Basic Principles of Deterministic Quantum Physics

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Abstract

In this paper the following results are achieved: at first we demonstrate that Heisenberg’s indeterminacy principle is based on a mathematical model that is unsound because of an inadequate use of the Fourier development and we prove a new mathematical model. In second place we show that with regard to physical quantities there isn’t theoretical indeterminacy and only the inadequacy of measuring instruments can cause working indeterminacy. Then we prove within the ambit of the new theory: the stability of orbital motions of atomic electrons, the quantized wave equation, the fine and hyperfine structure of atoms, the Lamb shift as a consequence of the relativistic correction. Results on these subjects are concordant with experimental data and are obtained without making use of probabilistic concepts. We think that only the most adequate use of mathematical models can allow a further evolution of our scientific knowledges that must be adapted to the examined physical event and supported with more valid analyses.

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Introduction

Contemporary physics in its standard models and theories is based on the concept of indeterminacy: this is no doubt manifest with regard to quantum mechanics (QM) whose theoretical foundation is just Heisenberg' s indeterminacy principle.
Also in Special Relativity nevertheless there is an indeterminacy because the speed of light is invariant with respect to all inertial reference systems and this introduces an indeterminacy just on the reference. The Theory of Reference Frames (TR)[1] eliminates this indeterminacy and defines the peculiarity of the preferred reference frame that coincides with the physical system where event happens.
The concrete possibility then to explain in deterministic way the working of some systems that in scientific literature are described normally by probabilistic models[2] has strengthened my aim to contribute to eliminate the indeterminacy in physical knowledge as far as it is possible. The probability theory is a formidable mathematical instrument that we can command in order to foresee a future event or to describe a not well-known physical situation in all its specifics but we cannot think that probability and indeterminacy are the theoretical foundations of nature.
The prime purpose of this paper is to show that quantum physics doesn' t require necessarily a probabilistic approach and therefore atomic structure and behavior of elementary particles can be explained by deterministic reasonings and methods.

1. The indeterminacy principle and new mathematical models

1.1 Probability and determinism

Classical mechanics allows to know and verify by a correct method of measure for every physical quantity the precise value with an error that is generally small and due to measuring instruments. Moreover if initial conditions are well-known, for example initial position and initial velocity of a moving system, it is possible to know in the absence of external noises by the equation of motion the values that these quantities assume at any time. On this account classical physics is able also to foresee events that happen in the same physical conditions. This possibility is called "determinism", it is based on causality which assers that in the absence of external noises equal causes determine always same effects.
When physical science began to study microphysical systems serious problems were born: in the first place measures on elementary particles were very complicated and therefore the concepts of position and velocity lost meaning. Moreover it was hard to define the concept of trajectory whether for elementary particles or photons and also to reconcile the quantum nature of photon with the continuous feature of the luminous intensity. All these and other difficulties (like stability of atomic systems) encouraged scientists to search for new ways: Heisenberg' s indeterminacy principle and Schrodinger' s wave equation were the basic results of the new research.
Heisenberg' s indeterminacy principle defined a incompatibility between fundamental physical quantities like position and velocity, wave number and position, energy and time, and underlined the theoretical impossibility, and not only experimental, to know simultaneously both those physical magnitudes whether for photons or for elementary particles. On this account physicists abandoned the classical model of

microphysical systems and developed a new model based on the method of matrix algebra due above all to Heisenberg, Born and Jordan.

Schrodinger's wave equation was born almost simultaneously to the matrix method and it is the foundation of wave mechanics where elementary particles are considered wave systems. In this viewpoint Schrodinger's wave equation is the new mathematical model that allows to describe the behavior of elementary particles. Later Schrodinger proved that matrix method and wave mechanics were equivalent and reached the same results.

The method of matrix algebra and the wave mechanics are the foundations of quantum mechanics that are based on the following reasoning: it is impossible to know and foresee at any time for particles and for photons the simultaneous values of two incompatible quantities, like for example position and velocity. On this account we can only define all the possible results of these quantities and allocate a value of probability to every result. This reasoning represents the transition from the deterministic philosophy of nature to the probabilistic philosophy.

Schrodinger's wave general equation, derived from d'Alembert's wave equation, is[3]

$$\Delta \psi = \frac{1}{V^2} \frac{\delta^2 \psi}{\delta t^2} \qquad (1.1)$$

where Δ is Laplace's operator, V is the speed of De Broglie's equivalent wave, ψ is the amplitude of probability and $P=|\psi|^2$ is the density of probability. In this viewpoint Pdxdydz represents the probability at any time t to find the particle in the element of volume dxdydz.

1.2 Mathematical models

1.2.1 Periodic functions and domains

Let us consider the f(t) function defined only in the interval (a,b) and representable by a graph in the system of two Cartesian axes (Fig.1.1).

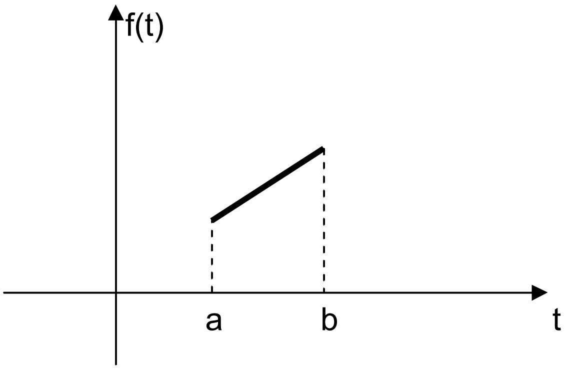

Figure 1.1 The f(t) function is defined only in the (a,b) interval. Out of this interval the function doesn't exist. The (a,b) interval is the domain of the f(t) function.

Let us define "periodic function $f_p(t)$" the function obtained by f(t) expanding its domain over the entire axis of time (Fig.1.2)

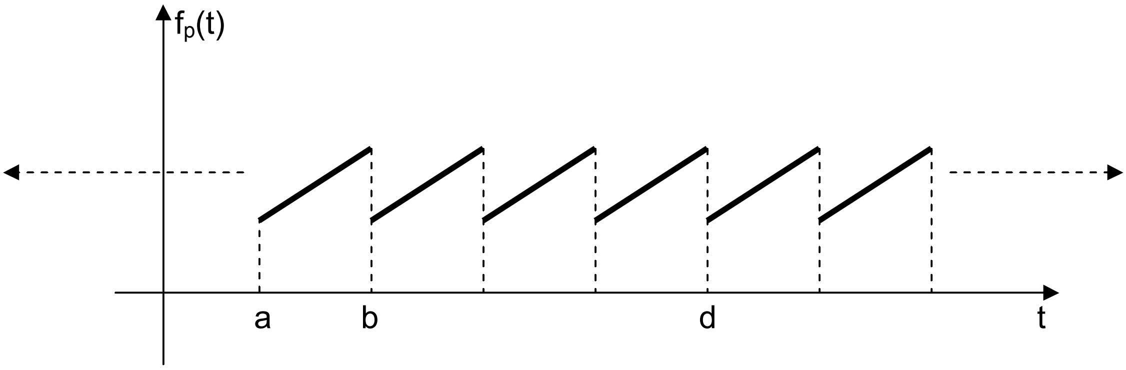

Figure 1.2 Expansion of the f(t) function throughout the axis of t time.

Let us define then " bounded periodic function $f_{pk}(t)$ " the f(t) function expanded only over a subset k of the t axis. In Fig.1.2 an example of bounded periodic function is that defined, for example, in the (a,d) interval with k=4.

1.2.2 Bounded sine function

Let us consider now the V vector whose vertex makes one complete round with ω_o constant angular velocity along the circumference (with r=1) in the positive direction (anticlockwise) as from the P point (Fig.1.3). If we consider the vertical projection of the vector vertex at any time and transfer this projection along the direction of the t axis we obtain the graph like in figure where $T_o=2\pi/\omega_o$ is the time that the vector spends to cover a complete round.

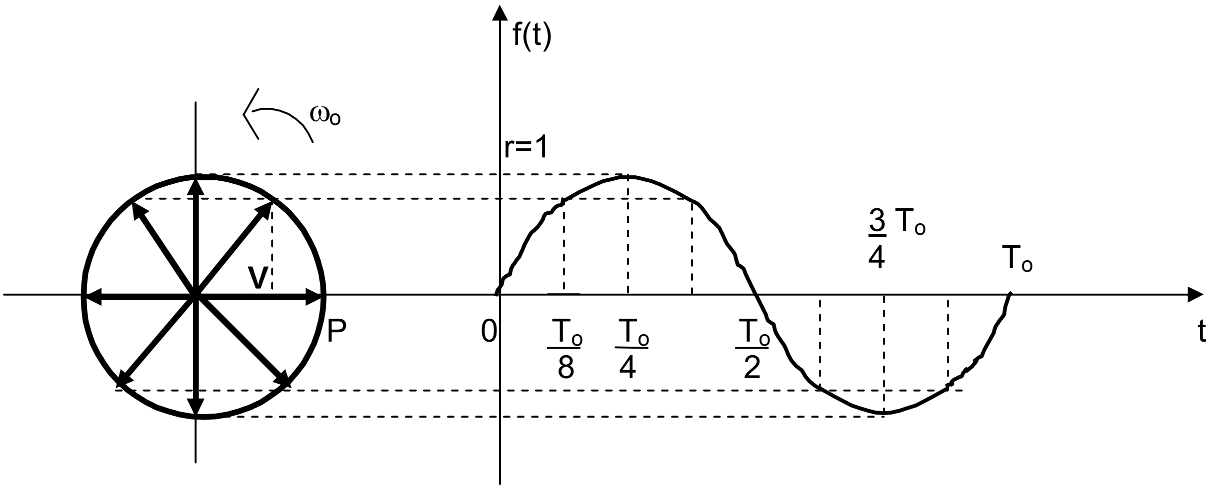

Figure 1.3 A complete rotation of the V vector generates one cycle of the sine function.

The obtained function has the trend like in figure 1.3 in the $(0,T_o)$ interval and it isn't defined out of this domain. We call this function " bounded sine function " with k=1. If we expand the bounded sine function over the entire axis of time we have the classical trigonometric sine function.

1.2.3 Development in series of functions

Let us apply the development in Fourier series to the following bounded sine function represented in fig. 1.3

$$\begin{array}{lll} f(t) = \sin\omega_o t & \text{for} & 0 \leq t \leq T_o \\ f(t) \ \text{non defined} & \text{for} & t<0 \ \text{and} \ t>T_o \end{array} \qquad (1.2)$$

The f(t) function respects the following conditions of development in Fourier series: " f(t) must be generally continuous and generally derivable in the interval $(0,T_o)$ with generally continuous first derivative ".
In order to develop f(t) in Fourier series it is necessary to expand the f(t) function over the entire axis of time and to apply the development in Fourier series to the $f_p(t)$ obtained function which is periodic and coincides with the trigonometric sine.
Representing the Fourier series with FS we have

$$FS[f(t)] = FS[f_p(t)] = \sin\omega_o t \qquad (1.3)$$

We deduce that the bounded sine and the trigonometric sine have the same frequency spectrum according to Fourier represented only by one line (Fig.1.4).

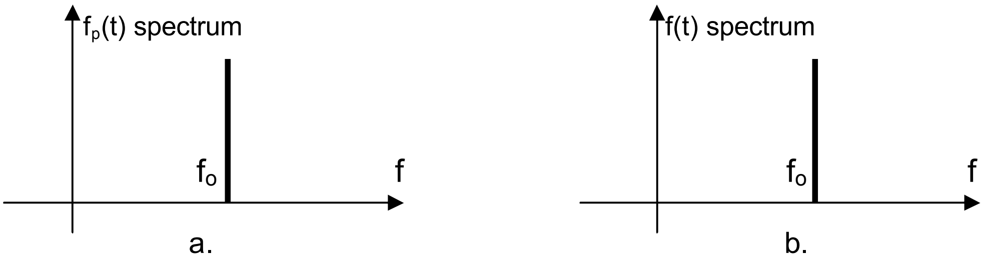

Figure 1.4 Frequency spectra with $f_o = \omega_o/2\pi$
a. Frequency spectrum of the $f_p(t)$ function obtained expanding f(t) over the entire axis of time. The $f_p(t)$ function coincides with the trigonometric sine function.
b. Frequency spectrum of the f(t) bounded function

If in place of the considered previously f(t) function we consider now the function

$$\begin{array}{lll} f_1(t) = \sin\omega_o t & \text{for} & 0 \leq t \leq T_o \\ f_1(t) = 0 & \text{for} & t>T_o \\ f_1(t) \ \text{not defined} & \text{for} & t<0 \end{array} \qquad (1.4)$$

defined and null for $t>T_o$, it isn' t possible to expand the function over the entire axis of time and therefore the $f_1(t)$ function isn' t developable in Fourier series. It is possible however to develop the $f_1(t)$ function according to the Fourier integral and because $f_1(t)$ is an odd function we have

$$F_1(\omega) = -j\,T_o \left[\frac{\sin(\omega_o - \omega)\,T_o}{(\omega_o - \omega)\,T_o} - \frac{\sin(\omega_o + \omega)\,T_o}{(\omega_o + \omega)\,T_o} \right] \qquad (1.5)$$

Considering only real positive frequencies and overlooking not real negative frequencies the frequency spectrum $|F_1(\omega)|$ of the $f_1(t)$ function has the trend like in Fig.1.5.

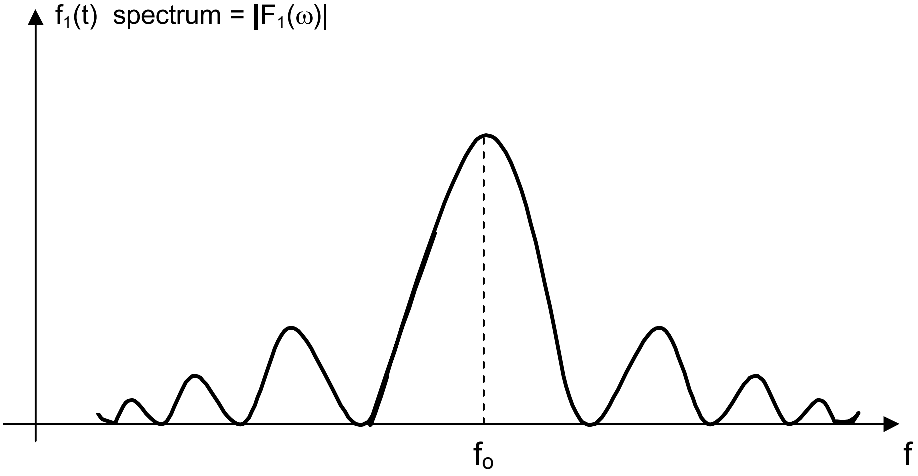

Figure 1.5 Frequency spectrum of the $f_1(t)$ function defined and null for $t>T_o$

We can see that the two functions, f(t) and $f_1(t)$, have very different frequency spectra: the f(t) function, defined only in the interval $(0,T_o)$ has only one f_o frequency line, the $f_1(t)$ function defined and null for $t>T_o$ has a rich spectrum in harmonics round the fundamental frequency f_o. The two functions are equal in the interval $(0,T_o)$ but they have different domains of existence and this causes very different frequency spectra.

1.3 Critical analysis of the indeterminacy principle

1.3.1 Mathematical representation of photon

Photon is an electromagnetic nanowave[4][5] which can be represented in the space at any time t by a nanowave function, for instance sinusoidal, (Fig. 1.6)

$$
\begin{aligned}
&f(x,t) = \sin 2\pi k_o x && \text{for } 0 \le x \le \lambda_o \\
&f(x,t) = \text{non defined} && \text{for } x<0 \text{ and } x>\lambda_o \qquad (1.6)
\end{aligned}
$$

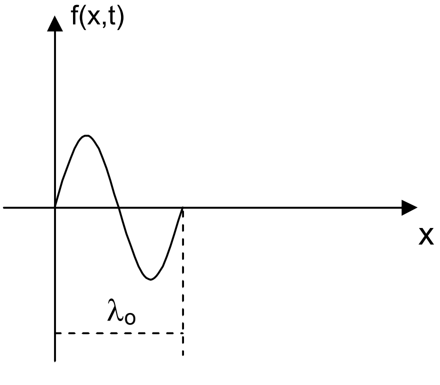

Figure 1.6 Graphic representation of photon: λ_o is the length of one nanowave, $k_o = 1/\lambda_o$ is the nanowave number in one metre.

According to the preceding considerations, expanding the function f(x,t) over the x entire axis and developing in Fourier series we obtain that the spectrum of f(x,t) at any time t is characterized only by one line k_o (Fig.1.7).

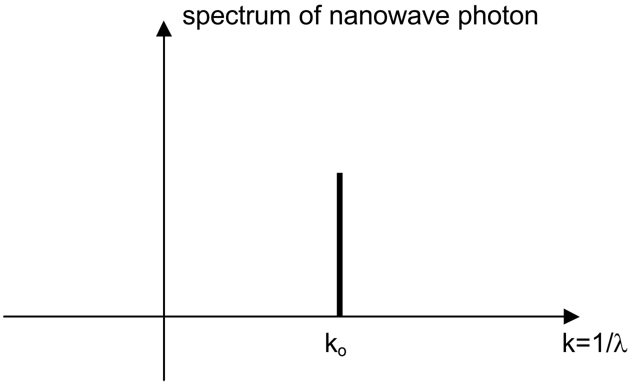

Figure 1.7 Spectrum of nanowave photon that is represented by the relationship (1.6).

In Heisenberg' s theory the mathematical representation of photon is defined null for $x>\lambda_o$ and the following mathematical model is considered

$$
\begin{array}{lll}
f(x,t) = \sin 2\pi k_o x & \text{for } 0 \leq x \leq \lambda_o & \\
f(x,t) = 0 & \text{for } x>\lambda_o & (1.7) \\
f(x,t) \text{ not defined} & \text{for } x<0 &
\end{array}
$$

On this account in Heisenberg' s theory the spectrum of photon is rich in harmonics like in Fig.1.8.

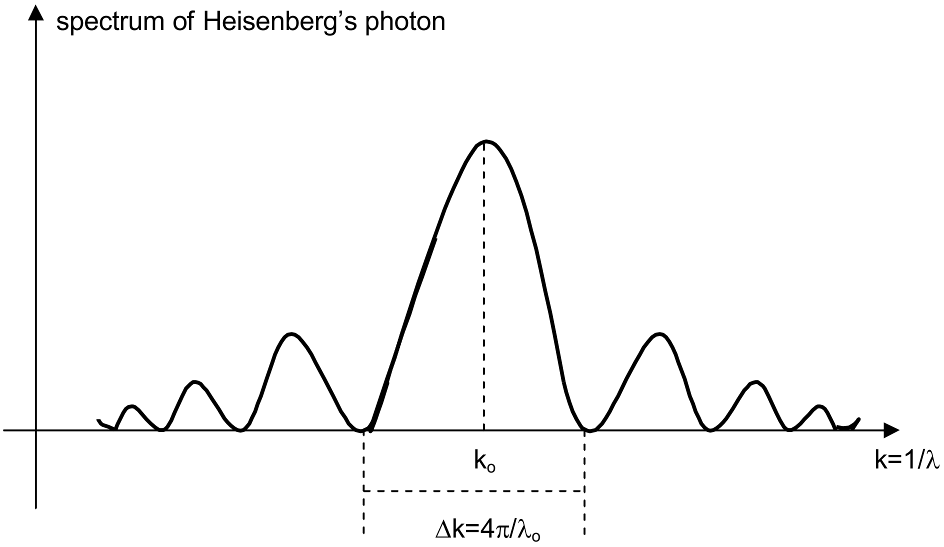

Figure 1.8 Spectrum of Heisenberg's photon that is represented by relationship (1.7). The k quantity represents the wave number.

The difference between the two spectra represented in fig.7 and in fig.8 depends exclusively on the different domains used by the two mathematical and physical models in the (1.6) and (1.7) relationships.

1.3.2 Photons, elementary particles and indeterminacy

The indeterminacy principle asserts that it isn' t possible theoretically to know and to measure simultaneously whether for photons or for elementary particles two characteristic quantities, like position and momentum, position and wave number. Heisenberg proved the following conditions of indeterminacy for photons[3] with respect to position (x,y,z) and momentum (p_x, p_y, p_z)

$$\Delta x \, \Delta p_x \geq \sim h \qquad \Delta y \, \Delta p_y \geq \sim h \qquad \Delta z \, \Delta p_z \geq \sim h \qquad (1.8)$$

and with respect to position (x,y,z) and wave number k (k_x, k_y, k_z)

$$\Delta x \, \Delta k_x \geq \sim 1 \qquad \Delta y \, \Delta k_y \geq \sim 1 \qquad \Delta z \, \Delta k_z \geq \sim 1 \qquad (1.9)$$

Heisenberg proved these conditions assuming for photon the mathematical model (1.7) with rich spectrum in harmonics. If we assume the new mathematical model (1.6) in which photon has a spectrum with only one line of wave number, Heisenberg' s conditions lose validity because $\Delta k=0$ ($\Delta k_x= \Delta k_y= \Delta k_z=0$) and $\Delta p=0$ ($\Delta p_x=\Delta p_y=\Delta p_z=0$) being p=hk. Indeterminacy therefore isn' t in the physical nature of photons but depends only on the used mathematical model.
The same considerations can be applied as per De Broglie' s equivalence principle also to all the other elementary particles, like electrons, and therefore also for them theoretical indeterminacy loses meaning and at the most it is possible to suppose a working indeterminacy which now we want to point out. Let us consider in fact an electron beam with fixed momentum $p(p_x,0,0)$ and fixed velocity $v(v_x,0,0)$ that moves in parallel direction to axis x. In order to measure the y position of electrons we interpose a S surface provided with a slot that has d width (Fig.1.9).

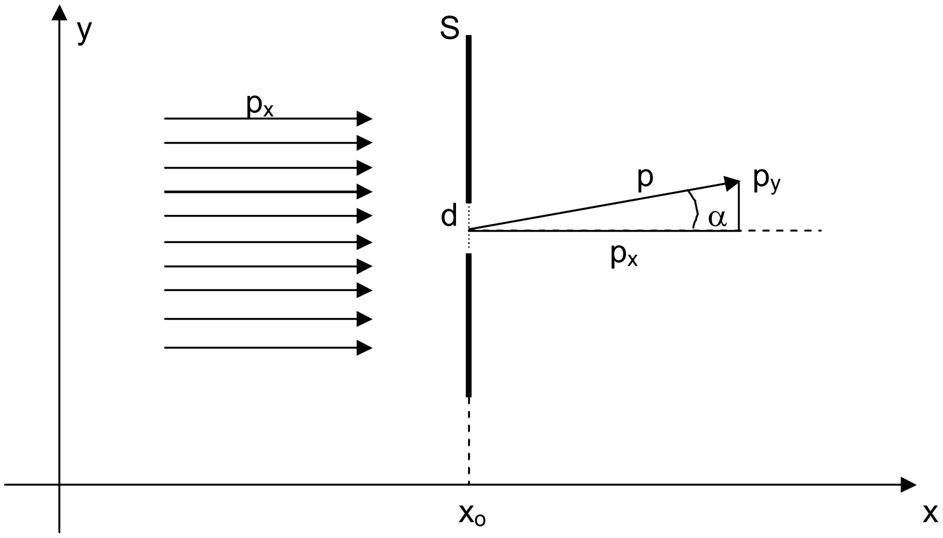

Figure 1.9 There is working indeterminacy relatively to position and momentum of electron only if the d width of the slot is comparable to De Broglie' s wavelength ($d \approx \lambda$).
If $d>>\lambda$ after the slot $p_y=0$, $\Delta p_y = 0$, $\Delta y \Delta p_y = 0$ and therefore there isn' t indeterminacy.

For every particle passing through the slot the coordinate y is determined with an uncertainty $\Delta y=\sim d$. According to De Broglie' s equivalence relation[3][5] a wavelength

λ=h/mv is associated to every electron and for d>>λ the electron isn' t subjected to diffraction and deflection. On this account after the slot p_y=0 and there isn't indeterminacy. In order to reduce the uncertainty relatively to y it is necessary to reduce the d width but for very small widths (d≈λ) the particle is subjected to a phenomenon of diffraction and deflection. In that case every particle, provided with fixed momentum p(p_x,0,0) before the slot, passing through the slot suffers an angular deflection α.

Before the slot p_y=0 and after the slot p_y=psinα=mv_y and for De Broglie' s equivalence relation (p=mv=h/λ) we have

$$p_y = \frac{h\sin\alpha}{\lambda} \qquad (1.10)$$

Because d≈λ and because sinα varies between 0 and 1, the p_y component of momentum after the slot is characterized by an uncertainty

$$\Delta p_y =\sim \frac{h}{d} \qquad (1.11)$$

and consequently

$$\Delta y\, \Delta p_y =\sim h \qquad (1.12)$$

This proves that indeterminacy is only working because in order to measure the coordinate y of the particle it has been necessary to use a screen with slot which generates a phenomenon of diffraction from which indeterminacy in momentum and in position derives when d≈λ.

Electron diffraction can be explained by the Fourier integral likewise radiant energy[4][5]. In fact if N(x,y,z,t) is the number of electrons in the beam at x_o abscissa, where the slot is positioned, we have

$$\begin{aligned} N(x_o,y,z,t) &= N \qquad \text{inside the slot} \\ N(x_o,y,z,t) &= 0 \qquad \text{outside the slot} \end{aligned} \qquad (1.13)$$

For d>>λ there isn' t practically diffraction, electrons don' t undergo deflection and therefore after the slot we have Δp_y=0 and $\Delta y \Delta p_y$=0: there isn' t indeterminacy. Decreasing the d width of the slot the diffraction increases and it has the maximum intensity when d≈λ. In these conditions the equation (1.12) is valid and we have working indeterminacy. The same working indeterminacy can be pointed out also for photons when they produce diffraction[5].

In conclusion we have proved that theoretical indeterminacy can be eliminated using a more adequate mathematical model and therefore it isn' t in the physical nature of photons and particles. Inadequacy of measuring instruments (in our case the use of a surface with slot) then can involve a working indeterminacy when we intervene on the microphysical event in order to execute a measurement of not observable directly physical quantities.

We see in conclusion that Heisenberg' s indeterminacy is the fruit of both the use inadequate of the mathematical model (that generates theoretical indeterminacy) and the use of not osservable directly physical quantities like position (that generates working indeterminacy).

2. Stability of atomic systems in DQP

2.1 Wave model of electron orbital motions

Classical physics wasn' t able to explain the stability of the atomic orbital structure, in fact as per fundamentals of classical physics moving orbital electron around the nucleus is an accelerated electric charge and on this account it must irradiate electromagnetic energy. Electron therefore loses energy, its orbit narrows gradually as far as the fall of electron in nucleus with annihilation and instability of atom. Because this doesn' t happen Bohr introduced some postulates that were out of classical physics, specifically Bohr assumed that orbital electrons don' t irradiate energy and orbits are quantized. These concepts were concordant with experience but in disagreement with classical scientific theories for which many scientists left the classical theory and accepted the foundations of the new theory of quantum mechanics based on a probabilistic approach of physics.
We will show now that theoretical problem of the atom stability can be completely solved out of the probabilistic model of quantum mechanics and consequently it is possible to formulate the theory of Deterministic Quantum Phisics (DQP).
To that end let us assume d' Alembert' s wave model for describing the electron orbital motion in the central field of the atomic nucleus. In this model the electron motion is described by the following wave equation[3], derived from the (1.1),

$$\Delta u + C\,(E - E_p)\,u = 0 \qquad (2.1)$$

where $\Delta = \delta^2/\delta x^2 + \delta^2/\delta y^2 + \delta^2/\delta z^2$ is Laplace' s operator, u(x,y,z) is a space function independent of time, C is a quantity to calculate, E is the electron total energy, E_p is its potential energy and $E_c = E - E_p$ is the kinetic energy.
Suppose still that electron motion is circular and periodic with radius r and its constant angular speed is ω_o (Fig.2.1). In the considered model the u(x,y,z) wave function represents the projection of orbital motion on the axis y and therefore it is function only of the x variable.

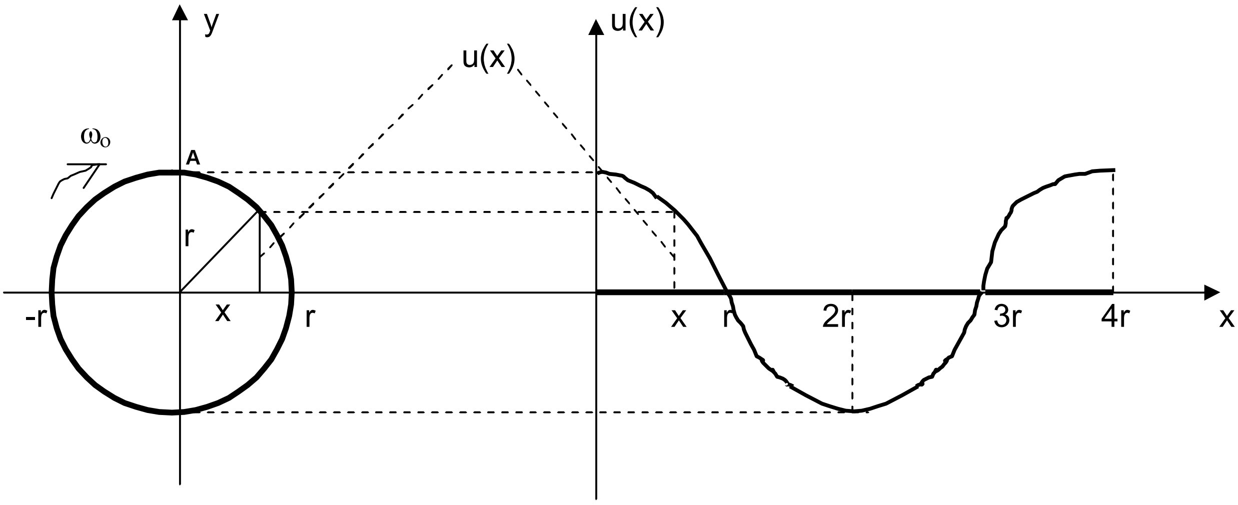

Figure 2.1 The orbital motion of the electron around nucleus generates the u(x) function that is the projection of the orbital motion on the y axis. Suppose that the electron motion starts from the A point.

Assuming $W=CE_c$ where C is a quantity to calculate, the wave equation of motion is a linear differential equation of second order

$$\frac{d^2u}{dx^2} + W u = 0 \qquad (2.2)$$

that respects the initial conditions $u(0)=r$ and $[du/dx]_0=0$. In terms of algebra the equation (2.2) has two solutions $p_1= -\sqrt{-W}$ and $p_2= +\sqrt{-W}$ and therefore the wave function u(x) has the expression

$$u(x) = A_1e^{-x\sqrt{-W}} + A_2e^{+x\sqrt{-W}} \qquad (2.3)$$

It is important to underline that in Schrodinger' s wave mechanics the u wave function is associated to the probability concept: in fact $|u|^2=|\psi|^2$ is the density of probability; in our model the same function has no probabilistic meaning.

2.2 Stability of orbital motions

The orbital motion is stable if it maintains the same orbit in the course of time and consequently if the maximum amplitude of the u(x) wave function is constant. Let us consider two possible situations according as W is negative or positive and assume the quantity C is positive. Let us reminder that in classical physics the kinetic energy cannot be negative while in quantum mechanics this is possible as also it is possible that a particle with energy E is able to pass a barrier of potential energy larger than E.

2.2.1 $W<0$ and therefore $E_c<0$.

In that case the two solutions are real, p_1 is certainly negative while p_2 is certainly positive. In the theory of stability of linear systems[6] a positive real solution produces instability because the u(x) amplitude of orbital motion tends to increase in exponential way (Fig.2.2).

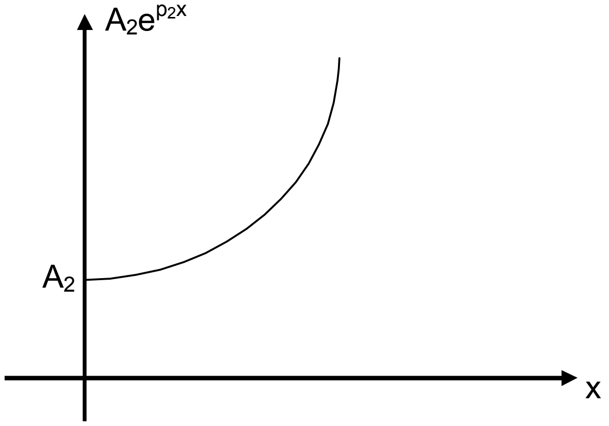

Figure 2.2 A positive real solution generates an unstable orbital motion with increasing amplitude

A negative solution instead produces an exponential decrease of the u(x) amplitude (Fig.2.3) and this causes a gradual narrowing of the orbit until the fall of electron in nucleus with manifest instability of the orbital motion.

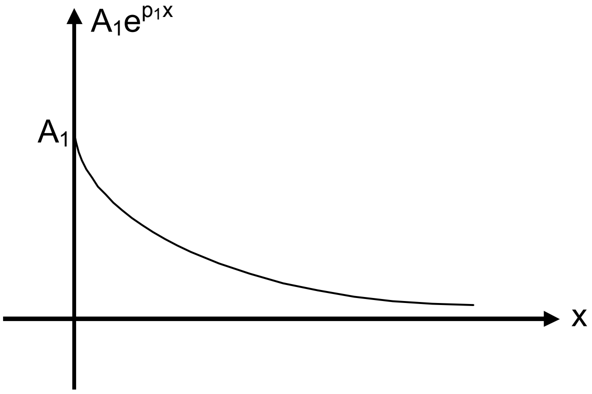

Figure 2.3 A negative real solution generates an unstable orbital motion with decreasing amplitude

In our wave model the E_c negative kinetic energy involves necessarily instability of orbital motion and on this account decomposition of atom. Consequently while in QM kinetic energy can be negative in DQP it cannot be negative.

2.2.2 W > 0 and $E_c > 0$.

The two algebraic solutions are conjugate imaginary and assuming $W=Z^2$ the wave function has the expression

$$u(x) = A_1 e^{-iZx} + A_2 e^{iZx} \qquad (2.4)$$

Applying Eulero' s formulas we have

$$u(x) = (A_1+A_2) \cos Zx - i (A_1- A_2) \text{sen } Zx \qquad (2.5)$$

Because u(x) is a real function, it is necessarily $A_1=A_2$ and therefore

$$u(x) = (A_1+A_2) \cos Zx \qquad (2.6)$$

This result is concordant with the theory of stability of linear systems[6] which asserts that imaginary solutions produce trends with constant maximum amplitude; in that case in fact u(x) has constant maximum amplitude (Fig.2.4). Therefore imaginary solutions deriving from positive kinetic energy assure the stability of orbital trajectories. In our model the atom stability is the result of deterministic considerations and no probabilistic hypothesis is necessary.

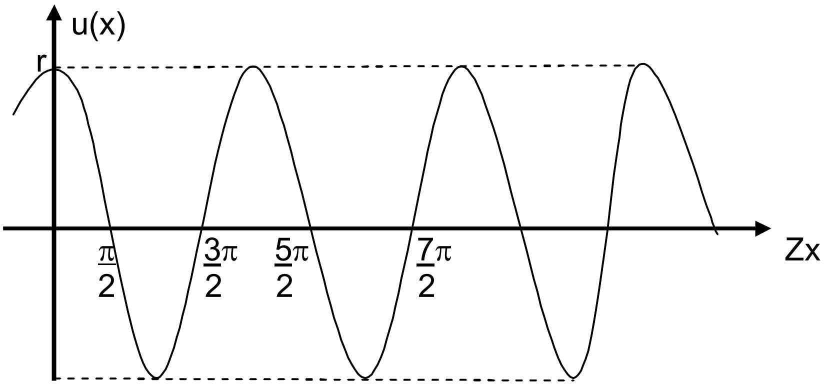

Figure 2.4 Imaginary solutions generate an orbital motion with stable and constant maximum amplitude

In concordance with the initial condition u(r)=0 we have

$$\cos Z r = 0 \qquad (2.7)$$

The preceding relation is fulfilled only if

$$Zr = (2l - 1)\frac{\pi}{2} \qquad \text{with } l=1, 2, 3, \ldots\ldots. \qquad (2.8)$$

for wich being $W=Z^2$ we have

$$W = \frac{(2l - 1)^2\pi^2}{4\, r^2} \qquad (2.9)$$

and for the kinetic energy

$$E_c(r) = \frac{(2l - 1)^2\, \pi^2}{4\, C\, r^2} \qquad (2.10)$$

where C is a quantity to calculate. We will see the l integer number doesn' t have influence on the kinetic energy.

2.3 Equivalence between matter particles and energy particles

Photon is an energy particle with null real mass, representable mathematically at any time by the limited sine function (nanowave)[4][5],

$$f(x,t) = \text{sen } 2\pi kx \qquad \text{for} \quad 0 \le x \le \lambda$$
$$f(x,t) = \text{non defined} \qquad \text{for} \quad x<0 \text{ and } x>\lambda \qquad (2.11)$$

where $k=1/\lambda$ is the nanowave number (Fig.2.5).

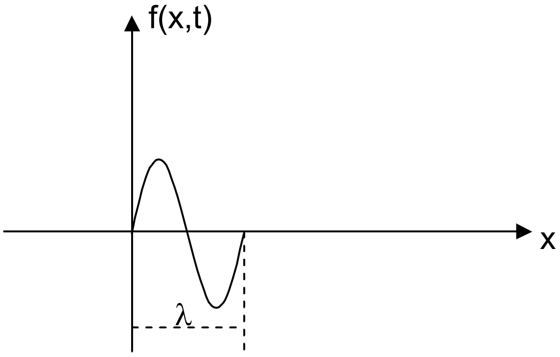

Figure 2.5. Space mathematical representation of photon with one nanowave.

For the photon the following well-known relationships are valid: $p=h/\lambda$, $W=hf$, $\lambda f=c$ in which p is the momentum of photon, W is the energy associated with photon, f is the frequency, λ is the nanowavelength and c is the physical speed of photon. Because a matter elementary particle with at rest m_o electrodynamic mass[7] has an intrinsic energy $W=c^2m_o$, we can associate a virtual electrodynamic mass $m_f = hf/c^2$ with photon and therefore we can set out the equivalence between the energy photon and a virtual elementary particle with mass m_f. Similarly according

to De Broglie' s idea, we can associate an energy virtual quantum with a matter elementary particle that has electrodynamic mass m and constant speed v, where wavelength λ is related to momentum of particle by the relationship $p=mv=h/\lambda$ from which $\lambda=h/mv$: it' s manifest the wavelength of the virtual photon depends on the speed of matter elementary particle while the wavelength of the real photon is independent from its speed which is constant and equals c.
It needs to underline that photons and matter particles are different physical entities, in fact photon is a moving energy quantum with the constant physical speed of light while elementary particles are matter quanta that have an electrodynamic mass that decreases when speed increases. Therefore the fixed equivalence is only an useful instrument in order to describe and study some physical situations concerning elementary particles and photons.
Using De Broglie's equivalence an electron with m electrodynamic mass and v velocity is representable with a virtual photon with wavelength $\lambda=h/p=h/mv$.
In the fixed equivalence a moving electron along an orbit with r radius in the central field of atomic nucleus, in order to maintain the stability , must cover an integer number of wavelengths in order for it to be in phase at the end of every orbit and therefore

$$2\pi r=n\lambda \qquad (2.12)$$

where n is an integer number with n=1, 2, ….
From (2.12) we obtain the momentum of orbital electron

$$p = mv = \frac{h}{\lambda} = \frac{n\,h}{2\pi r} \qquad (2.13)$$

and the angular momentum

$$q = mvr = \frac{n}{2\pi}\,h \qquad (2.14)$$

We deduce therefore from equations (2.12), (2.13) and (2.14) that the orbital radius $r=n\lambda/2\pi$, the momentum and the angular momentum of electron in atom are quantized, depending on the quantum number n: it proves that atom has a quantum nature. The n integer number is the " quantum number of state ".

2.4 The quantized wave equation of electron in the central field of nucleus

Let us calculate now the quantity C. From the relationship (2.10) and from the well-known expression of kinetic energy $E_c(r)=mv^2/2$ we infer

$$C = \frac{(2l-1)^2\,\pi^2}{2mv^2r^2} \qquad (2.15)$$

From (2.14) we derive

$$v^2r^2 = \frac{n^2\,h^2}{4\pi^2m^2} \qquad (2.16)$$

and therefore the quantity C is

$$C = \frac{2\pi^4 m\,(2l-1)^2}{h^2n^2} \qquad (2.17)$$

Replacing (2.17) in (2.10) the values of kinetic energy of atomic electron are

$$E_c(r) = \frac{h^2n^2}{8\,\pi^2\,m\,r^2} \qquad (2.18)$$

The important relationship (2.18) proves that for every r quantized value of orbital radius, depending on n as per (2.12), there are n values of kinetic energy into the orbit with r radius and n quantum state.
In order to avoid confusion with regard to the n quantum number it is convenient to introduce a second quantum number k=1, 2, ….n that is the " quantum number of energy " and therefore the (2.18) can be rewrited in the following

$$E_c(r) = \frac{h^2k^2}{8\,\pi^2\,m\,r^2} \qquad k=1, 2, \ldots.n \qquad (2.19)$$

It is time now to calculate the r quantized value of the orbital radius of atomic electron.
To that end let's suppose that atom is an isolated system and the central nucleus has a positive charge Q=+Ze where Z is the atomic number and "e" is the absolute value of electron charge. For Z=1 we have hydrogen atom.
Let's suppose too that atom has lost Z-1 electrons. The residual electron is attracted by Coulomb's central force $F=Ze^2/4\pi\varepsilon_o r^2$ (*). This force is balanced in the orbital motion by the centrifugal force $F_c=mv^2/r$ and therefore the radius of the orbit is $r=Ze^2/4\pi\varepsilon_o mv^2$. Deriving v^2 from the (2.16) we have

$$r = \frac{\varepsilon_o\,h^2n^2}{\pi m e^2 Z} \qquad (2.20)$$

Relationship (2.20) gives the radius of orbital trajectory and establishes that orbital radius is quantized depending on the quantum number n. Consequently electron in atom cannot assume all the orbits for any value of the r radius but only orbits with radius fixed by (2.20) are permitted. We deduce from this that orbital electron can emit energy only through quanta and not with continuity. It isn' t possible therefore that electron loses energy with continuity until its collapse in nucleus.
The stability of orbital electrons in atom is fully explained in DQP by the theory of stability of linear systems. We observe moreover that orbital radius depends also on the electrodynamic mass of electron.
Replacing before (2.20) in (2.19) and after (2.17) and (2.19) in the relationship (2.1) we obtain the quantized wave equation of electron in the central field of atomic nucleus

$$\Delta u + \frac{\pi^4e^4m^2Z^2(2l-1)^2k^2}{4\varepsilon_o^2h^4n^6}\,u = 0 \qquad (2.21)$$

which has the following solutions

$$u(x) = \frac{\varepsilon_o h^2n^2}{\pi e^2 m Z}\cos\frac{\pi^2e^2mZ(2l-1)x}{2\varepsilon_o h^2n^2} \qquad (2.22)$$

(*) In scientific literature generally the $1/4\pi\varepsilon_o$ term is omitted but this omission generates often confusion and errors.

2.5 Energy levels, frequency spectra and fine structure

Continuing in calculation we deduce that for every value of the radius depending on n there are n levels of kinetic energy of atom electron. In fact according to (2.20) the (2.19) becomes

$$E_{cnk} = \frac{e^4 m Z^2 k^2}{8\varepsilon_o^2 h^2 n^4} \qquad \begin{matrix} n=1, 2,\ldots\ldots \\ k=1, 2,\ldots.n \end{matrix} \qquad (2.23)$$

Assuming for the E_p potential energy the expression $-Ze^2/4\pi\varepsilon_o r$ we obtain that also the potential energy in atom is quantized

$$E_{pn} = - \frac{e^4\, m\, Z^2}{4\varepsilon_o^2\, h^2\, n^2} \qquad (2.24)$$

and similarly also the total energy $E_{nk} = E_{cnk} + E_{pn}$ is quantized

$$E_{nk} = - \frac{e^4 m Z^2}{4\varepsilon_o^2\, h^2 n^2} \left(1 - \frac{k^2}{2n^2} \right) \qquad \begin{matrix} n=1, 2,\ldots.. \\ k=1, 2, \ldots n \end{matrix} \qquad (2.25)$$

Levels of total energy are quantized and concordant with energy levels of Bohr-Sommerfeld’ s theory for k=n.
Let us consider now an atomic electron which is in the n quantum state and in the k energy level. Acquiring energy it jumps for instance into the n’ quantum state and the k’ energy level with n’>n. The same electron going back to the initial quantum state emits an energy quantum with intensity

$$\Delta E = E_{n'k'} - E_{nk} = \frac{e^4\, m\, Z^2}{4\varepsilon_o^2\, h^2} \left(\frac{1}{n^2} - \frac{1}{n'^2} + \frac{k'^2}{2n'^4} - \frac{k^2}{2n^4} \right) \qquad (2.26)$$

and with frequency $f=\Delta E/h$

$$f = \frac{E_{n'k'} - E_{nk}}{h} = \frac{e^4\, m\, Z^2}{4\varepsilon_o^2\, h^3} \left(\frac{1}{n^2} - \frac{1}{n'^2} + \frac{k'^2}{2n'^4} - \frac{k^2}{2n^4} \right) \qquad (2.27)$$

Equations (2.25) and (2.27) are the fundamentals of Deterministic Quantum Physics (DQP). Equations (2.25) and (2.27) through the introduction of k (called also “quantum number of fine structure”) furnish the fine structure of frequency spectra. For k’=n’ and k=n the equation (2.27) gives Bohr and Sommerfeld’ s frequency spectra which are therefore a particular case of (2.27). In that case for k’=n’ , k=n and for Z=1 (hydrogen atom) we obtain Lyman’ s series (n=k=1, n’=k’>1), Balmer’ s series (n=k=2, n’=k’>2), Paschen’ s series (n=k=3, n’=k’>3), Brackett’ s series (n=k=4, n’=k’>4).
Complex ions similar to hydrogen atom (Z>1 and with only one orbital electron) have spectra that are obtained with good reliability from the spectrum of hydrogen atom multiplying the frequency of emission of hydrogen atom for Z^2.
For example with regard to helium ion He^+ with Z=2, for k=n=4, k’=n’>4, we obtain Pickering’s series

$$f = \frac{e^4\, m}{2\varepsilon_o^2\, h^3} \left(\frac{1}{16} - \frac{1}{n'^2} \right) \qquad (2.28)$$

3. Relativistic Deterministic Quantum Physics (RDQP)

3.1 Relativistic correction and Lamb shift

In the equation (2.27) the emitted frequency depends on m electrodynamic mass of electron and we know the relativistic variation of electrodynamic mass with speed[7] is $m=m_o(1-v^2/2c^2)$ where m_o is the electrodynamic mass at rest. In order to consider this variation let' s calculate the quantized values of velocity. From (2.13) we derive

$$v = \frac{nh}{2\pi mr} \qquad (3.1)$$

For every value r of orbital radius depending on n we have n different values of speed. In order to avoid confusions we make still use of the k quantum number like in (2.19) and according to (2.20) we have

$$v_{nk} = \frac{e^2Z}{2\varepsilon_o h}\,\frac{k}{n^2} \qquad (3.2)$$

where n=1, 2, … is the quantum number of state and k=1, 2, …n is the “quantum number of energy ” or “quantum number of fine structure”.
We can use the relativistic correction of mass into the expression (2.25) for which

$$E_{nk} = E_{nko}\left(1 - \frac{1}{2}\,\frac{v_{nk}^2}{c^2}\right) \qquad (3.3)$$

where

$$E_{nko} = -\,\frac{e^4 m_o Z^2}{4\varepsilon_o^2 h^2 n^2}\left(1 - \frac{k^2}{2n^2}\right) \qquad (3.4)$$

and replacing (3.2) and (3.4) in (3.3) we have

$$E_{nk} = -\,\frac{e^4 m_o Z^2}{4\varepsilon_o^2\, h^2\, n^2}\left(1 - \frac{k^2}{2n^2}\right)\left(1 - \frac{1}{2}\,\frac{e^4 Z^2}{4\varepsilon_o^2\, c^2 h^2}\,\frac{k^2}{n^4}\right) \qquad (3.5)$$

Considering Rydberg’s constant $R=e^4m_o/8\varepsilon_o^2ch^3$ and “ Lamb' s constant “ (known like constant of fine structure) $\alpha=e^2/2\varepsilon_o ch$ we have

$$E_{nk} = -\,\frac{2RhcZ^2}{n^2}\left(1 - \frac{k^2}{2n^2}\right)\left(1 - \frac{1}{2}\,\frac{\alpha^2 Z^2 k^2}{n^4}\right) \qquad (3.6)$$

Through Lamb' s constant the relativistic correction explains completely the Lamb shift of energy levels. For a different quantum state n’>n we have

$$E_{n'k'} = -\,\frac{2RhcZ^2}{n'^2}\left(1 - \frac{k'^2}{2n'^2}\right)\left(1 - \frac{1}{2}\,\frac{\alpha^2 Z^2 k'^2}{n'^4}\right) \qquad (3.7)$$

where k’=1, 2, …n’ is the quantum number of energy relative to the state n’.
Energy difference between the two energy levels according to relativistic correction is consequently

$$\Delta E=E_{n'k'}-E_{nk}=2RhcZ^2\left(\left(\frac{1}{n^2}-\frac{k^2}{2n^4}\right)\left(1-\frac{1}{2}\frac{\alpha^2Z^2k^2}{n^4}\right)-\left(\frac{1}{n'^2}-\frac{k'^2}{2n'^4}\right)\left(1-\frac{1}{2}\frac{\alpha^2Z^2k'^2}{n'^4}\right)\right) \qquad (3.8)$$

and therefore frequencies emitted by atom are

$$f=\frac{E_{n'k'}-E_{nk}}{h}=2RcZ^2\left(\left(\frac{1}{n^2}-\frac{k^2}{2n^4}\right)\left(1-\frac{1}{2}\frac{\alpha^2Z^2k^2}{n^4}\right)-\left(\frac{1}{n'^2}-\frac{k'^2}{2n'^4}\right)\left(1-\frac{1}{2}\frac{\alpha^2Z^2k'^2}{n'^4}\right)\right) \qquad (3.9)$$

For $\alpha=0$, $k=n$ and $k'=n'$ we have again Bohr and Sommerfeld' s frequency spectra. The quantum number k and the relativistic correction due to the variation of electrodynamic mass with the velocity are able to explain completely the fine structure of frequency spectra and the Lamb shift. Equations (3.6) and (3.9) are the foundations of the Relativistic Deterministic Quantum Physics (RDQP).

3.2 Spin quantum number and hyperfine structure

We deduce from (3.2) and (2.20) that the electron with momentum $p_{nk}=mv_{nk}$ has a quantized angular momentum $q_k=kh/2\pi$ dependent only on k and independent from n. This angular momentum is a mechanical momentum due to the orbital motion of electron around nucleus. An orbital electron is also equivalent to a moving electric charge and hence to an electric current that has a quantized magnetic moment[8]

$$M_k = \frac{\mu_o ehk}{4\pi m} \qquad (3.10)$$

and in Gauss units

$$M_k = \frac{ehk}{4\pi mc} \qquad (3.11)$$

For k=1 we have the Bohr magneton that is the magnetic moment in the energy fundamental level $M_1=eh/4\pi mc$.

Orbital electron is supplied in addition with a rotary motion around its axis for which it has an intrinsic angular momentum q_s (spin) that can be calculated easily. In fact if J is moment of inertia of rotating electron, ω_s is its angular speed, the spin momentum is $p_s=J\omega_s$ and considering for convenience of calculation a punctiform electron with mass m at distance r_s from the axis of rotation its spin angular momentum is $q_s=r_sp_s= r_sJ\omega_s$. We can associate a wavelength $\lambda_s=h/p_s$ with the rotating electron for which for stability' s sake $2\pi r_s=s\lambda_s$ where s is spin quantum number. Continuing in calculation we have

$$q_s = r_sp_s = \frac{s\lambda_s}{2\pi}\,\frac{h}{\lambda_s} \qquad (3.12)$$

$$q_s = s\,\frac{h}{2\pi} \qquad (3.13)$$

An elementary particle rotating around its axis is also equivalent to moving electric charge that produces a M_s spin magnetic moment equal to orbital magnetic moment M_k. From this we deduce in Gauss units for intrinsic motion

$$\frac{M_s}{q_s} = \frac{ek}{2mcs} \qquad (3.14)$$

and for orbital motion

$$\frac{M_k}{q_k} = \frac{e}{2mc} \qquad (3.15)$$

A few experiments (for example Zeeman effect[3]) prove that

$$\frac{M_s}{q_s} = 2\,\frac{M_k}{q_k} \qquad (3.16)$$

from which

$$s = \frac{1}{2}k \qquad (3.17)$$

We deduce therefore that electron has a spin angular momentum

$$q_s = s\,\frac{h}{2\pi} = \frac{1}{2}k\,\frac{h}{2\pi} = \frac{1}{2}q_k \qquad (3.18)$$

The spin angular momentum is the half of the orbital angular momentum. Because the spin angular momentum can have the same direction or opposite direction of the orbital angular momentum we can write

$$q_s = \pm\frac{1}{2}q_k \qquad (3.19)$$

Orbital electron in the n quantum state and in the k energy level has a total angular momentum $q_{kt} = q_k + q_s$ from which

$$q_{kt} = k\,\frac{h}{2\pi} \pm s\,\frac{h}{2\pi} \qquad (3.20)$$

$$q_{kt} = \frac{h}{2\pi}(k \pm s) \qquad (3.21)$$

It is manifest the spin decomposes every energy level in other two sublevels very close between them and therefore energy complete levels of atom are

$$E_{nks} = -\frac{2RhcZ^2}{n^2}\left(1 - \frac{k^2}{2n^2}\right)\left(1 - \frac{1}{2}\,\frac{\alpha^2 Z^2 (k \pm s)^2}{n^4}\right) \qquad (3.22)$$

where n=1, 2,….. ; k=1, 2, …….n; and s=k/2.

For $\alpha=0$ and k=n we have again Bohr and Sommerfeld' s energy levels.

The equation (3.22) defines the hyperfine structure of frequency atomic spectra with the splitting of energy levels through the agency of the s quantum number.

3.3 Complex atomic systems

The above explained theory is relative to the hydrogen atom and to simple ions with only one orbital electron, complex ions have over one orbital electron. In that case every electron fills the most inside free energy level. The state with n quantum number contains n levels of energy (k=1, 2, ..n) because of the fine structure of atom and energy levels belonging to the same quantum state contain the same number of electrons. Every k energy level with n quantum number can contain n electrons and the quantum state n can contain a total of n^2 electrons. Taking account of the hyperfine structure every electron can have two values of spin and the n quantum state contains therefore at the maximum $2n^2$ electrons.

Quantum state n	Maximum number of electrons for energy level =2n					maximum number of electrons for the quantum state n $=2n^2$
	k=1 level s	k=2 level p	k=3 level d	k=4 level f	k=5 level q	
n=1	2e					2e
n=2	4e	4e				8e
n=3	6e	6e	6e			18e
n=4	8e	8e	8e	8e		32e
n=5	10e	10e	10e	10e	10e	50e

table1. In the classical representation the levels (s, p, d, f, q) have always the same maximum number of electrons for any n quantum state: s=2, p=6, d=10, f=14, q=18. In our representation energy levels of every n state are equivalent and the number of electrons for level is determined only by the quantum number of state.

Calculating the energy levels through the equation (3.22) we see that electron energies relative to the quantum state n=4 (energy level k=1) are in absolute value greater than electron energies relative to the quantum state n=3 (energy level k=3) in accordance with the experimental results. Similarly electron energies relative to the quantum states n=5 (energy levels k=1, k=2, k=3) are in absolute value greater than electron energies relative to the quantum state n=4 (energy level k=4).

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