To mitigate this problem, we can move both the poles and zeros from the unit circle to a circle just inside the unit circle, say at radius $r = 1 - \epsilon$, where ϵ is a very small number. Thus the system function of the linear-phase FIR filter becomes

$$H(z) = \frac{1 - r^M z^{-M} e^{j2\pi\alpha}}{M} \sum_{k=0}^{M-1} \frac{H(k+\alpha)}{1 - re^{j2\omega\pi(k+\alpha)/M} z^{-1}} \tag{10.2.42}$$

The corresponding two-pole filter realization given in Section 9.2.3 can be modified accordingly. The damping provided by selecting $r < 1$ ensures that roundoff noise will be bounded and thus instability is avoided.

10.2.4 Design of Optimum Equiripple Linear-Phase FIR Filters

The window method and the frequency-sampling method are relatively simple techniques for designing linear-phase FIR filters. However, they also possess some minor disadvantages, described in Section 10.2.6, which may render them undesirable for some applications. A major problem is the lack of precise control of the critical frequencies such as ω_p and ω_s.

The filter design method described in this section is formulated as a Chebyshev approximation problem. It is viewed as an optimum design criterion in the sense that the weighted approximation error between the desired frequency response and the actual frequency response is spread evenly across the passband and evenly across the stopband of the filter minimizing the maximum error. The resulting filter designs have ripples in both the passband and the stopband.

To describe the design procedure, let us consider the design of a lowpass filter with passband edge frequency ω_p and stopband edge frequency ω_s. From the general specifications given in Fig. 10.1.2, in the passband, the filter frequency response satisfies the condition

$$1 - \delta_1 \le H_r(\omega) \le 1 + \delta_1, \qquad |\omega| \le \omega_p \tag{10.2.43}$$

Similarly, in the stopband, the filter frequency response is specified to fall between the limits $\pm\delta_2$, that is,

$$-\delta_2 \le H_r(\omega) \le \delta_2, \qquad |\omega| > \omega_s \tag{10.2.44}$$

Thus δ_1 represents the ripple in the passband and δ_2 represents the attenuation or ripple in the stopband. The remaining filter parameter is M, the filter length or the number of filter coefficients.

Let us focus on the four different cases that result in a linear-phase FIR filter. These cases were treated in Section 10.2.2 and are summarized below.

Case 1: Symmetric unit sample response. $h(n) = h(M-1-n)$ **and** M **odd.** In this case, the real-valued frequency response characteristic $H_r(\omega)$ is

$$H_r(\omega) = h\left(\frac{M-1}{2}\right) + 2 \sum_{n=0}^{(M-3)/2} h(n) \cos\omega\left(\frac{M-1}{2} - n\right) \tag{10.2.45}$$

If we let $k = (M-1)/2 - n$ and define a new set of filter parameters $\{a(k)\}$ as

$$a(k) = \begin{cases} h\left(\dfrac{M-1}{2}\right), & k = 0 \\ 2h\left(\dfrac{M-1}{2} - k\right), & k = 1, 2, \ldots, \dfrac{M-1}{2} \end{cases} \tag{10.2.46}$$

then (10.2.45) reduces to the compact form

$$H_r(\omega) = \sum_{k=0}^{(M-1)/2} a(k) \cos \omega k \tag{10.2.47}$$

Case 2: Symmetric unit sample response. $h(n) = h(M-1-n)$ **and** M **even.** In this case, $H_r(\omega)$ is expressed as

$$H_r(\omega) = 2 \sum_{n=0}^{(M/2)-1} h(n) \cos \omega \left(\frac{M-1}{2} - n\right) \tag{10.2.48}$$

Again, we change the summation index from n to $k = M/2 - n$ and define a new set of filter parameters $\{b(k)\}$ as

$$b(k) = 2h\left(\frac{M}{2} - k\right), \qquad k = 1, 2, \ldots, M/2 \tag{10.2.49}$$

With these substitutions (10.2.48) becomes

$$H_r(\omega) = \sum_{k=1}^{M/2} b(k) \cos \omega \left(k - \frac{1}{2}\right) \tag{10.2.50}$$

In carrying out the optimization, it is convenient to rearrange (10.2.50) further into the form

$$H_r(\omega) = \cos \frac{\omega}{2} \sum_{k=0}^{(M/2)-1} \tilde{b}(k) \cos \omega k \tag{10.2.51}$$

where the coefficients $\{\tilde{b}(k)\}$ are linearly related to the coefficients $\{b(k)\}$. In fact, it can be shown that the relationship is

$$\begin{aligned} \tilde{b}(0) &= \tfrac{1}{2} b(1) \\ \tilde{b}(k) &= 2b(k) - \tilde{b}(k-1), \qquad k = 1, 2, 3, \ldots, \frac{M}{2} - 2 \\ \tilde{b}\left(\frac{M}{2} - 1\right) &= 2b\left(\frac{M}{2}\right) \end{aligned} \tag{10.2.52}$$

Case 3: Antisymmetric unit sample response. $h(n) = -h(M-1-n)$ **and** M **odd.** The real-valued frequency response characteristic $H_r(\omega)$ for this case is

$$H_r(\omega) = 2 \sum_{n=0}^{(M-3)/2} h(n) \sin \omega \left(\frac{M-1}{2} - n \right) \tag{10.2.53}$$

If we change the summation in (10.2.53) from n to $k = (M-1)/2 - n$ and define a new set of filter parameters $\{c(k)\}$ as

$$c(k) = 2h \left(\frac{M-1}{2} - k \right), \qquad k = 1, 2, \ldots, (M-1)/2 \tag{10.2.54}$$

then (10.2.53) becomes

$$H_r(\omega) = \sum_{k=1}^{(M-1)/2} c(k) \sin \omega k \tag{10.2.55}$$

As in the previous case, it is convenient to rearrange (10.2.55) into the form

$$H_r(\omega) = \sin \omega \sum_{k=0}^{(M-3)/2} \tilde{c}(k) \cos \omega k \tag{10.2.56}$$

where the coefficients $\{\tilde{c}(k)\}$ are linearly related to the parameters $\{c(k)\}$. This desired relationship can be derived from (10.2.55) and (10.2.56) and is simply given as

$$\begin{aligned}
\tilde{c}\left(\frac{M-3}{2}\right) &= c\left(\frac{M-1}{2}\right) \\
\tilde{c}\left(\frac{M-5}{2}\right) &= 2c\left(\frac{M-3}{2}\right) \\
\vdots \qquad & \qquad \vdots \\
\tilde{c}(k-1) - \tilde{c}(k+1) &= 2c(k), \qquad 2 \le k \le \frac{M-5}{2} \\
\tilde{c}(0) + \tfrac{1}{2}\tilde{c}(2) &= c(1)
\end{aligned} \tag{10.2.57}$$

Case 4: Antisymmetric unit sample response. $h(n) = -h(M-1-n)$ **and** M **even.** In this case, the real-valued frequency response characteristic $H_r(\omega)$ is

$$H_r(\omega) = 2 \sum_{n=0}^{(M/2)-1} h(n) \sin \omega \left(\frac{M-1}{2} - n \right) \tag{10.2.58}$$

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A change in the summation index from n to $k = M/2 - n$ combined with a definition of a new set of filter coefficients $\{d(k)\}$, related to $\{h(n)\}$ according to

$$d(k) = 2h\left(\frac{M}{2} - k\right), \qquad k = 1, 2, \ldots, \frac{M}{2} \tag{10.2.59}$$

results in the expression

$$H_r(\omega) = \sum_{k=1}^{M/2} d(k) \sin \omega \left(k - \frac{1}{2}\right) \tag{10.2.60}$$

As in the previous two cases, we find it convenient to rearrange (10.2.60) into the form

$$H_r(\omega) = \sin \frac{\omega}{2} \sum_{k=0}^{(M/2)-1} \tilde{d}(k) \cos \omega k \tag{10.2.61}$$

where the new filter parameters $\{\tilde{d}(k)\}$ are related to $\{d(k)\}$ as follows:

$$\begin{aligned} \tilde{d}\left(\frac{M}{2} - 1\right) &= 2d\left(\frac{M}{2}\right) \\ \tilde{d}(k-1) - \tilde{d}(k) &= 2d(k), \qquad 2 \le k \le \frac{M}{2} - 1 \\ \tilde{d}(0) - \tfrac{1}{2}\tilde{d}(1) &= d(1) \end{aligned} \tag{10.2.62}$$

TABLE 10.5 Real-Valued Frequency Response Functions for Linear-Phase FIR Filters

Filter type	$Q(\omega)$	$P(\omega)$
$h(n) = h(M-1-n)$ M odd (Case 1)	1	$\sum_{k=0}^{(M-1)/2} a(k) \cos \omega k$
$h(n) = h(M-1-n)$ M even (Case 2)	$\cos \frac{\omega}{2}$	$\sum_{k=0}^{(M/2)-1} \tilde{b}(k) \cos \omega k$
$h(n) = -h(M-1-n)$ M odd (Case 3)	$\sin \omega$	$\sum_{k=0}^{(M-3)/2} \tilde{c}(k) \cos \omega k$
$h(n) = -h(M-1-n)$ M even (Case 4)	$\sin \frac{\omega}{2}$	$\sum_{k=0}^{(M/2)-1} \tilde{d}(k) \cos \omega k$

The expressions for $H_r(\omega)$ in these four cases are summarized in Table 10.5. We note that the rearrangements that we made in Cases 2, 3, and 4 have allowed us to express $H_r(\omega)$ as

$$H_r(\omega) = Q(\omega)P(\omega) \tag{10.2.63}$$

where

$$Q(\omega) = \begin{cases} 1 & \text{Case 1} \\ \cos\dfrac{\omega}{2} & \text{Case 2} \\ \sin\omega & \text{Case 3} \\ \sin\dfrac{\omega}{2} & \text{Case 4} \end{cases} \tag{10.2.64}$$

and $P(\omega)$ has the common form

$$P(\omega) = \sum_{k=0}^{L} \alpha(k)\cos\omega k \tag{10.2.65}$$

with $\{\alpha(k)\}$ representing the parameters of the filter, which are linearly related to the unit sample response $h(n)$ of the FIR filter. The upper limit L in the sum is $L = (M-1)/2$ for Case 1, $L = (M-3)/2$ for Case 3, and $L = M/2 - 1$ for Case 2 and Case 4.

In addition to the common framework given above for the representation of $H_r(\omega)$, we also define the real-valued desired frequency response $H_{dr}(\omega)$ and the weighting function $W(\omega)$ on the approximation error. The real-valued desired frequency response $H_{dr}(\omega)$ is simply defined to be unity in the passband and zero in the stopband. For example, Fig. 10.2.15 illustrates several different types of characteristics for $H_{dr}(\omega)$. The weighting function on the approximation error allows us to choose the relative size of the errors in the different frequency bands (i.e., in the passband and in the stopband). In particular, it is convenient to normalize $W(\omega)$ to unity in the stopband and set $W(\omega) = \delta_2/\delta_1$ in the passband, that is,

$$W(\omega) = \begin{cases} \delta_2/\delta_1, & \omega \text{ in the passband} \\ 1, & \omega \text{ in the stopband} \end{cases} \tag{10.2.66}$$

Then we simply select $W(\omega)$ in the passband to reflect our emphasis on the relative size of the ripple in the stopband to the ripple in the passband.

With the specification of $H_{dr}(\omega)$ and $W(\omega)$, we can now define the weighted approximation error as

$$\begin{aligned} E(\omega) &= W(\omega)[H_{dr}(\omega) - H_r(\omega)] \\ &= W(\omega)[H_{dr}(\omega) - Q(\omega)P(\omega)] \\ &= W(\omega)Q(\omega)\left[\frac{H_{dr}(\omega)}{Q(\omega)} - P(\omega)\right] \end{aligned} \tag{10.2.67}$$

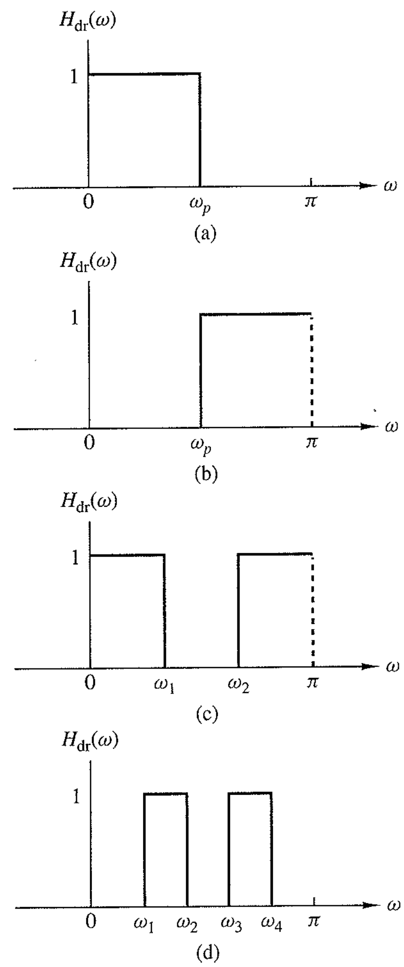

Figure 10.2.15
Desired frequency response characteristics for different types of filters.

For mathematical convenience, we define a modified weighting function $\hat{W}(\omega)$ and a modified desired frequency response $\hat{H}_{dr}(\omega)$ as

$$\hat{W}(\omega) = W(\omega)Q(\omega)$$
$$\hat{H}_{dr}(\omega) = \frac{H_{dr}(\omega)}{Q(\omega)} \tag{10.2.68}$$

Then the weighted approximation error may be expressed as

$$E(\omega) = \hat{W}(\omega)[\hat{H}_{dr}(\omega) - P(\omega)] \tag{10.2.69}$$

for all four different types of linear-phase FIR filters.

Given the error function $E(\omega)$, the Chebyshev approximation problem is basically to determine the filter parameters $\{\alpha(k)\}$ that minimize the maximum absolute value of $E(\omega)$ over the frequency bands in which the approximation is to be performed. In mathematical terms, we seek the solution to the problem

$$\min_{\text{over } \{\alpha(k)\}} \left[\max_{\omega \in S} |E(\omega)| \right] = \min_{\text{over } \{\alpha(k)\}} \left[\max_{\omega \in S} |\hat{W}(\omega)[\hat{H}_{\text{dr}}(\omega) - \sum_{k=0}^{L} \alpha(k) \cos \omega k]| \right] \tag{10.2.70}$$

where S represents the set (disjoint union) of frequency bands over which the optimization is to be performed. Basically, the set S consists of the passbands and stopbands of the desired filter.

The solution to this problem is due to Parks and McClellan (1972a), who applied a theorem in the theory of Chebyshev approximation. It is called the *alternation theorem*, which we state without proof.

Alternation Theorem. Let S be a compact subset of the interval $[0, \pi)$. A necessary and sufficient condition for

$$P(\omega) = \sum_{k=0}^{L} \alpha(k) \cos \omega k$$

to be the unique, best weighted Chebyshev approximation to $\hat{H}_{\text{dr}}(\omega)$ in S, is that the error function $E(\omega)$ exhibit at least $L + 2$ extremal frequencies in S. That is, there must exist at least $L + 2$ frequencies $\{\omega_i\}$ in S such that $\omega_1 < \omega_2 < \cdots < \omega_{L+2}$, $E(\omega_i) = -E(\omega_{i+1})$, and

$$|E(\omega_i)| = \max_{\omega \in S} |E(\omega)|, \qquad i = 1, 2, \ldots, L + 2$$

We note that the error function $E(\omega)$ alternates in sign between two successive extremal frequencies. Hence the theorem is called the alternation theorem.

To elaborate on the alternation theorem, let us consider the design of a lowpass filter with passband $0 \le \omega \le \omega_p$ and stopband $\omega_s \le \omega \le \pi$. Since the desired frequency response $H_{\text{dr}}(\omega)$ and the weighting function $W(\omega)$ are piecewise constant, we have

$$\begin{aligned} \frac{dE(\omega)}{d\omega} &= \frac{d}{d\omega}\{W(\omega)[H_{\text{dr}}(\omega) - H_r(\omega)]\} \\ &= -\frac{dH_r(\omega)}{d\omega} = 0 \end{aligned}$$

Consequently, the frequencies $\{\omega_i\}$ corresponding to the peaks of $E(\omega)$ also correspond to peaks at which $H_r(\omega)$ meets the error tolerance. Since $H_r(\omega)$ is a trigonometric polynomial of degree L, for Case 1, for example,

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$$\begin{aligned} H_r(\omega) &= \sum_{k=0}^{L} \alpha(k) \cos \omega k \\ &= \sum_{k=0}^{L} \alpha(k) \left[\sum_{n=0}^{k} \beta_{nk} (\cos \omega)^n \right] \\ &= \sum_{k=0}^{L} \alpha'(k) (\cos \omega)^k \end{aligned} \tag{10.2.71}$$

it follows that $H_r(\omega)$ can have at most $L - 1$ local maxima and minima in the open interval $0 < \omega < \pi$. In addition, $\omega = 0$ and $\omega = \pi$ are usually extrema of $H_r(\omega)$ and, also, of $E(\omega)$. Therefore, $H_r(\omega)$ has at most $L + 1$ extremal frequencies. Furthermore, the band-edge frequencies ω_p and ω_s are also extrema of $E(\omega)$, since $|E(\omega)|$ is maximum at $\omega = \omega_p$ and $\omega = \omega_s$. As a consequence, there are at most $L+3$ extremal frequencies in $E(\omega)$ for the unique, best approximation of the ideal lowpass filter. On the other hand, the alternation theorem states that there are at least $L + 2$ extremal frequencies in $E(\omega)$. Thus the error function for the lowpass filter design has either $L+3$ or $L+2$ extrema. In general, filter designs that contain more than $L+2$ alternations or ripples are called *extra ripple filters*. When the filter design contains the maximum number of alternations, it is called a *maximal ripple filter*.

The alternation theorem guarantees a unique solution for the Chebyshev optimization problem in (10.2.70). At the desired extremal frequencies $\{\omega_n\}$, we have the set of equations

$$\hat{W}(\omega_n)[\hat{H}_{\mathrm{dr}}(\omega_n) - P(\omega_n)] = (-1)^n \delta, \qquad n = 0, 1, \ldots, L+1 \tag{10.2.72}$$

where δ represents the maximum value of the error function $E(\omega)$. In fact, if we select $W(\omega)$ as indicated by (10.2.66), it follows that $\delta = \delta_2$.

The set of linear equations in (10.2.72) can be rearranged as

$$P(\omega_n) + \frac{(-1)^n \delta}{\hat{W}(\omega_n)} = \hat{H}_{\mathrm{dr}}(\omega_n), \qquad n = 0, 1, \ldots, L+1$$

or, equivalently, in the form

$$\sum_{k=0}^{L} \alpha(k) \cos \omega_n k + \frac{(-1)^n \delta}{\hat{W}(\omega_n)} = \hat{H}_{\mathrm{dr}}(\omega_n), \qquad n = 0, 1, \ldots, L+1 \tag{10.2.73}$$

If we treat the $\{a(k)\}$ and δ as the parameters to be determined, (10.2.73) can be expressed in matrix form as

$$\begin{bmatrix} 1 & \cos\omega_0 & \cos 2\omega_0 & \cdots & \cos L\omega_0 & \dfrac{1}{\hat{W}(\omega_0)} \\ 1 & \cos\omega_1 & \cos 2\omega_1 & \cdots & \cos L\omega_1 & \dfrac{-1}{\hat{W}(\omega_1)} \\ \vdots & & & & & \\ \vdots & & & & & \\ 1 & \cos\omega_{L+1} & \cos 2\omega_{L+1} & \cdots & \cos L\omega_{L+1} & \dfrac{(-1)^{L+1}}{\hat{W}(\omega_{L+1})} \end{bmatrix} \begin{bmatrix} \alpha(0) \\ \alpha(1) \\ \vdots \\ \alpha(L) \\ \delta \end{bmatrix} = \begin{bmatrix} \hat{H}_{\mathrm{dr}}(\omega_0) \\ \hat{H}_{\mathrm{dr}}(\omega_1) \\ \vdots \\ \vdots \\ \hat{H}_{\mathrm{dr}}(\omega_{L+1}) \end{bmatrix} \tag{10.2.74}$$

Initially, we know neither the set of extremal frequencies $\{\omega_n\}$ nor the parameters $\{\alpha(k)\}$ and δ. To solve for the parameters, we use an iterative algorithm, called the *Remez exchange algorithm* [see Rabiner et al. (1975)], in which we begin by guessing at the set of extremal frequencies, determine $P(\omega)$ and δ, and then compute the error function $E(\omega)$. From $E(\omega)$ we determine another set of $L+2$ extremal frequencies and repeat the process iteratively until it converges to the optimal set of extremal frequencies. Although the matrix equation in (10.2.74) can be used in the iterative procedure, matrix inversion is time consuming and inefficient.

A more efficient procedure, suggested in the paper by Rabiner et al. (1975), is to compute δ analytically, according to the formula

$$\delta = \frac{\gamma_0 \hat{H}_{\mathrm{dr}}(\omega_0) + \gamma_1 \hat{H}_{\mathrm{dr}}(\omega_1) + \cdots + \gamma_{L+1} \hat{H}_{\mathrm{dr}}(\omega_{L+1})}{\dfrac{\gamma_0}{\hat{W}(\omega_0)} - \dfrac{\gamma_1}{\hat{W}(\omega_1)} + \cdots + \dfrac{(-1)^{L+1}\gamma_{L+1}}{\hat{W}(\omega_{L+1})}} \tag{10.2.75}$$

where

$$\gamma_k = \prod_{\substack{n=0 \\ n\neq k}}^{L+1} \frac{1}{\cos\omega_k - \cos\omega_n} \tag{10.2.76}$$

The expression for δ in (10.2.75) follows immediately from the matrix equation in (10.2.74). Thus with an initial guess at the $L+2$ extremal frequencies, we compute δ.

Now since $P(\omega)$ is a trigonometric polynomial of the form

$$P(\omega) = \sum_{k=0}^{L} \alpha(k) x^k, \qquad x = \cos\omega$$

and since we know that the polynomial at the points $x_n \equiv \cos\omega_n$, $n = 0, 1, \ldots, L+1$, has the corresponding values

$$P(\omega_n) = \hat{H}_{\mathrm{dr}}(\omega_n) - \frac{(-1)^n \delta}{\hat{W}(\omega_n)}, \qquad n = 0, 1, \ldots, L+1 \tag{10.2.77}$$

we can use the Lagrange interpolation formula for $P(\omega)$. Thus $P(\omega)$ can be expressed as [see Hamming (1962)]

$$P(\omega) = \frac{\sum_{k=0}^{L} P(\omega_k)[\beta_k/(x - x_k)]}{\sum_{k=0}^{L} [\beta_k/(x - x_k)]} \tag{10.2.78}$$

where $P(\omega_n)$ is given by (10.2.77), $x = \cos\omega$, $x_k = \cos\omega_k$, and

$$\beta_k = \prod_{\substack{n=0 \\ n\neq k}}^{L} \frac{1}{x_k - x_n} \tag{10.2.79}$$

Having the solution for $P(\omega)$, we can now compute the error function $E(\omega)$ from

$$E(\omega) = \hat{W}(\omega)[\hat{H}_{\mathrm{dr}}(\omega) - P(\omega)] \tag{10.2.80}$$

on a dense set of frequency points. Usually, a number of points equal to $16M$, where M is the length of the filter, suffices. If $|E(\omega)| \geq \delta$ for some frequencies on the dense set, then a new set of frequencies corresponding to the $L+2$ largest peaks of $|E(\omega)|$ are selected and the computational procedure beginning with (10.2.75) is repeated. Since the new set of $L+2$ extremal frequencies is selected to correspond to the peaks of the error function $|E(\omega)|$, the algorithm forces δ to increase in each iteration until it converges to the upper bound and hence to the optimum solution for the Chebyshev approximation problem. In other words, when $|E(\omega)| \leq \delta$ for all frequencies on the dense set, the optimal solution has been found in terms of the polynomial $H(\omega)$. A flowchart of the algorithm is shown in Fig. 10.2.16 and is due to Remez (1957).

Once the optimal solution has been obtained in terms of $P(\omega)$, the unit sample response $h(n)$ can be computed directly, without having to compute the parameters $\{\alpha(k)\}$. In effect, we have determined

$$H_r(\omega) = Q(\omega)P(\omega)$$

which can be evaluated at $\omega = 2\pi k/M$, $k = 0, 1, \ldots, (M-1)/2$, for M odd, or $M/2$ for M even. Then, depending on the type of filter being designed, $h(n)$ can be determined from the formulas given in Table 10.3.

A computer program written by Parks and McClellan (1972b) is available for designing linear-phase FIR filters based on the Chebyshev approximation criterion and implemented with the Remez exchange algorithm. This program can be used to design lowpass, highpass, or bandpass filters, differentiators, and Hilbert transformers. The latter two types of filters are described in the following sections. A number of software packages for designing equiripple linear-phase FIR filte. now available.

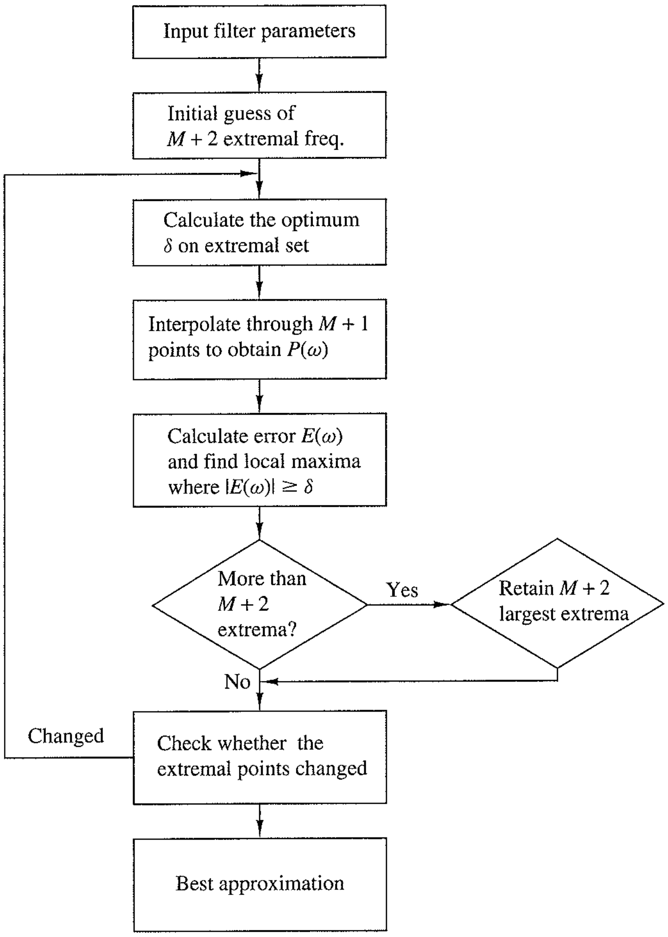

Figure 10.2.16 Flowchart of Remez algorithm.

The Parks–McClellan program requires a number of input parameters which determine the filter characteristics. In particular, the following parameters must be specified:

NFILT: The filter length, denoted above as M.

JTYPE: Type of filter:
JTYPE = 1 results in a multiple passband/stopband filter.
JTYPE = 2 results in a differentiator.
JTYPE = 3 results in a Hilbert transformer.

NBANDS: The number of frequency bands from 2 (for a lowpass filter) to a maximum of 10 (for a multiple-band filter).

LGRID: The grid density for interpolating the error function $E(\omega)$. The default value is 16 if left unspecified.

EDGE: The frequency bands specified by lower and upper cutoff frequencies,

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EXAMPLE 10.2.3

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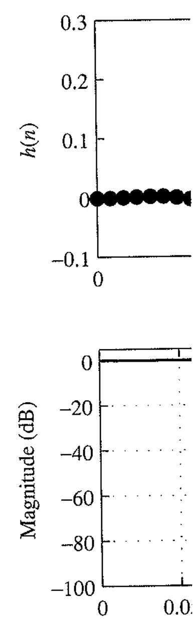

Figure 10.2.17 I
Example 10.2.3.

up to a maximum of 10 bands (an array of size 20, maximum). The frequencies are given in terms of the variable $f = \omega/2\pi$, where $f = 0.5$ corresponds to the folding frequency.

FX: An array of maximum size 10 that specifies the desired frequency response $H_{dr}(\omega)$ in each band.

WTX: An array of maximum size 10 that specifies the weight function in each band.

The following examples demonstrate the use of this program to design a lowpass and a bandpass filter.

EXAMPLE 10.2.3

Design a lowpass filter of length $M = 61$ with a passband edge frequency $f_p = 0.1$ and a stopband edge frequency $f_s = 0.15$.

Solution. The lowpass filter is a two-band filter with passband edge frequencies $(0, 0.1)$ and stopband edge frequencies $(0.15, 0.5)$. The desired response is $(1, 0)$ and the weight function is arbitrarily selected as $(1, 1)$.

$$61, 1, 2$$
$$0.0, 0.1, 0.15, 0.5$$
$$1.0, 0.0$$
$$1.0, 1.0$$

The impulse response and frequency response are shown in Fig. 10.2.17. The resulting filter has a stopband attenuation of −56 dB and a passband ripple of 0.0135 dB.

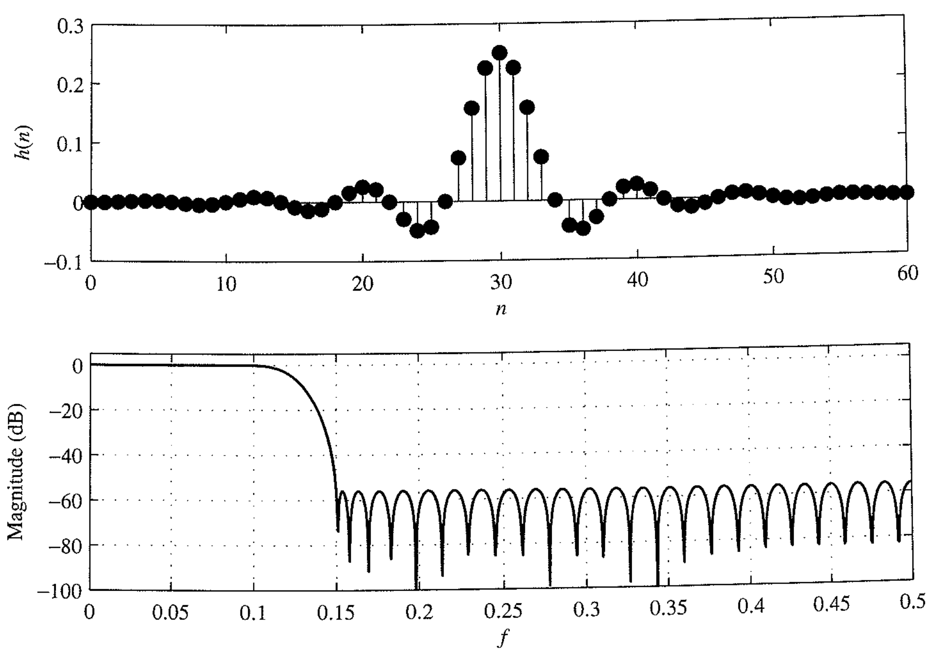

Figure 10.2.17 Impulse response and frequency response of $M = 61$ FIR filter in Example 10.2.3.

If we increase the length of the filter to $M = 101$ while maintaining all the other parameters given above the same, the resulting filter has the impulse response and frequency response characteristics shown in Fig. 10.2.18. Now, the stopband attenuation is -85 dB and the passband ripple is reduced to 0.00046 dB.

We should indicate that it is possible to increase the attenuation in the stopband by keeping the filter length fixed, say at $M = 61$, and decreasing the weighting function $W(\omega) = \delta_2/\delta_1$ in the passband. With $M = 61$ and a weighting function $(0.1, 1)$, we obtain a filter that has a stopband attenuation of -65 dB and a passband ripple of 0.049 dB.

EXAMPLE 10.2.4

Design a bandpass filter of length $M = 32$ with passband edge frequencies $f_{p1} = 0.2$ and $f_{p2} = 0.35$ and stopband edge frequencies of $f_{s1} = 0.1$ and $f_{s2} = 0.425$.

Solution. This passband filter is a three-band filter with a stopband range of $(0, 0.1)$, a passband range of $(0.2, 0.35)$, and a second stopband range of $(0.425, 0.5)$. The weighting function is selected as $(10.0, 1.0, 10.0)$, or as $(1.0, 0.1, 1.0)$, and the desired response in the three bands is $(0.0, 1.0, 0.0)$. Thus the input parameters to the program are

32, 1, 3
0.0, 0.1, 0.2, 0.35, 0.425, 0.5
0.0, 1.0, 0.0
10.0, 1.0, 10.0

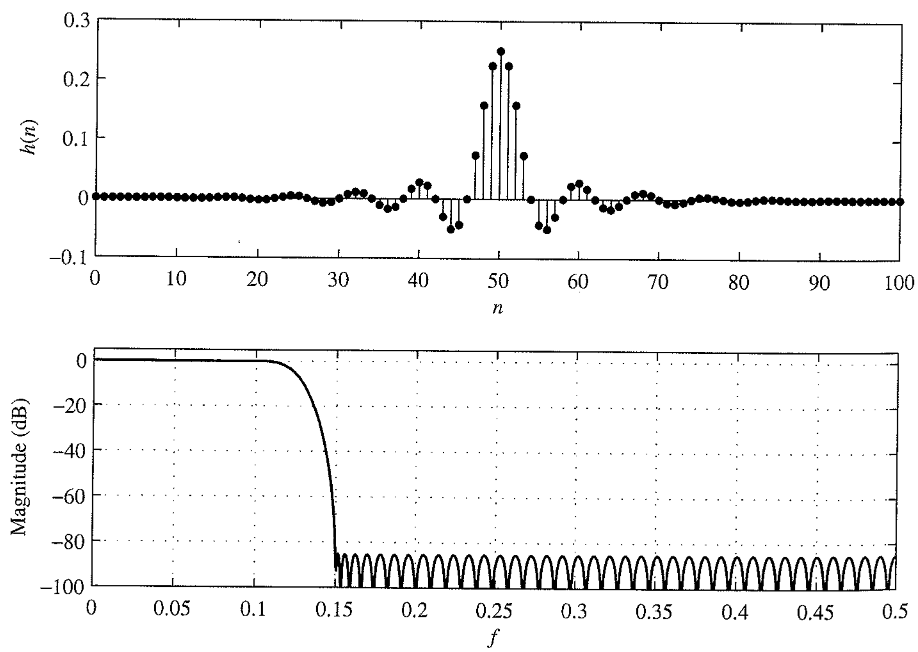

Figure 10.2.18 Impulse response and frequency response of $M = 101$ FIR filter in Example 10.2.3.

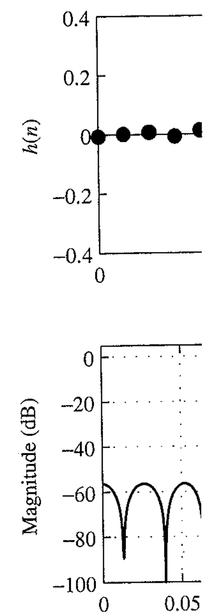

Figure 10.2.19 In
Example 10.2.4.

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