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Subtraction and Division Operations of Simplified Neutrosophic Sets

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Abstract: A simplified neutrosophic set is characterized by a truth-membership function, an indeterminacy-membership function, and a falsity-membership function, which is a subclass of the neutrosophic set and contains the concepts of an interval neutrosophic set and a single valued neutrosophic set. It is a powerful structure in expressing indeterminate and inconsistent information. However, there has only been one paper until now—to the best of my knowledge—on the subtraction and division operators in the basic operational laws of neutrosophic single-valued numbers defined in existing literature. Therefore, this paper proposes subtraction operation and division operation for simplified neutrosophic sets, including single valued neutrosophic sets and interval neutrosophic sets respectively, under some constrained conditions to form the integral theoretical framework of simplified neutrosophic sets. In addition, we give numerical examples to illustrate the defined operations. The subtraction and division operations are very important in many practical applications, such as decision making and image processing.

Keywords: subtraction operation; division operation; simplified neutrosophic set; interval neutrosophic set; single valued neutrosophic set

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1. Introduction

To handle uncertainty, imprecise, incomplete, and inconsistent information, Smarandache [1] proposed the concept of a neutrosophic set from philosophical viewpoint, which is a powerful general formal framework and generalizes the concepts of the classic set, fuzzy set [2], intuitionistic fuzzy set (IFS) [3], interval-valued intuitionistic fuzzy set (IVIFS) [4]. In the neutrosophic set, a truth-membership function $\rho(x)$, an indeterminacy-membership function $\sigma(x)$, and a falsity-membership function $\tau(x)$ are characterized independently, where $\rho(x)$, $\sigma(x)$, and $\tau(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$, such that $\rho(x)$: $X \to]^-0, 1^+[$, $\sigma(x)$: $X \to]^-0, 1^+[$, and $\tau(x)$: $X \to]^-0, 1^+[$. Thus, the sum of $\rho(x)$, $\sigma(x)$, and $\tau(x)$ satisfies the condition $^-0 \leq \sup\rho(x) + \sup\sigma(x) + \sup\tau(x) \leq 3^+$. Then, the obvious advantage of the neutrosophic set is that its components are best fit in the representation of indeterminacy and inconsistent information, because IFSs and IVIFSs cannot represent indeterminacy and inconsistent information. However, the defined range of $\rho(x)$, $\sigma(x)$, and $\tau(x)$ in a neutrosophic set is the non-standard unit interval $]^-0, 1^+[$, the neutrosophic set is merely used for philosophical applications, while its engineering applications are difficult. Hence, the defined range of $\rho(x)$, $\sigma(x)$, and $\tau(x)$ can be constrained to the real standard unit interval [0, 1] for convenient engineering applications. Thus, a single valued neutrosophic set (SVNS) [5] and an interval neutrosophic set (INS) [6] were introduced as the subclasses of a neutrosophic set. Further, Ye [7] introduced a simplified neutrosophic set (SNS), which is a subclass of the neutrosophic set and contains a SVNS and an INS, and defined some basic operational laws over SNSs, such as addition and multiplication. Then, Zhang et al. [8] indicated some unreasonable phenomena of the basic operational laws over SNSs in [7] and improved some basic operational laws of INSs. Recently, Smarandache [9] defined the subtraction and division operators in the basic operational laws of neutrosophic single-valued numbers and presented some restrictions for these operations of neutrosophic single-valued numbers and neutrosophic single-valued overnumbers/undernumbers/offnumbers. However, as far as we know, there has not been any investigation on the subtraction and division operations over SNSs (SVNSs and INSs) until now. Since SNSs are the extension of IFSs and IVIFSs, the subtraction and division operations are very important in forming the integral theoretical framework of SNSs. Then, the subtraction and division operations of IFSs and IVIFSs have been investigated by some researchers [10–14]. Motivated by the subtraction and division operations for IFSs and IVIFSs, this paper will try to develop the two new subtraction and division operations for SNSs to form the integral theoretical framework of SNSs.

The remainder of this paper is arranged as follows. Section 2 describes some basic knowledge on SNSs and their basic operations. Section 3 proposes the subtraction operation and division operation over SNSs. Some remarks are contained in Section 4.

2. Some Basic Knowledge of SNSs and Their Basic Operations

For the science and engineering applications of neutrosophic sets, Ye [7] introduced the SNS concept, which is a subclass of the neutrosophic set, and gave the following definition of a SNS.

Definition 1 [7]. *Let X be a universal of discourse. A SNS N in X is characterized by a truth-membership function $\rho_N(x)$, an indeterminacy-membership function $\sigma_N(x)$, and a falsity-membership function $\tau_N(x)$, where the functions $\rho_N(x)$, $\sigma_N(x)$ and $\tau_N(x)$ are singleton subintervals/subsets in the real standard interval [0, 1], such that $\rho_N(x)$: $X \to [0, 1]$, $\sigma_N(x)$: $X \to [0, 1]$, and $\tau_N(x)$: $X \to [0, 1]$. Thus, a SNS N is denoted by*

$$N = \{\langle x, \rho_N(x), \sigma_N(x), \tau_N(x)\rangle | x \in X\}.$$

The SNS is a subclass of the neutrosophic set and contains the concepts of INS and SVNS [7].

If the values of the three functions $\rho_N(x)$, $\sigma_N(x)$ and $\tau_N(x)$ in the SNS N are taken as three real numbers, i.e., $\rho_N(x)$, $\sigma_N(x)$, $\tau_N(x) \in [0, 1]$, then the SNS N is reduced to the SVNS N. Thus, the sum of $\rho_N(x)$, $\sigma_N(x)$, and $\tau_N(x)$ satisfies the condition $0 \leq \rho_N(x) + \sigma_N(x) + \tau_N(x) \leq 3$.

For SVNSs $N_1 = \{\langle x, \rho_{N_1}(x), \sigma_{N_1}(x), \tau_{N_1}(x)\rangle | x \in X\}$ and $N_2 = \{\langle x, \rho_{N_2}(x), \sigma_{N_2}(x), \tau_{N_2}(x)\rangle | x \in X\}$, there are the following relations [5]:

(1) Complement: $N_1^c = \{\langle x, \tau_{N_1}(x), 1 - \sigma_{N_1}(x), \rho_{N_1}(x)\rangle | x \in X\}$;
(2) Inclusion: $N_1 \subseteq N_2$ if and only if $\rho_{N_1}(x) \leq \rho_{N_2}(x)$, $\sigma_{N_1}(x) \geq \sigma_{N_2}(x)$, and $\tau_{N_1}(x) \geq \tau_{N_2}(x)$ for any x in X;
(3) Equality: $N_1 = N_2$ if and only if $N_1 \subseteq N_2$ and $N_2 \subseteq N_1$.

Since SVNSs are the special case of INSs, the operational laws of the SVNSs N_1 and N_2 are introduced as follows [8]:

(1) $N_1 + N_2 = \{\langle x, \rho_{N_1}(x) + \rho_{N_2}(x) - \rho_{N_1}(x)\rho_{N_2}(x), \sigma_{N_1}(x)\sigma_{N_2}(x), \tau_{N_1}(x)\tau_{N_2}(x)\rangle | x \in X\}$;
(2) $N_1 \times N_2 = \left\{\left\langle \begin{array}{c} x, \rho_{N_1}(x)\rho_{N_2}(x), \sigma_{N_1}(x) + \sigma_{N_2}(x) - \sigma_{N_1}(x)\sigma_{N_2}(x), \\ \tau_{N_1}(x) + \tau_{N_2}(x) - \tau_{N_1}(x)\tau_{N_2}(x) \end{array}\right\rangle | x \in X\right\}$;
(3) $\lambda N_1 = \left\{\left\langle x, 1 - (1 - \rho_{N_1}(x))^\lambda, \sigma_{N_1}^\lambda(x), \tau_{N_1}^\lambda(x)\right\rangle | x \in X\right\}, \lambda > 0$;
(4) $N_1^\lambda = \left\{\left\langle x, , \rho_{N_1}^\lambda(x), 1 - (1 - \sigma_{N_1}(x))^\lambda, 1 - (1 - \tau_{N_1}(x))^\lambda\right\rangle | x \in X\right\}, \lambda > 0$.

If the values of the three functions $\rho_N(x)$, $\sigma_N(x)$, and $\tau_N(x)$ in the SNS N are taken as three interval numbers, i.e., $\rho_N(x)$, $\sigma_N(x)$, $\tau_N(x) \subseteq [0, 1]$, then the SNS N is reduced to the INS N. Thus, the sum of $\rho_N(x)$, $\sigma_N(x)$, and $\tau_N(x)$ satisfies the condition $0 \leq \sup\rho_N(x) + \sup\sigma_N(x) + \sup\tau_N(x) \leq 3$.

For INSs $N_1 = \{\langle x, \rho_{N_1}(x), \sigma_{N_1}(x), \tau_{N_1}(x)\rangle | x \in X\}$ and $N_2 = \{\langle x, \rho_{N_2}(x), \sigma_{N_2}(x), \tau_{N_2}(x)\rangle | x \in X\}$, there are the following relations [6]:

(1) Complement: $N_1^c = \left\{ \left\langle \begin{array}{l} x, [\inf\tau_{N_1}(x), \sup\tau_{N_1}(x)], \\ [1 - \sup\sigma_{N_1}(x), 1 - \inf\sigma_{N_1}(x)], \\ [\inf\rho_{N_1}(x), \sup\rho_{N_1}(x)] \end{array} \right\rangle | x \in X \right\}$;

(2) Inclusion: $N_1 \subseteq N_2$ if and only if $\inf\rho_{N_1}(x) \leq \inf\rho_{N_2}(x)$, $\sup\rho_{N_1}(x) \leq \sup\rho_{N_2}(x)$, $\inf\sigma_{N_1}(x) \geq \inf\sigma_{N_2}(x)$, $\sup\sigma_{N_1}(x) \geq \sup\sigma_{N_2}(x)$, $\inf\tau_{N_1}(x) \geq \inf\tau_{N_2}(x)$, and $\sup\tau_{N_1}(x) \geq \sup\tau_{N_2}(x)$ for any x in X;

(3) Equality: $N_1 = N_2$ if and only if $N_1 \subseteq N_2$ and $N_2 \subseteq N_1$.

Whereas, the operational laws of the INSs N_1 and N_2 are introduced as follows [8]:

(1) $N_1 + N_2 = \left\{ \left\langle \begin{array}{l} x, [\inf\rho_{N_1}(x) + \inf\rho_{N_2}(x) - \inf\rho_{N_1}(x)\inf\rho_{N_2}(x), \\ \sup\rho_{N_1}(x) + \sup\rho_{N_2}(x) - \sup\rho_{N_1}(x)\sup\rho_{N_2}(x)], \\ [\inf\sigma_{N_1}(x)\inf\sigma_{N_2}(x), \sup\sigma_{N_1}(x)\sup\sigma_{N_2}(x)], \\ [\inf\tau_{N_1}(x)\inf\tau_{N_2}(x), \sup\tau_{N_1}(x)\sup\tau_{N_2}(x)] \end{array} \right\rangle | x \in X \right\}$;

(2) $N_1 \times N_2 = \left\{ \left\langle \begin{array}{l} x, [\inf\rho_{N_1}(x)\inf\rho_{N_2}(x), \sup\rho_{N_1}(x)\sup\rho_{N_2}(x)], \\ [\inf\sigma_{N_1}(x) + \inf\sigma_{N_2}(x) - \inf\sigma_{N_1}(x)\inf\sigma_{N_2}(x), \\ \sup\sigma_{N_1}(x) + \sup\sigma_{N_2}(x) - \sup\sigma_{N_1}(x)\sup\sigma_{N_2}(x)] \\ [\inf\tau_{N_1}(x) + \inf\tau_{N_2}(x) - \inf\tau_{N_1}(x)\inf\tau_{N_2}(x), \\ \sup\tau_{N_1}(x) + \sup\tau_{N_2}(x) - \sup\tau_{N_1}(x)\sup\tau_{N_2}(x)] \end{array} \right\rangle | x \in X \right\}$;

(3) $\lambda N_1 = \left\{ \left\langle \begin{array}{l} x, [1 - (1 - \inf\rho_{N_1}(x))^\lambda, 1 - (1 - \sup\rho_{N_1}(x))^\lambda] \\ [\inf\sigma_{N_1}^\lambda(x), \sup\sigma_{N_1}^\lambda(x)], [\inf\tau_{N_1}^\lambda(x), \sup\tau_{N_1}^\lambda(x)] \end{array} \right\rangle | x \in X \right\}, \lambda > 0$;

(4) $N_1^\lambda = \left\{ \left\langle \begin{array}{l} x, [\inf\rho_{N_1}^\lambda(x), \sup\rho_{N_1}^\lambda(x)], \\ [1 - (1 - \inf\sigma_{N_1}(x))^\lambda, 1 - (1 - \sup\sigma_{N_1}(x))^\lambda], \\ [1 - (1 - \inf\tau_{N_1}(x))^\lambda, 1 - (1 - \sup\tau_{N_1}(x))^\lambda] \end{array} \right\rangle | x \in X \right\}, \lambda > 0.$

However, there is little any investigation on the subtraction and division operations over SNSs (SVNSs and INSs) in existing literature. Therefore, we should investigate the subtraction and division operations of SNSs to form the integral theoretical framework of SNSs.

3. Subtraction and Division Operations over SNSs

3.1. Subtraction and Division Operations over SVNSs

For any two given SVNSs A and B, the problem is how to find the unknown SVNS $C = A - B$, which satisfies $\rho_C(x)$, $\sigma_C(x)$, $\tau_C(x) \in [0, 1]$ and $0 \leq \rho_C(x) + \sigma_C(x) + \tau_C(x) \leq 3$ for $x \in X$. Based on the addition operation of SVNSs for $A = C + B$, there are $\rho_A(x) = \rho_C(x) + \rho_B(x) - \rho_C(x)\rho_B(x)$, $\sigma_A(x) = \sigma_C(x)\sigma_B(x)$, and $\tau_A(x) = \tau_C(x)\tau_B(x)$. Then, we can obtain the following results:

$$\rho_C(x) = \frac{\rho_A(x) - \rho_B(x)}{1 - \rho_B(x)} \tag{1}$$

$$\sigma_C(x) = \frac{\sigma_A(x)}{\sigma_B(x)} \tag{2}$$

$$\tau_C(x) = \frac{\tau_A(x)}{\tau_B(x)} \tag{3}$$

Then, the functions $\rho_C(x)$, $\sigma_C(x)$, $\tau_C(x)$ in C must take values in the interval [0, 1] and satisfy the following conditions:

$$0 \leq \frac{\rho_A(x) - \rho_B(x)}{1 - \rho_B(x)} \leq 1 \ \text{ for } \rho_B(x) \leq \rho_A(x) \text{ and } \rho_B(x) \neq 1 \tag{4}$$

$$0 \leq \frac{\sigma_A(x)}{\sigma_B(x)} \leq 1 \ \text{ for } \sigma_B(x) \geq \sigma_A(x) \text{ and } \sigma_B(x) \neq 0 \tag{5}$$

$$0 \leq \frac{\tau_A(x)}{\tau_B(x)} \leq 1 \text{ for } \tau_B(x) \geq \tau_A(x) \text{ and } \tau_B(x) \neq 0 \tag{6}$$

Obviously, the inequalities (4)–(6) hold only if $A \geq B$, $\rho_B(x) \neq 1$, $\sigma_B(x) \neq 0$, and $\tau_B(x) \neq 0$. Therefore, we can give the following definition of subtraction operation for SVNSs.

Definition 2. *For any two given SVNSs A and B, the subtraction operation of the SVNSs A and B are defined as*

$$A - B = \left\{ \left\langle x, \frac{\rho_A(x) - \rho_B(x)}{1 - \rho_B(x)}, \frac{\sigma_A(x)}{\sigma_B(x)}, \frac{\tau_A(x)}{\tau_B(x)} \right\rangle | x \in X \right\} \tag{7}$$

which is valid under the conditions $A \geq B$, $\rho_B(x) \neq 1$, $\sigma_B(x) \neq 0$, and $\tau_B(x) \neq 0$.

Example 1. *Let us consider two SVNSs A and B in the universe of discourse X = {x}: A = {<x, 0.7, 0.4, 0.3>* | *$x \in X$} and B = {<x, 0.5, 0.6, 0.5>* | *$x \in X$}. Then, we find the unknown SVNS C = A − B.*

By using Equation (7), we can obtain that

$$\begin{aligned} C &= A - B = \left\{ \left\langle x, \tfrac{0.7-0.5}{1-0.5}, \tfrac{0.4}{0.6}, \tfrac{0.3}{0.5} \right\rangle | x \in X \right\} \\ &= \{ \langle x, 0.4, 0.6667, 0.6 \rangle | x \in X \}. \end{aligned} \tag{8}$$

For any two given SVNSs A and B, the problem is how to find the unknown SVNS $D = A/B$, which satisfies $\rho_D(x)$, $\sigma_D(x)$, $\tau_D(x) \in [0, 1]$ and $0 \leq \rho_D(x) + \sigma_D(x) + \tau_D(x) \leq 3$ for $x \in X$. Based on the multiplication operation of SVNSs for $A = D \times B$, there are $\rho_A(x) = \rho_D(x)\rho_B(x)$, $\sigma_A(x) = \sigma_D(x) + \sigma_B(x) - \sigma_D(x)\sigma_B(x)$, and $\tau_A(x) = \tau_D(x) + \tau_B(x) - \tau_D(x)\tau_B(x)$. Then, we can obtain the following results:

$$\rho_D(x) = \frac{\rho_A(x)}{\rho_B(x)} \tag{9}$$

$$\sigma_D(x) = \frac{\sigma_A(x) - \sigma_B(x)}{1 - \sigma_B(x)} \tag{10}$$

$$\tau_D(x) = \frac{\tau_A(x) - \tau_B(x)}{1 - \tau_B(x)} \tag{11}$$

Then, the functions $\rho_D(x)$, $\sigma_D(x)$, $\tau_D(x)$ in D must take the values in the interval [0, 1] and satisfy the following conditions:

$$0 \leq \frac{\rho_A(x)}{\rho_B(x)} \leq 1 \text{ for } \rho_B(x) \geq \rho_A(x) \text{ and } \rho_B(x) \neq 0 \tag{12}$$

$$0 \leq \frac{\sigma_A(x) - \sigma_B(x)}{1 - \sigma_B(x)} \leq 1 \text{ for } \sigma_B(x) \leq \sigma_A(x) \text{ and } \sigma_B(x) \neq 1 \tag{13}$$

$$0 \leq \frac{\tau_A(x) - \tau_B(x)}{1 - \tau_B(x)} \leq 1 \text{ for } \tau_B(x) \leq \tau_A(x) \text{ and } \tau_B(x) \neq 1 \tag{14}$$

Obviously, the inequalities (11)–(13) hold only if $B \geq A$, $\rho_B(x) \neq 0$, $\sigma_B(x) \neq 1$, and $\tau_B(x) \neq 1$. Thus, we can give the following definition of division operation for SVNSs A and B.

Definition 3. *For any two given SVNSs A and B, the division operation of the SVNSs A and B is defined as*

$$A/B = \left\{ \left\langle x, \frac{\rho_A(x)}{\rho_B(x)}, \frac{\sigma_A(x) - \sigma_B(x)}{1 - \sigma_B(x)}, \frac{\tau_A(x) - \tau_B(x)}{1 - \tau_B(x)} \right\rangle | x \in X \right\} \tag{15}$$

which is valid under the conditions $B \geq A$, $\rho_B(x) \neq 0$, $\sigma_B(x) \neq 1$, and $\tau_B(x) \neq 1$.

Example 2. *Let us consider two SVNSs A and B in the universe of discourse X = {x}: A = {<x, 0.6, 0.3, 0.2>* | *x* ∈ *X} and B = {<x, 0.8, 0.2, 0.1>* | *x* ∈ *X}. Then, we need to find the unknown SVNS D = A/B.*

According to Equation (14), we have that

$$\begin{aligned} D &= A/B = \left\{ \left\langle x, \tfrac{0.6}{0.8}, \tfrac{0.3-0.2}{1-0.2}, \tfrac{0.2-0.1}{1-0.1} \right\rangle | x \in X \right\} \\ &= \{ \langle x, 0.75, 0.125, 0.1111 \rangle | x \in X \}. \end{aligned} \quad (16)$$

3.2. Subtraction and Division Operations of INSs

Based on the aforementioned discussions in the subtraction and division operations of SVNSs, we can give the subtraction and division operations of INSs as the extension of the subtraction and division operations of SVNSs.

If we only consider $\rho_A(x)$, $\sigma_A(x)$, $\tau_A(x) \subseteq [0, 1]$ and $\rho_B(x)$, $\sigma_B(x)$, $\tau_B(x) \subseteq [0, 1]$ in two SNSs A and B as interval numbers, i.e., two INSs A and B, the problem is how to find the unknown INS $C = A - B$, which satisfies $\rho_C(x)$, $\sigma_C(x)$, $\tau_C(x) \subseteq [0, 1]$ and $0 \le \sup\rho_C(x) + \sup\sigma_C(x) + \sup\tau_C(x) \le 3$ for $x \in X$. Based on the addition operation of INSs for $A = C + B$, there are $\rho_A(x) = [\inf\rho_C(x) + \inf\rho_B(x) - \inf\rho_C(x)\inf\rho_B(x), \sup\rho_C(x) + \sup\rho_B(x) - \sup\rho_C(x)\sup\rho_B(x)]$, $\sigma_A(x) = [\inf\sigma_C(x)\inf\sigma_B(x), \sup\sigma_C(x)\sup\sigma_B(x)]$, and $\tau_A(x) = [\inf\tau_C(x)\inf\tau_B(x), \sup\tau_C(x)\sup\tau_B(x)]$. Then, we can obtain

$$\rho_C(x) = \left[\frac{\inf\rho_A(x) - \inf\rho_B(x)}{1 - \inf\rho_B(x)}, \frac{\sup\rho_A(x) - \sup\rho_B(x)}{1 - \sup\rho_B(x)} \right] \quad (17)$$

$$\sigma_C(x) = \left[\frac{\inf\sigma_A(x)}{\inf\sigma_B(x)}, \frac{\sup\sigma_A(x)}{\sup\sigma_B(x)} \right] \quad (18)$$

$$\tau_C(x) = \left[\frac{\inf\tau_A(x)}{\inf\tau_B(x)}, \frac{\sup\tau_A(x)}{\sup\tau_B(x)} \right] \quad (19)$$

Then, the functions $\rho_C(x)$, $\sigma_C(x)$, $\tau_C(x)$ in C must take the subintervals in the real standard interval [0, 1]:

$$\begin{gathered} \rho_C(x) = \left[\tfrac{\inf\rho_A(x) - \inf\rho_B(x)}{1 - \inf\rho_B(x)}, \tfrac{\sup\rho_A(x) - \sup\rho_B(x)}{1 - \sup\rho_B(x)} \right] \subseteq [0, 1] \\ \text{for } \inf\rho_A(x) \ge \inf\rho_B(x), \sup\rho_A(x) \ge \sup\rho_B(x) \text{ and } \rho_B(x) \ne [1, 1] \end{gathered} \quad (20)$$

$$\begin{gathered} \sigma_C(x) = \left[\tfrac{\inf\sigma_A(x)}{\inf\sigma_B(x)}, \tfrac{\sup\sigma_A(x)}{\sup\sigma_B(x)} \right] \subseteq [0, 1] \\ \text{for } \inf\sigma_B(x) \ge \inf\sigma_A(x), \sup\sigma_B(x) \ge \sup\sigma_A(x) \text{ and } \sigma_B(x) \ne [0, 0] \end{gathered} \quad (21)$$

$$\begin{gathered} \tau_C(x) = \left[\tfrac{\inf\tau_A(x)}{\inf\tau_B(x)}, \tfrac{\sup\tau_A(x)}{\sup\tau_B(x)} \right] \subseteq [0, 1] \\ \text{for } \inf\tau_B(x) \ge \inf\tau_A(x), \sup\tau_B(x) \ge \sup\tau_A(x) \text{ and } \tau_B(x) \ne [0, 0] \end{gathered} \quad (22)$$

Obviously, Equations (18)–(20) hold only if $A \ge B$, $\rho_B(x) \ne [1, 1]$, $\sigma_B(x) \ne [0, 0]$, and $\tau_B(x) \ne [0, 0]$. Thus, we can give the following definition of subtraction operation for INSs A and B.

Definition 4. *If we consider two INSs A and B, the subtraction operation of the INSs A and B is defined as*

$$A - B = \left\{ \left\langle \begin{array}{c} x, \left[\frac{\inf\rho_A(x) - \inf\rho_B(x)}{1 - \inf\rho_B(x)}, \frac{\sup\rho_A(x) - \sup\rho_B(x)}{1 - \sup\rho_B(x)} \right], \\ \left[\frac{\inf\sigma_A(x)}{\inf\sigma_B(x)}, \frac{\sup\sigma_A(x)}{\sup\sigma_B(x)} \right], \left[\frac{\inf\tau_A(x)}{\inf\tau_B(x)}, \frac{\sup\tau_A(x)}{\sup\tau_B(x)} \right] \end{array} \right\rangle | x \in X \right\} \quad (23)$$

which is valid under the conditions $A \ge B$, $\rho_B(x) \ne [1, 1]$, $\sigma_B(x) \ne [0, 0]$, *and* $\tau_B(x) \ne [0, 0]$.

Example 3. *Let us consider two INSs A and B in the universe of discourse X = {x}: A = {<x, [0.7, 0.9], [0.4, 0.5], [0.3, 0.4]>∣ x ∈ X} and B = {<x, [0.5, 0.7], [0.5, 0.6], [0.5, 0.6]>∣ x ∈ X}. Then, we need to obtain the unknown INS C = A − B.*

By using Equation (21), we can obtain that

$$\begin{aligned} C &= A - B = \left\{ \left\langle \begin{array}{c} x, \left[\frac{0.7-0.5}{1-0.5}, \frac{0.9-0.7}{1-0.7}\right], \\ \left[\frac{0.4}{0.5}, \frac{0.5}{0.6}\right], \left[\frac{0.3}{0.5}, \frac{0.4}{0.6}\right] \end{array} \right\rangle | x \in X \right\} \\ &= \{\langle x, [0.4, 0.6667], [0.8, 0.8333], [0.6, 0.6667]\rangle | x \in X\}. \end{aligned} \tag{24}$$

If we only consider $\rho_A(x)$, $\sigma_A(x)$, $\tau_A(x) \subseteq [0, 1]$ and $\rho_B(x)$, $\sigma_B(x)$, $\tau_B(x) \subseteq [0, 1]$ in two SNSs A and B as interval numbers, i.e., two INSs A and B, the problem is how to find the unknown INS $D = A/B$, which satisfies $\rho_D(x)$, $\sigma_D(x)$, $\tau_D(x) \subseteq [0, 1]$ and $0 \leq \sup\rho_D(x) + \sup\sigma_D(x) + \sup\tau_D(x) \leq 3$ for $x \in X$. Based on the multiplication operation of INSs for $A = D \times B$, there are $\rho_A(x) = [\inf\rho_D(x)\inf\rho_B(x), \sup\rho_D(x)\sup\rho_B(x)]$, $\sigma_A(x) = [\inf\sigma_D(x) + \inf\sigma_B(x) - \inf\sigma_D(x)\inf\sigma_B(x), \sup\sigma_D(x) + \sup\sigma_B(x) - \sup\sigma_D(x)\sup\sigma_B(x)]$, and $\tau_A(x) = [\inf\tau_D(x) + \inf\tau_B(x) - \inf\tau_D(x)\inf\tau_B(x), \sup\tau_D(x) + \sup\tau_B(x) - \sup\tau_D(x)\sup\tau_B(x)]$. Then, we can obtain the following:

$$\rho_D(x) = \left[\frac{\inf\rho_A(x)}{\inf\rho_B(x)}, \frac{\sup\rho_A(x)}{\sup\rho_B(x)}\right] \tag{25}$$

$$\sigma_D(x) = \left[\frac{\inf\sigma_A(x) - \inf\sigma_B(x)}{1 - \inf\sigma_B(x)}, \frac{\sup\sigma_A(x) - \sup\sigma_B(x)}{1 - \sup\sigma_B(x)}\right] \tag{26}$$

$$\tau_D(x) = \left[\frac{\inf\tau_A(x) - \inf\tau_B(x)}{1 - \inf\tau_B(x)}, \frac{\sup\tau_A(x) - \sup\tau_B(x)}{1 - \sup\tau_B(x)}\right] \tag{27}$$

Then, the functions $\rho_D(x)$, $\sigma_D(x)$, $\tau_D(x)$ in D must take the subintervals in the real standard interval [0, 1]:

$$\begin{array}{c} \rho_D(x) = \left[\frac{\inf\rho_A(x)}{\inf\rho_B(x)}, \frac{\sup\rho_A(x)}{\sup\rho_B(x)}\right] \subseteq [0,1] \\ \text{for } \inf\rho_B(x) \geq \inf\rho_A(x), \sup\rho_B(x) \geq \sup\rho_A(x) \text{ and } \rho_B(x) \neq [0,0] \end{array} \tag{28}$$

$$\begin{array}{c} \sigma_D(x) = \left[\frac{\inf\sigma_A(x)-\inf\sigma_B(x)}{1-\inf\sigma_B(x)}, \frac{\sup\sigma_A(x)-\sup\sigma_B(x)}{1-\sup\sigma_B(x)}\right] \subseteq [0,1] \\ \text{for } \inf\sigma_B(x) \leq \inf\sigma_A(x), \sup\sigma_B(x) \leq \sup\sigma_A(x) \text{ and } \sigma_B(x) \neq [1,1] \end{array} \tag{29}$$

$$\begin{array}{c} \tau_D(x) = \left[\frac{\inf\tau_A(x)-\inf\tau_B(x)}{1-\inf\tau_B(x)}, \frac{\sup\tau_A(x)-\sup\tau_B(x)}{1-\sup\tau_B(x)}\right] \subseteq [0,1] \\ \text{for } \inf\tau_B(x) \leq \inf\tau_A(x), \sup\tau_B(x) \leq \sup\tau_A(x) \text{ and } \tau_B(x) \neq [1,1] \end{array} \tag{30}$$

Obviously, the inequalities (25)–(27) hold only if $B \geq A$, $\rho_B(x) \neq [0, 0]$, $\sigma_B(x) \neq [1, 1]$, and $\tau_B(x) \neq [1, 1]$. Thus, we can give the following definition of division operation for the INSs A and B.

Definition 5. *If we consider two INSs A and B, the division operation of the SNSs A and B is defined as*

$$A/B = \left\{ \left\langle \begin{array}{c} x, \left[\frac{\inf\rho_A(x)}{\inf\rho_B(x)}, \frac{\sup\rho_A(x)}{\sup\rho_B(x)}\right], \\ \left[\frac{\inf\sigma_A(x)-\inf\sigma_B(x)}{1-\inf\sigma_B(x)}, \frac{\sup\sigma_A(x)-\sup\sigma_B(x)}{1-\sup\sigma_B(x)}\right], \\ \left[\frac{\inf\tau_A(x)-\inf\tau_B(x)}{1-\inf\tau_B(x)}, \frac{\sup\tau_A(x)-\sup\tau_B(x)}{1-\sup\tau_B(x)}\right] \end{array} \right\rangle | x \in X \right\} \tag{31}$$

which is valid under the conditions $B \geq A$, $\rho_B(x) \neq [0, 0]$, $\sigma_B(x) \neq [1, 1]$, *and* $\tau_B(x) \neq [1, 1]$.

Example 4. *Let us consider two INSs A and B in the universe of discourse X = {x}, which are given by A = {<x, [0.4, 0.6], [0.3, 0.5], [0.2, 0.4]>| x ∈ X} and B = {<x, [0.6, 0.8], [0.2, 0.3], [0.1, 0.3]>| x ∈ X}. Then, we need to obtain the unknown INS D = A/B.*

According to Equation (28), we can yield that

$$\begin{aligned} D &= A/B = \left\{ \left\langle x, \left[\tfrac{0.4}{0.6}, \tfrac{0.6}{0.8}\right], \left[\tfrac{0.3-0.2}{1-0.2}, \tfrac{0.5-0.3}{1-0.3}\right], \left[\tfrac{0.2-0.1}{1-0.1}, \tfrac{0.4-0.3}{1-0.3}\right] \right\rangle | x \in X \right\} \\ &= \{\langle x, [0.6667, 0.75], [0.125, 0.2857], [0.1111, 0.1429]\rangle | x \in X\}. \end{aligned} \tag{32}$$

4. Conclusions

In this paper, we proposed the subtraction and division operations of the SNSs (SVNSs and INSs) with corresponding constrained conditions. Meantime, numerical examples were provided to show the subtraction and division operations over SNSs (SVNSs and INSs). The subtraction and division operations are very important in forming the integral theoretical framework of SNSs and may have many practical applications, such as decision making and image processing.

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