Core Professional Assignment:
Theories of Learning

A critical examination and comparison of two theories of learning and their implications for classroom teachers.

In order to become effective classroom teachers, it is imperative that we have an understanding of how learning occurs, and how it can be engendered in pupils. Much research has been done into how we assimilate new information, how we develop cognition and what constitutes effective learning as well as how we foster understanding in learners. I intend to examine two major learning theories, discuss their relative merits and compare their relevance and value for the classroom teacher.

Naturally, for such a broad area of study as theories of learning, it is important to develop an appreciation of the field as a whole, and thus the context of the theories to be examined in detail. The first major paradigm of learning theories is Behaviourism. Encompassing classical and operant conditioning, the premise here is that the learner is essentially a passive respondent whose behaviour depends on the environment. In classical conditioning, learners develop cognitive links between otherwise unconnected stimuli, and react accordingly (as in the case of Pavlov's dogs). Operant conditioning is slightly more complex, and relies upon either a punishment or reward based on a particular action, the theory being that behaviour that results in a punishment is less likely to occur, and behaviour that attracts rewards is reinforced. The scope of behaviourist theory in the teaching of mathematics is, I feel, somewhat limited. While it may have some value in examining how we encourage pupils to work or discourage them from unhelpful behaviour, it does not pretend to unlock the intricacies of how we come to understand new theory or learn to apply it.

For this, cognitivist theories may prove more valuable. Cognitivism acknowledges that people are capable of more than merely a trained response to stimuli, and attempts to examine the process of learning by studying how rational thought brings about understanding in the human mind. Cognitivism involves the study of the 'schema' – the mental framework within which we develop understanding.

Constructivism can be said in some ways to follow on from Cognitivism, and is also a reaction to the behaviourist model. Constructivism can be divided into Trivial and Radical, based on the premise by Ernst Von Glasersfeld:

1. Knowledge is not passively received but is actively built up by the cognising subject.
2. The function of cognition is adaptive and serves the organisation of the experiential world, not the discovery of ontological reality.

(Glasersfeld, 1983)

I would suggest that in the context of an application to teaching, the distinction between the two types is unimportant, since it is primarily the first statement which affects the way we think about learning and teaching. Whether or not the knowledge we impart in, for examples, mathematics is an ontological reality in its own right outside our own minds is not particularly relevant to how we learn the subject.

In a more blatant contrast to Behaviourism, Humanism places a much greater emphasis on the individual's potential, their values and personal motivation. Instead of, as in the case of Constructivism, taking the construction of new knowledge to be the central point, Humanism focuses on the whole person and studies the motivation and self-actualisation of the learner, with the assimilation of knowledge and understanding put within this context. Maslow's Hierarchy of Needs falls within this category, giving us a useful model for breaking down issues of a lack of motivation in learners by means of his pyramid model of Physiological, Safety, Belongingness, Esteem and Self-Actualisation categories.

There are many more learning theories and paradigms, and it would be impossible to do justice to them here, but I intend to examine in more detail the Constructivist theory set out by Glasersfeld and developed or used in a number of other prominent works, and

Kolb's theory of Experiential Learning, which comes under the umbrella of Humanist learning theories.

As described, Constructivism theory stems from the concept that knowledge is actively constructed within the mind of the learner. This standpoint lends a very interesting perspective to how we think about learning. Firstly, it is clear that, since building up knowledge is an active process, the primary role of a teacher should be to provide experiences which enable the learner to adapt and expand their own construct of reality, rather than merely presenting new information.

It is important to recognise how Constructivism was devised and where it has its roots. It seems to have grown from a Piagetian view of learning. Piaget came up with four stages of growth which tally well with the idea of Constructivism. He labels them as follows:

- Sensorimotor stage
- Preoperational stage
- Concrete operational stage
- Formal operational stage

The first is the transition from sensory information being merely a disconnected flow and being rationalised by the construction of ideas such as permanence (things still exist when out of sight) and perspective (things retain their characteristics regardless of where we view them from). This is something we do normally without even considering it, but it is, in a fundamental way, the basis of our constructs.

The second stage develops this further by bringing in representation of what is perceived. Learning to talk is one example – we use a series of sounds to represent objects we see or actions upon those objects. Using this representation, children can respond to speech by quite complex procedures. For example, "Pick up the blue ball and bring it to me" involves the identification of a round object, distinguishing it from similar objects by a specific characteristic – in this case colour – and moving it from one place to another. Along with representation comes a better understanding of shape and size as objects are manipulated.

The period of concrete operations refers to the point at which the child will begin to carry out logical operations on objects. Instead of randomly trying to fit objects through a hole, they will examine their relative properties, choosing likely candidates based on shape and size. This suggests that some simple model of the world is starting to develop, allowing, to some extent, sub-conscious thought-experiments to determine the outcome of events before they are carried out.

The period of formal operations is possibly the most relevant to Constructivism, since it concerns the way a child will use their brain to 'imagine' procedures and objects in order to solve a problem without reference to the objects themselves. The child can begin to consider "what if..?" questions, and use the symbolism they have learnt to come to reasoned outcomes. Even adults will find talking out loud is an aid to understanding, since it is one of the most powerful tools of symbolism we have. It is vital to note that this fourth stage cannot come about without the basis of the previous three. A child cannot begin to consider hypothetical situations with objects and procedures with which they are not familiar, and similarly cannot work logically with them until they have created a form of representation.

According to Piaget, a child starts to become confident with formal operations around the age of 7, which is believed to be the age at which their brains are developing faster, and they are learning faster than at any other time in their lives. I would suggest this has to do with the way they are reacting to new experiences and creating their own constructs of the world. Before this age, they are not properly familiar with formal operations, and beyond this period of growth their ideas and constructs will be more difficult to change. While they are constantly being fed new experiences which they can analyse and draw conclusions from, their construct is constantly being changed and adapted. (Inhelder & Piaget, 1958)

Whenever we have new experiences, we tend to do one of three things; either we distort what we seem to have experienced to fit in with our existing ideas, or we ignore the experience altogether, or we can choose to alter our construct to accommodate the new experience. If I see an arrow shot at close range, I might assume it travels in a straight line. My new experience might be to attempt a long range shot, and notice that the arrow lands lower than expected. I could distort the experience by assuming I must have aimed too low; I could ignore the experience, possibly by suggesting the arrow wasn't straight or there was something wrong with the bow; or I could examine the experience truthfully and maybe expand my range of experiences by trying different ranges and different angles of elevation. By assimilating these experiences, I could accommodate them in my construct by altering my image of the trajectory of the arrow from a straight line to a parabola, and do considerably better.

Of course, if we need to use the results of some experience which we haven't assimilated with our own construct, we often create a different construct. This need not be conflicting with the rest of your mental image, but it isn't necessarily compatible either. For example, when we learn how to solve n simultaneous equations in m variables, we might use a matrix and produce a result from that. However, if there were two equations in x and y, we would go back to a method we used at GCSE or A-level. The two methods are both valid, but may not be linked in the construct of the individual.

It is important to note the links between the idea of Constructivism and various other ideas introduced by mathematical educationalists. For instance, the idea of a Concept Image and Concept Definition (Vinner, 1983) distinguishes between what is known as the formal definition of a new concept, and the image or construct associated with it in the subject's mind. This idea may seem a little simplistic – we can, for instance, create temporary concept images, or build up more than one concept image for the same object presented under different conditions, but it links well with Constructivism since it concerns the building up of an image to represent what you know about a particular topic.

Another useful way of looking at our construct is in terms of three types of mental structure: frame, script and network. The frame is similar to the concept image; it is the set of expectations about an object or procedure – everything we associate with the concept in our heads. The script is equivalent to the concept definition; not necessarily the same as the formal definition, but the boundaries to which a concept is confined as perceived by the individual. The network is then the set of links between different concepts, and how ideas relate to each other. Just as our frame is not static, but changes with new information and examples, so the network alters as we assimilate new ideas. When we first learn algebra, and solve simple simultaneous equations, there is no link whatsoever to the x and y we see on a graph, yet later on we can see how a plot of each equation would result in lines in the x-y plane, and how that links to a set of equations having no solutions, a finite set of solutions, or an infinite number of solutions. The concepts of a function and simultaneous equations have been linked by the network.

One important concept for understanding how Constructivism links to the learning of mathematics is the process-object theory, referred to as APOS theory (Cottrill & Dubinsky, 1992). This appears to take the idea of concept definition and concept image even further. Essentially, it suggests that an Action we perform becomes a Process we can repeat, and later an Object we can examine and involve in further computations, until finally a Schema, which is similar to the concept image discussed earlier – a comprehensive set of ideas about the concept which, taken together, make up all of what we understand by it, including examples and images that help us to visualise what it means. To understand this properly, I will use the example of x^2:

Action:We see finding the square of a number as an action, or something we do to a number. For instance, we know that 3^2 is 9, and whenever we are given a number to square, we simply multiply it by itself.

Process: At this point we can write down 3^2 without feeling the need to 'work it out'. It is seen as something we recognise as a process we can carry out. We can see how it would be worked out, and some properties of the function, such as the square always being positive.

Object: As an object, we can now look at x^2 in its own right instead of thinking of what we get when we multiply the number that is x by itself. x^2 is something that can be manipulated – it is a number just like x was, though we now have a clearer understanding of how the concept of x^2 is rooted, as it were, in the concept of x. We will have a clearer idea of what it means to square a number, such as the knowledge that $x^2 < x$ only for $x < 1$, and recognise that as x increases, x^2 increases faster. We could also have some concept of quadratic equations.

Schema: Finally we have a comprehensive understanding of x^2 in terms of what it does and how we can manipulate it. As part of the image attached to it, we may have a parabolic graph. We might include ideas of how quadratics behave, and how to perform transformations on the graph. We may even see how the solutions of quadratics link to a geometric square.

It is important to note that transforming the concept of x^2 into an object or schema is not necessary in order to use it. Quadratic equations can be solved by memorizing a formula or a method of factorization. This is known as instrumental understanding (Skemp, 1976), and although it is a valid way of learning, it does not progress through the stages described above, and hence does not give a full and comprehensive, or relational, understanding.

Before we go further, we must be clear about exactly what we mean by understanding in this context. In a recent visit to a primary school, I observed an exchange between the pupils of a year 5 class and their teacher which goes some way to demonstrating the broadness of what we mean by understanding. The question asked by the teacher was part of a class discussion on the myth of Pandora's box: "Why do you think we grow old?" I still don't know what kind of response the teacher was looking for, but four different answers were suggested by the class, and each of them betrayed a very different interpretation of the question:

1) "So you can have birthdays." The sense in which this pupil took the question was that of "What's the benefit of growing old?"
2) "Coz you can't grow young." This pupil put the emphasis on what, quite rightly, appeared to him a clear logical consequence of growing – if we grow, then clearly we must grow old.
3) "The insides of our bodies change, so we grow older." The 'why' of the question is here interpreted more as 'how', and begins to explain the mechanism by which age manifests itself.
4) "If you didn't grow older, no-one would get married and have children, and where would those people come from?" This final answer hides a level of cognitive reasoning which we are still developing in learners at A-level. In another form, it might be called a 'proof by contradiction'. Taking the question to be a challenge of the validity of the fact, it tentatively accepts the premise of the question – that we don't grow old – and follows it to its logical conclusion; namely that we could never reach maturity and thence procreate, and since the people who exist had to come from somewhere, this presents us with a contradiction.

Now, of course such an analysis would not consciously occur to these young pupils, but the wide and unrestricted application of their knowledge and reasoning skills is something that seems to have been largely restrained, or at least narrowed considerably, later on in their education. The understanding we aim for in maths is something that approaches the active, imaginative, conceptual or relational understanding described here:

> "When a mathematician says he understands mathematical theory he possesses much more knowledge than that which concerns the deductive aspects of theorems and proofs. He knows about examples and heuristics and how they are related. He has a sense of what to use and when to use it, and what is worth remembering. He has an intuitive feeling for the subject, how it hangs together, and how it relates to other theories. He knows how not to be swamped by details, but also to reference them when he needs them."
>
> (Michener, 1978)

It is this kind of understanding that creates the best kind of mental construct. If we can learn to understand a concept to the extent Michener describes, we can then move on, using our knowledge as a base for further thinking and study. We will recognise how a new idea links with previous ones, and be able to explore potential links with concepts we already understand.

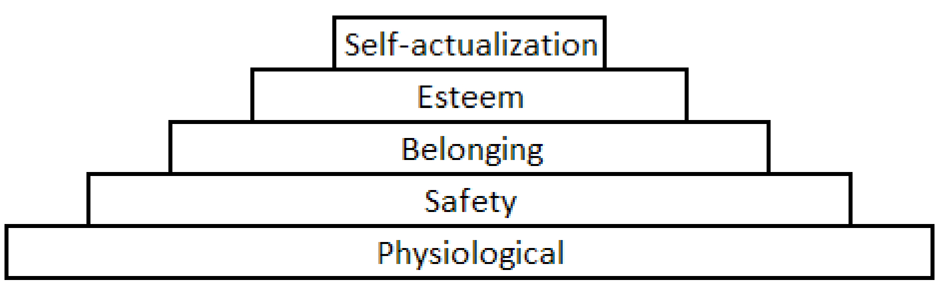
If a topic is learnt relationally, branch-off topics are much easier to learn, since they can be added to an extended schema. With a primarily instrumental understanding, each new topic is learned in isolation, with its own specific method. For example, an instrumental understanding of how to calculate the height of a tree from distance and angle of elevation will work fine for that example, and be quicker to learn, but a relational understanding of trigonometry of a right triangle will enable the learner to expand their knowledge of this problem to, say, calculating the length of rope needed to brace the tree to a given point on the ground. With a merely instrumental understanding, every new version of the problem will require a different method which has to be assimilated individually. Method learning is open to mistakes, and without a relational understanding to give the whys and hows, they will not be noticed. If you understand how a method has been generated, you can correct inaccuracies in your own memory, and you can even adapt a method to new variants of the same problem, or extend beyond the original problem by building on the internalised knowledge and comprehension you have achieved.

We must be aware that however desirable a relational understanding might be to the pursuit of mathematics, and, indeed, any rigorous subject, there are disadvantages to learning in this way compared to an instrumental stance. Relational understanding is much harder to come by. Learning a formula or step-by-step method without understanding its roots will be simpler, and it will get you the right answers much faster. What can be said for a relational understanding is that, once achieved, it makes concepts and techniques easier to remember. It is possible to learn all the formulas you need to calculate the area of common shapes for an exam, and do just as well as a relational learner, but a year or two on the instrumental learner will have forgotten some, and time will have bred inaccuracies in others. The relational learner may or may not take the time to memorize the formulae, but they will understand how each one is a direct result of what we mean by the concept of area, and therefore will be able to construct them when needed, from first principles. Although it has taken longer to learn, the knowledge will be much more enduring.

Another point in favour of relational understanding is its adaptability to new tasks. Taking the example of basic shape areas, an instrumental learner may need to find a book to tell them a new formula for finding the area of a hexagon, while the relational learner can use a similar method to that used to deduce the area of a pentagon, and come up with the formula independently. Also, relational knowledge can be thought of as a goal in itself. The feeling all mathematicians are familiar with when a new topic finally ‘clicks’, and the enlightening which enables you to see how all the theorems and examples go together is very satisfying, and, as I explained, is crucial to enable branching out into new fields.

This last point is one of the more important in the argument for Constructivism as a way of thinking. Constructivism enables us to see how the way we assimilate new information affects the way we learn new concepts. Particularly in maths, by creating a comprehensive concept image and cultivating a relational understanding, we can ensure a full knowledge of the new concept, a good idea of the network that can be formed between it and others, and generate a firm basis on which to build for further, more advanced, mathematical thinking. The process of learning mathematics is all about assimilating information, reifying it into a schema, and then building up new knowledge and understanding from these newly constructed building blocks. An appreciation of Constructivism and the active role of the learner in this task is a very useful tool in the learning of mathematics.

"Why do we need to know this?" is a common refrain in most mathematics classrooms. The National Curriculum content is by now sufficiently divorced from teachers' input or analysis that too often we may not ourselves know the motivation for presenting the particular topics we do. The all-too-narrow standard response, "because it'll be on the exam", demonstrates a critical lack of understanding in teachers of the value and application of the subject. Kolb's theory of Experiential Learning provides us with something like a wide-angle view of education. Instead of homing in on either the behaviour or even the cognitive processes of the learner, it embraces the concepts of intentionality and personal goal attainment. The theory is rooted in the Humanist paradigm, and as such has links with Maslow's Hierarchy of Needs, illustrated here:

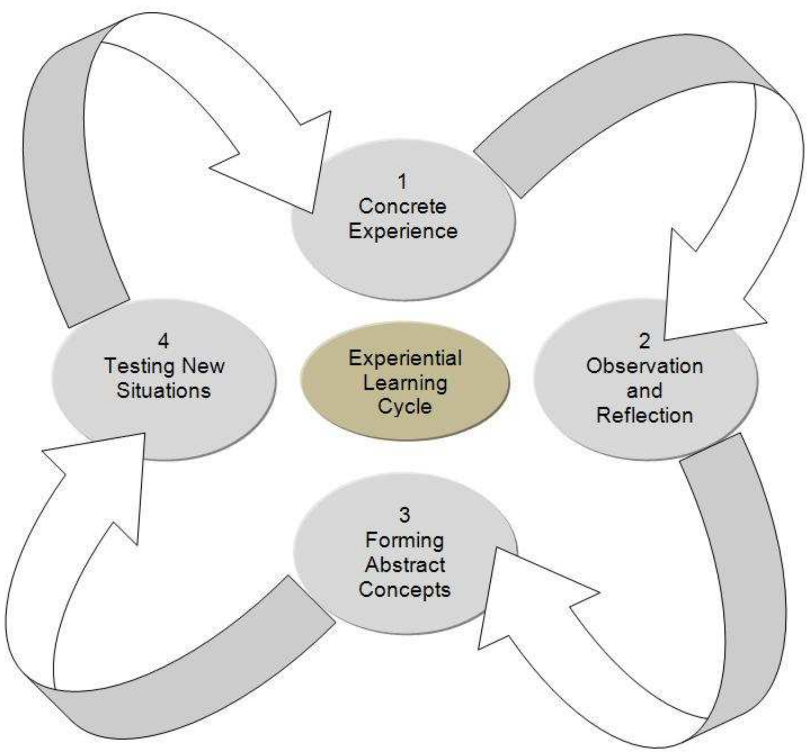


Maslow's Hierarchy of Needs

(Maslow, 1943)

The lowest level contains fundamental needs such as food and water, moving on to safety needs such as health and security. The next level deals with love and belonging, and esteem deals with the respect of others and of self. Finally self-actualization encompasses creativity, morality, problem-solving and so forth. The concept of the pyramid illustrates that there are many components that build towards self-actualization, and the higher needs depend upon the lower needs being met in order to be fully realised. Clearly the implications of this model extend well beyond formal education, but it serves to illustrate the point that school children, like everyone else, require specific motivation for their learning, and in the case of basic needs going unfilled this will have a knock-on effect upon the appropriateness of higher-level motivation.

Experiential Learning takes into account the reasons pupils may have for learning, and encourages us to develop further motivation, making the classroom not a place of dry study, but of vibrant and stimulating activity and investigation. This learning theory is perhaps best illustrated by a diagram:



(Kolb, 1984)

The concrete experience involves some form of practical activity in which the learner is actively involved, and which therefore has the potential to affect their understanding. Reflective observation is the next important element, and this involves the learner looking back over the experience and considering its implications and what they may learn from it. Abstract conceptualization is the cognitive process whereby the learner begins to build a mental model of the situation which can then be referred back to in similar situations in the future. Finally, abstract experimentation covers the testing of any theories or models developed by the learner, and will by its very nature result in more concrete experiences. Through the use of this cycle, learners may develop and refine their own understanding, and the role of a teacher within this framework is not that of an imparter of knowledge, first and foremost, but of a facilitator to the active learning process of their students.

To illustrate this, I will use the example of learning to calculate the area of a circle. Now, there are a number of methods in use by mathematics teachers for illustrating the relationship between the radius and the area of a circle, $A = \pi r^2$. However, most begin with the teacher providing the formula and then immediately moving on to studying examples of using it to convert between areas and radii. If Kolb's model is to be used, the teacher is responsible not for providing answers, but for directing inquiry and investigation into the given problem.

For instance, students may start by building a concrete experience – measuring the radius of a circle, and then estimating the area based on a counting-the-squares method which is commonly used for irregular shapes. This involves overlaying a grid of centimetre squares on the shape to be measured, and counting the number enclosed within the boundary. This initial experience or observation will provide pupils with a pair of numbers which they will then go on to analyze, considering what they may already know about area. Next, a model will be tentatively developed to explain the pattern observed. Pupils will be aware that one single example is not sufficient to have much confidence in an initial model, but hopefully will recognize the value of using such models during the process of development. Say the student decides that the area of a circle is always 12 times the radius, based on an example with radius 4 units. They will then go on to test and refine their model by trying the same thing on a different sized circle. With some guidance and prompting by a teacher as and when necessary, students may start to investigate a relationship of the area to the square of the radius, in keeping with the concept of area they will be familiar with from squares and rectangles. By following the cycle of experiential learning, students have the opportunity to develop for themselves a sense of π, and by virtue of this relationship being discovered independently, it is much more likely to stick in the memory of the learner, and be more easily applied when necessary. This parallels with the relational understanding discussed earlier.

Kolb also identified four distinct learning styles which may be useful in choosing the most appropriate way of introducing a new topic to students:
Focusing on the active experimentation of the concrete experiences phase and the abstract conceptual phase, the 'converger' is often a very concentrated learner, good at reasoning through problems, but also strong in relating theories to real problems and new experiences. The 'diverger' has strengths both in concrete experiences and reflection, having a strong imagination and the ability to come up with and develop ideas, including lateral thinking. A diverger will tend to have broader interests than the converger, with a greater emphasis on the big picture rather than focusing on the details. The 'assimilator' is strong with creating and applying logical theories. They will typically focus on the conceptualization and reflection stages of the cycle. Finally, the 'accommodator' learns best when provided with hands-on experience, often has strengths in the experimentation stage, and tends to be a more intuitive problem solver, keen to try out new things and good at reacting quickly to situations. (Smith, 2001)

In contrast with Constructivism, this examination of the learning preferences of individuals with respect to the cycle of learning provides us with a tool for differentiation between students. While Constructivism develops a model of cognitive development, Experiential Learning pans out from the mechanics of understanding to discern the different methods by which we access that understanding. As such, it has considerable significance in the way we apply the theory of Constructivism.

One argument against either of these theories is that in many cases pupils are quite content to halt learning at a somewhat superficial, instrumental understanding – the level needed to gain the required grades on exams, and no more.

> "The predominant didactic teaching style which is oriented towards developing and practising techniques, and then applying them to structured standard problems, may be successful in enabling students to obtain the examination grades they require. However, it seems to be much less effective in developing both the understanding and fluency and the thought and persistence that are needed for successful problem-solving." (Haggarty, 2002)

I believe that to assume students do not desire a relational understanding is very unhelpful within the context of teaching. Unfortunately, much of the experience they may have received throughout their school education will have given the impression that mastery in maths depends upon the memorisation of formulae and being able to recall and use procedures and rules. Glib responses like "two negatives make a positive" and "the rule is: flip it and multiply" are tempting for teachers to use, especially in a room of 30 pupils all struggling to grasp a new topic, but in the long run they reinforce the idea that maths is about being good at blindly recalling and using facts and results that are understood poorly if at all. As teachers we must be constantly reminded, and constantly remind learners, of the reason behind pursuing understanding, both in a general sense and specifically for each new topic. Newton describes what it is that makes understanding a worthwhile goal: "its flexibility in application, its durability, the way it facilitates further learning, and the way it enables critical abilities." (Newton, 2000)

If we are to teach for a thorough, broad and applicable understanding of mathematics, we will produce learners who are not only equipped to excel in formal examinations, but are able and eager to apply their knowledge and insight to new and unfamiliar situations or problems. These are the kind of mathematicians, and indeed the kind of learners in general, who are needed in today's workforce. The reason mathematics qualifications command such respect in the professional world as well as in higher education is that our society is increasingly in need of people with keen, inquiring minds, accustomed to meeting new and challenging problems and dealing with them in a rigorous, logical and insightful manner. It is precisely these skills which are required increasingly in every area of development from medical research to automobile design, and from computer science to engineering. In fact, as computing power increases exponentially, the tasks of recalling mathematical theorems and proofs, and even carrying out algorithms to solve standard problems can be delegated to a computer for a fraction of the cost and a substantial increase in efficiency and reliability.

Therefore, while a knowledge of mathematical theory and practice is undoubtedly of some value, it is essential that this goes hand in hand with the adaptability, mathematical intuition and problem solving ability that makes us more than 'thinking machines'. The same principle holds true, to one degree or another, in all school subjects; students are not taught about past events for the sake of producing historians capable of regurgitating what anyone could learn on the internet, but in order to develop a spirit of inquiry, and a clear, impartial approach to the study of sources in order to draw valid conclusions. Some subjects are more informal in the development they engender, but even in the study of woodwork, it is not the blind ability to replicate artefacts that is the primary aim, but rather an understanding of the material's properties – 'getting a feel for it' – which will put learners in a position to create or adapt designs for manufacture. Even though the non-creative element of this process is beginning to be exploited by computers for simpler materials such as polymers and metal alloys, the design input is still very much in the domain of the human mind.

In conclusion, while it is apparent that no single learning theory can provide answers to all the questions posed by teachers and learners, there are a number of carefully developed, empirically sound theories which have the potential, when appropriately applied, to empower teachers to not only bring about effective learning in their students, but develop them into active, imaginative and explorative learners in their own right, taking charge of their own learning and consolidating the kind of broad and deep conceptual understanding that will stand them in good stead throughout their adult lives.

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