EEL 3420
Engineering Analysis

Fall 2005

Home Work Solutions

<u>Homework Assignment #1</u>

EEL 3420 Engineering Analysis

Error Analysis

1. Use zero through third order Taylor series expansions to predict $f(3)$ for

$$f(x) = 25x^3 - 6x^2 + 7x - 88$$

   using a base point of x = 2. Compute the percent relative error and the number of significant digits for each approximation.

2. Use zero through fourth order Taylor series expansions to predict $f(3)$ for

$$f(x) = e^{-x}$$

   using a base point of x = 1. Compute the percent relative error and the number of significant digits for each approximation.

3. Recall that the velocity of a falling parachutist can be computed by

$$v = \frac{gm}{c}(1 - e^{-\frac{c}{m}t})$$

   Use a first-order error analysis to estimate the error of v at

$$t = 7$$
$$g = 9.8$$
$$m = 68.1 \pm 0.5$$
$$c = 12.5 \pm 2$$

4. Let x = 37.5678 and y = 37.5318. If our computer can only store 4 digits, what is the relative error for x and y when we represent the numbers using our computer. Compute z = (x - y) + 0.01946. What is the relative error of z. How many digits of significance does z have.

5. What is the <u>rate of convergence</u> for f(n). f(n) converges to 0 but at what rate O(?). Recall α_n converges to α with rate β_n if $|\alpha_n - \alpha| \le |\beta_n| \cdot k$ for some constant k

$$f(n) := \frac{(n-2)\cdot(n+4)}{n^3}$$

HW #1 Sol

1) $f(x) = 25x^3 - 6x^2 + 7x - 88$

$a = 2 \quad f(3) = 554$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2} +$$

$$\frac{f'''(a)(x-a)^3}{3!} + \frac{f^{(4)}(a)(x-a)^4}{4!} + \frac{f^{(5)}(\delta)(x-a)^5}{5!}$$

where δ between x and a.

$f(x) = 25x^3 - 6x^2 + 7x - 88 \qquad f(2) = 102$

$f'(x) = 75x^2 - 12x + 7 \qquad f'(2) = 283$

$f''(x) = 150x - 12 \qquad f''(2) = 288$

$f'''(x) = 150 \qquad f'''(2) = 150$

$f^{(4)}(x) = \emptyset$

$P_0(x) = f(2) = 102$

$P_1(x) = P_0(x) + f'(2)(x-2) = P_0(x) + 283(x-2)$

$P_2(x) = P_1(x) + f''(2)(x-2)^2 \frac{1}{2} = P_1(x) + 288(x-2)^2 \frac{1}{2}$

$P_3(x) = P_2(x) + f^{(3)}(x)(x-2)^3 \frac{1}{6} = P_2(x) + 150(x-2)^3 \frac{1}{6}$

for $x=2$ we get

$P_0(3) = 102$

$P_1(3) = 102 + 283(1) = 385$

$P_2(3) = 385 + 288(1)\frac{1}{2} = 529$

$P_3(3) = 529 + 150\frac{1}{6} = 554$

error $P_0 = \dfrac{f(2) - P_0(2)}{f(2)} = \dfrac{|554 - 102|}{554} = .8159$

error $P_1 = \dfrac{|385 - 554|}{554} = .3051$

error $P_2 = \dfrac{|529 - 554|}{554} = .04513$

error $P_3 = \dfrac{|554 - 554|}{554} = \emptyset$

$P_0(3)$ has $\emptyset$ sig digits $\quad .8159 < 5 = 5 \times 10^{-\emptyset}$

$P_1(3)$ has 1 sig digit $\quad .3051 < .5 = 5 \times 10^{-1}$

$P_2(3)$ has 2 sig digit $\quad .04513 < .05 = 5 \times 10^{-2}$

$P_3(3)$ has 3 sig digits $\quad$ error $= \emptyset$

Problem 1

$$f(a) := 25\cdot a^3 - 6\cdot a^2 + 7\cdot a - 88$$

$a := 2 \quad x := 3$

$f(a) = 102 \qquad f(x) = 554$

$$\frac{d}{da}f(a) = 283$$

$$\frac{d^2}{da^2}f(a) = 288$$

$$\frac{d^3}{da^3}f(a) = 150$$

$$p0(x) := f(a) \qquad e0(x) := \frac{|f(x) - p0(x)|}{|f(x)|}$$

$p0(3) = 102 \qquad e0(3) = 0.816$

$$p1(x) := f(a) + \left(\frac{d}{da}f(a)\right)\cdot(x-a) \qquad e1(x) := \frac{|f(x) - p1(x)|}{|f(x)|}$$

$p1(3) = 385 \qquad e1(3) = 0.305$

$$p2(x) := f(a) + \left(\frac{d}{da}f(a)\right)\cdot(x-a) + \frac{d^2}{da^2}f(a)\cdot(x-a)^2\cdot\frac{1}{2} \qquad e2(x) := \frac{|f(x) - p2(x)|}{|f(x)|}$$

$p2(3) = 529 \qquad e2(3) = 0.045$

$$p3(x) := f(a) + \left(\frac{d}{da}f(a)\right)\cdot(x-a) + \frac{d^2}{da^2}f(a)\cdot(x-a)^2\cdot\frac{1}{2} + \frac{d^3}{da^3}f(a)\cdot(x-a)^3\cdot\frac{1}{6} \qquad e3(x) := \frac{|f(x) - p3(x)|}{|f(x)|}$$

$p3(3) = 554 \qquad e3(3) = 3.694 \times 10^{-15}$

Problem 2

$f(a) := e^{-a}$

$a := 1 \quad x := 3$

$f(a) = 0.368 \quad f(x) = 0.05$

$\frac{d}{da}f(a) = -0.368$

$\frac{d^2}{da^2}f(a) = 0.368$

$\frac{d^3}{da^3}f(a) = -0.368$

$p0(x) := f(a)$

$e0(x) := \frac{|f(x) - p0(x)|}{|f(x)|}$

$p0(3) = 0.368 \quad e0(3) = 6.389$

$p1(x) := f(a) + \left(\frac{d}{da}f(a)\right)\cdot(x - a)$

$e1(x) := \frac{|f(x) - p1(x)|}{|f(x)|}$

$p1(3) = -0.368 \quad e1(3) = 8.389$

$p2(x) := f(a) + \left(\frac{d}{da}f(a)\right)\cdot(x - a) + \frac{d^2}{da^2}f(a)\cdot(x - a)^2\cdot\frac{1}{2}$

$e2(x) := \frac{|f(x) - p2(x)|}{|f(x)|}$

$p2(3) = 0.368 \quad e2(3) = 6.389$

$p3(x) := f(a) + \left(\frac{d}{da}f(a)\right)\cdot(x - a) + \frac{d^2}{da^2}f(a)\cdot(x - a)^2\cdot\frac{1}{2} + \frac{d^3}{da^3}f(a)\cdot(x - a)^3\cdot\frac{1}{6}$

$e3(x) := \frac{|f(x) - p3(x)|}{|f(x)|}$

$p3(3) = -0.123 \quad e3(3) = 3.463$

$p4(x) := p3(x) + \frac{d^4}{da^4}f(a)\cdot(x - a)^4\cdot\frac{1}{4!}$

$e4(x) := \frac{|f(x) - p4(x)|}{|f(x)|}$

da^{4} $4!$ $|f(x)|$

$p4(3) = 0.123 \qquad e4(3) = 1.463$

$$p5(x) := p4(x) + \frac{d^5}{da^5} f(a) \cdot (x-a)^5 \cdot \frac{1}{5!} \qquad e5(x) := \frac{|f(x) - p5(x)|}{|f(x)|}$$

$p5(3) = 0.025 \qquad e5(3) = 0.507$

$$p6(x) := p5(x) + \frac{d^6}{da^6} f(a) \cdot (x-a)^6 \cdot \frac{1}{6!} \qquad e6(x) := \frac{|f(x) - p6(x)|}{|f(x)|}$$

$p6(3) = \blacksquare \qquad e6(3) = \blacksquare$

3) $V = \frac{gm}{c}\left(1 - e^{-\frac{c}{m}t}\right)$

$t = 7$

$g = 9.8$

$m = 68.1 \pm .5$

$c = 12.5 \pm 2$

$$\Delta V = \left|\frac{\partial V}{\partial t}\right|\Delta t + \left|\frac{\partial V}{\partial g}\right|\Delta g + \left|\frac{\partial V}{\partial m}\right|\Delta m + \left|\frac{\partial V}{\partial c}\right|\Delta c$$

$$\frac{\partial V}{\partial m} = \frac{g}{c}\left(1 - e^{-\frac{c}{m}t}\right) + \frac{gm}{c}\left(-\frac{ct}{m^2}e^{-\frac{c}{m}t}\right)$$

$$= \frac{g}{c}\left(1 - e^{-\frac{c}{m}t}\right) - \frac{gt}{m}e^{-\frac{c}{m}t}$$

$$\left.\frac{\partial V}{\partial m}\right|_{t,g,m,c} = .288$$

error in V due to error in m is

$.288(.5) = .144$

$$\frac{\partial v}{\partial c} = -\frac{gm}{c^2}\left(1-e^{-\frac{c}{m}t}\right) + \frac{gm}{c}\left(\frac{t}{m}e^{-\frac{c}{m}t}\right)$$

$$= -\frac{gm}{c^2}\left(1-e^{-\frac{c}{m}t}\right) + \frac{gt}{c}e^{-\frac{c}{m}t}$$

$$\left.\frac{\partial v}{\partial c}\right|_{g,t,c,m} = \left|-1.571\right| = 1.571$$

$$\left|\frac{\partial v}{\partial c}\right|\Delta c = 1.571(2) = 3.142$$

$$\Delta V = \left|\frac{\partial V}{\partial m}\right|\Delta m + \left|\frac{\partial V}{\partial c}\right|\Delta c = .144 + 3.142$$

$$= 3.286$$

$$V = 38.618 \pm 3.286$$

Problem 3

$t := 7$

$$f(t,g,m,c) := \frac{g \cdot m}{c} \cdot \left(1 - e^{\frac{-c}{m} \cdot t}\right)$$

$g := 9.8$

$m := 68.1 \quad em := .5$

$c := 12.5 \quad ec := 2$

$$\left|\frac{d}{dm} f(t,g,m,c)\right| \cdot em + \left|\frac{d}{dc} f(t,g,m,c)\right| \cdot ec = 3.286$$

$$f(t,g,m,c) = 38.618$$

$$\frac{d}{dm} f(t,g,m,c) = 0.288$$

$$\frac{d}{dc} f(t,g,m,c) = -1.571$$

4)

$X = 37.5678 \qquad fl(x) = 37.56$

$Y = 37.5318 \qquad fl(y) = 37.53$

$$\text{error } fl(x) = \frac{x - fl(x)}{x} = \frac{.0078}{37.5678} = .2076 \times 10^{-3}$$

$$\text{error } fl(y) = \frac{y - fl(y)}{y} = \frac{.0018}{37.5318} = .48 \times 10^{-4}$$

$z = (x - y) + .01946 \qquad fl(z) = fl(x) - fl(y) + .01946$

$z = .05546 \qquad fl(z) = .04946$

$$\text{error } z = \frac{z - fl(z)}{z} = .10819$$

$.10819 < .5 = 5 \times 10^{-1} \Rightarrow 1$ sig digit.

5)

$$f(n) = \frac{(n-2)(n+4)}{n^3}$$

try $\frac{1}{n}$ Since $f(n) \to \phi$ for $n \to \infty$

So $\alpha = \phi$

$$|f(n) - \phi| \leq \frac{1}{n} K$$

$$f(n) \leq \frac{1}{n} K \Rightarrow f(n) n \leq K$$

$$\frac{(n-2)(n+4)}{n^3} n \leq K \Rightarrow \frac{n^2 + 4n - 2n - 8}{n^2} \leq K$$

$$\Rightarrow 1 + \frac{2}{n} - \frac{8}{n^2} \leq K$$

for $n = 1 \Rightarrow 1 + 2 - 8 = -5$

$$\lim_{n \to \infty} \left(1 + \frac{2}{n} - \frac{8}{n^2}\right) = 1 \Rightarrow K = 2 \text{ and } \beta = \frac{1}{n}$$

= rate of convergence

Homework Assignment #2

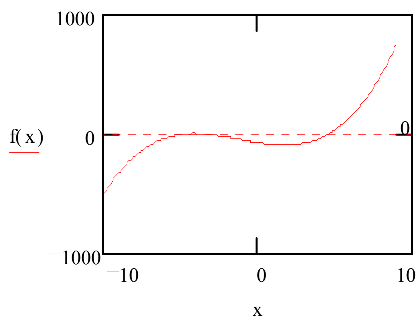
EEL 3420 Engineering Analysis

Root Finding Techniques

1. Find the positive root of the function

$$f(x) := x^3 + 3.5 \cdot x^2 - 21.0 \cdot x - 67.5.$$

Use a bracket method first to find an initial approximation then use Newton's method to further improve your estimate to 2 digits of accuracy.



2. Find the root of the function

$$f(x) = \frac{1}{x-5} - 5.$$

3. Find the value of $\sqrt{3}$ to 4 digits of accuracy.

4. Using a computer language of your choice implement Newton's method to find the roots of arbitrary polynomials to 4 significant digits. The function will have as its inputs the coefficients of the input function and its degree, the coefficients of the derivative function and an initial guess of the root. In each iteration, the function is to compute the values of $f(p_k)$ and $f^{'}(p_k)$ using Horner's method as discussed in class. Recall that Newton's method uses

$$p_{k+1} = p_k - \frac{f(p_k)}{f^{'}(p_k)}$$

Horner'method: The value of the polynomial

$$p(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_n$$

evaluated at x = c may be found by using Horner's rule:

$$p(x) = (((a_0 c + a_1)c + a_2)c + a_3)c + \cdots$$

Using the following algorithm:

```
Ans = A[0];
For (x = 0; x < N; x++)
     Ans = Ans *c + A[x];
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Homework Assignment #3

EEL 3420 Engineering Analysis

Matrices

1. Given

$$A = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$$

show that

$$(A+B)(A-B) \neq A^2 - B^2$$

2. Find all matrices A such that

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot A = A \cdot \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \qquad \begin{bmatrix} 3 & 0 \\ 5 & 0 \end{bmatrix} \cdot A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

3. Find the determinant of the two matrices

$$\begin{bmatrix} -3 & 1 & 1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{bmatrix} \qquad \begin{bmatrix} 2 & 3 & 1 \\ 1 & 4 & -3 \\ -1 & 2 & 0 \end{bmatrix}$$

4. Prove that

$$|AB| = |A||B|$$

for any 2 square matrices. Assume the matrices are 2x2. Note this is true for all square matrices of size nxn.

5. Find the multiplicative inverse, if it exist, of A where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$$

6. Find the left multiplicative inverse, if it exist, of

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \\ 0 & 0 \end{bmatrix}$$

Show that the right inverse does not exist.

7. Determine the rank of the following matrix.

$$\begin{bmatrix} 0 & 2 & 4 & 6 \\ 3 & -1 & 4 & -2 \\ 6 & -1 & 10 & -1 \end{bmatrix}$$

Use the 3 elementary row transformations to simplify the work.

8. Show that the following 3 equations are linearly dependant:

$$2x + z$$
$$x + y$$
$$2y - z$$

9. Find the characteristic equation, the eigenvalues, and a set of eigenvectors for the given 4 matrices.

$$\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 0 & 2 \\ 0 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

10. Use the Hamilton-Cayley theorem to find

$$A^{-3}$$

for the matrix

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$$

1)

$$A = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 3 & 4 \\ -3 & 2 \end{bmatrix} \qquad A-B = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ -2 & -6 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix} \qquad A^2 - B^2 = \begin{bmatrix} -5 & 3 \\ -2 & -6 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix} =$$

$$= \begin{bmatrix} -8 & -1 \\ 2 & -9 \end{bmatrix}$$

$$(A+B)(A-B) = \begin{bmatrix} 3 & 4 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -7 & -2 \\ 1 & -10 \end{bmatrix}$$

$$\begin{bmatrix} -8 & -1 \\ 2 & -9 \end{bmatrix} \neq \begin{bmatrix} -7 & -2 \\ 1 & -10 \end{bmatrix}$$

2)

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A = A \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \qquad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$2a + c = 2a + 3b \Rightarrow c = 3b$

$2b + d = a + 2b \Rightarrow a = d$

$3a + 2c = 2c + 3d \Rightarrow 3a = 3d \Rightarrow a = d$

$3b + 2d = c + 2d \Rightarrow 3b = c \Rightarrow b = \frac{c}{3}$

So we only have $a = d$ and $c = 3b$

$$\begin{bmatrix} a & \frac{c}{3} \\ c & a \end{bmatrix} \text{ or } = \begin{bmatrix} a & b \\ 3b & a \end{bmatrix}$$

2) cont

$$\begin{bmatrix} 3 & 0 \\ 5 & 0 \end{bmatrix} A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$3a = 2$ $\qquad$ $5a = 1$ $\qquad$ $b = \emptyset,\ c = \emptyset$

$3b = 0$ $\qquad$ $5b = 0$ $\qquad$ $a = 2/3$ and $a = 1/5$

$3c = \emptyset$ $\qquad$ $5c = \emptyset$ $\qquad$ no Sol.

also if A exist then

$$A = \begin{bmatrix} 3 & 0 \\ 5 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$ but since $\begin{vmatrix} 3 & 0 \\ 5 & 0 \end{vmatrix} = \emptyset$

$\begin{bmatrix} 3 & 0 \\ 5 & 0 \end{bmatrix}^{-1}$ Can not exist $\Rightarrow$ A Can not exist.

3)

$$\begin{vmatrix} -3 & 1 & 1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{vmatrix} = (-3)(-3)(-3) + (1)(1)(1) + (1)(1)(1) - (1)(-3)(1) - (1)(1)(-3) - (-3)(1)(1) =$$

$$= -27 + 2 + 3 + 3 + 3 = -27 + 11 = -16$$

$$\begin{vmatrix} 2 & 3 & 1 \\ 1 & 4 & -3 \\ -1 & 2 & 0 \end{vmatrix} = 2(4)(0) + 3\cdot(-3)\cdot(-1) + (1)(1)(2) - (-1)4\cdot 1 - 2\cdot(-3)\cdot 2 - 0(1)3 =$$

$$0 + 9 + 2 + 4 + 12 = 27$$

4) $|AB| = |A||B|$ $\quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad B = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} aw+by & ax+bz \\ cw+dy & cx+dz \end{bmatrix} =$$

$$(aw+by)(cx+dz) - (cw+dy)(ax+bz) =$$

$$\cancel{awcx} + awdz + bycx + \cancel{bydz} - (\cancel{cwax} + cwbz + dyax + \cancel{dybz})$$

$$= awdz + bycx - cwbz - dyax$$

$|A| = ad - cb \qquad |B| = wz - yx$

$|A||B| = (ad-cb)(wz-yx) = adwz - adyx - cbwz + cbyx$

$= awdz + bycx - cwbz - dyax$ ✓

5)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix} \qquad A^{-1}A = I, \quad A^{-1} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$a + b + c = 1 \qquad d + e + f = \emptyset \qquad g + h + i = \emptyset$

$2a + 3b + 5c = \emptyset \qquad 2d + 3e + 5f = 1 \qquad 2g + 3h + 5i = 0$

$3a + 5b + 12c = 0 \qquad 3d + 5e + 12f = \emptyset \qquad 3g + 5h + 12i = 1$

$\Rightarrow a = 3\frac{2}{3}, \quad b = -3, \quad c = \frac{1}{3}, \quad d = -2\frac{1}{3}, \quad e = 3$

$f = -\frac{2}{3}, \quad g = \frac{2}{3}, \quad h = -1, \quad i = \frac{1}{3}$

6) find left Multiplicative inverse

$$A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \\ 0 & 0 \end{bmatrix} \quad \text{find } A^{-1}: \; A^{-1}A = I$$

$$A^{-1} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$A^{-1}A = I \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 4 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$a + 3b = 1 \rightarrow -4b + 3b = 1 \Rightarrow b = -1$

$a + 4b = 0 \rightarrow a = -4b \Rightarrow a = 4$

$d + 3e = 0 \rightarrow d = -3e \rightarrow d = -3$

$d + 4e = 1 \rightarrow -3e + 4e = 1 \rightarrow e = 1$

So $A^{-1} = \begin{bmatrix} 4 & -1 & c \\ -3 & 1 & f \end{bmatrix}$ where $c \,\&\, f \in \mathbb{R}$

ex: $A^{-1} = \begin{bmatrix} 4 & -1 & 0 \\ -3 & 1 & 0 \end{bmatrix}$

6) Cont Right inverse

$AA^{-1} = I \qquad A^{-1} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

get 9 equations but equation yealds

$0c + 0f = 1 \Rightarrow 0(c+f) = 1$

this is imposible so A^{-1} here does not exist

7) rank

$$A = \begin{array}{c} \begin{bmatrix} 0 & 2 & 4 & 6 \\ 3 & -1 & 4 & -2 \\ 6 & -1 & 10 & -1 \end{bmatrix} \end{array} \begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array}$$

swap e_1, e_2

$$A = \begin{bmatrix} 3 & -1 & 4 & -2 \\ 0 & 2 & 4 & 6 \\ 6 & -1 & 10 & -1 \end{bmatrix}$$

7) cont

$e_3 = e_3 - 2e_1 = [0 \quad -1-2(-1) \quad 10-8 \quad -1-(2)(-2)]$

$= [0 \quad 1 \quad 2 \quad 3]$

$$A = \begin{bmatrix} 3 & -1 & 4 & -2 \\ 0 & 2 & 4 & 6 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$e_3 = e_3 - \frac{1}{2}e_2 = [0 \quad 0 \quad 0 \quad 0]$

$$A = \begin{bmatrix} 3 & -1 & 4 & -2 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ try rank = 2

$$\left| \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix} \right| = 6 \quad \text{rank} = 2$$

8) $2x + z$

$x + y$

$2y - z$

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

if $\text{Rank}(A) < 3$ then A does not have full rank so A has less than 3 linearly independant equations.

$|A| = 2\cdot 1\cdot(-1) + \cancel{0} + 1\cdot 1\cdot 2 - \cancel{0} - 0 - 0 = -2+2 = \emptyset$

So equations are linearly dependent.

9)

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix} \qquad \Delta(\lambda) = \det(A - \lambda I) = \phi$$

$$(A - \lambda I) = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 5-\lambda & 3 \\ 2 & 4-\lambda \end{bmatrix}$$

$$\det = (5-\lambda)(4-\lambda) - 6 = 20 - 5\lambda - 4\lambda + \lambda^2 - 6$$

$$= \lambda^2 - 9\lambda + 14 = (\lambda - 7)(\lambda - 2) = \phi$$

eigenvalues are $\lambda = 7, \lambda = 2$

for $\lambda = 7$:

$$\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 7 \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{array}{l} 5x + 3y = 7x \\ 2x + 4y = 7y \end{array}$$

$$3y = 2x \Rightarrow y = \tfrac{2}{3}x \quad \text{so} \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 2/3\, x \end{bmatrix}$$

ex $\begin{bmatrix} 1 \\ 2/3 \end{bmatrix}$

9) cont

for $\lambda = 2$

$$\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 2\begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \quad 5x + 3y = 2x$$

$3y = -3x$

$y = -x$

So $\begin{bmatrix} x \\ -x \end{bmatrix}$ or $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

for $A = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & 0 \\ 0 & -\lambda \end{bmatrix}$$

$$\det(A) = (2-\lambda)(-\lambda) = \lambda^2 - 2\lambda = \lambda(\lambda - 2) = 0$$

$\lambda = 0, \ \lambda = 2$

for $\lambda = 0$

$$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0 \qquad 2x = 0 \Rightarrow x = 0$$

So $\begin{bmatrix} 0 \\ n \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\lambda = 2$ 9) Cont

$$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

$2x = 2x$

$0x + 0y = 2y \Rightarrow 2y = 0$

$y = 0$

So $\begin{bmatrix} k \\ 0 \end{bmatrix}$

10) $A = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$ $\Delta(\lambda) = \det(A - \lambda I) = 0$

$\Delta(A) = 0$

$$\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & 4 \\ 1 & 1-\lambda \end{bmatrix}$$

$\det(A) = (2-\lambda)(1-\lambda) - 4 = 0$

$2 - 2\lambda - \lambda + \lambda^2 - 4 = 0$

$\lambda^2 - 3\lambda - 2 = 0$

$\Delta(A) = 0 \Rightarrow A^2 - 3A - 2I = 0$

$A^2 = 3A + 2I$

10) Cont

$$A^{-1}A^2 - 3A^{-1}A - 2A^{-1} = \emptyset$$

$$A - 3I - 2A^{-1} = \emptyset$$

$$-2A^{-1} = -A + 3I$$

$$A^{-1} = \frac{1}{2}A - \frac{3}{2}I$$

$$A^{-2} = A^{-1}A^{-1} = A^{-1}\left(\frac{1}{2}A - \frac{3}{2}I\right) = \frac{1}{2}I - \frac{3}{2}A^{-1}$$

$$= \frac{1}{2}I - \frac{3}{2}\left(\frac{1}{2}A - \frac{3}{2}I\right) = \frac{1}{2}I - \frac{3}{4}A + \frac{9}{4}I$$

$$= -\frac{3}{4}A + \left(\frac{9}{4} + \frac{1}{2}\right)I = -\frac{3}{4}A + \frac{11}{4}I$$

$$A^{-3} = A^{-1}A^{-2} = A^{-1}\left(-\frac{3}{4}A + \frac{11}{4}I\right)$$

$$= -\frac{3}{4}I + \frac{11}{4}A^{-1} = -\frac{3}{4}I + \frac{11}{4}\left(\frac{1}{2}A - \frac{3}{2}I\right)$$

$$= -\frac{3}{4}I + \frac{11}{8}A - \frac{33}{8}I = \frac{11}{8}A - \frac{39}{8}I$$

So

$$A^{-3} = \frac{11}{8}A - \frac{39}{8}I = \frac{11}{8}\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} - \frac{39}{8}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{11}{4} & \frac{11}{2} \\ \frac{11}{8} & \frac{11}{8} \end{bmatrix} - \begin{bmatrix} \frac{39}{8} & 0 \\ 0 & \frac{39}{8} \end{bmatrix} = \begin{bmatrix} \frac{11}{4} - \frac{39}{8} & \frac{11}{2} \\ \frac{11}{8} & \frac{11}{8} - \frac{39}{8} \end{bmatrix}$$

So

$$A^{-3} = \begin{bmatrix} \frac{-17}{8} & \frac{11}{2} \\ \frac{11}{8} & \frac{-7}{2} \end{bmatrix}$$

Optional Compute A^{-n}:

from the derivation we see ↘

$A^{-1} = \frac{1}{2}A - \frac{3}{2}I$

$A^{-2} = \frac{-3}{2}A + \frac{11}{4}I \qquad \frac{-3}{2} = \frac{1}{2}\cdot\frac{-3}{2}, \quad \frac{11}{4} = \frac{-3}{2}\cdot\frac{-3}{2} + \frac{1}{2}$

$A^{-3} = \frac{11}{8}A - \frac{39}{8}I \qquad \frac{11}{8} = \frac{1}{2}\cdot\frac{11}{4}, \quad \frac{39}{8} = \frac{11}{4}\cdot\frac{-3}{2} - \frac{3}{4}$

We see recursively that if $A^{-1} = aA + bI$

$$A^{-n} = a_1 b_{n-1} A + (b_1 b_{n-1} + a_{n-1})I$$

with $a_1 = \frac{1}{2}$ and $b_1 = \frac{-3}{2}$

So

$$A^{-4} = \frac{1}{2}b_3 A + \left(\frac{-3}{2}b_3 + a_3\right)I = \frac{-39}{8}\cdot\frac{1}{2}A + \left[\frac{-3}{2}\left(\frac{-39}{8}\right) + \frac{11}{8}\right]I$$

$$= \frac{-39}{16}A + \frac{129}{16}I$$

Homework Assignment #4
EEL 3420 Engineering Analysis

Systems of Equations

1. Solve, by hand, the following linear systems using Gaussian-elimination.

$$
\begin{aligned}
x_1 + x_2 - x_3 &= 3 \\
2x_1 - x_2 - 3x_3 &= 0 \\
-x_1 - 2x_2 + x_3 &= -5
\end{aligned}
$$

2. Write a MATLAB program to solve the following linear systems using all three Gaussian algorithms and the Gauss-Jordan algorithm.

$$
\begin{aligned}
x_1 + 2\cdot x_2 + 3\cdot x_3 &= 1 \\
2\cdot x_1 + 3\cdot x_2 + 4\cdot x_3 &= -1 \\
3\cdot x_x + 4\cdot x_2 + 6\cdot x_3 &= 2
\end{aligned}
\qquad
\begin{aligned}
2\cdot x_1 + 4\cdot x_2 - x_3 &= -5 \\
x_1 + x_2 - 3\cdot x_3 &= -9 \\
4\cdot x_1 + x_2 + 2\cdot x_3 &= 9
\end{aligned}
\qquad
\begin{aligned}
.832\cdot x_1 + .448\cdot x_2 + .193\cdot x_3 &= 1.00 \\
.784\cdot x_1 + .421\cdot x_2 - .207\cdot x_3 &= 0.00 \\
.784\cdot x_1 - .421\cdot x_2 + 279\cdot x_3 &= 0.00
\end{aligned}
$$

3. Use the Gaussian-Jordan method, by hand, to find the inverse of the following matrix.

$$
\begin{bmatrix} 10 & -3 & 6 \\ 1 & 8 & -2 \\ -2 & 4 & -9 \end{bmatrix}
$$

4. Modify the MATLAB Gaussian-Jordan program that you implemented so that it can be used to find the inverse of the matrix in the previous problem.

5. Find the fixed points for the following system of nonlinear equations. You may use MATLAB as a tool for the matrix manipulation.

$$
\begin{aligned}
x &= g_1(x,y,z) = 9x^2 + 36y^2 + 4z^2 - 36 \\
y &= g_2(x,y,z) = x^2 - 2y^2 - 20z \\
z &= g_3(x,y,z) = 16x - x^3 - 2y^2 - 16z^2
\end{aligned}
$$

egn 3420 HW #4 F05

1)

$$A = \begin{bmatrix} 1 & 1 & -1 & 3 \\ 2 & -1 & -3 & 0 \\ -1 & -2 & 1 & -5 \end{bmatrix}$$

$$e_2 = e_2 - 2e_1 = [\,0 \ -3 \ -1 \ -6\,]$$

$$e_3 = e_3 + e_1 = [\,0 \ -1 \ 0 \ -2\,]$$

$$e_3 = e_3 - \tfrac{1}{3}e_2 = [\,0 \ 0 \ 1/3 \ 0\,]$$

$$A = \begin{bmatrix} 1 & 1 & -1 & 3 \\ 0 & -3 & -1 & -6 \\ 0 & 0 & 1/3 & 0 \end{bmatrix}$$

$$\tfrac{1}{3}x_3 = 0 \Rightarrow x_3 = 0$$

$$-3x_2 = -6 + x_3 \Rightarrow x_2 = 2$$

$$x_1 = 3 + x_3 - x_2 = 3 - 2 = 1$$

$$x = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

3)

$$A \begin{bmatrix} 10 & -3 & 6 & 1 & 0 & 0 \\ 1 & 8 & -2 & 0 & 1 & 0 \\ -2 & 4 & -9 & 0 & 0 & 1 \end{bmatrix}$$

$e_2 = e_2 - .1 e_1 = [0 \ \ 8.3 \ \ -2.6 \ \ -.1 \ \ 1 \ \ 0]$

$e_3 = e_3 + .2 e_1 = [0 \ \ 3.4 \ \ -7.8 \ \ .2 \ \ 0 \ \ 1]$

$e_3 = e_3 - .41 e_2 = [0 \ \ 0 \ \ -6.735 \ \ .241 \ \ -.41 \ \ 1]$

$e_1 = e_1 + .361 e_2 = [10 \ \ 0 \ \ 5.06 \ \ .964 \ \ .361 \ \ 0]$

$e_1 = e_1 + .751 e_3 = [10 \ \ 0 \ \ 0 \ \ 1.145 \ \ .054 \ \ .751]$

$e_2 = e_2 - .386 e_3 = [0 \ \ 8.3 \ \ 0 \ \ -.193 \ \ 1.158 \ \ -.386]$

$e_1 = e_1 (.1) = [1 \ \ 0 \ \ 0 \ \ .114 \ \ .0054 \ \ .0751]$

$e_2 = e_2 \frac{1}{8.3} = [0 \ \ 1 \ \ 0 \ \ -.023 \ \ .14 \ \ -.047]$

$e_3 = e_3 \frac{-1}{6.735} = [0 \ \ 0 \ \ 1 \ \ -.036 \ \ .061 \ \ -.148]$

$$A = \begin{bmatrix} 1 & 0 & 0 & .114 & .0054 & .0751 \\ 0 & 1 & 0 & -.023 & .14 & -.047 \\ 0 & 0 & 1 & -.036 & .061 & -.148 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} .114 & .0054 & .0751 \\ -.023 & .14 & -.047 \\ -.036 & .061 & -.148 \end{bmatrix}$$

5)

$$Q = 9x^2 + 36y^2 + 4z^2 - 36$$

$$Q = x^2 - 2y^2 - y - 20z$$

$$Q = 16x - x^3 - 2y^2 - 16z^2 - z$$

5)

$$F(x,y,z) = \begin{bmatrix} 9x^2 + 36y^2 + 4z^2 - 36 \\ x^2 - 2y^2 - 20z \\ 16x - x^3 - 2y^2 - 16z^2 \end{bmatrix}$$

$$J(x,y,z) = \begin{bmatrix} 18x & 72y & 8z \\ 2x & -4y & -20 \\ -3x^2+16 & -4y & -32z \end{bmatrix}$$

$$p_0 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$p_1 = p_0 - J^{-1}(p_0)\,F(p_0)$$

$$J(p_0) = \begin{bmatrix} 0 & 72 & 0 \\ 0 & -4 & -20 \\ 16 & -4 & 0 \end{bmatrix} \quad J^{-1}(p_0) = \begin{bmatrix} .00347 & 0 & .062 \\ .014 & 0 & 0 \\ -.00278 & -.05 & 0 \end{bmatrix}$$

$$F(p_0) = \begin{bmatrix} 0 \\ -2 \\ -2 \end{bmatrix}$$

$$p_1 = p_0 - J^{-1}(p_0)\,F(p_0) = \begin{bmatrix} .125 \\ 1 \\ -.1 \end{bmatrix}$$

$$p_2 = p_1 - J^{-1}(p_1)\,F(p_1) = \begin{bmatrix} .134 \\ .997 \\ -.099 \end{bmatrix}$$

$$p_3 = p_2 - J^{-1}(p_2)\,F(p_2) = \begin{bmatrix} .134 \\ .997 \\ -.099 \end{bmatrix}$$

Homework Assignment #5
EEL 3420 Engineering Analysis

Curve fitting

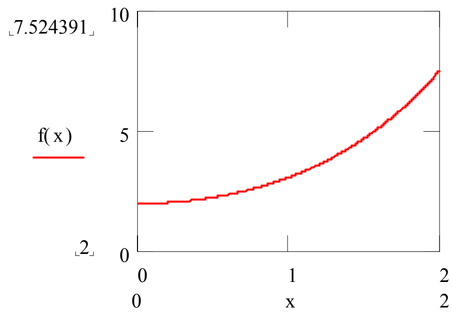
1. Given the following 3 sets of data, for each set, use regression to find a polynomial that best fits the data. For each set determine what percentage of the variation can be explained for. Try 1st order, 2nd order … until you get a good fit. Hint: put the data into two matrices so that you can easily manipulate them using MATLAB and generate

$$\sum_{i=1}^{n} x_i \qquad \sum_{i=1}^{n} (x_i)^2$$

and other terms.

y	g(y)	y	f(y)	y	g2(y)
-10	-351.962	-10	-355	-10	148.038
-9	-242.464	-9	-237.1	-9	85.586
-8	-139.437	-8	-145.4	-8	65.363
-7	-81.543	-7	-76.9	-7	38.507
-6	-26.771	-6	-28.6	-6	38.029
-5	4.074	-5	2.5	-5	35.324
-4	14.929	-4	19.4	-4	27.729
-3	31.028	-3	25.1	-3	35.078
-2	17.122	-2	22.6	-2	17.922
-1	18.164	-1	14.9	-1	18.214
0	5	0	5	0	5
1	-7.364	1	-4.1	1	-7.314
2	-3.922	2	-9.4	2	-3.122
3	-13.828	3	-7.9	3	-9.778
4	7.871	4	3.4	4	20.671
5	25.926	5	27.5	5	57.176
6	65.571	6	67.4	6	130.371
7	130.743	7	126.1	7	250.793
8	200.637	8	206.6	8	405.437
9	317.264	9	311.9	9	645.314
10	441.962	10	445	10	941.962

2. For the set of data points, find an interpolating polynomial that fits the data. You may use either Newton's or Lagrange interpolating polynomial. Do this by hand. Estimate the value at x = 1.5 and find the error.



$f(x) := e^{x} + e^{-x}$

x	f(x)
0	2
1	3.086
2	7.524

3. Write a program to generate 10 evenly spaced points from 0 to 2 using f(x) above and find a 9th order interpolating polynomial using these 10 points. Hint: Newton's algorithm produces the b's. Use this polynomial to estimate the value at x = 1.5 and find the error.

Egn 3420 F05 HW 5

1) for the 2nd set

X	f(X)
-10	-355
-9	-237.1
-8	-145.4
⋮	

1st order

$$y = \begin{bmatrix} -355 \\ -237.1 \\ -145.4 \\ \vdots \\ 445 \end{bmatrix} \quad z = \begin{bmatrix} 1 & -10 \\ 1 & -9 \\ 1 & -8 \\ & \vdots \\ 1 & 10 \end{bmatrix}$$

$$A = (Z^T Z)^{-1} Z^T y = \begin{bmatrix} 19.667 \\ 22.9 \end{bmatrix}$$

So

$P_1(x) = 19.667 + 22.9X$

$S_r = \sum (y_i - 19.667 - 22.9x_i)^2$

$S_t = \sum (y_i - \bar{y})^2$

$S_r = 1.593 \times 10^5$

$S_t = 5.631 \times 10^5$

$$r^2 = \frac{S_t - S_r}{S_t} = .717$$

71.7% fit

$$Y := \begin{bmatrix} -355 \\ -237.1 \\ -145.4 \\ -76.9 \\ -28.6 \\ 2.5 \\ 19.4 \\ 25.1 \\ 22.6 \\ 14.9 \\ 5 \\ -4.1 \\ -9.4 \\ -7.9 \\ 3.4 \\ 27.5 \\ 67.4 \\ 126.1 \\ 206.6 \\ 311.9 \\ 445 \end{bmatrix} \quad X := \begin{bmatrix} -10 \\ -9 \\ -8 \\ -7 \\ -6 \\ -5 \\ -4 \\ -3 \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{bmatrix} \quad O := \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$Z := \mathrm{augment}(O, X)$

$A := \left(Z^T \cdot Z\right)^{-1} \cdot Z^T \cdot Y$

$A = \begin{bmatrix} 19.667 \\ 22.9 \end{bmatrix}$

$L(x) := A_0 + A_1 \cdot x$

$k := 0, 1 .. n-1$

$n := \mathrm{rows}(Y)$

$n = 21$

$m := \mathrm{cols}(Z)$

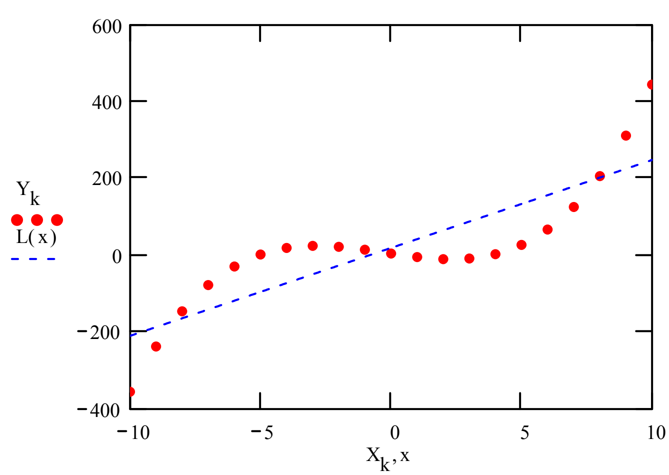
$m = 2$

$i := 0, 1 .. n-1$

$j := 0, 1 .. m-1$

$$sr := \sum_i \left(Y_i - \sum_j A_j \cdot Z_{i,j}\right)^2$$

First Order



$sr = 1.593 \cdot 10^5$

$$ybar := \frac{\sum_i Y_i}{n}$$

$$st := \sum_i \left(Y_i - ybar\right)^2$$

$st = 5.631 \cdot 10^5$

$$rsquare := \left(\frac{st - sr}{st}\right)$$

$rsquare = 0.717$

2^{nd} order

$$Z = \begin{array}{c} \begin{array}{ccc} & x & x^2 \end{array} \\ \begin{bmatrix} 1 & -10 & 100 \\ 1 & -9 & 81 \\ & \vdots & \\ 1 & 10 & 100 \end{bmatrix} \end{array}$$

y = Same as 1^{st} order

$$A = (Z^T Z)^{-1} Z^T y = \begin{bmatrix} 5 \\ 22.9 \\ .4 \end{bmatrix}$$

$$P_2(x) = 5 + 22.9x + .4x^2$$

$$S_r = 1.557 * 10^5$$

$$S_t = 5.631 * 10^5$$

$$r^2 = .723 \Rightarrow 72.3\% \text{ fit}$$

$X2_i := (X_i)^2$

$Z := \mathrm{augment}(O, X)$

$Z := \mathrm{augment}(Z, X2)$

$A := (Z^T \cdot Z)^{-1} \cdot Z^T \cdot Y$

$$A = \begin{bmatrix} 5 \\ 22.9 \\ 0.4 \end{bmatrix}$$

$L(x) := A_0 + A_1 \cdot x + A_2 \cdot x^2$

Z =

	0	1	2
0	1	-10	100
1	1	-9	81
2	1	-8	64
3	1	-7	49
4	1	-6	36
5	1	-5	25
6	1	-4	16
7	1	-3	9
8	1	-2	4
9	1	-1	1
10	1	0	0
11	1	1	1
12	1	2	4
13	1	3	9
14	1	4	16

$n := \mathrm{rows}(Y)$
$n = 21$
$m := \mathrm{cols}(Z)$
$m = 3$

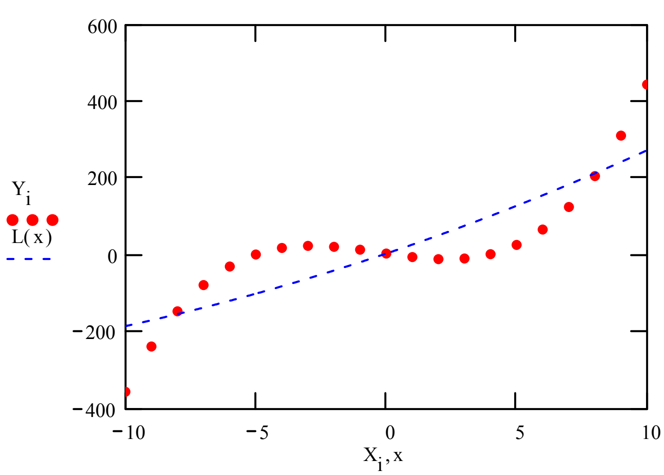
$i := 0, 1 .. n - 1$

$j := 0, 1 .. m - 1$

$$sr := \sum_i \left(Y_i - \sum_j A_j \cdot Z_{i,j} \right)^2$$

$i := 0, 1 .. 20$

Second Order



$sr = 1.557 \cdot 10^5$

$$ybar := \frac{\sum_i Y_i}{n}$$

$$st := \sum_i (Y_i - ybar)^2$$

$st = 5.631 \cdot 10^5$

$$rsquare := \left(\frac{st - sr}{st} \right)$$

$rsquare = 0.723$

3rd order

$$\begin{array}{cc} & \begin{array}{cccc} & x & x^2 & x^3 \end{array} \\ Z = & \begin{bmatrix} 1 & -10 & 100 & -1000 \\ 1 & -9 & 81 & -729 \\ & & \vdots & \\ 1 & 10 & 100 & 1000 \end{bmatrix} \end{array}$$

$$A = (Z^T Z)^{-1} Z^T Y = \begin{bmatrix} 5 \\ -10 \\ .4 \\ .5 \end{bmatrix}$$

$$P_3(x) = 5 - 10x + .4x^2 + .5x^3$$

$$S_r = \emptyset$$

$$S_t = 5.631 * 10^5$$

$r^2 = 1$ perfect fit $\Rightarrow$ original data was generated by a 3rd order polynomial.

$X3_i := (X_i)^3$

$Z := \mathrm{augment}(O, X)$

$Z := \mathrm{augment}(Z, X2)$

$Z := \mathrm{augment}(Z, X3)$

$A := (Z^T \cdot Z)^{-1} \cdot Z^T \cdot Y$

$$A = \begin{bmatrix} 5 \\ -10 \\ 0.4 \\ 0.5 \end{bmatrix}$$

$L(x) := A_0 + A_1 \cdot x + A_2 \cdot x^2 + A_3 \cdot x^3$

$i := 0, 1 .. 20$

Z =

	0	1	2	3
0	1	-10	100	$-1 \cdot 10^3$
1	1	-9	81	-729
2	1	-8	64	-512
3	1	-7	49	-343
4	1	-6	36	-216
5	1	-5	25	-125
6	1	-4	16	-64
7	1	-3	9	-27
8	1	-2	4	-8
9	1	-1	1	-1
10	1	0	0	0
11	1	1	1	1
12	1	2	4	8
13	1	3	9	27
14	1	4	16	64

$n := \mathrm{rows}(Y)$

$n = 21$

$m := \mathrm{cols}(Z)$

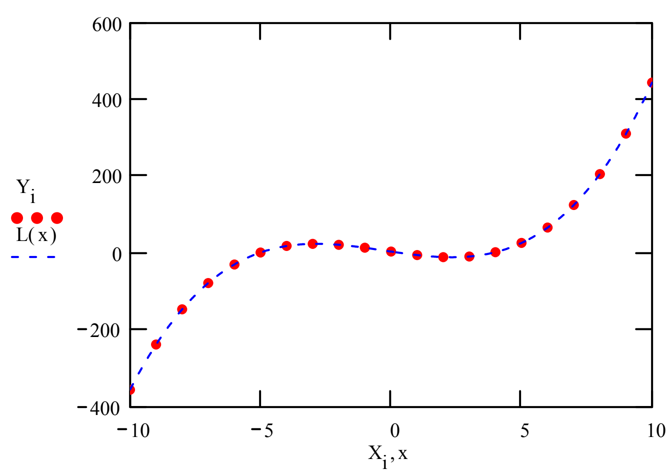
$m = 4$

$i := 0, 1 .. n - 1$

$j := 0, 1 .. m - 1$

$$sr := \sum_i \left(Y_i - \sum_j A_j \cdot Z_{i,j} \right)^2$$

Third Order



$sr = 0$

$$ybar := \frac{\sum_i Y_i}{n}$$

$$st := \sum_i (Y_i - ybar)^2$$

$st = 5.631 \cdot 10^5$

$$rsquare := \left(\frac{st - sr}{st} \right)$$

$rsquare = 1$

2)

x	$f(x)$
0	2
1	3.086
2	7.524

$$P_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1)$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$$= \frac{(x-1)(x-2)}{(0-1)(0-2)} 2 + \frac{(x-0)(x-2)}{(1-0)(1-2)} 3.086 +$$

$$\frac{(x-0)(x-1)}{(2-0)(2-1)} 7.524$$

$$= (x-1)(x-2) - x(x-2)3.086 + x(x-1)\frac{7.524}{2}$$

$$= x^2 - 2x - x + 2 - x^2 3.086 - 2x3.086 + \frac{7.524}{2}x^2$$

$$- \frac{7.524}{2}x$$

$\vdots$

$$= 1.676x^2 - .59x + 2 = P_2(x)$$

$$\text{error} = \frac{f(1.5) - P_2(1.5)}{f(1.5)} = .039 \Rightarrow 3.9\%$$

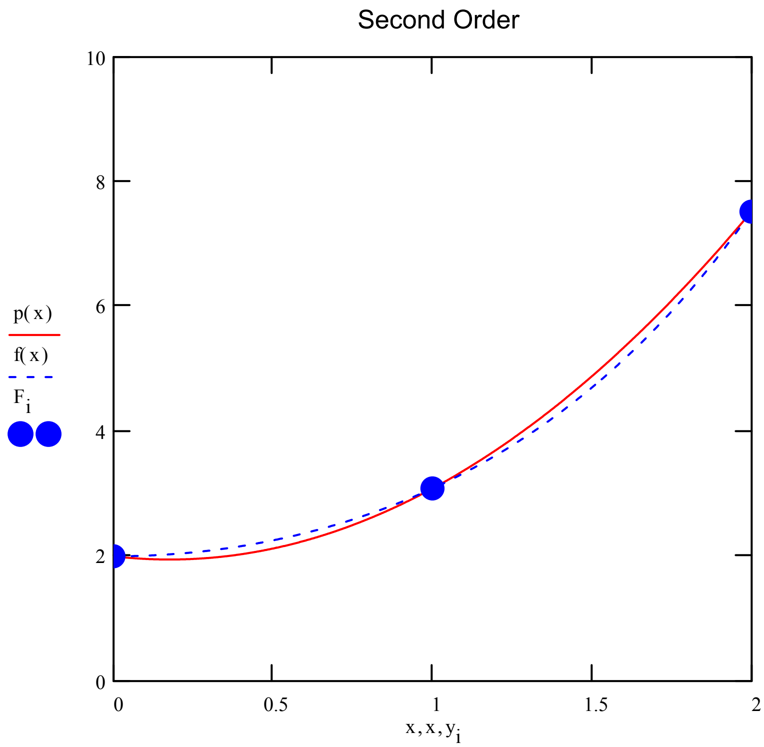
$f(x) = e^x + e^{-x}$ ~ points generated using this

$$p(x) := (x-1)\cdot(x-2) - x\cdot(x-2)\cdot 3.086 + x\cdot(x-1)\cdot\frac{7.524}{2} \qquad f(x) := e^{x} + e^{-x}$$

$$1.676\cdot x^{2} - .59\cdot x + 2.$$

$$y := \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \qquad F := \begin{bmatrix} 2 \\ 3.086 \\ 7.524 \end{bmatrix}$$

$$i := 0, 1 .. 2$$



$$Er := \frac{f(1.5) - p(1.5)}{f(1.5)} \qquad Er = -0.039$$

Homework Assignment #6
EEL 3420 Engineering Analysis

Numerical Integration, Differentiation, and Differential Equations

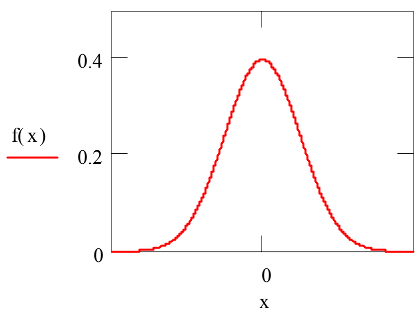
1. The following function corresponds to the normal distribution. There does not exist a closed form expression of the integral for this function so one must use numerical integration to find the probabilities. Use Simpson's 1/3 and 3/8 rule to integrate f(x) from 0 to 3. Partition the integrating interval into 10 evenly spaced segments and use the multiple application formula. You may use a computer if you like. Show all work or show the program listing.

$\mu := 0 \qquad \sigma := 1$

$$f(x) := \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma^2}} \cdot e^{\frac{-(x-\mu)^2}{2 \cdot \sigma^2}}$$

Note

$$\int_0^3 f(x)\,dx = 0.499$$



2. Use Romberg integration to evaluate $\int_0^4 xe^{2x}dx$ to an accuracy of 0.1%. Present your results as in the notes (showing the Richardson Extrapolations).

3. For the function $y = \sin x$ compute the first derivative at $x = \frac{\pi}{3}$ using the forward difference approximation of $O(h^2)$ and the centered difference approximations of $O(h^4)$ and using a value of $h = \frac{\pi}{12}$.

4. The following data was collected for the distance traveled versus time for a rocket.

T	0	1	2	3	4	5
Y	0	2.1	7.8	18.2	31.9	50.3

Use numerical differentiation to estimate the rocket's speed and acceleration at time T = 3.