\dot{W}_{shaft}/dD_1 = 0$

D_1=0.239ft

$$\dot{W}_{shaft} = \frac{1.04\times10^6 D_1^2}{(1+152D_1^4)^{3/2}} = -3.25\times10^4\,\text{ft}\cdot\text{lb/s} = -59.0\text{hp}$$

Example 12.6 Solution[3/3]

The rotor speed at the maximum power condition can be obtained from

$$U = \omega R = \frac{V_1}{2} \qquad \omega = \frac{V_1}{2R} = 295\text{rpm}$$

Example 12.7 Maximum Power Output for a Pelton Wheel Turbine

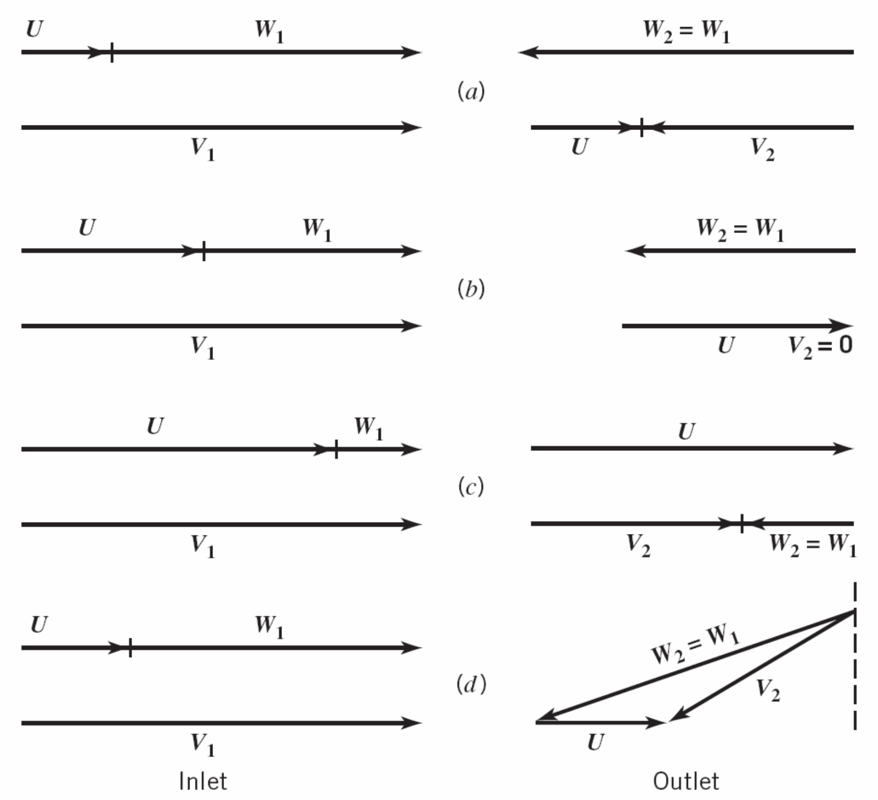
- Water flows through the Pelton wheel turbine shown in Fig. 12.24. For simplicity we assume that the water is turned 180º by the blade. Show, based on the energy equation, that the maximum power output occurs when the absolute velocity of the fluid exiting the turbine is zero.

Example 12.7 Solution[1/2]

(51) $$\dot{W}_{shaft} = \rho QU(U - V_1)(1 - \cos\beta) = 2\rho Q(U^2 - UV_1)$$

For this impulse turbine with $\beta = 180^{o}$, the velocity triangles simplify into the diagram types shown in Fig. E12.7. Three possibilities are indicated:

(a) The exit absolute velocity, V_2, is directed back toward the nozzle.



Example 12.7 Solution$^{2/2}$

(b) The absolute velocity at the exit is zero, or
(c) The exiting stream flows in the direction of the incoming stream.

The maximum power occurs when $U=V_1/2$.
If viscous effects are negligible, when $W_1=W_2$ and we have $U=W_2$, which gives $V_2=0$

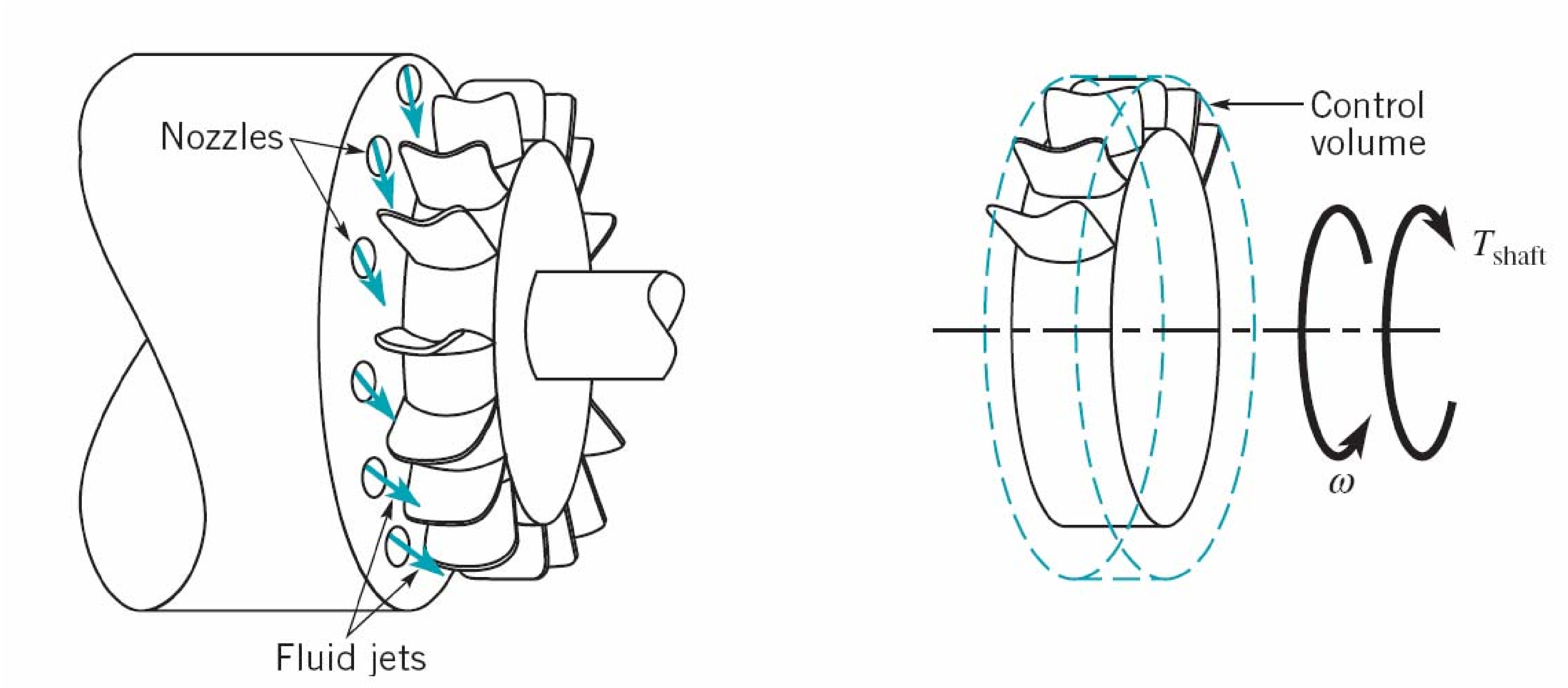
Consider the energy equation for flow across the rotor we have

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_T+h_L$$

$$\Rightarrow h_T=\frac{V_1^2-V_2^2}{2g}-h_L$$

$\Rightarrow$ $V_2=0$

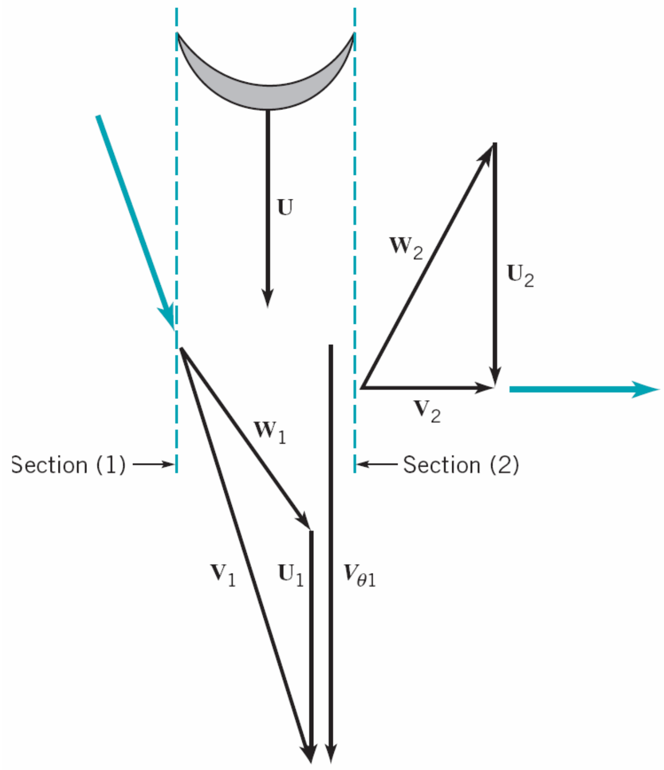
Second Type of Impulse Turbines 1/3



A multinozzle, non-Pelton wheel impulse turbine commonly used with air as the working fluid.

Second Type of Impulse Turbines 2/3

- A circumferential series of fluid jets strikes the rotating blades which, as with the Pelton wheel, alter both the direction and magnitude of the absolute velocity.
- The inlet and exit pressure are equal.
- The magnitude of the relative velocity is unchanged as the fluid slides across the blades.



Second Type of Impulse Turbines 3/3

- In order for the absolute velocity of the fluid to be changed as indicated during its passage across the blade, the blade must push on the fluid in the direction opposite of the blade motion.
  - ⇨ The fluid pushes on the blade in the direction f the blades motion – the fluid does work on the blade.

Example 12.8 Non-Pelton Wheel Impulse Turbine 1/2

- An air turbine used to drive the high-speed drill used by your dentist is shown in Fig. E12.8a. Air exiting from the upstream nozzle holes force the turbine blades to move in the direction shown. Estimate the shaft energy per unit mass of air flowing through the turbine under the following conditions. The turbine rotor speed is 300,000 rpm, the tangential component of velocity out of the nozzle is twice the blade speed, and the tangential component of the absolute velocity out of the rotor is zero.

Example 12.8 Non-Pelton Wheel Impulse Turbine 2/2

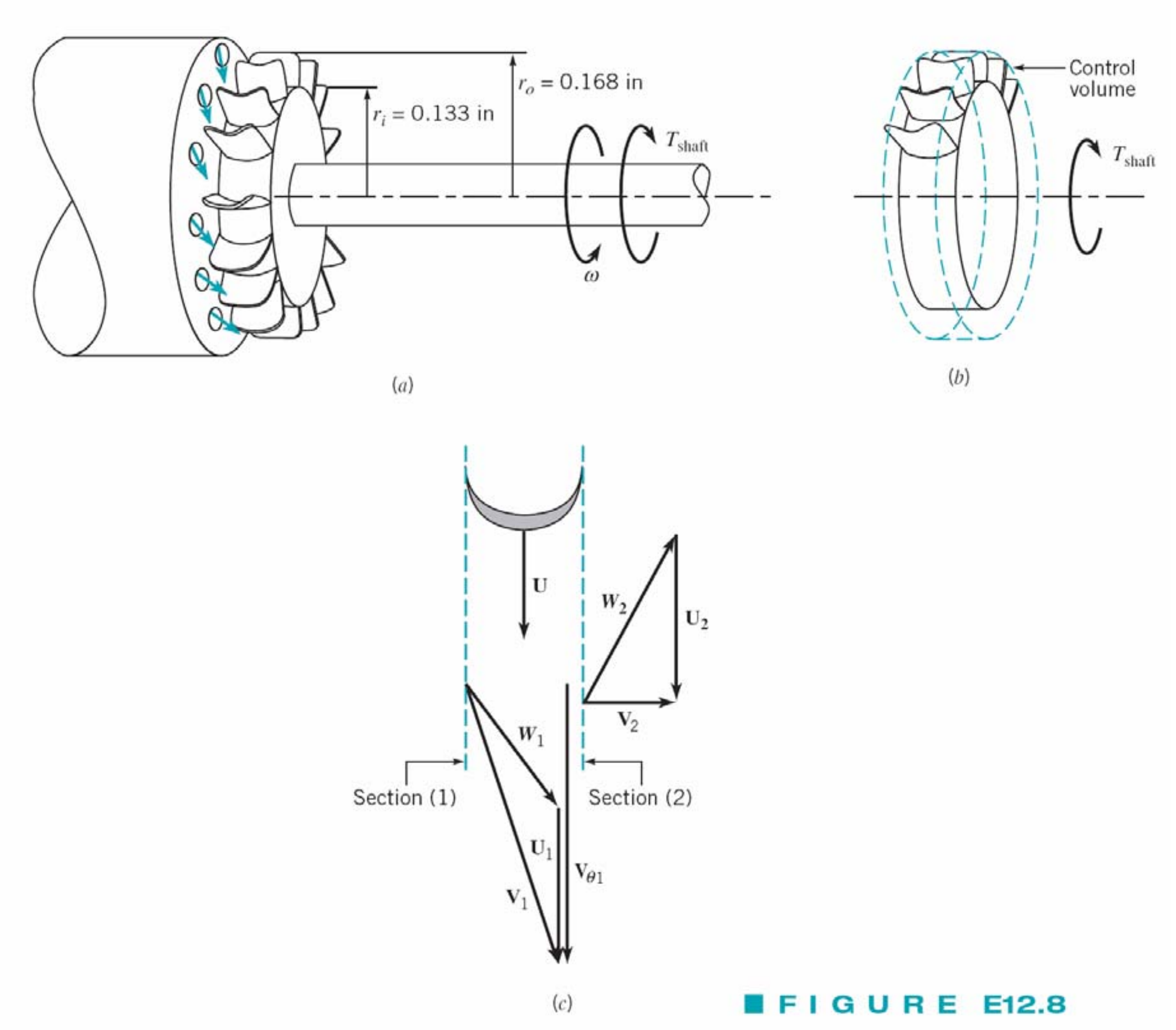


FIGURE E12.8

Example 12.8 Solution

For simplicity we analyze this problem using an arithmetic mean radius

$$r_m = \tfrac{1}{2}(r_o + r_i)$$

(5)

$$w_{shaft} = -U_1 V_{\theta 1} + U_2 V_{\theta 2}$$

$$V_{\theta 1} = 2U \qquad V_{\theta 2} = 0$$

$$U = \omega r_m = \ldots = 394\,\mathrm{ft/s}$$

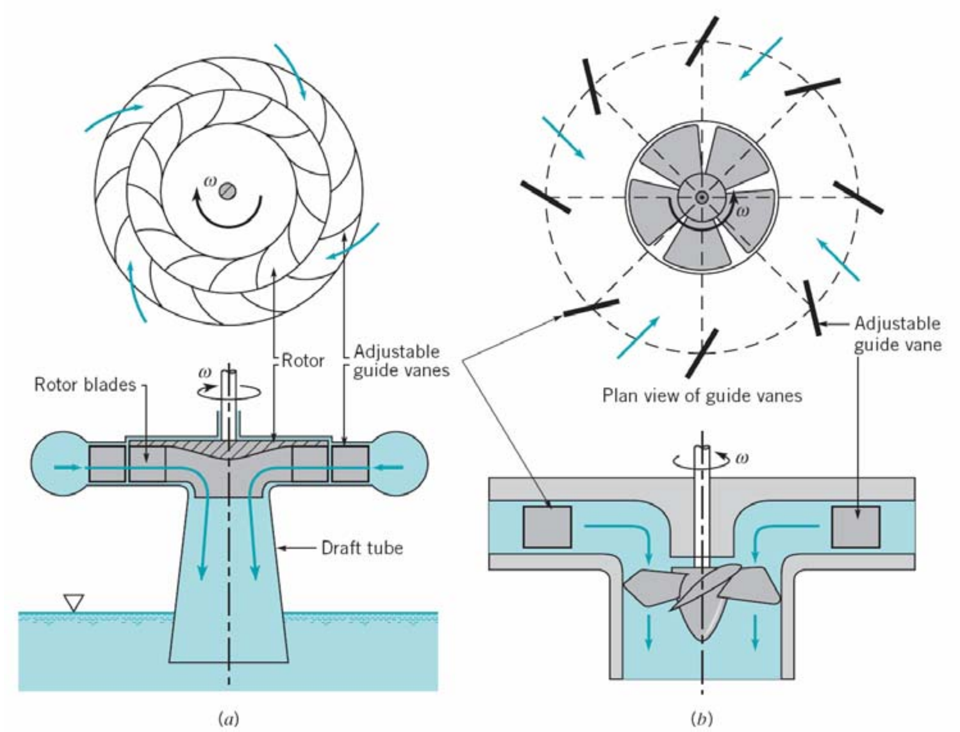
$$w_{shaft} = \ldots = -9640\,\mathrm{ft \cdot lb/lbm}$$

Reaction Turbines 1/2

- Best suited for higher flowrate abd lower head situations such as are often encountered in hydroelectric power plants associated with a dammed river.
- The working fluid completely fills the passageways through which it flows.
- The angular momentum, pressure, and the velocity of the fluid decrease as it flows through the turbine rotor – the turbine rotor extracts energy from the fluid.

Reaction Turbines 2/2

❖ The variety of configurations: radial-flow, mixed flow, and axial-flow.



(*a*) Typical radial-flow Francis turbine. (*b*) typical axial-flow Kaplan turbine.

Dimensionless Parameters for Turbines 1/2

- As with pumps, incompressible flow turbine performance is often specified in terms of appropriate dimensionless parameters

The flow coefficient $$C_Q = \frac{Q}{\omega D^3}$$

Head rise coefficient $$C_H = \frac{gh_T}{\omega^2 D^2}$$

Power coefficient $$C_p = \frac{\dot{W}_{shaft}}{\rho \omega^3 D^5}$$

Dimensionless Parameters for Turbines 2/2

- On the other head, turbine efficiency is the inverse of pump efficiency

$$\eta = \frac{\dot{W}_{shaft}}{\rho g Q h_T}$$

Similarity Laws for Turbines

- For geometrically similar turbines and for negligible Reynolds number and surface roughness difference effects, the relationship between the dimensionless parameters are given

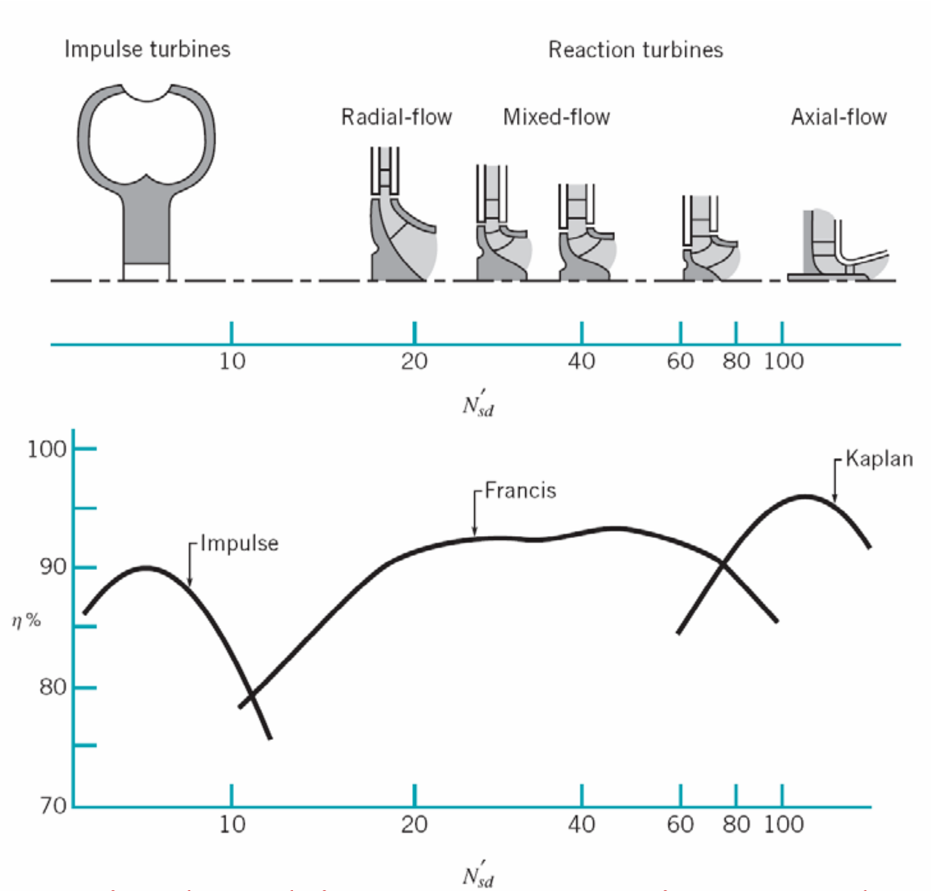
$$C_H = \phi_1(C_Q) \qquad C_p = \phi_2(C_Q) \qquad \eta = \phi_3(C_Q)$$

Power Specific Speed 1/2

- The design engineer has a variety of turbine types available for any given application.
- It is necessary to determine which type of turbine would best fit the job before detailed design work is attempted.
- As with pump, the use of a specific speed parameter can help provide this information

$$N'_s = \frac{\omega\sqrt{\dot{W}_{shaft}/\rho}}{(gh_T)^{5/4}} \Rightarrow N'_{sd} = \frac{\omega(rpm)\sqrt{\dot{W}_{shaft}(bhp)}}{[h_T(ft)]^{5/4}} \quad (53)$$

Power Specific Speed 2/2



- Provide a guide for turbine-type selection.
- The actual turbine efficiency for a given turbine depends very strongly on the detailed design of the turbine.

Typical turbine cross sections and maximum efficiencies as a function of specific speed.

Example 12.9 Use of Specific Speed to Select Turbine Type

- A hydraulic turbine is to operate at an angular velocity of 6 rev/s, a flowrate of 10 ft^3/s, and a head of 20 ft. What type of turbine should be selected? Explain.

Example 12.9 Solution

$$\omega = 6 \text{rev/s} = 360 \text{rpm}$$

Assumed efficiency $\eta = 94\%$

$$\dot{W}_{shaft} = \gamma Q z \eta = (62.4 \text{lb/ft}^3)(10 \text{ft}^3/\text{s}) \left[\frac{20 \text{ft}(0.94)}{550 \text{ft} \cdot \text{lb/s} \cdot \text{hp}} \right] = 21.3 \text{hp}$$

$$N'_{sd} = \frac{\omega \sqrt{\dot{W}_{shaft}}}{(h_T)^{5/4}} = 39.3$$

(Fig. 12.32) ⇒ **A mixed-flow Francis turbine would probably give the highest efficiency and an assumed efficiency of 0.94 is appropriate.**

Compressible Flow Turbomachines

Compressible Flow Turbomachines 1/2

- Compressible flow turbomachines are similar to the incompressible flow pumps and turbines.
- The main difference is that the density of the fluid changes significantly from the inlet to the outlet of the compressible flow machines.
- Compressor and pumps that add energy to the fluid, causing a significant pressure rise and a corresponding significant increase in density.

Compressible Flow Turbomachines 2/2

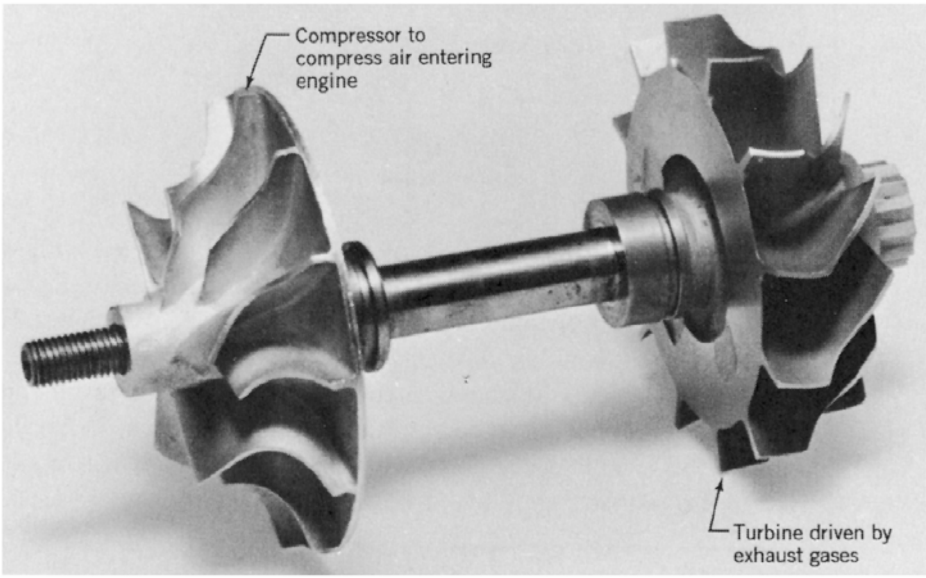
- Compressible flow turbomachines remove energy from the fluid, causing a lower pressure and a smaller density at the outlet than at the inlet.

Compressor

- Turbocompressors operate with the continuous compression of gas flowing through the device.
- Since there is a significant pressure and density increase, there is also a considerable temperature increase.
  - **Radial-flow (centrifugal) compressor.**
  - **Axial-flow compressor.**

Radial-Flow Compressor 1/3

- Use a gas as the working fluid.
- The typical high pressure rise, low flowrate, and axially compact turbomachine.



Photograph of the rotor from an automobile turbocharger.

Radial-Flow Compressor 2/3

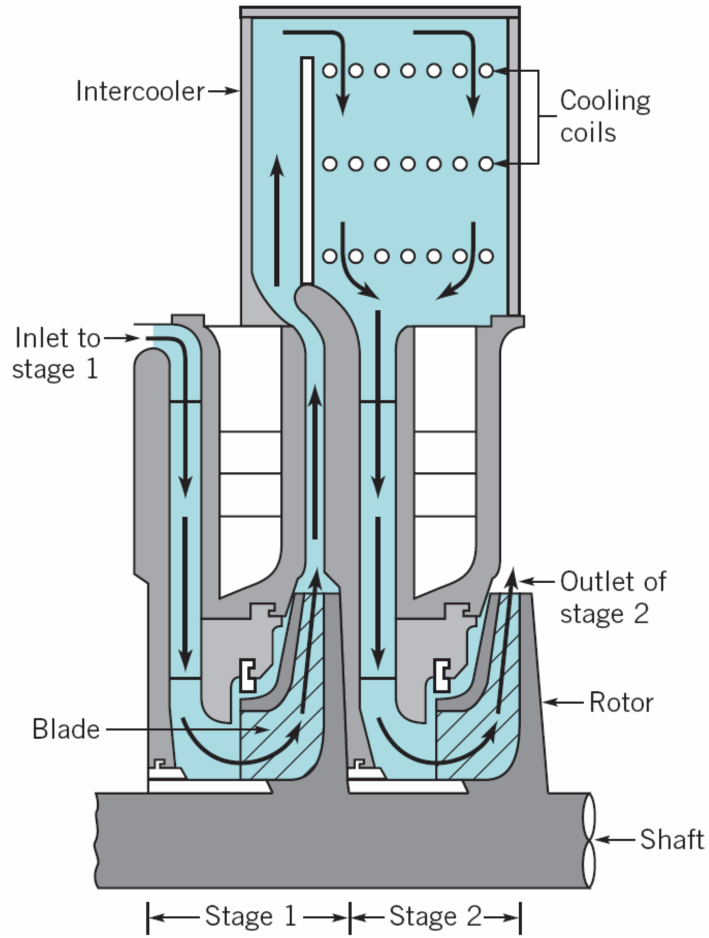
- The amount of compression is given in terms of the total pressure ratio

$$PR = \frac{p_{02}}{p_{01}}$$

- Higher pressure ratios can be obtained by using multiple stage device in which flow from the outlet of the proceeding stage proceeds to the inlet of the followwing stage.

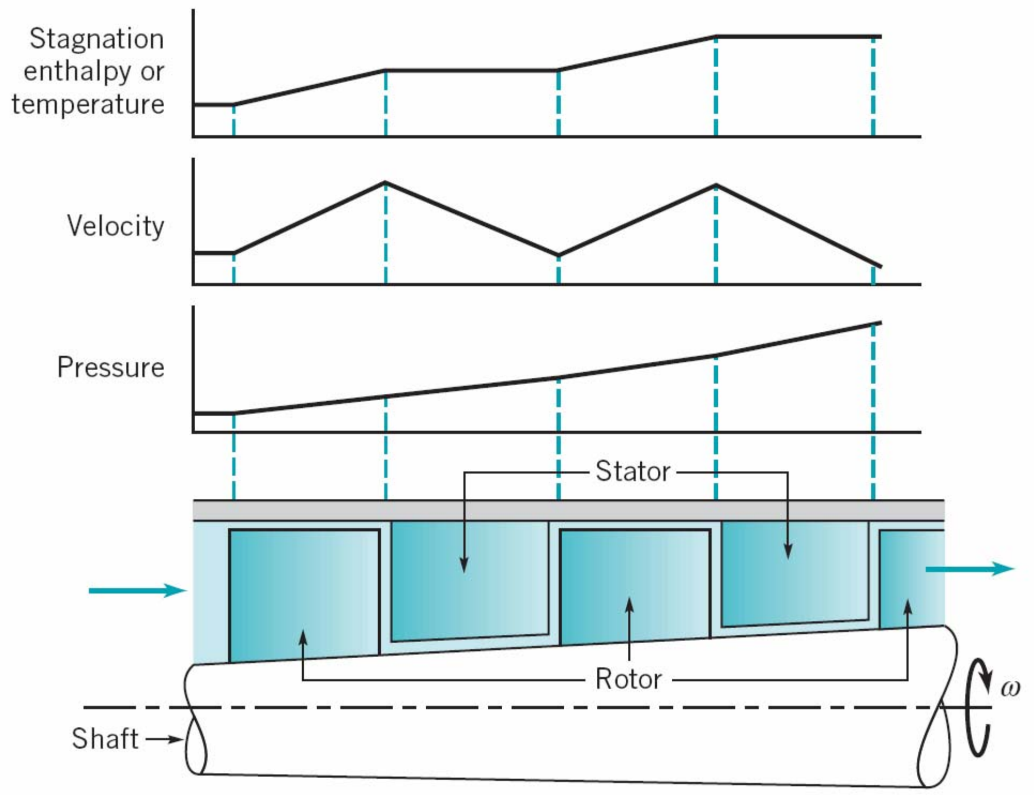
Radial-Flow Compressor 3/3

- Adiabatic compression of a gas causes an increase in temperature and requires more work than isothermal compression of a gas.
- An interstage cooler can be used to reduce the compressed gas temperature and thus the work required.



Axial-Flow Compressor 1/4

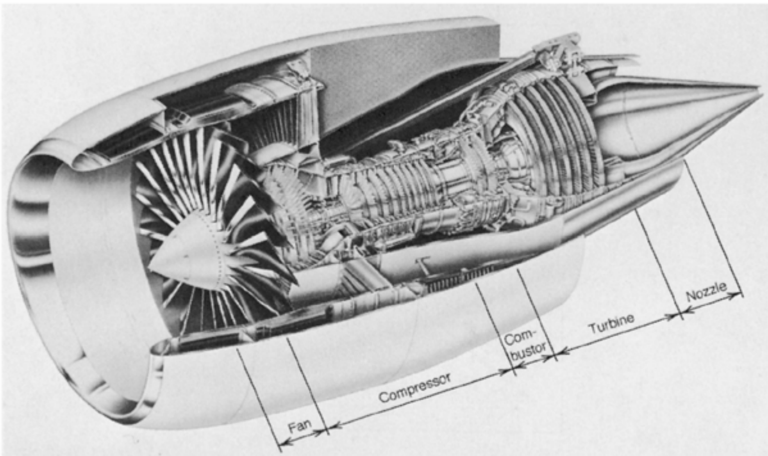
❖ The axial-flow compressor has a lower pressure rise per stage, a higher flowrate, and is more radially compact than a centrifugal compressor.



Enthalpy, velocity, and pressure distribution in an axial-flow compressor

Axial-Flow Compressor 2/4

- An axial-flow compressor usually consists of several stages, with each stage containing a rotor/stator row pair.
- As the gas is compressed and its density increases, a smaller annulus cross-sectional area is required and the flow channel size decreases from the inlet to the outlet of the compressor.



CE 90 propulsion system.
The typical jet aircraft engine uses an axial-flow compressor as one of its main components

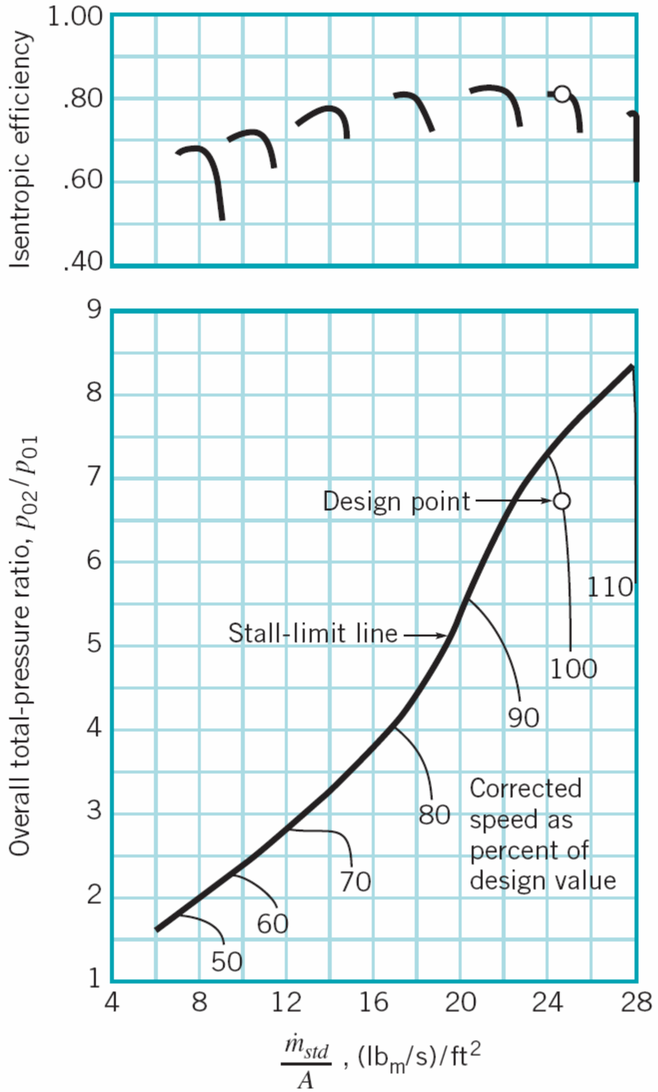
Axial-Flow Compressor 3/4

- An axial-flow compressor can include a set of inlet guide vanes upstream of the first rotor row. These guide vanes optimize the size of the relative velocity into the first rotor row by directing the flow away from the axial direction.
- Rotor blades push the gas in the direction of blade motion and to the rear, adding energy and moving the gas through the compressor.
- Stator blade rows act as diffusers, turning the fluid back toward the axial direction and increasing the static pressure.

Axial-Flow Compressor 4/4

- The stator blades cannot add energy to the fluid because they are stationary.
- The blades in an axial-flow compressor are airfoils carefully designed to produce appropriate lift and drag forces on the flowing gas.
- As occurs with airplane wings, compressor blades can stall.
- When the angle of incidence becomes too large, blade stall can occur and the result is compressor surge or stall – possible damage to the machine.

Performance Characteristics of Axial-Flow Compressor 1/2



- Either isentropic or polytropic efficiencies are used to characterize compressor performance.
- Each of these compressor efficiencies involves a ratio of ideal work to actual work required to accomplish the compression.

Performance Characteristics of Axial-Flow Compressor 2/2

- The isentropic efficiency involves a ratio of the ideal work required with an adiabatic and frictionless compression process to the actual work required to achieve the same total pressure rise.
- The polytropic efficiency involves a ratio of the ideal work required to achieve the actual end state of the compression with a polytropic and frictionless process between the actual beginning and end stagnation state across the compressor and the actual work involved between these same states.

Parameters for Compressor 1/3

- The common flow parameter used for compressor

$$\frac{R\dot{m}\sqrt{kRT_{01}}}{D^2 p_{01}}$$

T_{01} the stagnation temperature at the inlet
p_{01} the stagnation pressure at the inlet

- To account for variations in test conditions, the following strategy is employed.

$$\left(\frac{R\dot{m}\sqrt{kRT_{01}}}{D^2 p_{01}}\right)_{test} = \left(\frac{R\dot{m}\sqrt{kRT_{01}}}{D^2 p_{01}}\right)_{standard\ atmosphere}$$

std

Parameters for Compressor 2/3

❖ When we consider a given compressor operating on a given work fluid, the above equation reduces to

$$\dot{m}_{std} = \frac{\dot{m}_{test}\sqrt{kRT_{01test}/T_{0std}}}{p_{01test}/p_{0std}} \qquad (54)$$

The compressor-test mass flow "corrected" to the standard atmosphere inlet condition

The corrected compressor mass flowrate is used instead of flow coefficient.

T_0 and p_0 refer to the standard atmosphere

Parameters for Compressor 3/3

- While for pumps, blowers, and fan, rotor speed was accounted for in the flow coefficient, it is not in the corrected mass flowrate derived above.
- For compressors, rotor speed needs to be accounted for with an additional group. This dimensionless group us

$$\frac{ND}{\sqrt{kRT_{01}}} \Rightarrow \underset{\uparrow}{N_{std}} = \frac{N}{\sqrt{T_{01}/T_{std}}}$$

Corrected speed

Compressible Flow Turbines

Compressible Flow Turbines

- ❖ Turbines that use a gas or vapor as the working fluid are in many respects similar to hydraulic turbines.
- ❖ Compressible flow turbines may be impulse or reaction turbines, and mixed-, radial-, or axial-flow turbines.

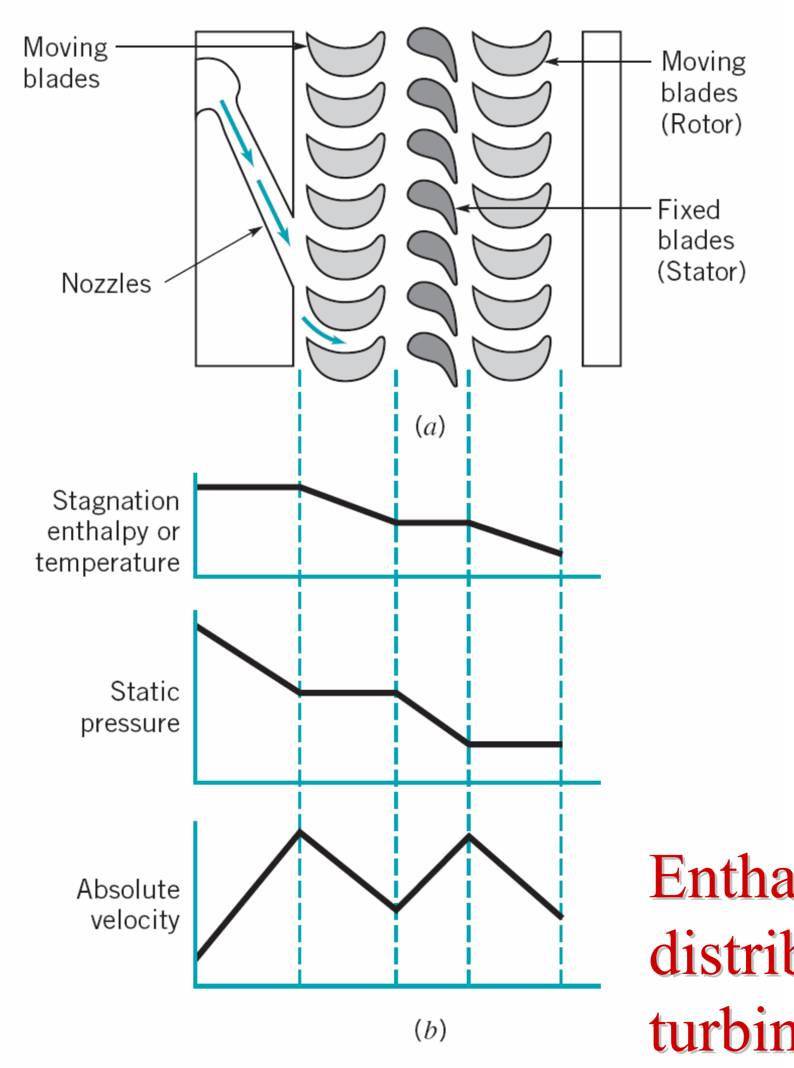
Radial-Flow Turbines

- Radial-flow turbine usually has a lower efficiency than an axial-flow turbine, but lower initial costs may be the compelling incentive in choosing a radial-flow turbine over an axial-flow one.
- The advantages of radial-flow turbines are: (1) It is robust and durable. (2) it is axially compact, and (3) it can be relatively inexpensive.

Axial-Flow Turbines 1/4

- Axial-flow turbines are widely used compressible flow turbines.
- Often they are multistage turbomachines, although single-stage compressible turbines are also produced.
- They may be either an impulse type or a reaction type.
- Steam engines used in electrical generating plants and marine propulsion and the turbines used in gas turbine engines are usually of the axial-flow type.

Axial-Flow Turbines 2/4



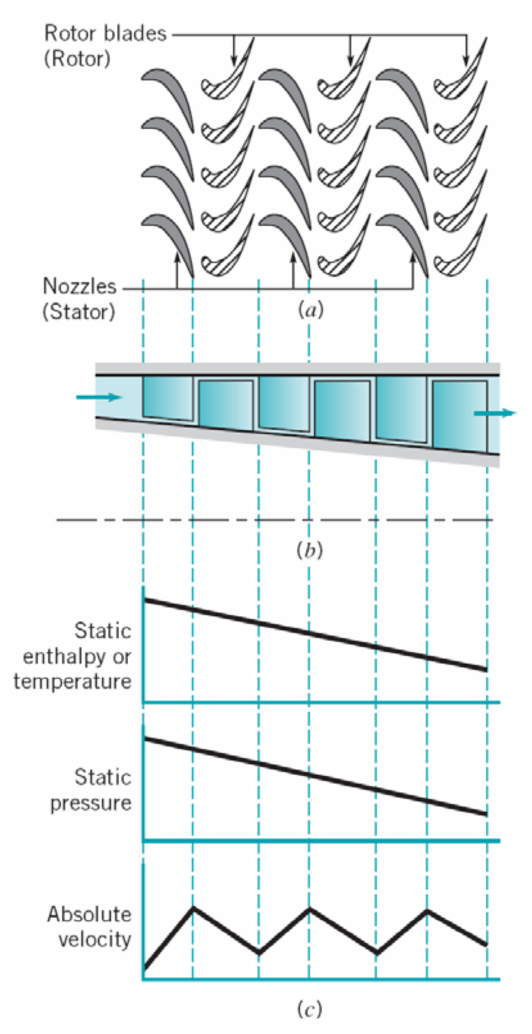
- The gas accelerates through the supply nozzles, has some of its energy removed by the first-stage rotor blades, accelerates again through the second-stage nozzle row, and has additional energy removed by the second-stage rotor blades.

Enthalpy, pressure, and velocity distribution in a two-stage impulse turbine

Axial-Flow Turbines 3/4

- The static pressure remains constant across the rotor rows.
- Across the second-stage nozzle row, the static pressure decreases, absolute velocity increases, and the stagnation enthalpy is constant.
- ….

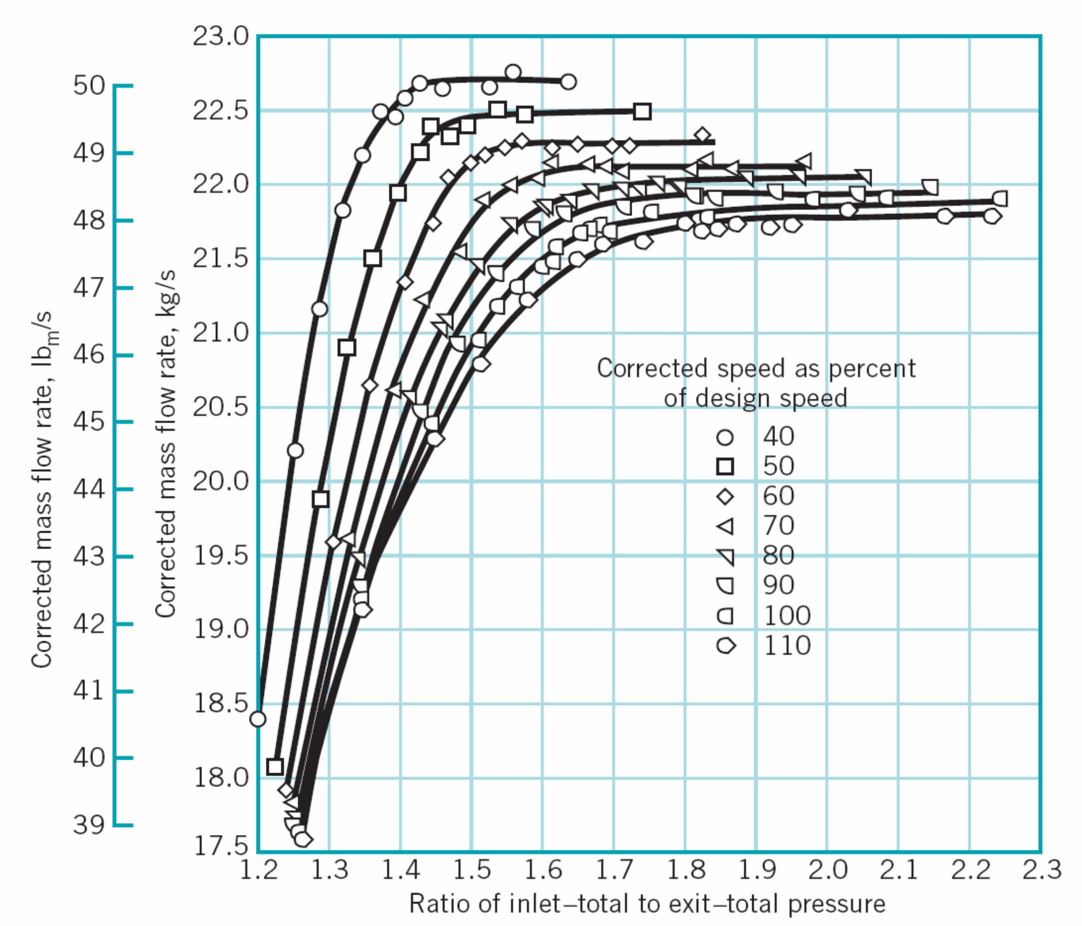
Axial-Flow Turbines 4/4



- ❖ Both the stationary and rotor blade act as flow-accelerating nozzles.
- ❖ The static pressure and enthalpy decrease in the direction of flow for both the fixed and the rotating blade rows.

Enthalpy, pressure, and velocity distribution in a three-stage reaction turbine

Performance Characteristics of Compressible Flow Turbines



- Isentropic and polytropic efficiencies are commonly used as are inlet-to-outlet total pressure ratios (p_{01}/p_{02}), corrected rotor speed (55), and corrected mass flowrate (54).