United Dipole Field

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The field of an electromagnetic (E) dipole has been examined using general relativistic (R) and quantum mechanical (Q) points of view, and an E=Q=R equivalence principle presented whereas the curvature of the electromagnetic streamlines of the field are taken to be evidence of the distortion of spacetime, and hence of the presence of a gravitational field surrounding the dipole. Using a quasi-refractive index function N, with the streamlines and equipotential surfaces as coordinates, a new dipole relativistic metric is described, replacing Schwarzschild's for a point mass. The same principle equates the curvature and other physical features of the field with fundamental quantum concepts such as the uncertainty principle, the probability distribution and the wave packet. The equations of the dipole field therefore yield the three fields emerging naturally one from the other and unified without resorting to any new dimensions. It is speculated whether this model can be extended to dipolar matter-antimatter pairs.

I. INTRODUCTION

The quest for a unified theory of Electromagnetism (E) Gravitational Relativity [R] and Quantum Mechanics (Q) is hampered by the abstractness of quantum concepts, compared to the other two. Feynman, in saying that there is no physical 'machinery' causing quantum effects [1] accepts Born's probabilistic interpretation of quantum events [2]. This lack of physical reality had always bothered Einstein, who advocated that a successful unification entails "starting all over again" [3]. Superstring unification theory [4] does indeed start afresh, but is not yet a complete theory, and there is still room to search for different starting points.

The present paper grew out of research in diffraction theory [5, 6], and no complete unification theory was attempted or is of course claimed from this elementary treatment. Nevertheless, it was found that certain physical features of the electromagnetic dipole field could have intriguing quantum probabilistic interpretations. And when it was discovered, through general relativistic analysis of the streamline curvature, that the dipole might also possess a gravitational field, the question arose whether the elements of a prototype unified field theory were now at hand.

The diffraction of a dipole-photon is examined in Section II as a more exact form of the Huygens-Fresnel principle. In Section III the equipotential surfaces ϕ, and the orthogonal streamlines S of a dipole field are pro-

posed as the coordinates of a $g_{\mu\nu}$ relativistic metric symmetric around the dipole axis and plane only, replacing the sphero-symmetric Schwarzschild metric for a particle: the severe curvature of the streamlines in the dipole origin is interpreted as an indication of an unexpectedly powerful gravitational field there. This field is characterized by a radial quasi-refractive index function $N(R,\theta)$ of the distortion of spacetime. In Section IV the dipole electric field is shown to coincide with the Gaussian probability function while the dipole's frequency and amplitude attenuations follow those of a quantum wave packet. An E=Q=R equivalence principle is stated in Section V, and it is speculated in Section VI whether the dipole unified field model may be adapted to other particles than the photon, perhaps through the concept of virtual pairs of matter-antimatter, and whether the strong force could be attributed to a dipole gravitational field.

II. ELECTROMAGNETIC DIPOLE DIFFRACTION

Using intuitive hydrodynamical arguments, Tamari has proposed that an expanding photon field must have both a foreword linear and a radial component [7], resembling the bow wave of a boat. Miller [8] has shown that Maxwell's equations can yield a more exact form of the Huygens-Fresnel principle for a point source, replacing it by a dipole with a given spatio-temporal relation of the phase and separation d of positive and negative charges a, emitting a field with wavelength $\lambda_0 = 2\pi/k$, and with a potential field:

$$\phi = \frac{ad}{4\pi R}\left[k\sin(kR)(1+\cos\theta) + \cos(kr)\frac{\cos\theta}{R}\right] \quad (1)$$

Both the bow-wave and the spatio-temporal dipole models of the photon have a characteristic geometry of expanding nested near-circular equipotential surfaces joined at the origin. For the sake of simplifying the analysis here, an approximate dipole field is taken from the

literature [9] whereby, for a dipole of moment $D = ad/4\pi$ aligned with the z-axis and symmetrical about the origin, with a time harmonic phase term H, the potential is:

$$\phi = \frac{DH\cos\theta}{R^2} \qquad (2)$$

with orthogonal streamlines:

$$S = \frac{R}{\sin^2\theta} \qquad (3)$$

as shown in Figure 1. These approximations also omit the magnetic and quadrupole components. While Eq. 2 and Eq. 3 are symmetrical about the $x-y$ plane, both the bow-wave and the spatio-temporal dipole field of Eq. 1 vanish in the $-z$ regions.

The curvature of the dipole streamlines S, along which the electromagnetic energy flows, is also a feature seen in Braunbek and Laukien's [10] analysis of Sommerfeld's rigorous solution of diffraction around an infinite half-plane [11]. In Tamari's Streamline Diffraction Theory [5] the various curved diffracting streamlines inclined at angles $-\frac{\pi}{2} \le \theta \le +\frac{\pi}{2}$ carry all the Fourier components of the field [12] in a fan-shaped flow pattern. Along the $+z$ axis the dipole streamline is straight and carries the electromagnetic field at a speed c, with equiphasals separated by the wavelength λ_0. But what of all the other curved S?
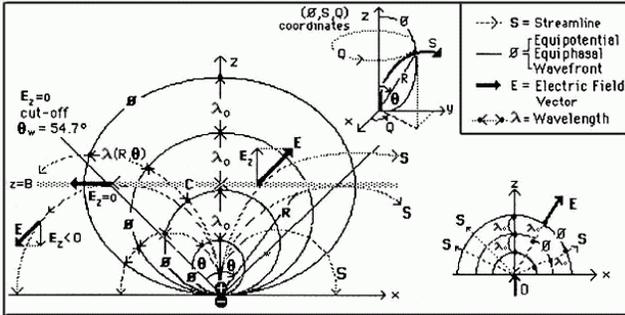

FIG. 1: Diagram of a dipole field. On the right, a Huygens-Fresnel wavelet.

III. RELATIVISTIC DIPOLE GRAVITATION

By definition no energy flows between streamlines, and each curved S can be considered a unique pathway carrying light. But according to Einstein's general theory of relativity it is the presence of a gravitational field that causes a ray of light to bend [13]. Postponing for the present the question of what the source of the dipole's gravitational field would be, the geometry of the streamlines themselves is considered. Since they carry light, S
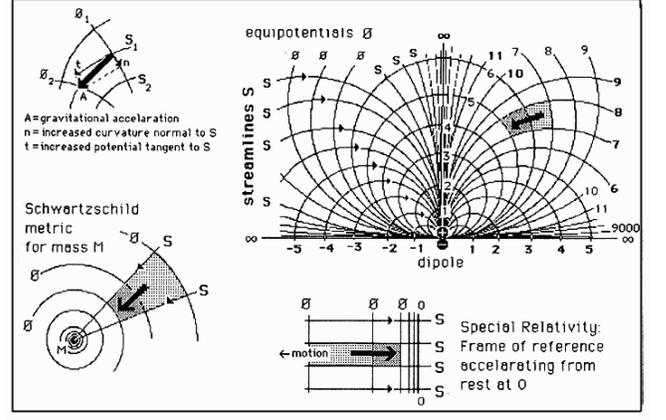

FIG. 2: Gravitational acceleration $\rightarrow$ acts along streamlines (S) towards increasing potential (ϕ): its tangential and normal components point towards increasingly curved spacetime.

must be *geodesics* of the field, *i.e.* following a minimum path through the dipole's spacetime [14]. Figure 2 illustrates the intuitive basis for regarding the dipole field as a gravitational field. A new (ϕ, S, Q, t) coordinate system based on Eq. 2 and Eq. 3 and symmetric about the z-axis is therefore defined, whereby, for a time $t = 0$, the distance element is:

$$ds^2 = d\phi^2 + dS^2 + R^2\sin^2\theta dQ^2 - \frac{c^2dt^2}{N} = g_{\mu\nu}x_1^2x_2^2 \;(4)$$

$$
\begin{aligned}
ds^2 = & \\
& \left[\frac{1}{\sin^4\theta} + \frac{4D^2H^2\cos^2\theta}{R^6}\right]dR^2 & \ldots\ldots g_{11} \\
+ & \left[\frac{4D^2H^2\cos\theta\sin\theta}{R^6} - \frac{4R\cos\theta}{\sin^5\theta}\right]dRd\theta & \ldots g_{12},\; g_{21} \\
+ & \left[\frac{4R^4\cos^2\theta}{\sin^6\theta} + \frac{D^2H^2\sin^2\theta}{R^2}\right]d\theta^2 & \ldots\ldots g_{22} \\
+ & \left[R^2\sin^2\theta\right]dQ^2 & \ldots\ldots g_{33} \\
+ & \left[\frac{c^2R^3}{DH\sqrt{3\cos^2\theta+1}}\right]dt^2 & \ldots\ldots g_{44}
\end{aligned}
$$

$$(5)$$

with all the other $g_{\mu\nu} = 0$. The N term is to be explained at length below. A full treatment using tensor algebra proving that the $g_{\mu\nu}$ obey Einstein's equations for general relativity, and finding the curvature tensor $g^{\mu\nu}G_{\mu\nu}$ although essential, is beyond the scope of this paper. Fortunately, there is a mathematically simple and exact device, which helps conceive the curvature of spacetime in vivid physical terms.

In dynamics the curvature of the path of a particle is evidence of acceleration, and hence a change in local velocity. According to Einstein, "A curvature of rays of light can only take place when the velocity of propagation of light varies with position" [15], a rather problematic statement because of the constancy of c in his relativity

theory. But as Eddington [16] explained, it is the "coordinate velocity" v of light, which diminishes in the presence of a gravitational field, and hence a quasi-refractive index of spacetime N can be defined where:

$$N = \frac{c}{v} \tag{6}$$

Since the local velocity defines N, it is a measure of *both* space contraction and time dilation, the more conventional relativistic terms. Eddington, Lodge, and others [17] used this concept of a variable density of spacetime, a gradient-index field N, akin to that in geometrical optics [18] to describe relativistic phenomena such as the properties of a black hole.

The use of N transforms a relativistic coordinate system into a classical one: At first S are considered "straight" geodesics in the vacuum surrounding the dipole. Alternately, the same S can be considered the curved rays of light in a glass-like medium with a gradient index of refraction function $N(R, \theta)$. While the two treatments in no way change the physical realities of the field, a variable speed of light v can now be unambiguously defined, and a geodesic can therefore be considered the solution to the eikonal equation of geometrical optics [18]:

$$N^2 = (\nabla \phi)^2 \tag{7}$$

In the absence of masses or charges, $N = 1$, but in gravitational fields light 'slows down' and $N > 1$, reaching infinity at the edge of a black hole. In dynamics, the curvature k of the path of a moving particle is related to its acceleration $A = (dv/dt)\boldsymbol{t} + kv^2\boldsymbol{n}$, where $\boldsymbol{t}$ and $\boldsymbol{n}$ are unit vectors tangent and normal to the path respectively [20]. In the present case, A becomes the *gravitational acceleration*, with its well-known relativistic dependence on the curvature of space, as illustrated in Figure 2. The streamline curvature k is derived from the standard equation for line curvature, and from Eq. 3:

$$k = \frac{R^2 + 2\left(\frac{dR}{d\theta}\right)^2 - R\left(\frac{d^2R}{d\theta^2}\right)}{\left(\left(\frac{dR}{d\theta}\right)^2 + R^2\right)^{\frac{3}{2}}} = \frac{3\left(1 + \frac{2}{\tan^2\theta}\right)}{R\left(1 + \frac{4}{\tan^2\theta}\right)^{\frac{3}{2}}} \tag{8}$$

Or, from a geometrical optical relation for a ray in a gradient index field, $|k| = \boldsymbol{n}\nabla \log N$ [21]. In Figure 3, rays curve towards regions of increasing N, which can be found directly from Eq. 7 and Eq. 2:

$$N_d = \frac{DH\sqrt{3\cos^2\theta + 1}}{R^3} \tag{9}$$

The path of any geodesic can now be traced through radial $d\theta$ segments of the N field with ease and precision by using Snell's law of refraction

$$N_1 \sin\theta_1 = N_2 \sin\theta_2 \tag{10}$$

as shown in Fig 3. In making these calculations Eq. 9 cannot be applied as it is because successive N_1, and N_2 segments share the same R but different angles. Therefore, from Eq. 2 and Eq. 9, and putting $H = \phi = D = 1$ since they cancel out in Eq. 10:

$$N_d = \frac{\sqrt{3\cos^2\theta + 1}}{(\cos\theta)^{\frac{3}{2}}} \tag{11}$$
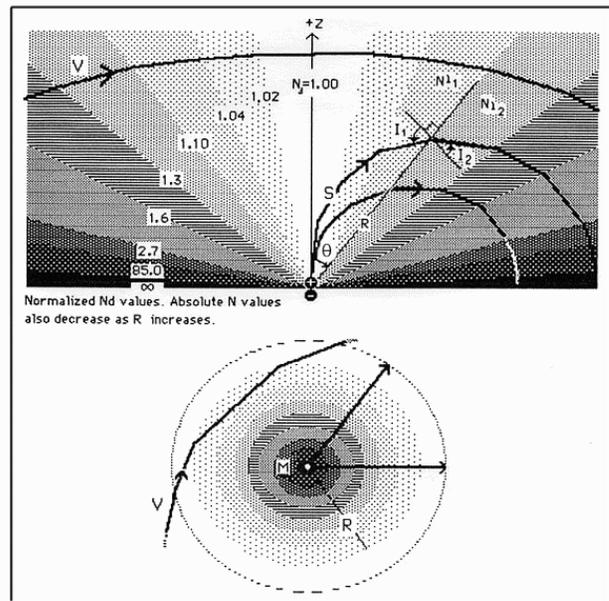
(normalized value for N calculations)

FIG. 3: Schematic representation of the path streamlines S and independent rays V through the quasi-refractive index fields $N_d(R, \theta)$ of a dipole (top) and in the Schwartschild $N_s(R)$ metric around a mass M (bottom). Typical refractive angles θ_1 and θ_2 show bending of light through increments of the N field.
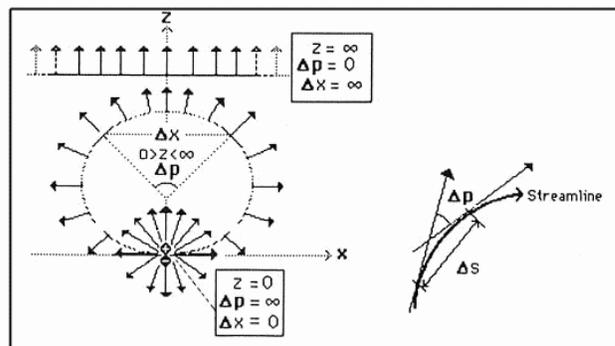
FIG. 4: Physical basis of the uncertainty of position Δx and momentum Δp in the dipole field and along a streamline.

Since a dipole's gravity causes the surrounding space to curve, bending its own streamlines, then any *other* particle or ray will similarly experience the same gravitational field. Based on this intuitive but unproven assumption, Eq. 10 and Eq. 11 were used to trace the path of various rays V initiated at given points $P(R, \theta)$ and angles of incidence I within the N field. The path of the streamlines S of Eq. 3, passing through a point P was confirmed if fine angular increments $< 0.1\deg$ were used to calculate the refracted rays.

But what of the Schwarzschild [22] metric:

$$ds^2 = \frac{dR^2}{F} + R^2 d\theta^2 - Fc^2 dt^2 \qquad (12)$$

with $F = (1 - 2GM/c^2R)$, where G is the constant of universal gravitation? Here, instead of a dipole at the origin, there is a mass M and space-time is distorted only radially. Putting $ds = 0$, the condition for a geodesic, and $d\theta = 0$ since the field is spherically symmetric, an expression for the local 'velocity' $v = dR/dt$ is obtained. And using Eq. 6, this give a quasi-refractive index function $N_s R$ for this metric [16]:

$$N_s = \frac{1}{F} \qquad (13)$$

indicating the 'density' of space-time due to a single *point mass* at the origin, the expression for a point charge being similar. Substituting Eq. 13 back into Eq. 12, this result can be generalized for all metrics: Space contracts by a factor of $\sqrt{N}$ along streamlines (due to the change in potential) and time dilates by $\sqrt{N^{-1}}$. But in the dipole case N was *derived* from the (θ, S) coordinates, and does not need to be included back in, which justifies Eq. 4.

The asymmetry of the dipole metric, compared to Schwarzschild's is due to the presence of *two opposite charges* at the origin. Another difference is that $N_s = \infty$ at the origin, but N_d has no singularities anywhere. Very near the charges, the dipole potential, and hence the N_d value gets quite complicated, and no longer follows that of Eq. 2. In general, the local N value at a point P is obtained by the the vector addition of the all electric fields at P caused by surrounding dipoles in different positions and orientations.

IV. THE DIPOLE QUANTUM FIELD

Some of the dipole field's physical features will now be interpreted from a quantum mechanical point of view. But as Hawking has remarked, "[in quantum mechanics] the unpredictable, random element comes in only when we try to interpret the wave in terms of the positions and velocities of particles. But maybe that is our mistake: maybe there are no particle positions and velocities, but only waves." [23] And Schrödinger always had reservations on the probabilistic interpretation of quantum mechanics. [24] Adopting these views is necessary here to justify the following physical interpretations:

A. The particle-wave duality

The dipole moment D is a quantum quantity since it depends on the distance between the charges and their quantified strength, and therein lies the 'particle' aspect of the field under consideration. The 'wave' aspect of the field is simply the time-harmonic classical dipole field itself. This in no way disputes the results of quantum mechanics, only the 'causes'. For example, an intensity distribution can be said to be the probabilistic accumulation of many whole particle photons. Here the photon will be described as a single continuous classical wave with local intensity fluctuations, which then cause random particle events during absorption or emission, and occurring *within the sensor*.

B. The uncertainty principle

Heisenberg himself cited diffraction [25] as one illustration of his uncertainty relations. But it will be argued here that *uncertainty relations exist precisely because waves diffract*. As in Figure 4, at the origin, the dipole field is basically concentrated in one point, so $\Delta x = 0$, but the streamline directions carrying the field's momentum p point in an infinite number of directions, hence $\Delta p = \infty$. Very far from the origin, the streamlines are basically parallel to the $+z$ axis and $\Delta p = 0$, but the wavefront has spread very widely and $\Delta x = \infty$. At intermediate points Δx and Δp can be mathematically related in several ways. For example, along any single streamline, Δp amounts to the curvature the streamline (when the streamline is straight, $\Delta p = 0$), while Δx is the distance element along the streamline. The physical basis of Planck's constant h in the relation $\Delta x \Delta p \geq h$ is elusive, but must be sought in the relation between charge quanta and the geometry of the dipole field at the origin.

C. The probability function

It is well known that $-\nabla \phi = E$, the electric field strength, the vectorial version of N. Using an alternative derivation [26] for the dipole's electric field intensity parallel to the $z-$axis, again with $D = H = 1$:

$$E_z = \frac{3\cos^2 \theta - 1}{R^3} \qquad (14)$$

Putting $3\cos^2 \theta = 1$ gives a cut-off angle $T_w = 54.73\deg$ beyond which $E_z = 0$ and there is no foreword momentum, as shown in Figure 1, where:

$$C = B \tan T_w = 1.4139B \qquad (15)$$

In Figure 5, the E_z values (normalized by an amplitude factor of $2.363B^{-2}$) of the dipole field along $z = B$,

was compared to the Gaussian probability distribution, related to the quantum probability function [27].

$$P(x) = \frac{1}{j\sqrt{2\pi}} e^{-\frac{x^2}{2j^2}} \qquad (16)$$

Where $j = C/3 = 0.4713B$, (the standard deviation, a third of the significant width before the curve dwindles asymptotically to zero), was chosen so that the Gaussian fits over $E_z(x)$. Figure 5 shows that there are only minor differences between $P(x)$ and $E_z(x)$ for $|x| < |C|$. For $|x| > |C|$, E_z becomes negative, and more significantly, no energy reaches a screen at B, since all the streamlines curve down before reaching it. What are the quantum implications of such a field, particularly the cut-off angle C? The probability amplitude can be directly related to the curvature of S: Increasing the curvature increases the angle of incidence of S on B, and hence the smaller the E_z vector.
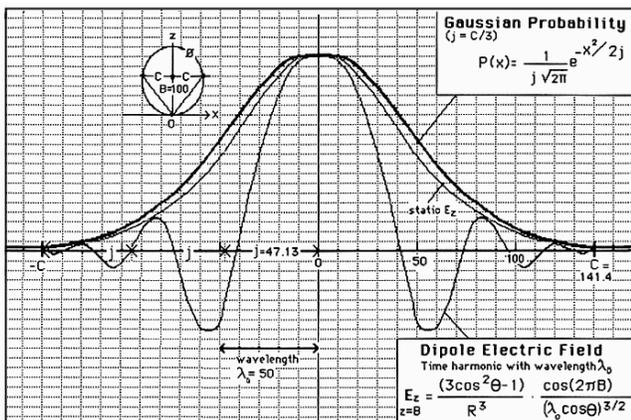

FIG. 5: Physical basis of probability in dipole field.

D. The Wave Packet

In the case of a time-harmonic dipole, using Eq. 2 and Eq. 9, and the fact that the wavelength is λ_0 along the $+z$ axis, it was found that along the line $z = B$:

$$E_{z(t=0)} = \cos\left(\frac{2\pi B}{\lambda_0(\cos\theta)^{\frac{3}{2}}}\right) \frac{3\cos^2\theta - 1}{R^3} \qquad (17)$$

so that the equipotentials occur at ever-decreasing distances. In quantum mechanics, the photon field is found to comprise a wave packet, with a spectral function $f(k)$ specifying the infinitely diminishing wavelengths and amplitudes fitting within a Gaussian envelope [27]. It is seen in Figure 5 how $E_{z(t=0)}$ resembles such a function in all details, as to amplitude and wavelength diminuation: as the wave packet expands, the equiphasals intersect B and S at ever shorter intervals, as in Fig. 1: there is a blue-shift in the field until the wavelength $I(R, \theta)$, and hence v, becomes zero on the x axis.

V. E=R=Q PRINCIPLE

According to the analysis above, an electromagnetic-relativistic-quantum-mechanical equivalence principle can now be generalized as follows: "in the field surrounding a system of electromagnetic charges, the gravitational field is equivalent to the curvature of the diffracting streamlines, while the local quantum mechanical state is equivalent to the gradient of the potential."

This of course has only been examined in the single case cited above, the first-order dipole approximation of Eq. 2. But other configurations produce curved streamlines, such as two point emitters of like charge, diffraction through an aperture in an opaque screen, diffraction from any continuous distribution of sources, and others.

VI. CONCLUSION

The electromagnetic dipole field has been examined from relativistic and quantum mechanical points of view, with preliminary evidence that it makes up a unified field in which an E=Q=R equivalence principle can be asserted: The three fields here appear to emerge, one from the other, in a satisfying and elegant way, and without the need for any new dimensions. Can Dirac's oppositely charged virtual pairs of particles (matter-antimatter) [28] be considered a kind of dipole, with electromagnetic, gravitational and quantum fields such as those discussed above? Is the atomic strong force the result of some kind of short-range gravitational field such as the one modelled above? These are some of the questions that emerge from the present study.

[1] R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics III*, Addison-Wesley, 1-10 (1965).

[2] M. Born, Zeit. Phyz. **38**, 803 (1926).

[3] A. Einstein, quoted in A. Pais, *Subtle is the Lord....*, Oxford Univ. Press, 461 (1982).

[4] J. Schwarz, "Superstring Unification", in *300 Years of Gravitation*, (S. Hawking and W. Israel, Eds.) Cambridge Univ. Press, 652 (1987).

[5] V. Tamari, *The Cancellation of Diffraction in Wave Fields*, Optoelectronics-Devices and Tech. Mita Press **2**, 59-82 (1987).

[6] V.F. Tamari, E-print: physics/0303073 (2003).

[7] V. Tamari, "Bow-Wave Geometry", post-deadline paper in *Huygens' Principle 1690-1990 Conference*, Scheveningen, The Netherlands (1990).

[8] D. A. B. Miller *Huygens' wave propagation principle corrected*, Optics Lett. **16**, 1370-72 (1991).

[9] R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics II* op. cit. II-6-3.

[10] W. Braunbek and 0. Laukien, Optik **9**, 174-179 (1952); see also M. Born and E. Wolf, *Principles of Optics*, 6th. Ed Pergamon, 575-77 ().

[11] A. Sommerfeld, Math. Ann. **47**, 317 (1896); see also M. Born and E. Wolf, *Principles of Optics*, op. cit. 556-575.

[12] J. Goodman, *Introduction to Fourier Optics*, McGraw-Hill (1968).

[13] A. Einstein, *The Meaning of Relativity*, Princeton Univ. Press, 93 (1988).

[14] B. Schutz, *A First Course in General Relativity*, Cambridge Univ. Press, 166 (1985).

[15] A. Einstein, *Relativity*, Bonanza Books, 76 (1961).

[16] A. Eddington, *Space, Time and Gravitation*, Cambridge Univ. Press, 107-109 (1990).

[17] W. Israel, "Dark stars: The Evolution of an Idea", in *300 Years of Gravitation*, op. cit. 204 (1992).

[18] M. Born and E. Wolf, *Principles of Optics*, op. cit. 13.

[19] M. Born and E. Wolf, *Principles of Optics*, op. cit. 112.

[20] C. Wylie and L. Barret, *Advanced Engineering Mathematics*, McGraw Hill, 775 (1983).

[21] M. Born and E. Wolf, *Principles of Optics*, op. cit. 124.

[22] B. Schutz, *A First Course in General Relativity*, op. cit. 258.

[23] S. Hawking, *A Brief History of Time*, Bantam, 183 (1988).

[24] W. Scott, *Erwin Schrödinger*, Univ. Mass. Press (1967); see A. Pais, *Subtle is the Lord...*, op. cit (Ref. 3 above), p.443f.

[25] W. Heisenberg, *The Physical Principles of the Quantum Theory*, Dover, 23 (1949).

[26] R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics*, op. cit. I-6-9.

[27] S. Brandt and H. D. Dahmen, *The Picture Book of Quantum Mechanics*, John Wiley & Sons, 36 (1985).

[28] G. Gamov, *Thirty Years That Shook Physics*, Dover, 131 (1966).