## SG2214 Fluid Mechanics
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Photo: Rochester Institute of Technology



## Teachers

Anders Dahlkild
Lectures

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Recitations

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## Fluid Mechanics: SG2214

***Course requirements (7.5 cr.)***
- **INL 1 (3 cr.)**
  - **3 sets of home work problems (for 10 p. on written exam)**
  - **1 laboration**
- **TEN1 (4.5 cr.)**
  - **1 written exam (50 p.)**

***The lab is scheduled ONCE a year (week 41)***

***Book:*** **Kundu & Cohen (5:th ed.)**
- **E-book via KTH Social/Course home page**
- **useful also in SG2218 Turbulence and SG2221 Wave motions and Hydrodynamic stability**



## Fluid Mechanics: SG2214

***"An in-depth introduction to fluid mechanics, with an emphasis of understanding fluid phenomena using the Navier-Stokes equations, the single set of partial differential equations governing all fluid flow".***

1. Introduction, tensors, kinematics
2. Conservation laws
3. Laminar viscous flow
4. Vorticity dynamics
5. 2D irrotational flow
6. Laminar boundary layers
7. Introduction to turbulent flow

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i$$

$$\frac{\partial u_i}{\partial x_i} = 0$$

## Why Fluid Mechanics ?

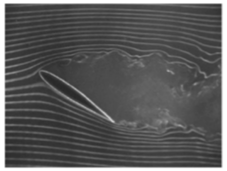
- **Atmosphere, Ocean**
- **Aerodynamics**
- **Energy conversion**
- **Transport of heat/other**
- **Numerous industrial processes**



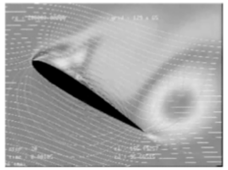
## Fluid Mechanics

### Fluid Engineers basic tools

**Experimental testing**



**Computational Fluid Dynamics**



**Theoretical estimates**

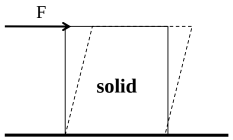
$$\text{Drag} \propto \rho U^2 A\, C_D(\alpha)$$

$$\text{Lift} \propto \rho U^2 A\, C_L(\alpha)$$

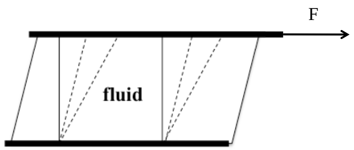


## Fluid Mechanics

### Definition of fluid




An elastic solid deforms a finite amount under the action of a shear force, and returns to its original shape as F is withdrawn.

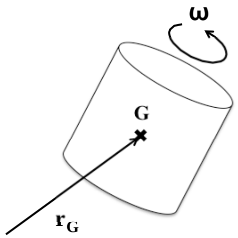



A fluid deforms continuously under the action of a shear force, ***however small***.



## Fluid Mechanics

### Newton's law of motion for solid body




$$m\, \mathbf{a}_G = \mathbf{F}$$

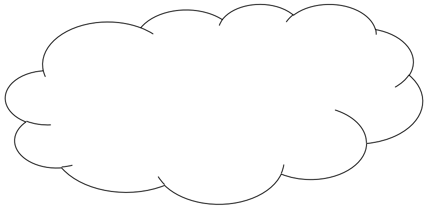
$$\frac{d}{dt}\mathbf{L}_G = \mathbf{M}_G$$

## Fluid Mechanics

*Continuum hypothesis:*

**Discrete molecular structure of matter is disregarded.**

**Replaced by *continuous* distribution of matter.**

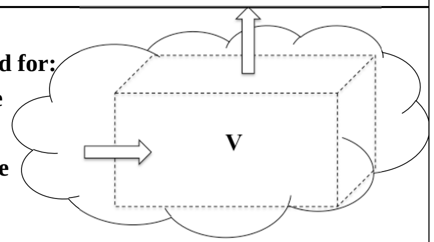




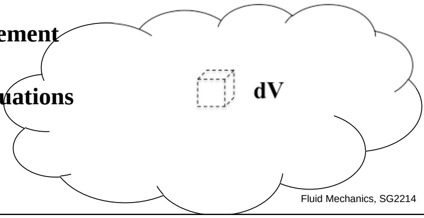
## Fluid Mechanics

**Newton's law of motion in fluid for:**

1. **Open fix control volume (integral form)**
   **=> Momentum principle**




2. **Infinitesimal fluid element (differential form)**
   **=> Navier-Stokes equations**

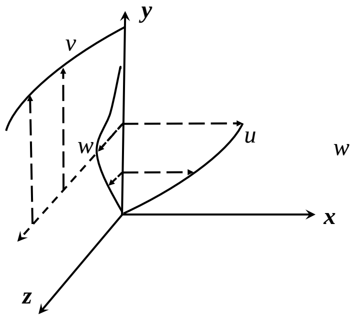


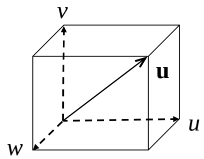



## Fluid Mechanics

Vector field $\mathbf{u}(x,y,z,t)$

Scalar field $p(x,y,z,t)$



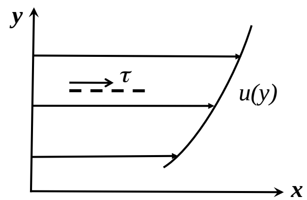






## Fluid Mechanics

**Newtons law of friction**

$$\tau = \mu \frac{du}{dy} \approx \mu \frac{U}{L}$$



# Fluid Mechanics

**Different regimes of flow:**

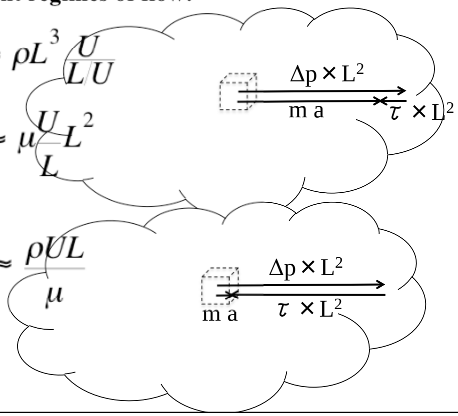
**Reynolds number**

$$m\,a \approx \rho L^3 \frac{U}{L/U}$$

$$\tau L^2 \approx \mu \frac{U}{L} L^2$$

$$\Downarrow$$

$$\frac{m\,a}{\tau L^2} \approx \frac{\rho U L}{\mu}$$

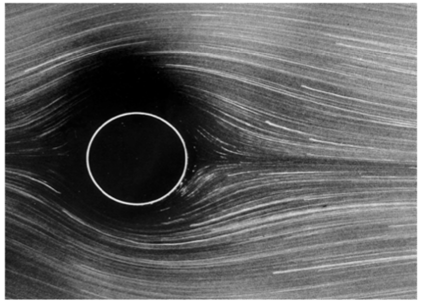



$$\frac{\rho UL}{\mu} >> 1$$

$$\frac{\rho UL}{\mu} << 1$$



# Viscous flow: circular cylinder



From An Album of Fluid Motion

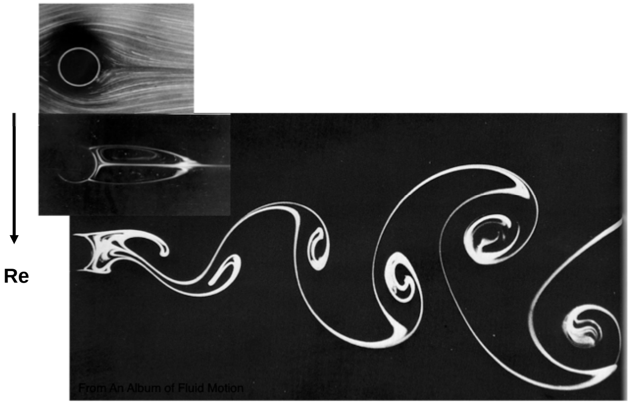
Stokes flow: $Re \to 0$
Slow flow or large viscosity

$$0 = -\frac{\partial p}{\partial x_i} + \mu \nabla^2 u_i$$

$$\frac{\partial u_i}{\partial x_i} = 0$$

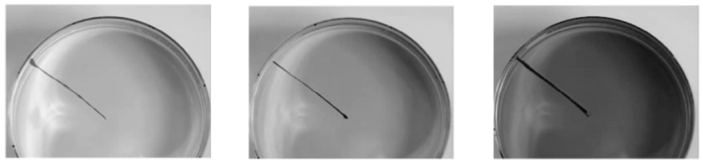


# Example: circular cylinder






# Viscous flow: rotating tank

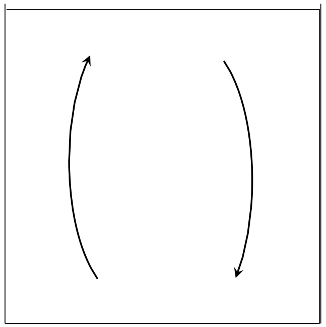


➔ Increasing viscosity ➔

← Increasing Re ←

## Thermal convection

Heated wall



Cooled wall



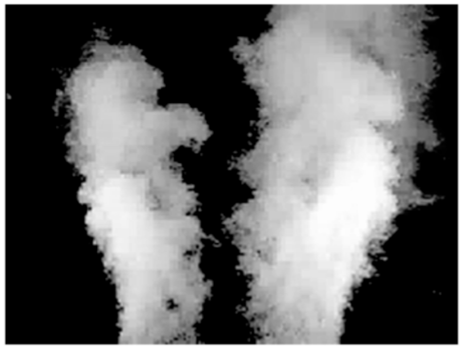
## Vorticies and vorticity: trailing-line vortices


Vorticity: $\omega_i = \varepsilon_{ijk}\frac{\partial u_j}{\partial x_k}$

$$\frac{\partial \omega_i}{\partial t} + u_j\frac{\partial \omega_i}{\partial x_j} = \omega_j\frac{\partial u_i}{\partial x_j} + \nu\nabla^2\omega_i$$

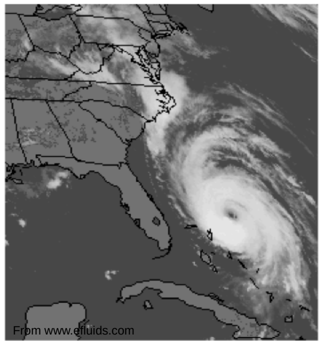


## Vorticies: wing tip vortex

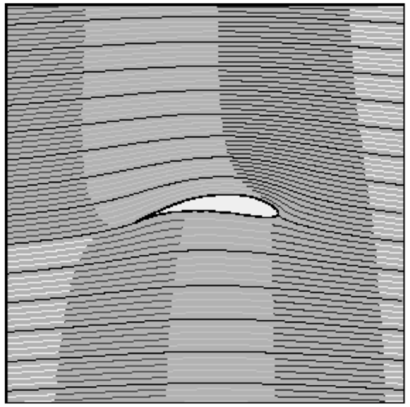




## Vorticies: hurricanes and tornados

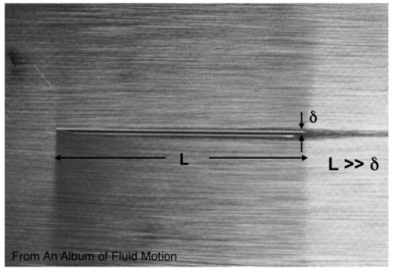



## Irrotational flow: 2D flow around airfoil



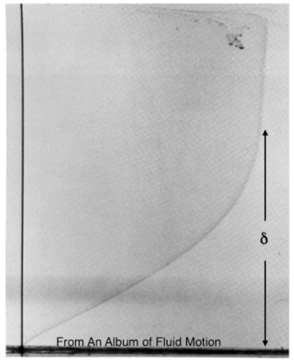


## Boundary layers



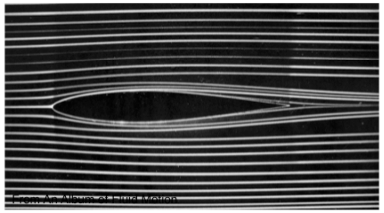

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

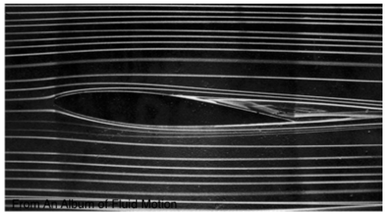





## Flow separation




Attached flow

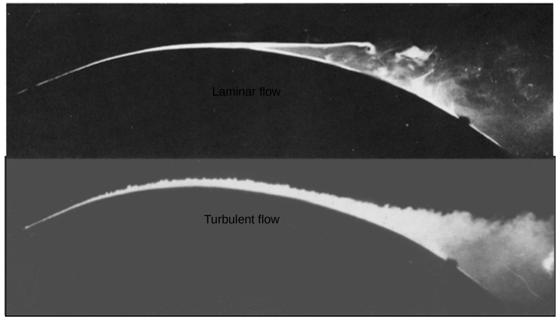



Separated flow



## Turbulent separation

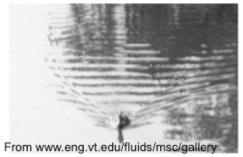
Laminar flow separates
faster than turbulent flow




From An Album of Fluid Motion

## Surface waves: potential flow with free surface

**SG2221: Wave motions and hydrodynamic stability, 7.5 cr.**



From www.eng.vt.edu/fluids/msc/gallery

Potential flow:

$$u_i = \frac{\partial \phi}{\partial x_i} \quad \Rightarrow \quad \nabla^2 \phi = 0$$

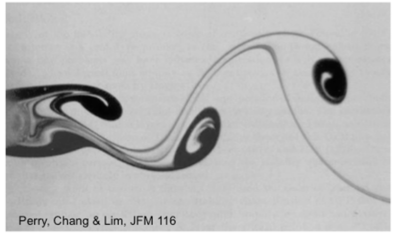
$$\frac{\partial \phi}{\partial t} + \frac{1}{2} u_i u_i + \frac{p}{\rho} = C$$

+ free surface condition

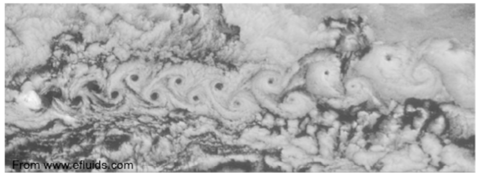
From An Album of Fluid Motion



## Flow instability: von Karman vortex street



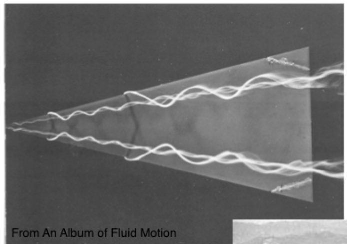
Perry, Chang & Lim, JFM 116



From www.efluids.com



## Flow instability: vortex breakdown



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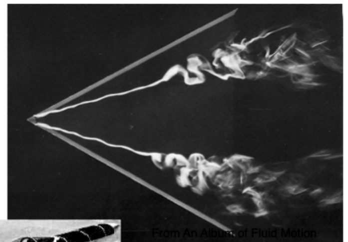
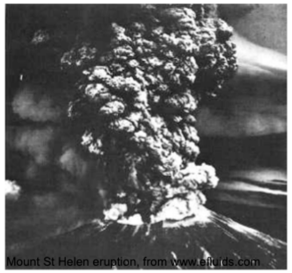


From An Album of Fluid Motion

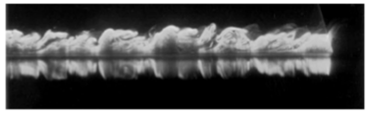
ww.efluids.com

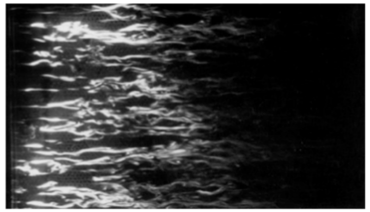


## Turbulence: jets and boundary layers



Mount St Helen eruption, from www.efluids.com

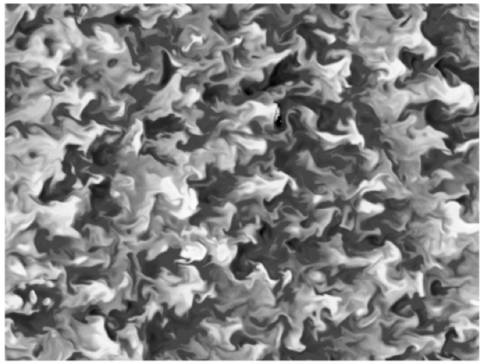




Average flow: $U_i = \frac{1}{N} \sum u_i^{(n)}$

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \nabla^2 U_i + \tau_{turb}$$

## SG2218 Turbulence, 7.5 cr.





## Compressible flow: shock-waves





## SG2215 Compressible Flow, 7.5 cr.

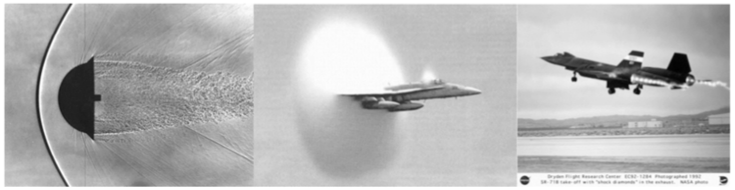
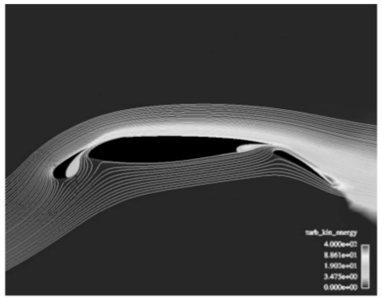


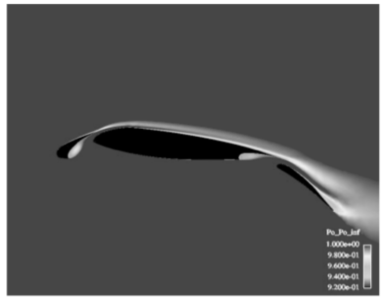
*Photo: NASA's Dryden Flight Research Center*



## Turbulent flow over high-lift profiles using RANS

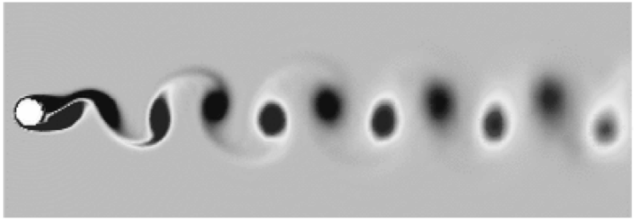


Turbulent kinetic energy and streamlines



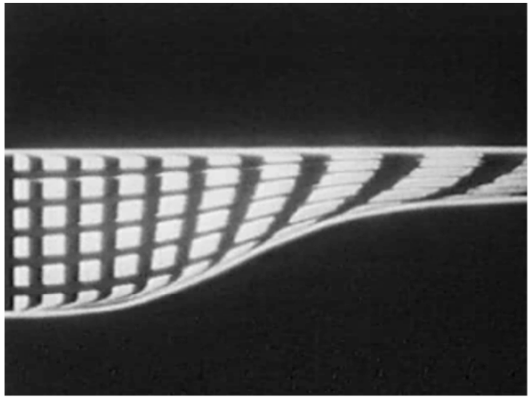
Total pressure loss p-p0

**SG2212 Computational Fluid dynamics, 7.5 cr.**
**SG2224 Applied Computational Fluid Dynamics, 5 cr.**



Fluid Mechanics, SG2214

## Incompressible fluid element



Fluid Mechanics, SG2214

## Navier-Stokes equations

| *Vector form* | *Cartesian tensor form* | |
|---|---|---|
| $\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u}$ | $\rho\left(\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 u_i}{\partial x_j \partial x_j}$ | ***i=1,2,3*** |
| $\nabla\cdot\mathbf{u} = 0$ | $\frac{\partial u_k}{\partial x_k} = 0$ | |

| | | | |
|---|---|---|---|
| ***Velocity*** | $\mathbf{u} = (u,v,w) = (u_1,u_2,u_3)$ | ***Dynamic viscosity*** | $\mu$ |
| ***Gradient operator*** | $\nabla = (\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}) = (\frac{\partial}{\partial x_1},\frac{\partial}{\partial x_2},\frac{\partial}{\partial x_3})$ | ***Density*** | $\rho$ |
| ***Laplace operator*** | $\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}$ | ***Pressure*** | $p$ |

Fluid Mechanics, SG2214