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Experimental Investigation in Fluid Mechanics - Its Role, Problems and Tasks

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Abstract. In this contribution, some problems and tasks of experimental fluid mechanics are presented. Paradoxes, basic laws and contemporary investigation approaches are discussed. Experimental results, together with theoretical knowledge and numerical simulations gradually form basis for solution of topical problems. The author of this contribution focuses his investigations into field of compressible fluid flow. Due to this, some results of high-speed aerodynamic research contributing to design and operation of machines, where flow velocities exceed speed of sound, are shown. Moreover, the author intends to show, that fluid mechanics is open field ready to describe complex interactions at fluid flows. Experimental fluid mechanics takes part in formulation and solution of tasks at flow field modelling, at explanation of phenomena taking place in nature and in technical works.

1 Introduction

Experiments in fluid mechanics are undoubtedly very important part of investigation. Their significance is dual - inspirational and proving. Experiments can give an impetus to theoretical studies, modelling of flow fields and flow effects, and preparation of numerical simulations.

An example of significance of experiment in fluid mechanics is the case of a colossus of science in ancient times, the founder of the science of fluid mechanics and its application in engineering, Greek scientist - Archimedes of Syracuse. His most important discovery of all concerns the force acting on a body immersed in a liquid. It is said that he discovered this principle while in the public baths in Syracuse when he immersed himself in a full tub. He related the force lifting him to the amount of water that overflowed from the tub. He stated that “Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object.” Thus Archimedes discovered this famous physical law of hydrostatics [1].

Another example of significance of experiment in fluid mechanics is the public demonstration of flow development performed by Osborne Reynolds in 1883. He proved by his experiment that a fluid movement can be realized in two different ways - as a laminar flow or a turbulent flow [2], [3]. Since that time, a lot of studies - theoretical solutions, experimental modelling and numerical simulations of laminar and turbulent flows were carried out to show its specific flow behaviour and effects.

Many other experiments can be mentioned to confirm exceptional and important inspirational role of experiment in fluid mechanics. It is possible to say that experimental fluid mechanics belongs to pillars of research attempting to reach the top of knowledge in fluid mechanics. We can show it in the schematic picture in figure 1. The other pillars are theory and numerical simulations. Their common advance gives possibility to achieve perfect, ideal model for solution of problems in fluid mechanics.

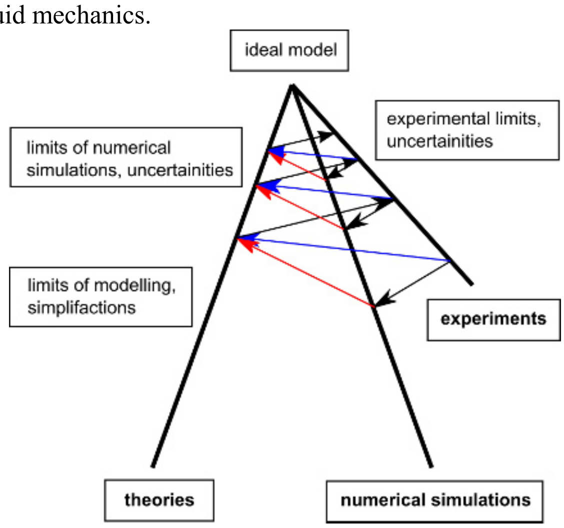

Fig. 1. A scheme of pillars of basic research in fluid mechanics.

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Of course, there are also experiments aiming to confirmation or verification of theoretical or numerical results. They are also important for development of general knowledge of fluid flow behaviour and they provide arguments for design and operation of technical and natural systems containing fluids. It is necessary to accept, that tenets contradicting reality appear in some conceptions or theories concerning fluid effects. We call them paradoxes. Presented contribution will show some of them.

2 Paradoxes in Fluid Mechanics

A paradox is a statement or situation which defies logic or reason. Its validity can be proved experimentally.

2.2 Hydrostatic Paradox

Hydrostatic forces acting on equal area bottoms of different vessels filled by the same liquid to the same level height under the same pressure of surroundings are always equal in spite of amount of liquid in vessels.

Figure 2 shows four different vessels filled by the same liquid having density ρ to level height h. Bottoms of vessels have area A. (g is acceleration of gravity.) Mass of liquids in those vessels is evidently different, but hydrostatic forces acting on the bottoms of vessels are equal. Generally, the volume of a loading figure is not the same as the real volume of liquid in the vessel [4].

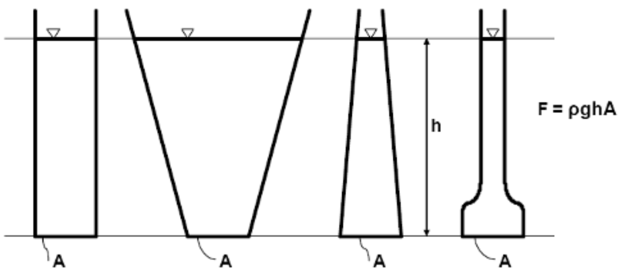

Fig. 2. Hydrostatic paradox.

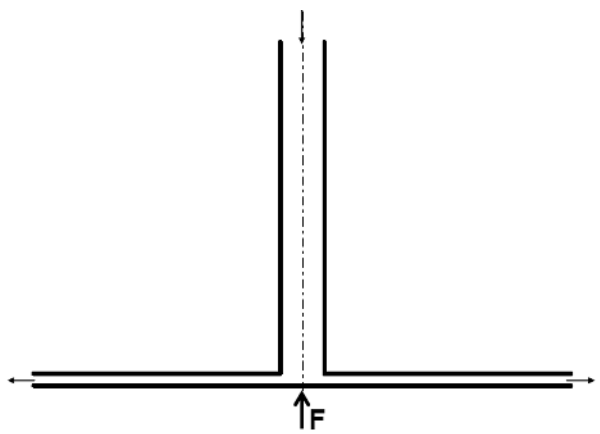

Fig. 3. Hydrodynamic paradox.

2.2 Hydrodynamic Paradox

Hydrodynamic paradox appears when resulting force acting on the flow channel is in contrary direction to prospective direction. This fact results in channels from the fact, that fluid flow pressure decrease is proportional to square of fluid flow velocity increase. In narrow part of the channel where liquid has higher velocity the pressure is lower.

Hydrodynamic paradox is shown in figure 3 and figure 4. The bottom plate in figure 3 is risen up in spite of flow acting in opposite direction. The curved side walls in figure 4 are forced towards each other since fluid flow is accelerated between them.

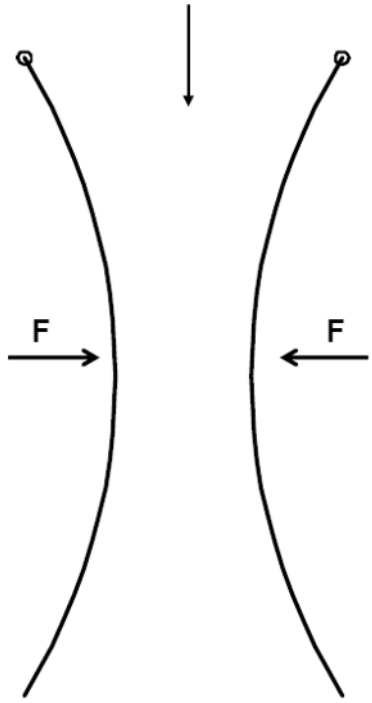

Fig. 4. Hydrodynamic paradox.

2.3 D´Alambert´s Paradox

Drag force on a body moving with constant velocity relative to incompressible inviscid fluid potential flow is zero. Zero drag is in direct contradiction to the observation of substantial drag on bodies moving relative to fluids. Nevertheless, theoretical model of incompressible inviscid fluid potential flow provides zero drag. D´Alambert´s paradox indicates flaws in the theory [5].

Figure 5 shows streamlines for incompressible and inviscid fluid potential flow around the circular cylinder in a uniform flow. Detail analysis proves zero drag in potential flow. Zero drag in potential flow can be also preliminary concluded from the symmetry of the flow field.

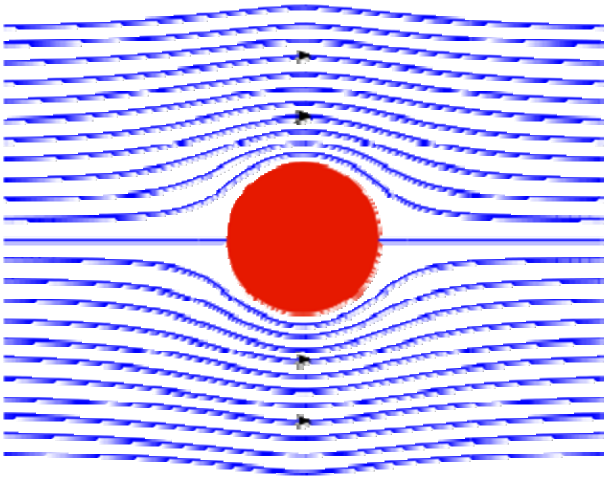

Fig. 5. Streamlines for incompressible inviscid fluid potential flow around the circular cylinder in a uniform onflow [5].

2.4 Loss of global stability of liquid flow through axisymmetric annular channel

Behaviour of flow can seem paradoxical when a liquid passes axisymmetric annular channel. Instead of flow along the channel axis, the liquid flow losses global stability and vortex movement of the liquid takes place and resulting angular momentum is produced.

On this principle, Sedlacek´s bladeless turbine operates. Figure 6 depicts a scheme of arrangement of a small Sedlacek´s turbine [6].

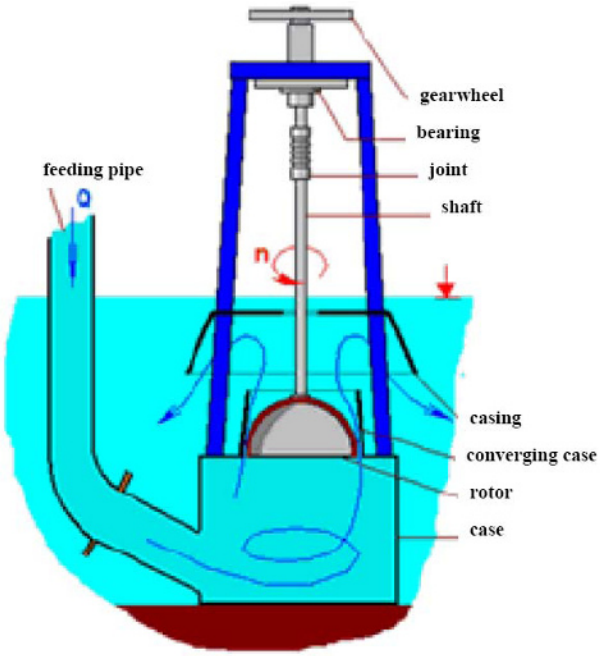

Fig. 5. A scheme of a small Sedlacek´s turbine [6].

3. Laws in Fluid Mechanics

Formulation of presented paradoxes in Sect.2 proves that it is necessary to perform thorough analysis of studied problems and obtained results in fluid mechanics. Laws in fluid mechanics have to be a basis for preparation of conclusions and statements. Basic laws and equations will be mentioned in following sections.

3.1 Archimedes´ Law

Archimedes´ law deals with buoyancy force acting on immersed body, and states: "Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object. " It can be expressed by equation

$$F = g \cdot \rho \cdot V_{fluid} \ , \tag{1}$$

where F is buoyancy force, ρ is density of fluid, g is acceleration of gravity, V_{fluid} is volume of displaced fluid.

3.2 Euler´s equation of hydrostatics

Euler´s equation of hydrostatics expresses principle of equilibrium fluids as a balance of forces due to intensity of mass forces and pressure gradient. This equation enables to solve basic task of hydrostatic – value of pressure in arbitrary position in stationary fluid. Another task can be solved – to determine density of a fluid. Euler equation of hydrostatics is expressed by

$$dp = \rho \left(K_x dx + K_y dy + K_z dz \right) \ , \tag{2}$$

where dp is total derivative of pressure, ρ is density of fluid, K_x, K_y, K_z are components of intensity of mass forces, dx, dy, dz are derivatives of coordinates of Cartesian coordinate system.

A consequential application of this principle would prevent formulation of hydrostatic paradox (Sect.2.1).

3.3 Law of Conservation of Mass

Conservation of mass is basic principle in mass balance and in analysis of flow systems. Law of conservation of mass states, that mass cannot disappear or be created spontaneously. Differential form of the principle of conservation of mass is expressed for unsteady flow of compressible fluid by continuity equation:

$$\frac{\partial \rho}{\partial t} + div(\rho \cdot v) = 0 \ , \tag{3}$$

where t is time, ρ is density of fluid, **v** is vector of velocity.

3.3 Law of Conservation of Energy

Conservation of energy is basic principle in energy balance and in analysis of physical systems. On this principle First law of thermodynamics is formulated. Law of conservation of energy states, that energy cannot be created or destroyed. Energy can only be transformed from one kind of energy to another. Bernoulli equation (4) balances mechanical energy of fluid flow in onedimensional tube (stream tube). For incompressible fluid it has form

$$g \cdot h_1 + \frac{p_1}{\rho} + \kappa_1 \cdot \frac{v_1^2}{2} = g \cdot h_2 + \frac{p_2}{\rho} + \kappa_2 \cdot \frac{v_2^2}{\rho} + e_{z\,1-2} \;,(4)$$

where subscript 1 represents values of quantities in position 1 in the tube and subscript 2 represents values of quantities in position 2 in tube downstream of the position 1, gh is specific potential energy, p/ρ is specific pressure energy, $\kappa v^2/2$ is specific kinetic energy modified by Coriolis coefficient κ, and e_z is irreversible part of specific mechanical energy which was transformed into another energy (mainly heat).

A consequential application of Bernoulli equation (4) would prevent formulation of hydrodynamic paradox (Sect.2.2).

On the principle of First Law of Thermodynamics under assumption of isentropic flow of ideal gas, Saint-Venant-Wantzel equation was derived. It has form

$$v = \sqrt{\frac{2\gamma}{\gamma - 1} r T_0 \left[1 - \left(\frac{p}{p_0} \right)^{\frac{\gamma - 1}{\gamma}} \right]} \;, \tag{5}$$

where γ is ratio of specific heat capacities, r is specific gas constant, T_0 is total temperature, p is static pressure, p_0 is total pressure. Both Bernoulli equation and Saint-Venant-Wantzel equation relate velocity v to static pressure p.

3.4 Law of Conservation of Momentum

Conservation of momentum is implied by Newton´s laws. In fluid mechanics, conservation of momentum is expressed to solution of fluid flow stream force effect on walls or channels by equation:

$$\mathbf{F} = \dot{\mathbf{H}}_1 - \dot{\mathbf{H}}_2 + \mathbf{F}_{p1} - \mathbf{F}_{p2} + \mathbf{G} \;, \tag{6}$$

where $\mathbf{F}$ is force acting on the control volume, subscript 1 represents values of quantities at volume entrance, and subscript 2 represents values of quantities at volume exit, $\mathbf{F}_p$ is vector of pressure force, $\mathbf{G}$ is vector of gravity force of the fluid in the control volume, $\mathbf{H}$ is vector of momentum flux defined as

$$\dot{\mathbf{H}} = \dot{m}\mathbf{v} \;, \tag{7}$$

where m is mass flux of the stream, and $\mathbf{v}$ is velocity vector.

Conservation of momentum is a basis for derivation of Navier-Stokes equations. These equations arise from applying Newton´s second law to fluid motion together with assumption that the fluid stress is the sum of viscous term (proportional to deformation rate), a pressure rate and term of mass forces. For incompressible fluid Navier-Stokes equations have following vector form:

$$\rho \frac{D\mathbf{v}}{Dt} = \rho \mathbf{K} - \nabla p + \eta \cdot \Delta \mathbf{v} \;, \tag{8}$$

where ρ is density of fluid, $D\mathbf{v}/Dt$ is the substantial derivative of velocity vector, $\mathbf{K}$ is intensity of mass forces, ∇p is gradient of pressure, η is dynamic viscosity, $\Delta\mathbf{v}$ is Laplace operator of velocity vector.

A consequential application of Navier-Stokes equations (8) and detail analysis of results would prevent formulation of d´Alambert´s paradox (Sect.2.3). But in the time of d´Alambert life, the Navier-Stokes equations were not known yet. Necessary to say, Navier-Stokes equations are still of great interest in a purely mathematical sense. Mathematicians have not yet proven that in three dimensions solutions always exist. The Clay Mathematics Institute has called solution of Navier-Stokes equations one of the seven most important problems in mathematics and offered 1,000,000 US$ prize for a solution or a counter-example [7].

3.5 Law of Conservation of Moment Momentum

Conservation of moment momentum (also called angular momentum or rotational) is an analogy of Newton´s second law for rotating bodies. In fluid mechanics, conservation of moment momentum is expressed by equation

$$\sum \mathbf{M} = \frac{d\mathbf{L}}{dt} \;, \tag{9}$$

or

$$\sum(\mathbf{r} \times \mathbf{F}) = \frac{d}{dt}\left[\sum(\mathbf{r} \times m\mathbf{v})\right] \;, \tag{9´}$$

where $\Sigma\mathbf{M}$ is a vector sum of moments of all external forces related to determined point or axis, $\mathbf{L}$ is a vector sum of moment momentums of all fluid particles in considered volume related to the same point or axis.

A consequential application of law of conservation of moment momentum, Navier-Stokes equations and detail analysis of results would prevent some ideas on paradox at loss of global stability of liquid flow through axisymmetric annular channel (Sect.2.4). Necessary to mention, that occurrence of vortices, their effects and breakdown are still topical problems of fluid mechanics, namely experimental fluid mechanics has to take part at investigations.

4. Selected experimental results of high-speed aerodynamic research

In this section, selected experimental results from modelling of high-speed flow in blade cascades representing sections of a rotor blading of last stage of large output steam turbine are presented. Cylindrical sections of rotor blading, as shown in figure 7, determined objects to be investigated at experimental aerodynamic tests. The blade cascades were manufactured and flow past them was measured in a high-speed aerodynamic win tunnel. Optical measurement techniques were used and some of these results – interferograms – are presented.

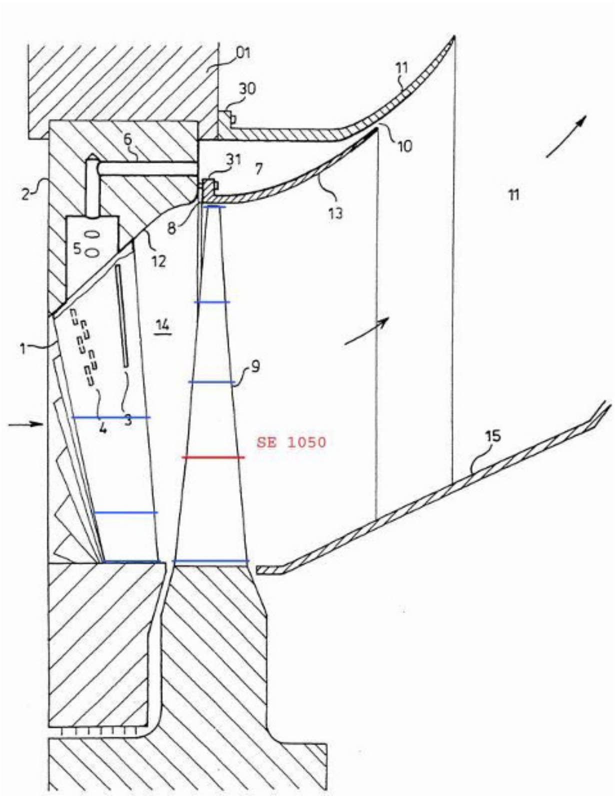

Fig. 7. Meridian section of the last stage of large output steam turbine.

4.1 The Root Section of Rotor Blading

The blade cascade representing the root section of rotor blading is characterized by very low relative pitch of blades and high turn angle of flow. This section operates at transonic velocities. Performed research proved an occurrence of aerodynamic choking in the inter blade channel. Recommendations were an impetus for development and design of new stages of large output steam turbine [8]. Interferogram in the figure 8 shows flow field in blade cascade representing the root section at conditions close to design.

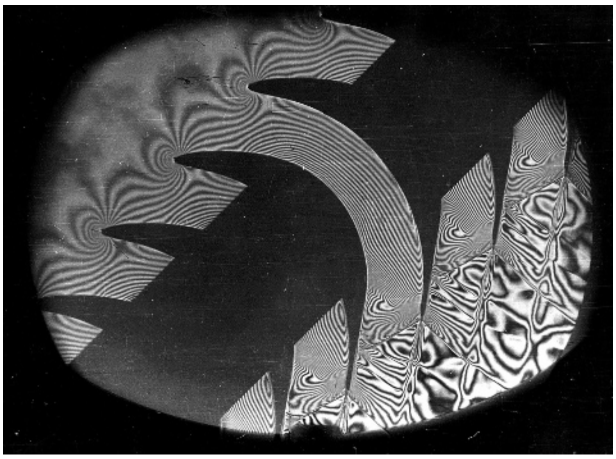

Fig. 8. Interferogram of flow field in the blade cascade representing the root section at exit isentropic Mach number M_{2is} = 1.097, inlet angle β_1 = 44.3°, and inlet Mach number M_1 = 0.628.

4.2 The Section in Rotor Blading at 320 mm from the Root

The blade cascade representing this section, in figure 7 denoted as SE 1050, has low relative pitch. Operation conditions range from low subsonic to transonic velocities. Measured flow field at nearly design regime is shown in interferogran in figure 9. This regime was evaluated and proved to be convenient, and was accepted by the ERCOFTAC as a benchmark for testing of numerical methods and experimental techniques [9]. A special effect in the flow field was studied theoretically and was named “a supersonic compression accompanying transonic expansion” [10].

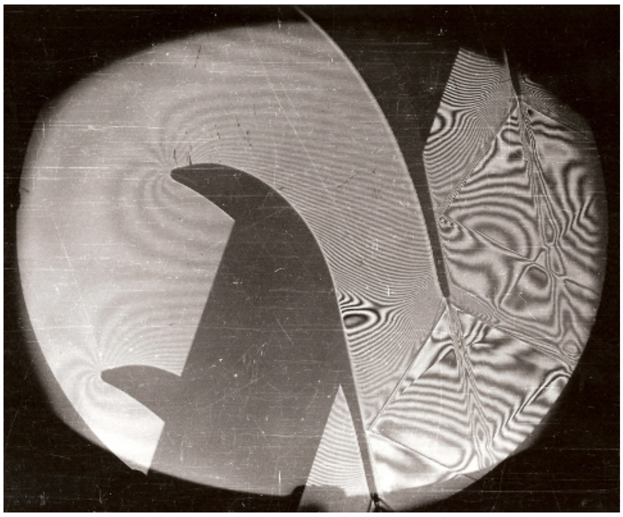

Fig. 9. Interferogram of flow field in the blade cascade representing the section 320 mm from the root at exit isentropic Mach number M_{2is} = 1.198, inlet angle β_1 = 70.7°, and inlet Mach number M_1 = 0.375.

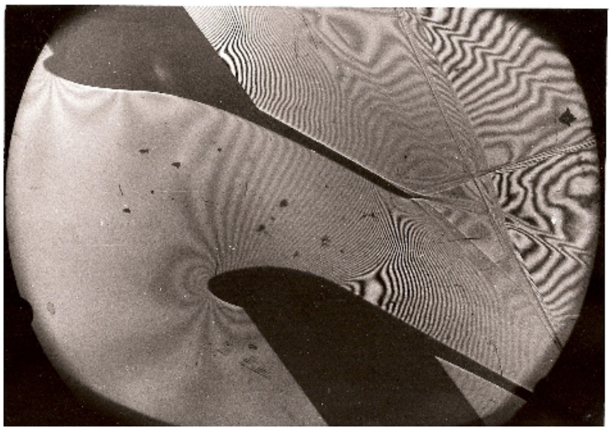

Fig. 10. Interferogram of flow field in the blade cascade representing the section 560 mm from the root at exit isentropic Mach number M_{2is} = 1.353, inlet angle β_1 = 155.0°, and inlet Mach number M_1 = 0.457.

4.3 The Section of Rotor Blading at 560 mm from the Root

The blade cascade representing this section found near middle of the blades is characteristic by low turning angle and due relative pitch. Operation conditions range from low subsonic to high transonic velocities. Measured flow field close to design conditions is shown in interferogram in figure 10. Recent analysis returned to this result and parasitic shock wave effect was studied. New modifications for experimental modelling are recommended [11].

4.4 The Section of Rotor Blading at 800 mm from the Root

The blade cascade representing section at 800 mm from the root has very low turning angle and high relative pitch. The profile has a special morphology in the first third of chord of the profile. It is due to structural and dynamic requirements. Operation conditions of the blade cascade range from high subsonic to low supersonic velocities. Measured flow field close to design conditions is shown in interferogram in figure 11. Performed investigation proved that this section has best value of kinetic energy loss coefficient of all sections at design conditions.

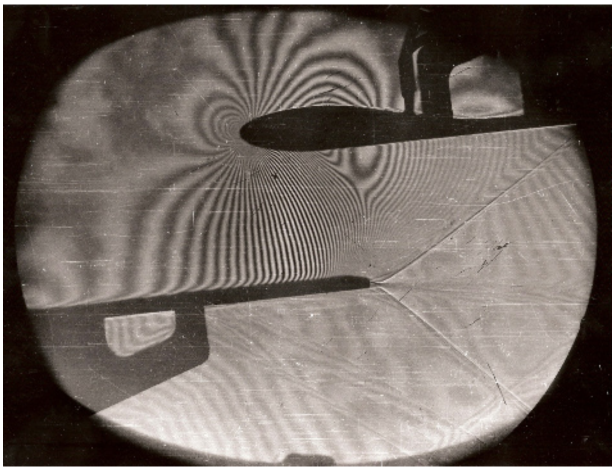

Fig. 11. Interferogram of flow field in the blade cascade representing the section 800 mm from the root at exit isentropic Mach number $M_{2is} = 1.503$, inlet angle $\beta_1 = 126.8°$, and inlet Mach number $M_1 = 0.529$.

4.5 The Section in Rotor Blading at the tip

The blade cascade representing section at the tip has zero turning angle and very high relative pitch. The profile is a plate essentially. Operation conditions of the blade cascade range from transonic to low supersonic velocities. Measured flow field at a little higher isentropic exit Mach number compared to design conditions is shown in interferogram in figure 12. Performed investigations proved very high sensitivity of flow field namely inlet velocity conditions on inlet angle, eventually incidence angle. In the exit part of the blade cascade, it is possible to observe declination of supersonic flow. This phenomenon has to be predicted at design of profiles. New investigations aim to aerodynamics of rotor blading tips because it is important for development of new generations of rotor blading of last stages of large output steam turbines [12].

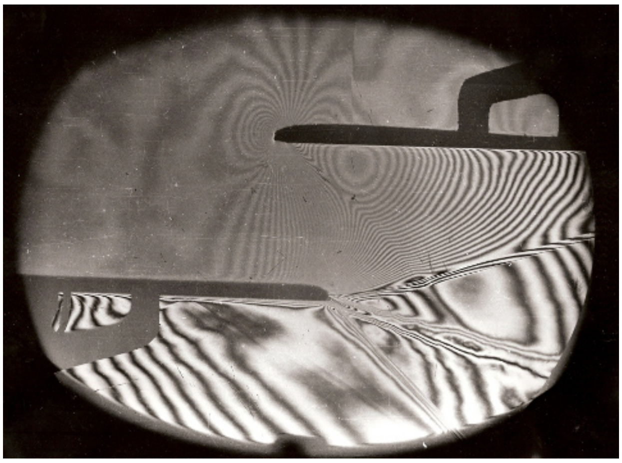

Fig. 12. Interferogram of flow field in the blade cascade representing the tip at exit isentropic Mach number $M_{2is} = 1.813$, inlet angle $\beta_1 = 165.3°$, and inlet Mach number $M_1 = 0.867$.

5. Overview of Distinctions and Topical Problems in High-Speed Aerodynamics

Distinctions of high-speed flow of compressible fluid in comparison to low-speed flows and flow of incompressible fluid can be summarized:

- thermodynamic properties:
  compressibility

$$\delta = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T , \tag{10}$$

  speed of sound

$$a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} , \tag{11}$$

- the existence of maximum mass flux

$$\frac{\partial(\rho v)}{\partial\left(\frac{p}{p_0}\right)} = 0 , \tag{12}$$

- an extreme sensitivity to boundary conditions,
- the supersonic deflection of supersonic flow in exit part of a blade cascade,
- a double solution of parameters of flow structures.

In equations (10) to (12), δ is isothermal compressibility, V is volume of gas, p is pressure, T is temperature, a is speed of sound, s is specific entropy, ρ is density, p_0 is total pressure, v is velocity of gas flow.

Topical problems of high-speed aerodynamics can be introduced:

- passing the speed of sound,
- aerodynamic choking,
- development of supersonic region,
- development of boundary layers,
- models of turbulence,

- flow past trailing edges,
- occurrence of shock waves,
- shock wave/boundary layer interaction,
- wakes, vortex structures,
- limit load of blade cascades,
- occurrence of parasite shock waves,
- unsteady high-speed flows,
- transonic instability,
- data reduction, uncertainty analysis,
- etc.

6. Conclusions

Experimental fluid mechanics is important pillar of research attempting to acquire new knowledge in wide branch of fluid mechanics. Experiments in fluid mechanics support and contribute to other two approaches – to theoretical investigation, and to numerical modelling.

Peculiarities of flow behaviour are mentioned as paradoxes. A consequential application of laws in fluid mechanics and attentive analysis can explain flow effects and behaviours and avoid formulation of paradoxes.

Selected results from high-speed aerodynamic research concerning the flow past blade cascades representing sections in a rotor blading of last stage of large output steam turbine are presented. Very brief overview of distinctions and topical problems in high-speed aerodynamics is introduced.

Acknowledgements

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