

# Connes NCG Physics and E8

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**Connes** has constructed a realistic physics model in 4-dim spacetime based on **NonCommutative Geometry (NCG) of  $M \times F$**  where  **$M = 4\text{-dim spacetime}$  and  $F = C \times H \times M3(C)$**  and  $C = \text{Complex Numbers}$ ,  $H = \text{Quaternions}$ , and  $M3(C) = 3 \times 3 \text{ Complex Matrices}$ .

**E8** has been used as a basis for physics models such as those by Lisi ( arXiv 1506.08073 ) and Smith ( viXra 1508.0157 ) so the purpose of this paper is to show a **connection between Connes NCG Physics and E8**.

**Connes NCG** is described by van den Dungen and van Suijlekom in arXiv 1204.0328 where they say: "... this review article is to present the applications of Connes' noncommutative geometry to elementary particle physics.

...

the noncommutative description of the Standard Model does not require the introduction of extra spacetime dimensions, its construction is very much like the original Kaluza-Klein theories. In fact, **one starts with a product  $M \times F$  of ordinary four-dimensional spacetime  $M$  with an internal space  $F$**  which is to describe the gauge content of the theory. Of course, spacetime itself still describes the gravitational part. The main difference with Kaluza-Klein theories is that the additional space is a discrete ... space whose structure is described by a ... noncommutative algebra ... This is very much like the description of spacetime  $M$  by its coordinate functions as usual in General Relativity, which form an algebra under pointwise multiplication:

$$(x^\mu x^\nu)(p) = x^\mu(p) x^\nu(p)$$

Such commutative relations are secretly used in any physics textbook. However, for a discrete space, ... propose to describe  $F$  by matrices ... yielding a much richer internal (algebraic) structure ... one can also describe a metric on  $F$  in terms of algebraic data, so that we can fully describe the geometrical structure of  $M \times F$ . This type of noncommutative manifolds are called almost-commutative (AC)

...

Given an AC manifold  $M \times F$  ... the group of diffeomorphisms ... generalized to such noncommutative spaces combines ordinary diffeomorphisms of  $M$  with gauge symmetries ... we obtain a combination of general coordinate transformations on  $M$  with the respective groups ...  $U(1) \times SU(2) \times SU(3)$  ... [whose] ... finite space is ...

**internal space  $F$  ... [ = ] ...  $C \times H \times M3(C)$**

... to construct a Lagrangian from the geometry of  $M \times F$ . This is accomplished by ... a simple counting of the eigenvalues of a Dirac operator on  $M \times F$  which are lower than a cutoff  $\Lambda$  ... we derive local formulas (integrals of Lagrangians) ... using heat kernel methods ...

The fermionic action is given as usual by an inner product.  
 The Lagrangians that one obtains in this way ... are the right ones,  
 and in addition minimally coupled to gravity.  
 This is unification with gravity of ... the full Standard Model. ...  
 We study conformal invariance ... with particular emphasis on the Higgs mechanism  
 coupled to the gravitational background

...  
 the Lagrangian derived ... from the relevant noncommutative space is not just the  
 Standard Model Lagrangian, but it implies that there are relations between some of the  
 Standard Model couplings and masses

...  
 If we would assume that the mass of the top quark is much larger than all other fermion  
 masses, we may neglect the other fermion masses. In that case ...

$$m_{\text{top}} \leq \sqrt{8/3} M_w [ = \sqrt{8/3} 80 = 130 \text{ GeV} ]$$

...  
 we shall evaluate the renormalization group equations (RGEs) for the Standard Model  
 from ordinary energies up to the ... GUT ... unification scale ...

The scale  $\Lambda_{12}$  ... is given by ...  $1.03 \times 10^{13} \text{ GeV}$  ...

The [scale]  $\Lambda_{23}$  is given by ...  $9.92 \times 10^{16} \text{ GeV}$  ...

we have ... included the simple case where we ignore the Yukawa coupling of the tau-  
 neutrino

[ as is realistic with no neutrino see-saw mechanism ] ... Numerical results [ are ]...

$$\Lambda_{\text{gut}} (10^{16} \text{ GeV}) \dots m_{\text{top}} (\text{GeV}) 186.0 \dots m_{\text{h}} (\text{GeV}) 188.1 \dots$$

$$\Lambda_{\text{gut}} (10^{13} \text{ GeV}) \dots m_{\text{top}} (\text{GeV}) 183.2 \dots m_{\text{h}} (\text{GeV}) 188.3 \dots".$$

If you do a naive extrapolation down to the Higgs  $\text{VeV } 250 \text{ GeV}$  energy scale where the  
 compositeness of a Higgs as Tquark condensate system might become evident (the  
 Non-perturbativity Boundary)

$$\Lambda_{\text{comp}} (250 \text{ GeV}) \dots m_{\text{top}} (\text{GeV}) 173.2 \dots m_{\text{h}} (\text{GeV}) 189$$

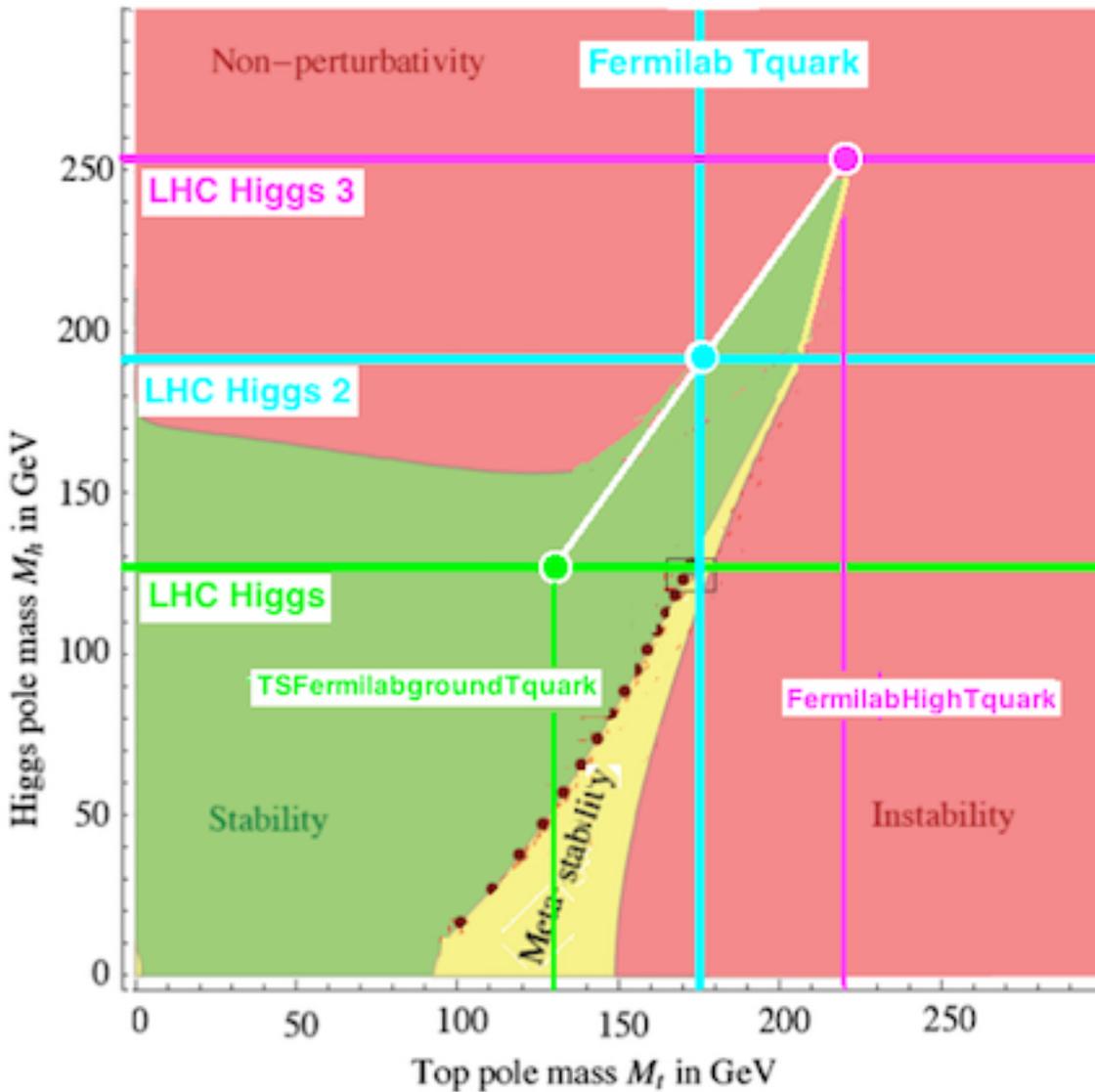
so the naively extrapolated

**NCG masses for the Tquark-Higgs Middle Mass States are consistent  
 with those of the E8 model of Smith ( viXra 1508.0157 )**

Further,

**the Basic Ground State NCG Tquark mass of 130 GeV is consistent  
 with that of the E8 model of Smith ( viXra 1508.0157 )**

Here is a chart showing the 3 Mass States of the Smith E8 model ( viXra 1508.0157 ): the green dot in the Stable region (green) has the 130 GeV Tquark mass state that is also calculated by NCG; the cyan dot on the Non-perturbativity Boundary has the 173 GeV Tquark and 189 GeV Higgs mass states that are also calculated by NCG; I have not seen where NCG may or may not calculate High-Mass (220 and 250 GeV) Tquark and Higgs mass states indicated by the magenta dot at the Critical Point.



## Structure of M and F of NCG

The **M of NCG** is 4-dim Spacetime, a discrete version of which is the Integral Domain of Integral Quaternions whose vertex figure ( nearest neighbors to the origin ) is the 24-cell Root Vector Polytope of the 28-dim D4 Lie Algebra which contains as a subalgebra the 15-dim D3 Lie Algebra of the Conformal Group  $\text{Spin}(2,4) = \text{SU}(2,2)$  for MacDowell-Mansouri Gravity plus Conformal Dark Energy.

4-dim Riemannian Spacetime can be Wick Rotated to 4-dim Euclidean Space which can be compactified to the 4-sphere  $S^4$  which can be discretized as the 600-cell



so the **M of NCG** can be locally represented as a 600-cell which has 120 vertices.

**F of NCG** is the 24-dim algebra  $C + H + M_3(C)$ .

Identify the 24 generators of F with the 24 elements of the Binary Tetrahedral Group and therefore **identify F with the Tetrahedron** of which it is the symmetry group. NCG, by using  $M \times F$  as its basic structure, puts a copy of F at each point of M.

Consider a flat 2-dim subspace of M, and add to it F Tetrahedra following this construction recipe from a Don Davis 8 Sep 1999 sci math post:

“... build ... a hollow torus of 300 cells ... as follows:

lay out a 5x10 grid of unit edges. omit the lefthand and lower boundaries' edges, because we're going to roll this grid into a torus later.

thus, the grid contains 100 edges: 50 running N-S, and 50 running EW.

attach one tetrahedron to each edge from above the grid.

the opposite edges of these tetrahedra will form a new 5x10 grid, whose vertices overlie the centers of the squares in the lower grid.

thus, these 100 tetrahedra now form an egg-carton shape, with 50 squarepyramid cups on each side.

divide each cup into two non-unit tetrahedra,

by erecting a right-triangular wall across the cup, corner-to-corner.

make the upper cups' dividers run NE/SW,

and make the upside-down lower cups' dividers run NW/SE.

note that the egg-carton is now a solid flat layer, one tetrahedron deep, containing 100 unit tetra- hedra and 200 non-unit tetrahedra.

when we shrink the right-triangular dividing walls into equilateral triangles, we distort each egg-cup into a pair of unit-tetrahedra.

at the same time,  
the opening of each egg-cup changes from a square to a bent rhombus.  
as the square openings bend,  
the flat sheet of 300 tetrahedra is forced to wrap around into a hollow torus with a one-unit-thick shell.

surprisingly,  
this bends each 5x10 grid into a toroidal sheet of 100 equilateral triangles.  
each grid's short edge is now a pentagon that threads through the donut hole.  
the grid's long edge is now a decagon that wraps around both holes in its donut.  
the two grids' long edges are now linked decagons.

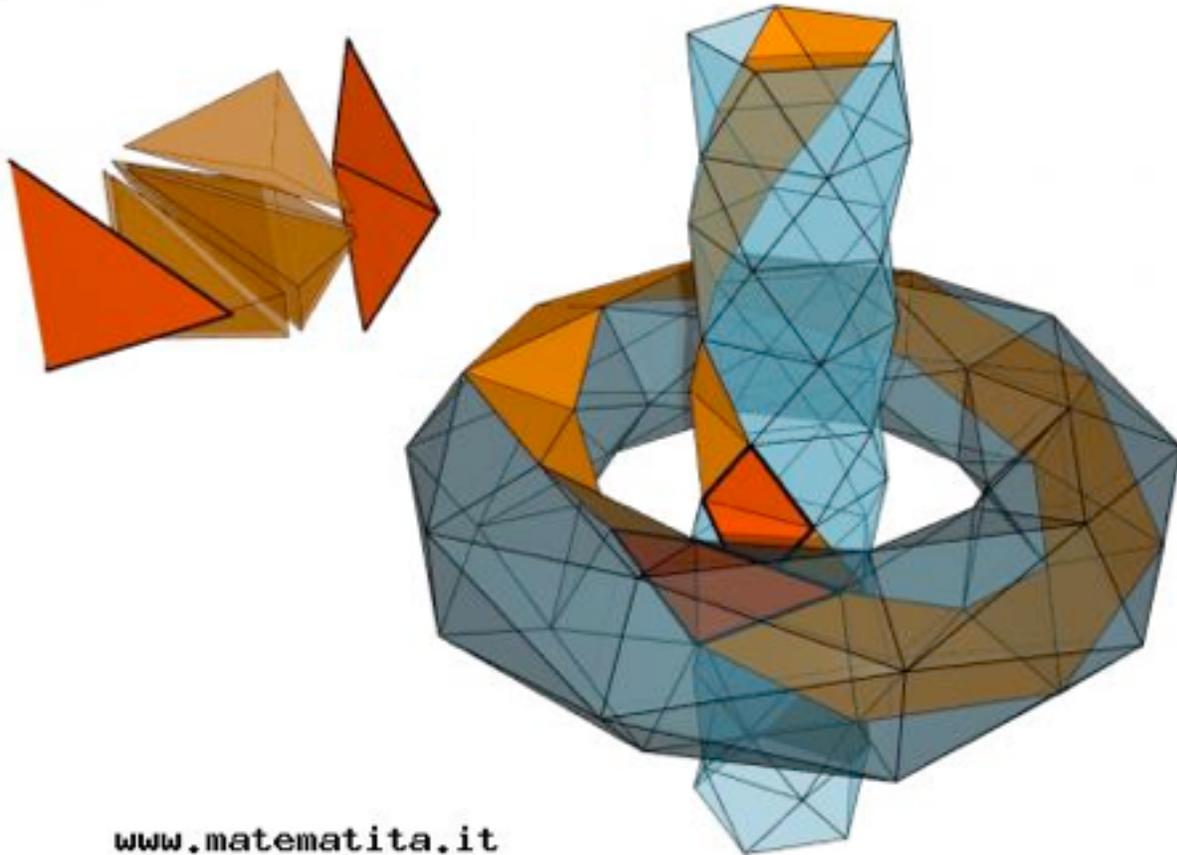
this wrapping cannot occur in R3, but it works fine in R4.  
I admit that this part of my presentation is not easy to visualize.  
perhaps a localized visualization image will help:  
as an upper egg-cup is squeezed in one direction,  
the edge-tetrahedra around it rotate,  
squeezing the nearby lower egg-cups in the other direction.  
this forces the flat sheet into a saddle-shape.  
in R4, when this saddle-bending happens across the whole egg-carton at once,  
the carton's edges can meet to make the toroidal sheet.

...

build each solid torus ....[of]... two solid tori of 150 cells each ... as follows:  
using 100 tetrahedra, assemble 5 solid icosahedra (this is possible in R4).  
daisy-chain five such icosahedra pole-to-pole ... between every pair of adjacent  
icosahedra, surround the common vertex with 10 tetrahedra.  
each solid torus has a decagonal "axis" running through the centers  
and poles of the icosahedra. each solid torus contains  $5 \cdot 20 + 5 \cdot 10 = 150$   
tetrahedra, and its surface is tiled with 100 equilateral triangles.  
on this surface, six triangles meet at every vertex.

...

we will link these solid tori, like two links of a chain. with the hollow torus acting as a glue layer between them ...[



]...

finally,  
put one solid torus inside the hollow toroidal sheet,  
attaching the 100 triangular faces of the solid  
to the 100 triangles of the sheet's inner surface.  
this gives us a fat solid torus,  
10 units around and 4 units thick, containing 450 tetrahedral cells.  
nevertheless, its surface has only 100 triangular faces.  
thread the second 150-cell solid torus through this fat torus,  
and attach the two solids' triangular faces. **this is the 600-cell polytope ...**".

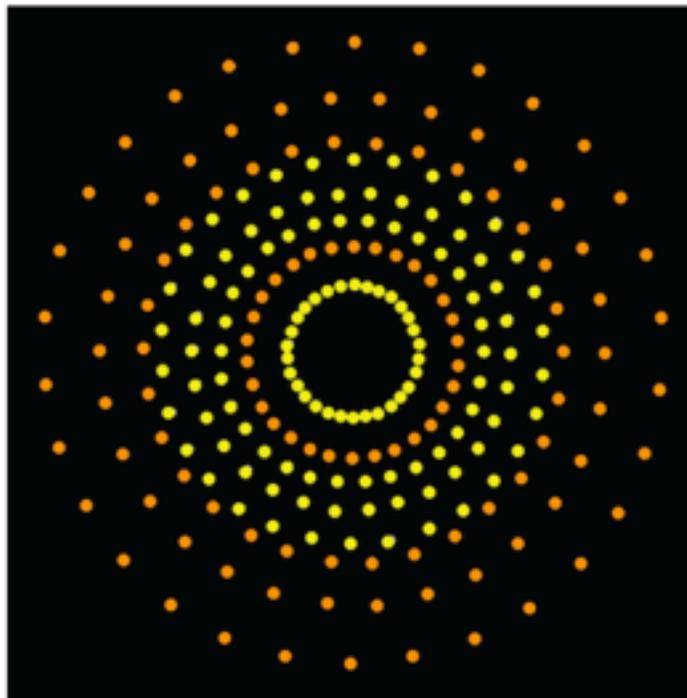


**Combine the M 600-cell (yellow)  
with the F 600-cell expanded by the Golden Ratio (orange)**

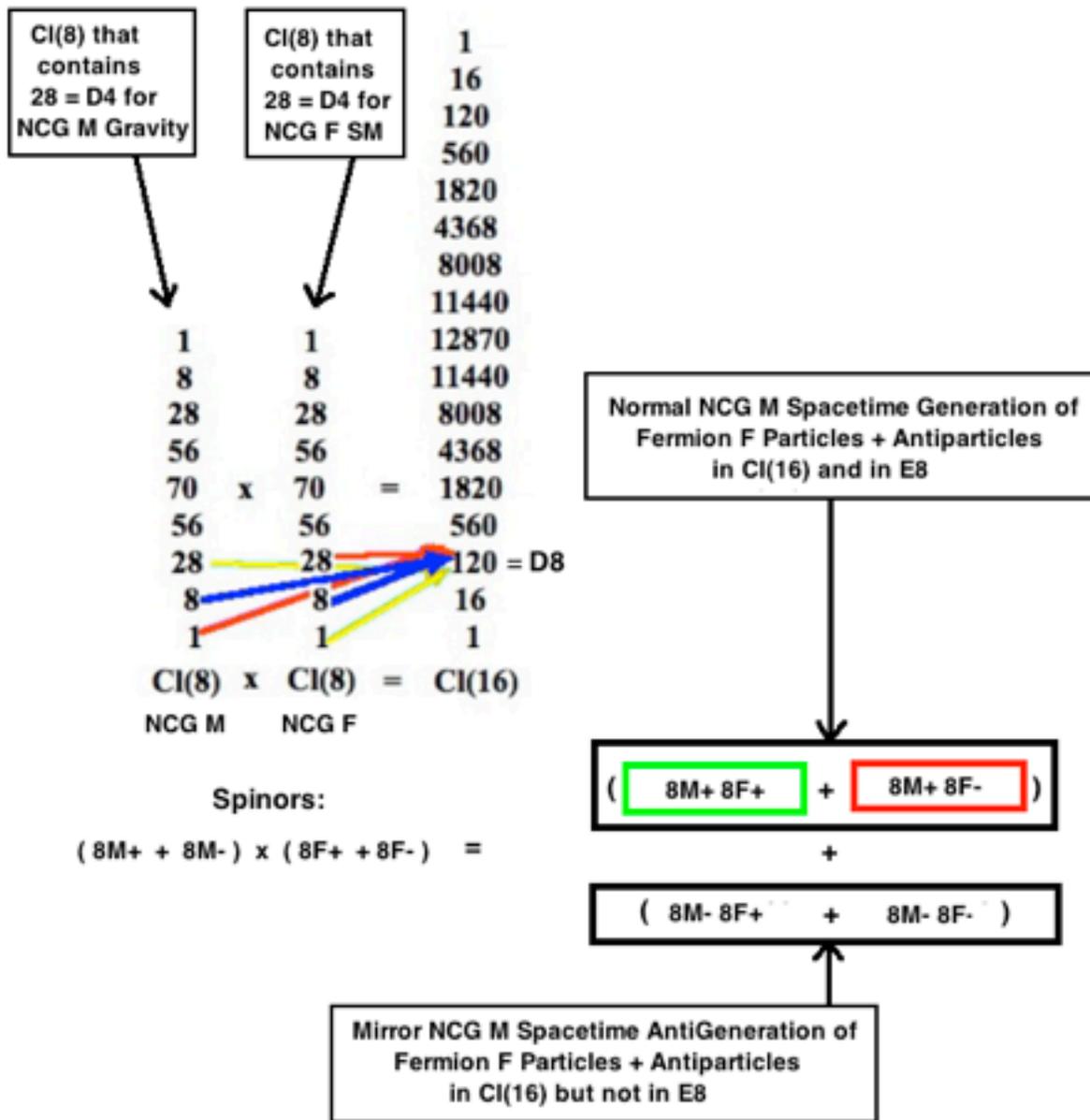


**to get the  $120+120 = 240$ -vertex 8-dim E8 polytope  
which is the Root Vector Polytope of the Lie Algebra E8**

In this way the 8-dim space of E8 Root Vectors is seen as  
being made up of two independent 4-dim spaces:  
a Rational Number 4-dim space of yellow M dots  
and  
an Algebraic Extension by the Golden Ratio 4-dim space of orange F dots



**The Lie Algebra E8 lives in the Clifford Algebra  $Cl(16) = Cl(8) \times Cl(8)$**



This is the basic structure of the E8 = Cl(16) Physics Model

described in viXra 1508.0157