Math Matters: Why Do I Need To Know This?

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Episode Three

1 Logical fallacies – Spotting bad arguments

Objective: To illustrate the relevance of symbolic logic by showing some common forms of "bad" logic in their symbolic representations.

Hello, and welcome to Math Matters, a program where we look at the mathematics in our entry level courses and try to show you, hopefully show you how that's useful to you in your everyday life. I've got some neat stuff to show you today, so let's get right after it. I want to talk to you about logic again. We've spoken a little bit about logic, but today I want to talk about logical fallacies and what I mean by that is incorrect logic, how to spot it, how not to use it, those kinds of things. The main purpose of all this stuff we do with logic in the first place is so that we can spot valid arguments, we can make valid arguments, and we can particularly keep ourselves from falling prey to one that, to an argument that's not valid, that's not believing what we see.

Now the word "valid," what I mean by that is that you take a concluding statement called a conclusion and that has to follow from all the initial statements which sometimes they are called hypotheses or premises, either one, they're both correct. But, the point is, that the argument itself has very little to do with the truthfulness of those statements. It's whether or not the conclusion has to follow from the statements. So, let me give you a couple of examples. Now this is a nonsense argument, but it's a good one. If we're just saying "All bears are cats," and then I say "I am a bear, therefore I am a cat." Well, those are nonsense statements, they're not true, none of them. However, if you take the first two statements as fact, the last statement, the conclusion "Therefore I am a cat" has to follow from those two so we call that a valid argument. (Figure 1)

Here's another example: "If you've ever owned a record player, then you are old" and then "If you are old, then you watch Matlock reruns," and "Therefore, if you have ever owned a record player, then you watch Matlock." Well again, those statements aren't necessarily true either. I had a record player and I'm not old. Well, not real old anyway, but I don't watch Matlock. But the point is that if you take each of those things for truth, then that last statement, the "therefore" statement has to follow from the previous two, so that's a valid argument. (Figure 1)

Now the common, kind of logical mistakes are called logical fallacies and there's a lot of these if you go to websites and you look up logical fallacies, you'll see a lot of them that aren't based on logic at all, they're based on behavior. Proof by celebrity: if the celebrity comes on and tells you to buy a certain product. Well, that doesn't mean the celebrity knows anything about the product, it just means that the product company was able to pay for the celebrity. That's what it means. So, the ones that I'm going to show you though are based in logic. These are logical fallacies in the sense that they mess up, logically they mess up, not because of our behavior.

Example:

"All bears are cats. I am a bear. Therefore, I am a cat." **valid**

Example:

"If you have ever owned a record player, then you are old. If you are old, then you watch *Matlock* reruns. Therefore, if you have ever owned a record player, then you watch *Matlock*."

The **statements** are **not** necessarily **true**, but the argument is **valid**.

Figure 1, Segment 1

The first one I'll mention is the fallacy of the converse. "If a person reads the daily news" for example, "then they are well-informed". "This person is well-informed." "Therefore, this person must read the daily news." Well this is, that's an argument that somebody might try to make. It's not valid though, because this says "if" the daily news, "then" well-informed. It doesn't tell us the other way. You could be well-informed because you watch CNN or because you get your news online or whatever it is. So that is an invalid type of argument. (Figure 2)

Another would be what is called a fallacy of the inverse and I'll give you another example: "If a person reads the Daily News, then they are well-informed. This person does not read the Daily News, therefore this person is not well-informed." Now that sounds maybe perhaps valid, but it's not. When we say the Daily News implies well-informed, that statement really tells us nothing about folks who do not read the Daily News. And then if we try to draw a conclusion about the folks who don't read the Daily News, that's not going to be a valid argument. That business of inverse, that's the inverse is when you reverse the "if" and "then" part, oh wait, I'm sorry, not reversal, but you negate them, not this one and not that one. So that's where the name comes from. (Figure 3)

And one last one I'll show you is the fallacy of the false chain. For example: "If you have ever owned a record player, then you are old. If you have ever owned a record player, then you watch Matlock reruns. Therefore, if you are old, then you watch Matlock." Okay, now what you've here are two kind of conditional statements, that "record player" imply both "old" and "Matlock". What does that tell us about the connection between the two? Well, it actually doesn't tell us anything. You know, if somehow "old" implied "record player" then "record player" would imply "Matlock" and we could make that, we could draw that conclusion, but

There are common logical mistakes that are often presented as valid arguments, called **logical fallacies**.

Fallacy of the Converse:

"If a person reads the *Daily News*, then they are well-informed. This person is well-informed. Therefore, this person must read the *Daily News*."

"*Daily News* $\Rightarrow$ well-informed", not "well-informed $\Rightarrow$ *Daily News*"

invalid argument

Figure 2, Segment 1

Fallacy of the Inverse:

"If a person reads the *Daily News*, then they are well-informed. This person does not read the *Daily News*. Therefore, this person is not well-informed."

"*Daily News* $\Rightarrow$ well-informed" tells us nothing about folks who do **not** read the *Daily News*. Any conclusion based on the assumption that it does will form an **invalid argument.**

Figure 3, Segment 1

that's not the case, and so this would be an invalid argument as well. (Figure 4)

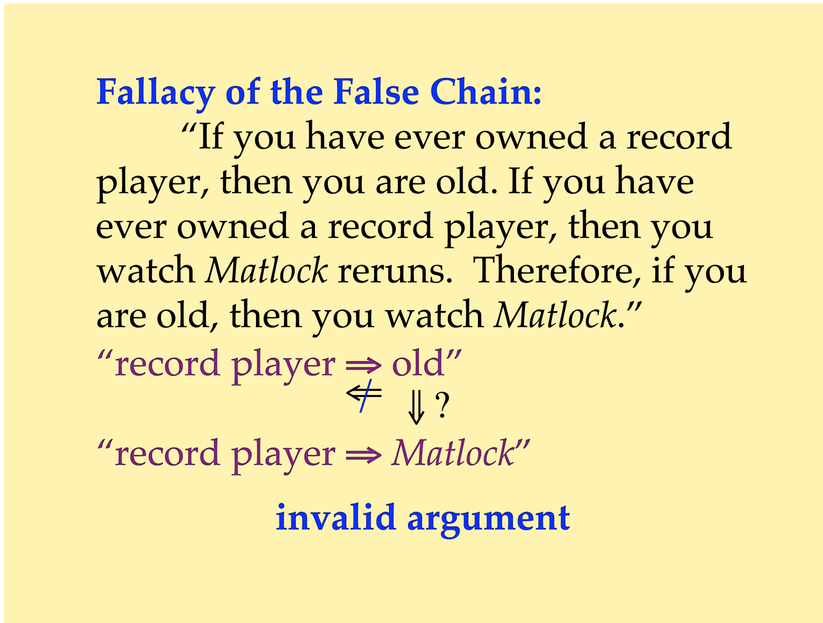
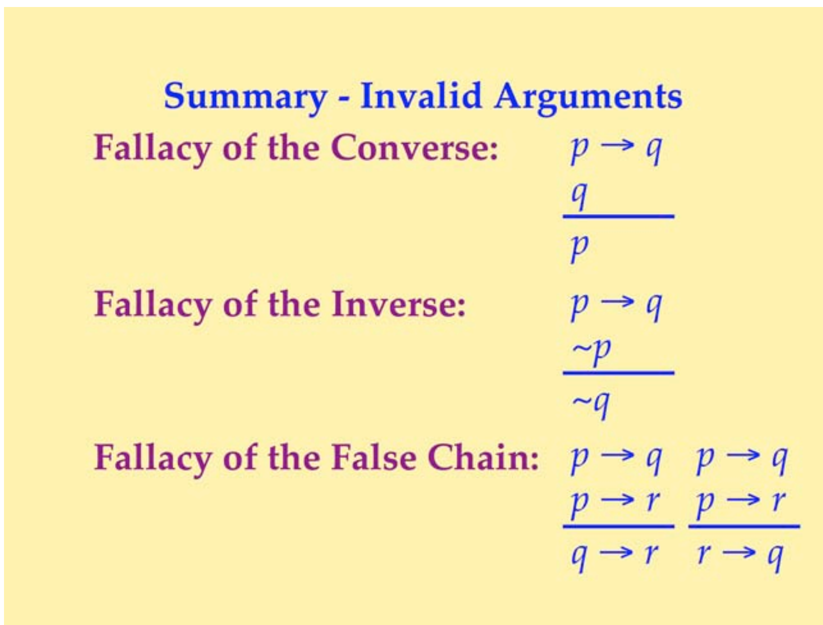


Figure 4, Segment 1

And so kind of in closing on this little topic, I'm going to flash up, what I've done is I've taken each of these things and kind of put them in symbolic form and I'll just kind of flash those up as a table right now.



Summary page, Segment 1

2 Tessellations, Part 1 – Home improvement projects with tile

<u>Objective</u>: To show how the geometric concepts of tessellations and angle measures apply to common home-improvement applications, specifically, laying tile. The concepts of polygons, regular and convex, are introduced graphically, and a formula is developed for showing the measure on an inside angle of a regular polygon.

The other thing I'd like to show you today is on a topic called tessellations and it actually involves a bit of geometry, but it also involves a bit of algebra and you'll see where the algebra comes into it. The way I'm going to kind of frame this is in a home improvement project. Let's say, for example, that you are a tile contractor or maybe you're a do-it-yourselfer who can do tile and you've got somebody that you're trying to make happy. It could be your client, your customer, spouse, whatever. You've got somebody paying the bills and they want a very creative kind of tile design laid out in their bathroom or something like that. And they're willing to pay for the creativity so you're willing to go some lengths to make them happy here. So they want to know, what can I, what can you do to make, you know, make me happy? So the question we have to answer is, okay, an eye-catchy kind of tile design that's innovative, but yet at the same time it's doable. It's got to be something that we can actually do.

Well, you can flash these up. These are the usual kind of tile patterns. We've got this in our bathroom where you just have squares arranged. (Figure 1) *You can juke that up by maybe doing different colors, maybe those kinds of things, getting a design in the tiles.* (Figure 2) *You can have hexagonal tiles and we've actually, I've had that in a bathroom too.* (Figure 3) *You could maybe juke that up, maybe get a soccer ball pattern going.* (Figure 4) *You can even do triangular tiles.* (Figure 5) *This is bit more work, there's more tiles, but you can do that or you can even get designs in that if you'd like, things like that. But that's not enough. "Yeah, but what else could you do?"* (Figure 6) *Let's be creative here, right? Alright, so what we've got to do here is decide what happens, what has to happen in order for shapes to fit together? And that's where we get into talking about tessellations.*

A tessellation is just a collection of shapes that cover a flat surface. And to do this, I want to talk to you a little bit about degree measure. Okay, the distance from all the way, well the distance all the way around a point is 360 degrees. If we cut that in half, a straight, what we call a straight angle is 180 degrees. Okay, I'm assuming that you, I'm hoping you've heard that before but I just want you to be familiar with those degree measures. Now, in order to make this doable, I'm going to use polygons. Closed in, now by that I mean a closed line made of shape, a closed shape made of line segments that don't cross. So for example, this is a polygon, a good example. These are not. The first one's not because they cross. The second one is not because it's a circle. It's not made of line segments. And we are restricted, if we're going to use polygons, the lowest number of sides we can have would be a triangle that would have three sides. (Figure 7)

I'm also going to talk about a word here, convex. That means that all the interior angles are less than 180 degrees. So for example, that's actually my example again, that is a convex polygon and there's one that's not convex that interior angle right here is bigger than 180.

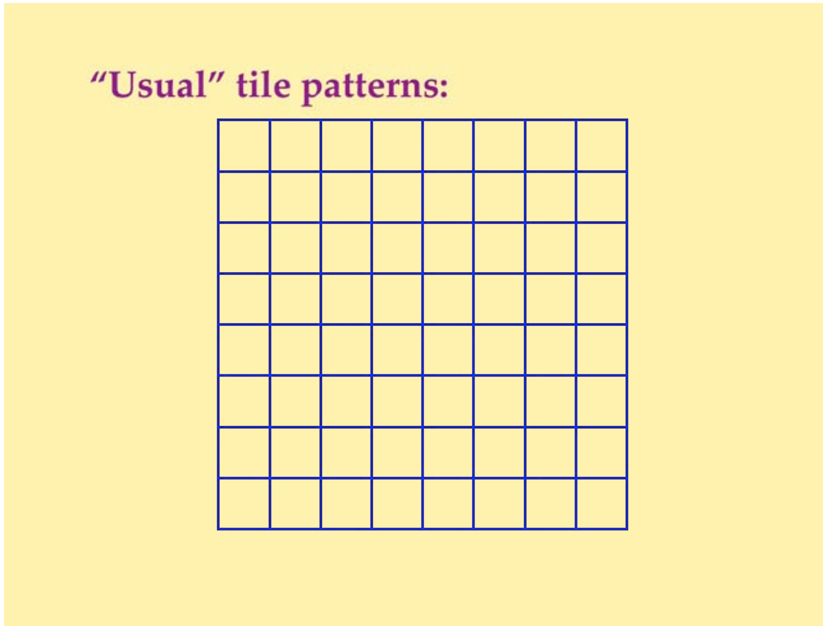


Figure 1, Segment 2

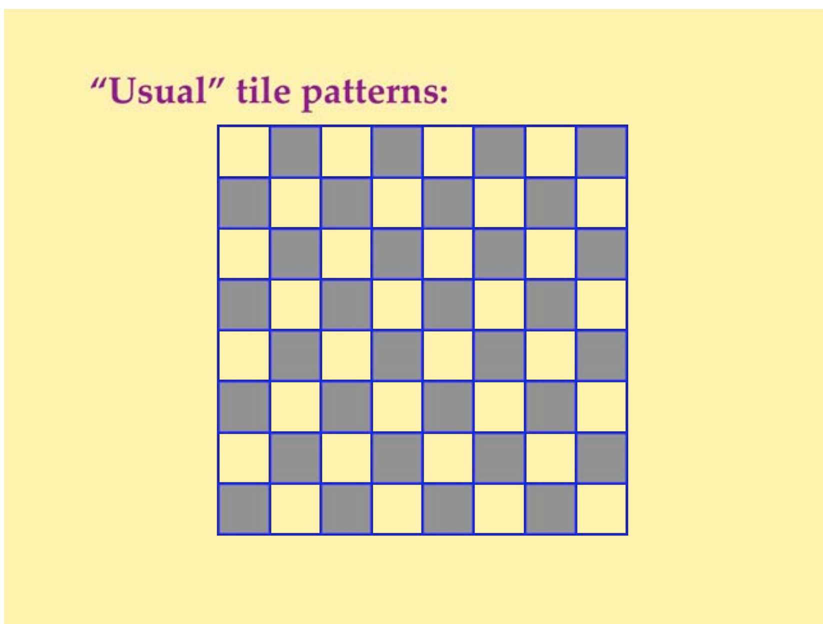


Figure 2, Segment 2

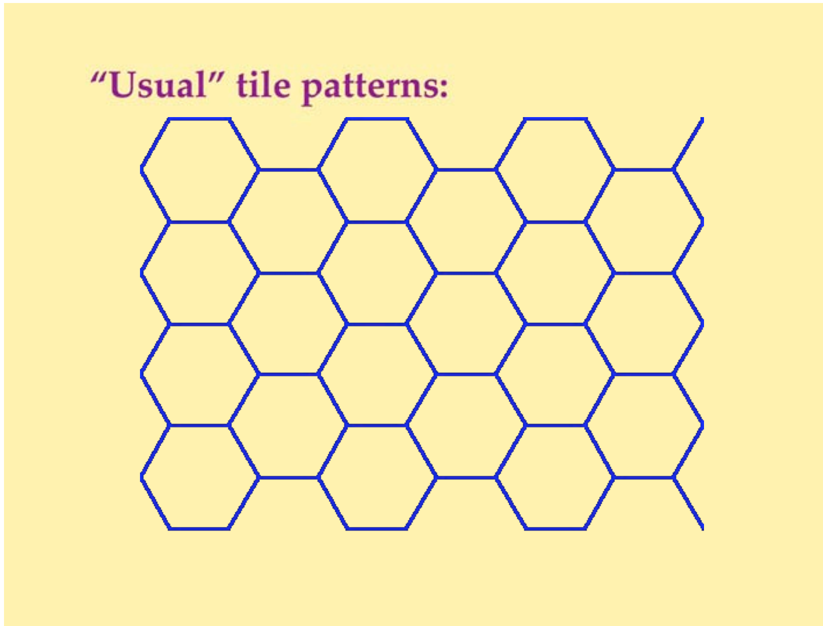


Figure 3, Segment 2

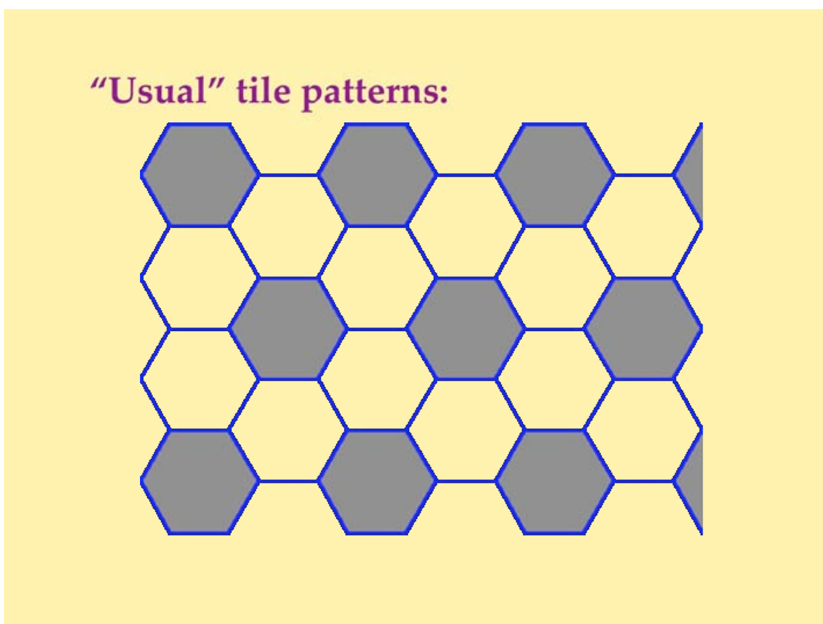


Figure 4, Segment 2

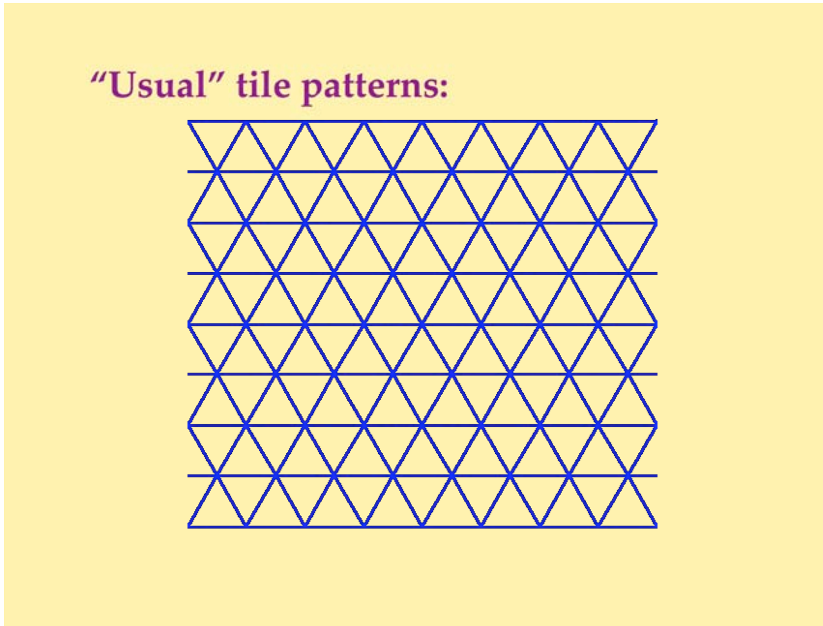


Figure 5, Segment 2

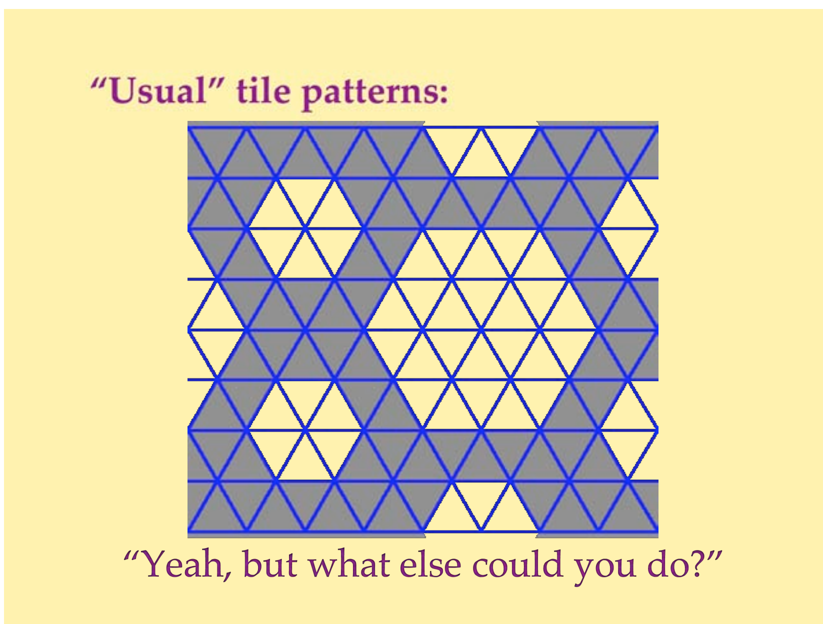


Figure 6, Segment 2

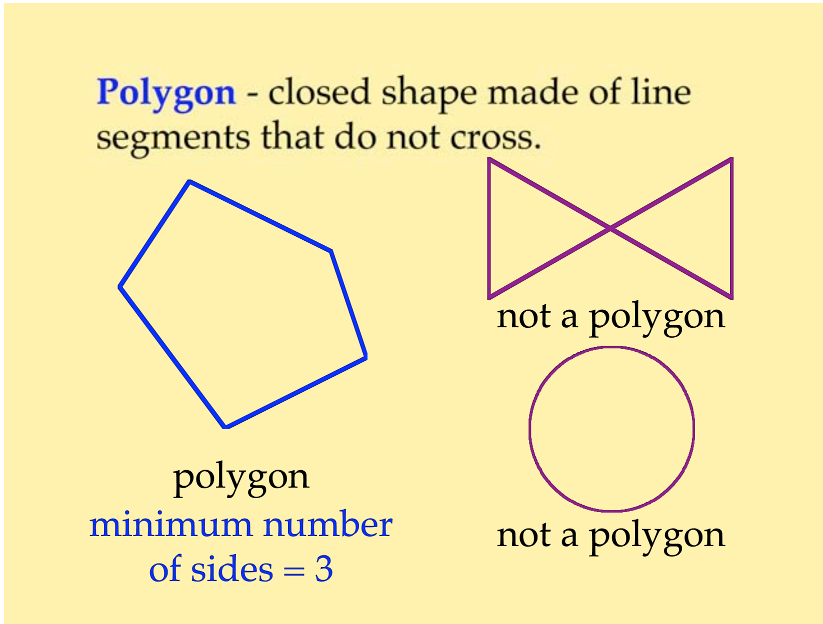


Figure 7, Segment 2

(Figure 8) *Now the sum of the measures of the inside angles of a triangle is 180 and I'm actually going to prove that to you. I've got a nice visual kind of demonstration of that. Here's a triangle, generic triangle, you've got angles one, two, and three and to kind of prove to you that these come together as 180 degrees, let me do a little apology here. This is where it gets a little hazy. One, two, and the last one, there you go. Does that look like a straight angle? It really is. You take those three angles, put them together and you've got a straight angle. So the sum of the measures of a triangle, angle measures in a triangle equals 180.*

Now, that actually helps us find the sum of the measures in any convex polygon. Now watch this trick. This is a five-sided polygon and what I'm going to do is I'm going to start at one vertex and split it up into triangles. Now those triangles all the angles of these triangles are interior angles or parts of interior angles. So this one is an interior angle, those can be added up to get an interior angle and I know that the sum of the measures of each of those is 180. So depending on the number of sides I have, I always get two less triangles so it'd be 180 times the number of sides minus two and that always will then give me the sum of the measure of the angles inside the convex polygon. (Figure 9)

Now, lastly, to make this doable, I want to use regular, what are called regular polygons and that is a polygon where all the sides are the same length, you know, that's going to be useful for putting things together, and all the inside angles have the same measure and that forces it to be convex and I can use that formula. For example, this is a regular polygon. It's a five-sided, all the angles are the same, all the lengths are the same. You have to have both. This is a rhombus, which means the sides have the same length, but the angles are different. That is not a regular polygon and neither is this rectangle. The angles are all 90

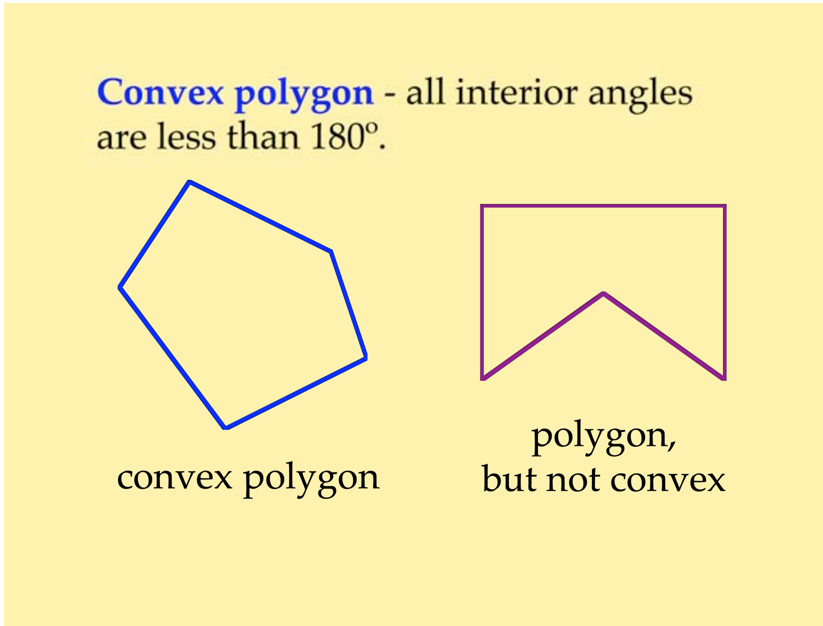


Figure 8, Segment 2

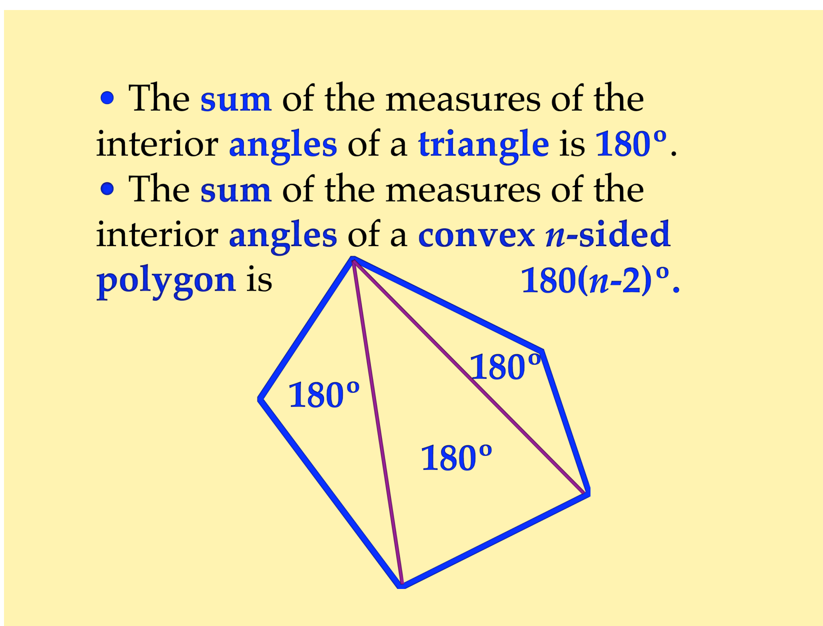


Figure 9, Segment 2

degrees, but the sides are different lengths. Those aren't regular, I'm going to use things like this. (Figure 10) *And so using that formula, well I haven't developed it. I take that total and because all those measures are the same, I can just divide that by "n" and that will tell me the measure of one angle. So for example with a triangle, 60 degrees, with a square, 90 degrees. I'm just using my formula now. For a pentagon, that's 108 degrees. There's a regular six-sided called a hexagon and those angles are 120 degrees. There's a heptagon, hepta stands for seven, weird angle measure, you know I've even got a fraction here, but that is the case. And then finally, an eight-sided polygon is an octagon, those angle measures are 135 degrees.* (Figure 11)

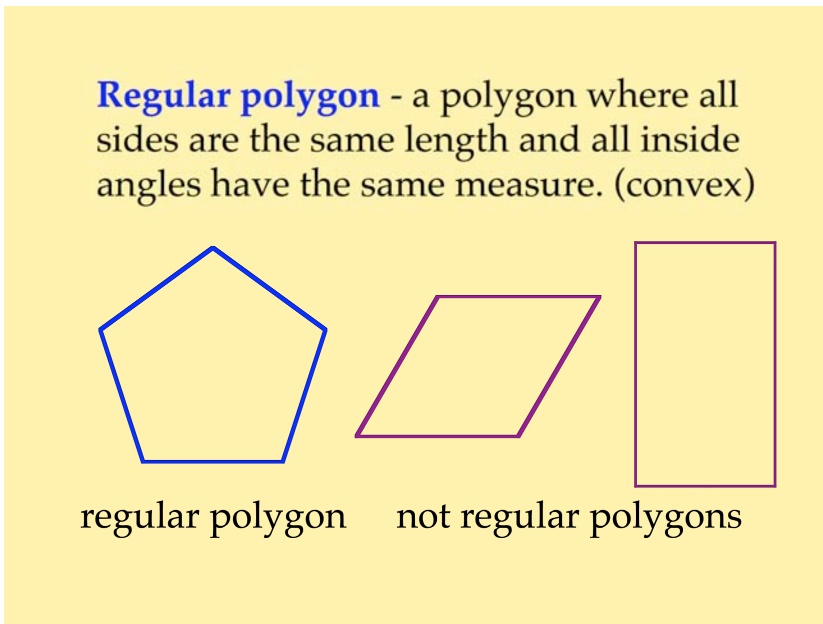
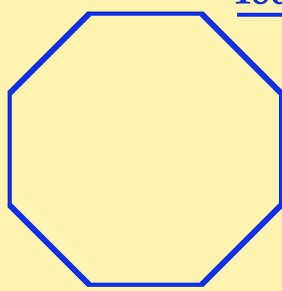


Figure 10, Segment 2

Now, what I've got to do is figure out how I'm going to put this together and I want to do that, I do want to spend just a minute kind of reviewing. Now I just talked about a bunch of stuff so I'm going to spend just a couple of seconds letting you digest that while I get ready to go on to the next piece so let's flash up, I've got some slides that show of the things I've kind of talked about here.

• Therefore, since the measures of the angles in a regular polygon are equal, the measure of any **one angle** of a **regular *n*-sided polygon** is $\dfrac{180(n\text{-}2)^\circ}{n}$.



sides	angle
3	60°
4	90°
5	108°
6	120°
7	$128\frac{4}{7}^\circ$
8	135°

Figure 11, Segment 2

Summary

Tessellations - shapes that cover a flat surface.

Polygon - closed shape made of line segments that do not cross.

Convex polygon - all interior angles are less than 180º.

• The **sum** of the measures of the interior **angles** of a **triangle** is **180º**.

Summary page 1, Segment 2

Summary

Regular polygon - a convex polygon where all sides are the same length and all inside angles have the same measure.

- The measure of any **one angle** of a **regular *n*-sided polygon** is $\dfrac{180(n-2)^\circ}{n}$.

sides	angle
3	60°
4	90°
5	108°
6	120°
7	$128\frac{4}{7}^\circ$
8	135°

Summary page 2, Segment 2

3 Tessellations, Part 2 – Home improvement projects with tile

Objective: This segment continues the problem started in the previous segment, and addresses the necessity of having the inside angle measures add to 360° and how that is insufficient to create a tessellation of the plane. We examine and illustrate all possible semiregular tessellations of the plane.

Okay, now we have not addressed the original problem and the original problem is how do I get a nice creative pattern, let me just kind of flash it up there for you again. How do I get a nice regular kind of design that's different, it's not something you'd find all the time in bathrooms around, but yet it's something that's doable and using regular polygons is going to help make that doable because all the sides will be the same length, I don't have to worry about these kinds of things and also I want designs that repeat, I want things that happen all the time. I don't want to have to change different designs as I go around, so let me flash up These are the usual kind of things that you see. These are what are called regular tessellations and that means that yes, they do use regular polygons to kind of cover a flat surface, but also, they use the same polygon, you know. Here I've got four squares around a point each time and here I've got three hexagons around a point, 120, 120, 120. And then I've got six triangles around a point each time. So those are called regular tessellations. And the thing I want to point out about this is the distance, I'm sorry not the distance, but the sum of the angles around each vertex, around each corner is 360 degrees. And so that's my goal: Can I find a mixture of regular polygons whose angles add up to 360 degrees? (Figure 1)

Now, there's my formula for calculating the angles of regular n-sided polygons. I do have some limitations. The number of sides obviously has to be more than three, we talked about that, and it has to be a whole number. That's important as well. The measure of the angles, well a three-sided and I showed you this just a second ago, a three-sided regular polygon, a triangle, has sides 60 degrees. So that's the lower bound on the measure of the angles. The upper bound, well if you look at that formula, I've got "n" minus two over "n", I'm putting in positive numbers so that number whatever it is, is a fraction less than one, and then when I multiply both sides of that by 180 that means that the upper bound on that angle would be 180 degrees. So I can't have an angle in these polygons bigger than 180 degrees. And then lastly, how many polygons can I put around a point? Well because the angle measures are less than 180, I have to have at least three. And because the smallest angle is 60, if you take 360 and divide by 60 you get six. So there is a limitation to the number, different types of combinations I can have around a point. I've got between three inclusive, three and six polygons around a point. (Figure 2)

So there turns out, it turns out that there are a finite number of ways to do this and you can find them systematically. You can start off with three polygon sets and then work up to four, then five, and then six and I'll show you the start of this. I don't want to do the whole thing but I'll show you how to do this and it actually helps if you know the inverse of the angle formula. The angle, I'm using a theta here, it looks like a zero, but it's a theta, it's a greek symbol. The angle is equal to this formula, right? So you can go through and solve for "n." That gives you the inverse formula. If you know the angle, then you can find the "n."

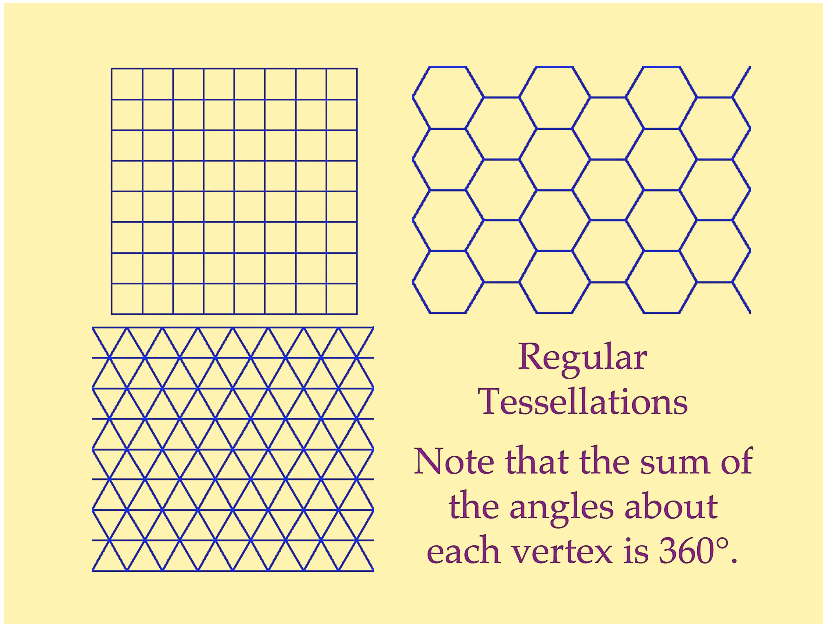


Figure 1, Segment 3

Can we find mixtures of regular polygons whose angle add to 360°?

Recall that the measure of any **one angle** of a **regular *n*-sided polygon** is

$$\frac{180(n-2)^\circ}{n}.$$

Number of sides: $3 \le n$, **whole number**

Measure of the angles: $60^\circ \le \theta < 180^\circ$

Number of polygons about a point:

$3 \le k \le 6$, **whole number**

Figure 2, Segment 3

It's just a little bit of algebra. You would clear the fractions multiplying through by "n" and then we're going to solve for "n" so I've got to get all my "n" business to the other side of the equation, right? And then you factor out the "n" and then if you don't like, I don't like the negatives here, so you could multiply through by a negative and that changes the order and gets rid of it here and then just divide by the junk. So you've got a nice little formula here once you know the angle that's left, this will tell you how many sides you have, the "n". So we'll just use that little formula. (Figure 3)

There are a **finite** number of ways to arrange regular polygons around a point. We can find them *systematically* by starting with **3**-polygon sets and working up to **4**, then **5**, and then **6**. Also, it helps to know the **inverse** of the angle formula:

$$\theta = \frac{180(n-2)}{n}$$

$$\theta n = 180n - 360$$

$$\theta n - 180n = -360$$

$$(\theta - 180)n = -360$$

$$(180 - \theta)n = 360$$

$$n = \frac{360}{180 - \theta}$$

Figure 3, Segment 3

So, let's start with three polygons. Let's do it systematically and kind of show you how you would kind of go about this. I would start with three and then seven. Why not three and three? Well, the first two have to add up to more than 180, right? The third one has to be less than 180. So the lowest number of sides I could use would be seven. And I go through and I calculate okay this is the sum, this is the measure for the equilateral triangle, this is for the heptagon. And so I'm left with an angle of $171\frac{3}{7}$ *degrees which sounds ridiculous, but if you plug that into the formula, that gives you a whole number, a 42-sided polygon. You could do that. It looks like a circle really because the angle's so shallow, but you could do that so there's one combination. If I go with three and eight, I'm just letting the second polygon get bigger. It works out to where I could use a 24-sided polygon. A triangle and a nine sided would give me an 18-sided polygon.* (Figure 4) *A three and ten gives me a 15-sided polygon. Three and eleven, this is kind of weird, 60 is the angle in the triangle. This is the angle in an eleven sided polygon. It gives me this left and when you plug that into a formula, you get thirteen and a half. Well I cannot have a thirteen and a half sided polygon, so I've got to bounce that one, that's not going to work at all. But you can just keep running* *through these. Three, twelve, and twelve will work. And then once these, I'm taking these in increasing order, so once I get to twelve, I'm done with the threes, and then I would move on to the fours.* (Figure 5)

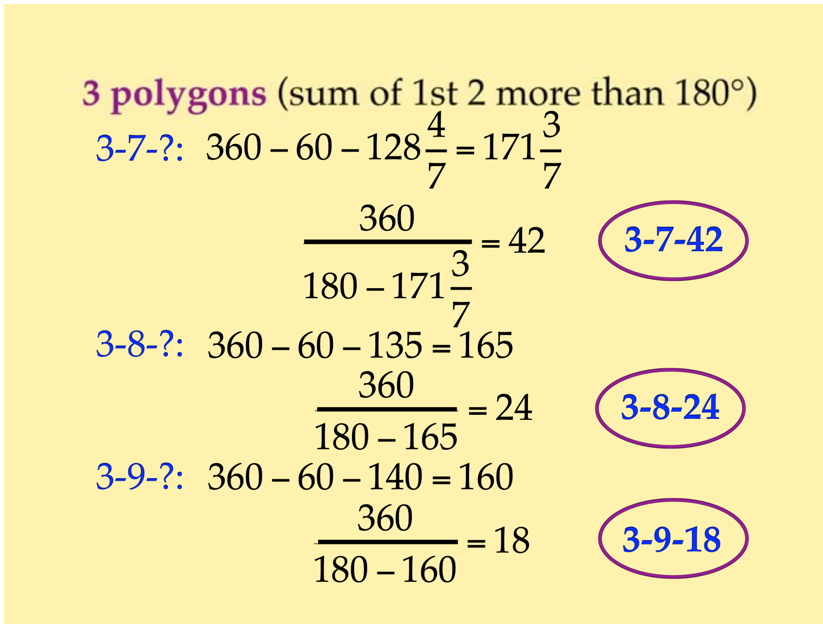


Figure 4, Segment 3

So what I've done, I've actually run through all of them, I'll list them for you. It's not worth going through, you know the process now, so you could do it. You start with the fours and you pick up a few more. You go to the fives and you actually have one and then go to the sixes and that's the one I showed you, the three hexagons. Then you go to four-sided things and now with four things, the order they appear matters. So you can split up the threes by putting the four between them. Okay, then three, three, six, six, you can split up the threes and sixes by intermingling those. Three, four, four, six and you can split up the fours by intermingling those and then the last one there and you can look at the fives and sixes. (Figure 6)

The real question is these are ways to get them to add up to 360 degrees, the question is, "Can these things be applied through the plane repeatedly? Just because they add up to 360 degrees does that, is that all that I need?" Okay, the way to kind of answer this is really to cut your patterns out and try this, but I can show you that it won't work in all cases. It will work in some obvious cases. The ones here where they're all the same, those are the regular tessellations I showed you earlier, okay? (Figure 6) *Some will work, some will not. The four, eight, eight will work. Here's a square with two octagons and I want this same pattern in each corner. Well I can extend that. I think that's pretty obvious that you can extend that. You just keep doing that and you've got the same mixture of polygons in every corner.* (Figure 7) *However, something like five, five, ten, here's a polygon and a polygon*

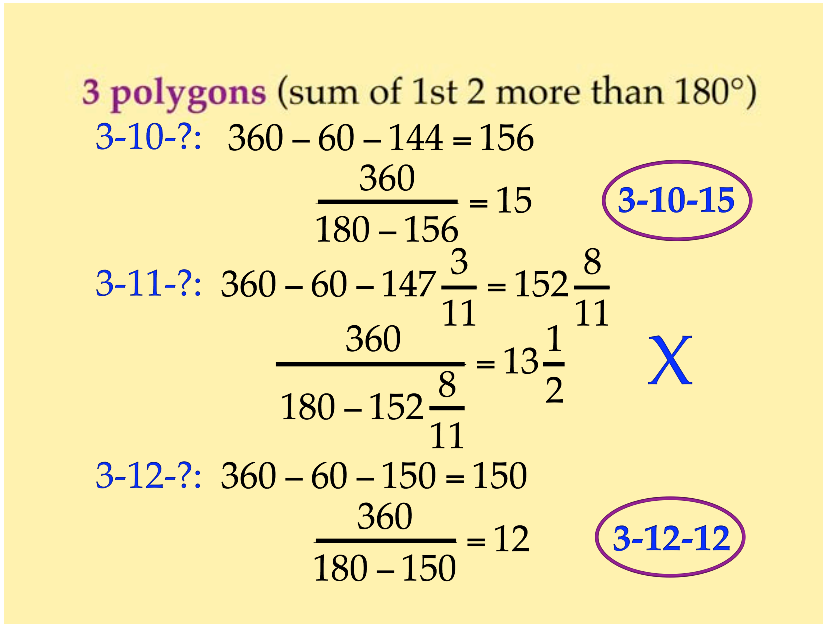


Figure 5, Segment 3

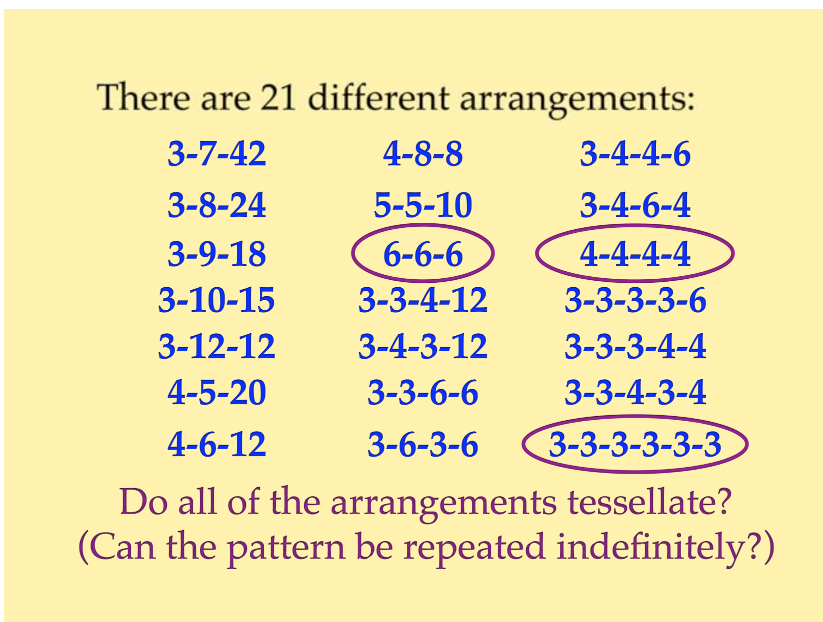


Figure 6, Segment 3

and a ten-sided polygon. Okay, and it'd have to be five, five, ten here. It has to be five, five, ten here and there, but you see at that blue point, you do not have enough space left, you do not have enough space left to put in another polygon. In fact, you don't have the pattern you need, you've got five, five, five. (Figure 8)

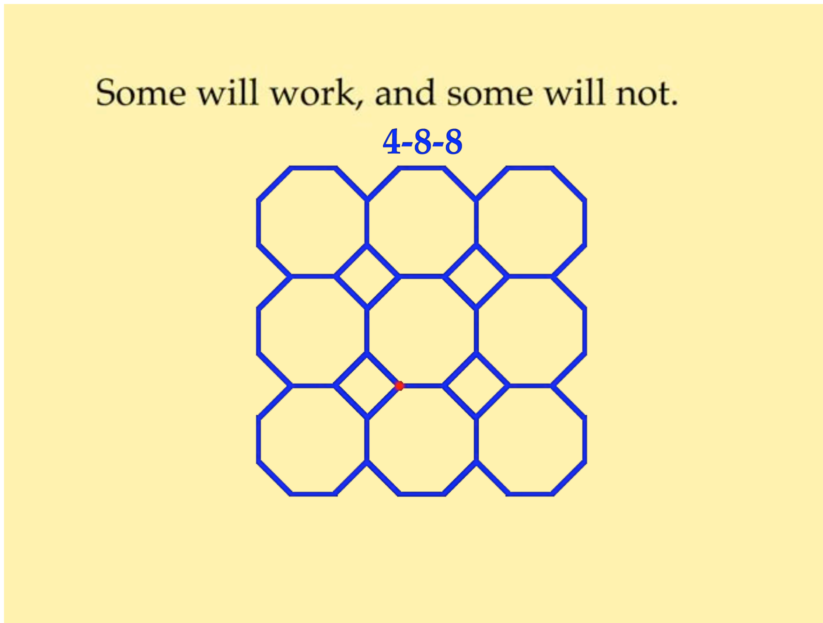


Figure 7, Segment 3

Okay, so some of these will work, some of them won't, and I think what I'll do is simply show you, rather than go through this exercise with all of them, I'll show you the ones that do work. The three, twelve, and twelve will work. Also, so does the four, six, and twelve. And these are some nice cool patterns. I have to say that these would look neat on a wall or a tile floor or something like that. (Figure 9) *Oh, and good luck finding a twelve-sided tile, regular polygon tile. That might be something that could be special made, but you might have trouble finding that at Home Depot or Lowe's or something like that. Four, eight, eight, I showed you it would tessellate, and I've actually seen this. This is one, I think octagonal tiles are something you could find at the store pretty easily and you could perhaps mix it up with some colors. Do the squares a different color than the octagons. Three, six, three, six, if you'll notice here this is very linear, you see a lot of lines through here, but you have triangle, hexagon, triangle, hexagon. At each point, triangle, hexagon, triangle, hexagon, triangle. That happens at every point. You could perhaps mix that up with some different color tiles.* (Figure 10) *These are the really cool ones. Three, four, six, four, if you'll notice, you get these almost circular shapes, now let me convince you that this is really going on at each point. Three, four, six, four and then we go to a different point. Three, four, six, four, three, four, six, four, it happens at every point. You could really do something cool with this. I think this would be a neat tile floor. And then also the three, three, three, three, six could*

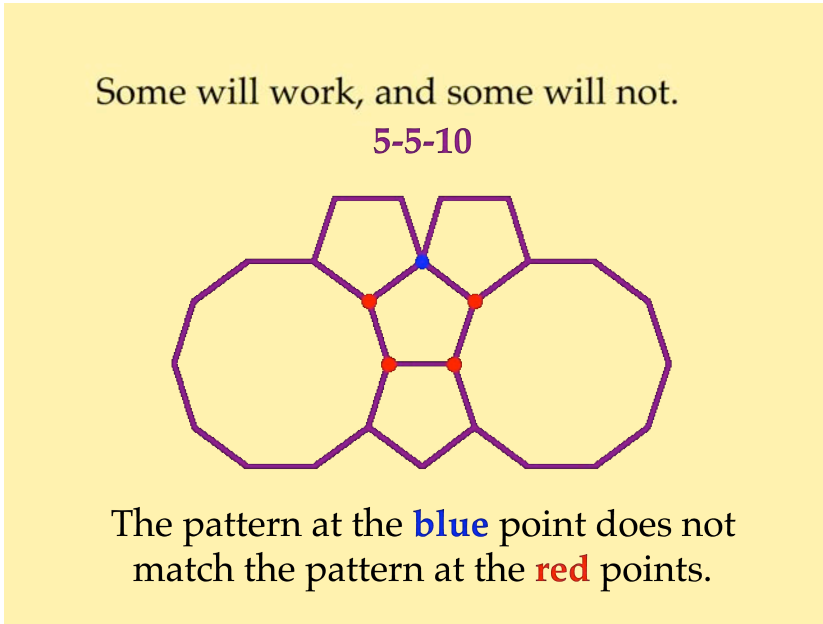


Figure 8, Segment 3

be put together as well. These are called semiregular tessellations and by that we're using regular polygons, so we keep the word regular, but we say semiregular to mean that it's not entirely one polygon. We're using different types of polygons to tile the plane. (Figure 11)

There's a lot of other things you can do if you abandon this business of using polygons and maybe we'll get a chance to talk about that, but you do some cool Escher things where you start with a polygon, you remove stuff from one side and you pack it on to the other side, and those kinds of things, when you do that still fit together. I guess what I'd like to do is just very quickly give you some slides here, some summary slides that show you then the different things that are possible. Oh wait, oh, I've got a couple left. I forgot these, there's eight. Scratch the summary thing, let me show you this real quick. I forgot these even were there. This is the coolest of the cool right here. The three, three, three, four, four is okay but it's very linear. That's my bad, I forgot these were even here. It's very linear, so I don't know if you'd like that or not. The three, three, four, three, four, this is this is bad. Three, three, four, three, four, and that happens at every point. Three, three, four, three, four, that would be a really interesting design. So I guess what I'll do in closing is, and you can doctor these up with any kind of colors you want. (Figure 12)

What I'd like to do is kind of leave you with these patterns. You can always go back and review these and look at them, but very quickly I'll flash up some summary slides that show you the different kind of regular tessellations and then also the semiregular tessellations that we talked about.

Closing

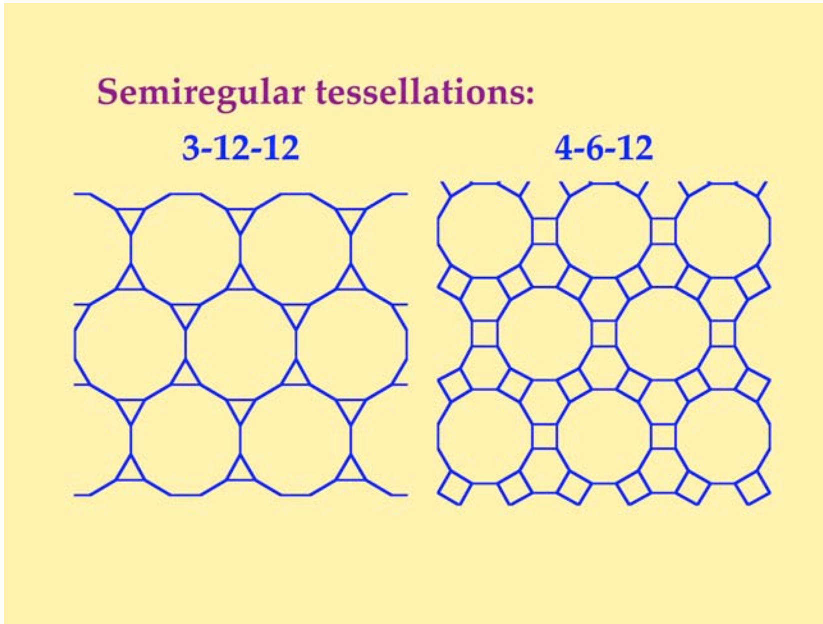


Figure 9, Segment 3

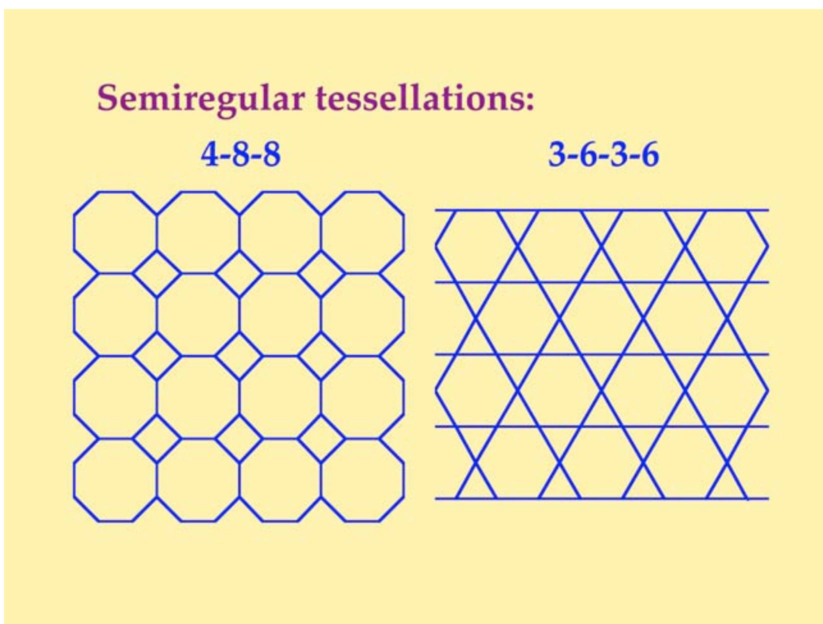


Figure 10, Segment 3

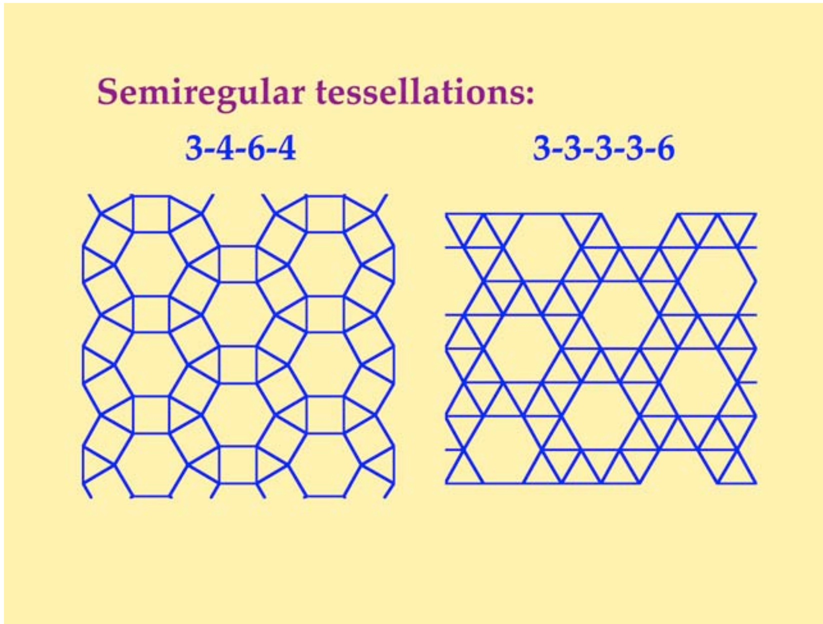


Figure 11, Segment 3

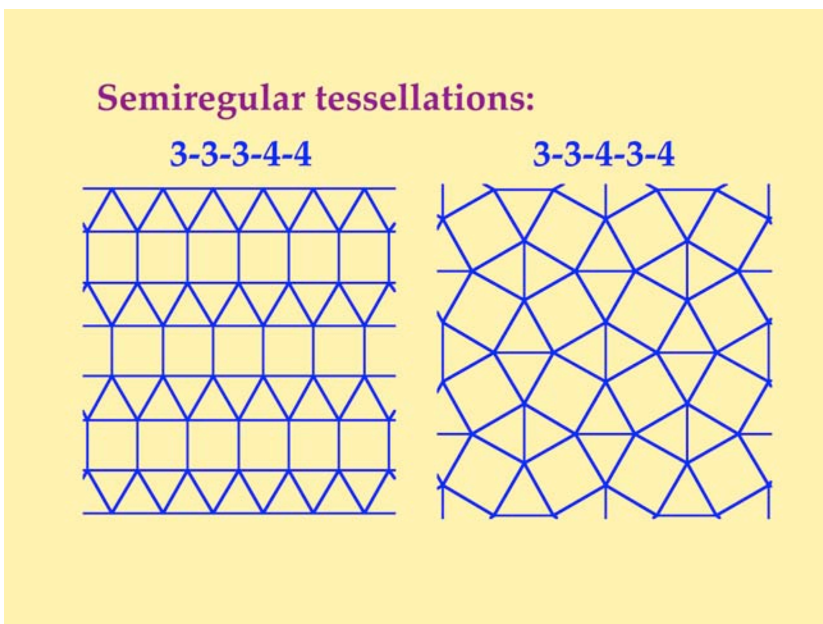
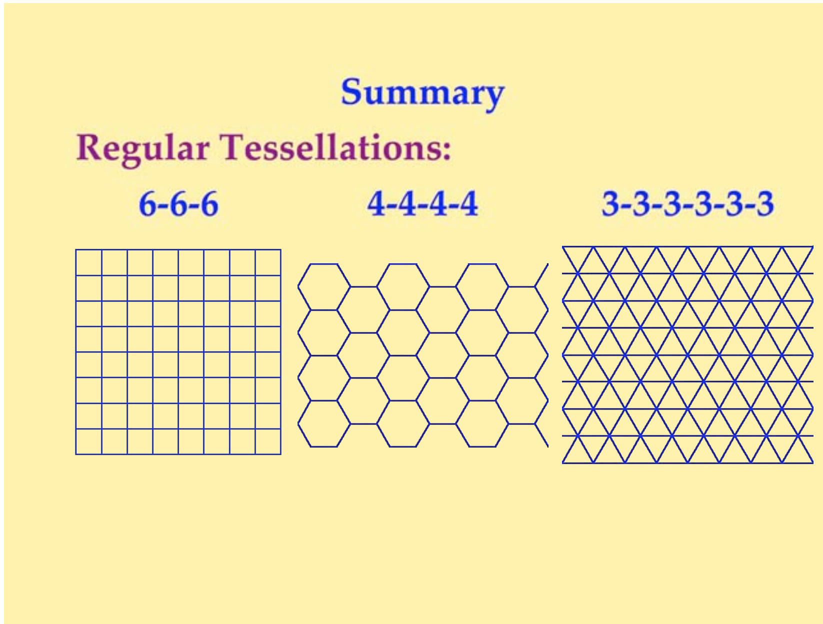
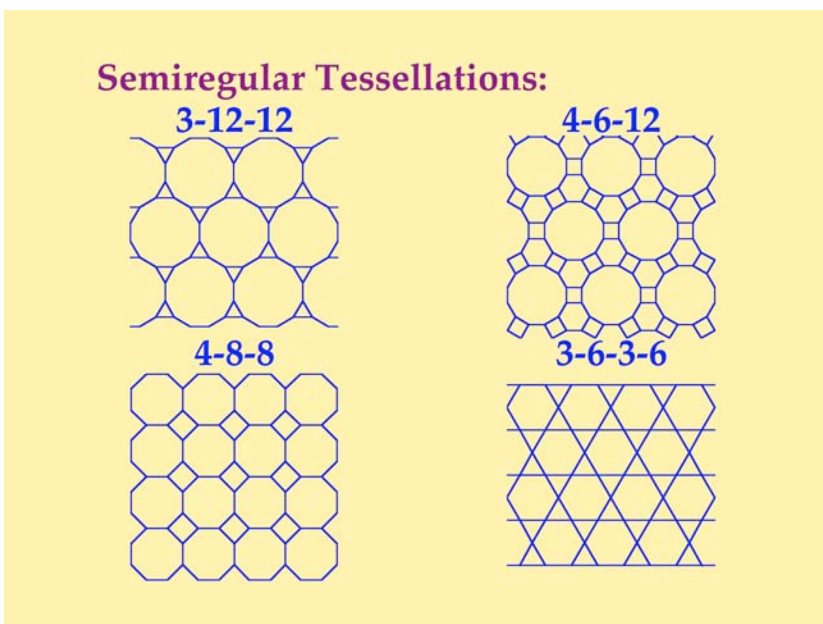


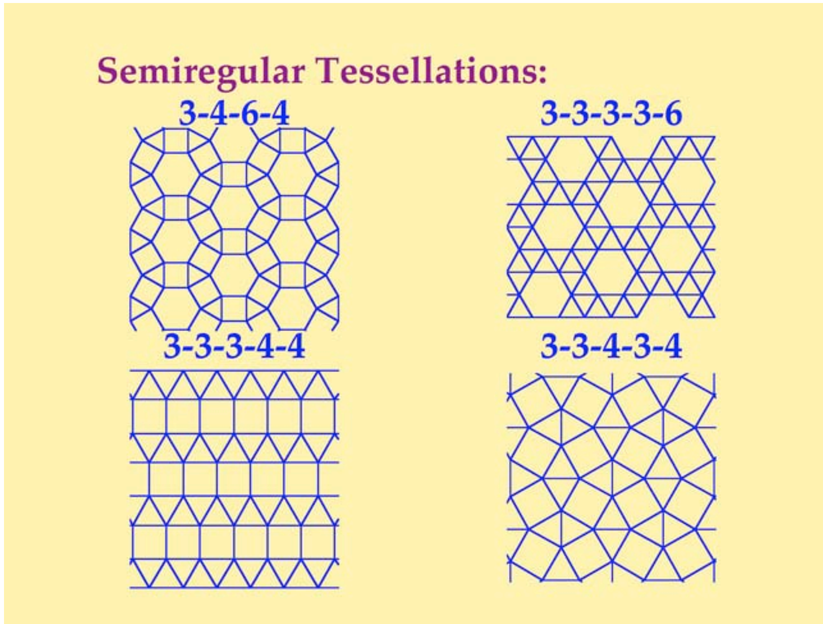
Figure 12, Segment 3



Summary page 1, Segment 3



Summary page 2, Segment 3



Summary page 3, Segment 3

Well, that's our program for this week. I hope you've enjoyed the things I've talked about and I hope you find that to be useful mathematics. Let me remind you that I realize I went through these things very quickly and so each of these episodes that we air appear on our webpage which we'll flash up that address here at the end. You can go there and download these programs and then review the kinds of things we've talked about. I know that it's really quick, so please do that. And please email me with your suggestions for the program. With that, I'm done. We'll see you next week, thanks.