Lesson 21

## Area and Circumference of a Circle

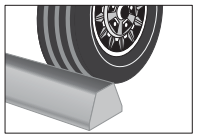
Name: _____

### Prerequisite: Finding Areas of Polygons

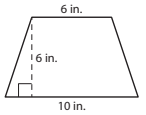
**Study the example showing how to find the area of a polygon. Then solve problems 1–9.**

**Example**

A concrete company makes blocks for parking lots. A construction worker who is installing the blocks needs to find the area of the trapezoid at the end of the block.

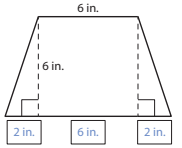


To find the area, the worker measures the trapezoid and draws this diagram showing the measurements.




He knows how to find the area of a square and the area of a triangle. He plans to divide the trapezoid into a square and two triangles to find the area.

B 1 Label the dimensions in the diagram at the right.




B 2 Find the area of the square in the diagram.

Area = $s \cdot s = 6 \cdot 6 = 36$ square inches

B 3 Find the area of one of the triangles in the diagram.

Area = $\frac{1}{2}bh = \frac{1}{2} \times 2 \times 6 = 6$ square inches

B 4 Write and solve an equation to find the area of the trapezoid.

***Show your work.*** Possible work:

$A = S + 2T$

$A = 36 + 2 \times 6$

$A = 36 + 12$

$A = 48$

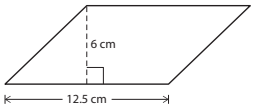
*Solution:* The total area of the trapezoid is 48 square inches.





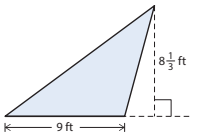
**Solve.**

B 5 Use the formula for the area of a parallelogram to find the area of the parallelogram at the right.




Area = $bh = 12.5 \times 6 = 75$ square centimeters

B 6 Use the formula for the area of a triangle to find the area of the shaded triangle at the right.




Area = $\frac{1}{2}bh = \frac{1}{2} \times 9 \times 8\frac{1}{3} = 37\frac{1}{2}$ square feet

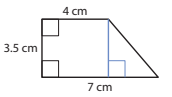
M 7 A rectangular yard is 18 feet long. The area of the yard is 400.5 square feet. What is the width of the yard? Explain.

Area = $bh$; $400.5 = 18h$; $h = 400.5 \div 18 = 22.25$ feet

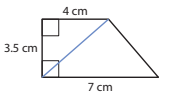
C 8 Divide these identical trapezoids into different sets of figures. Use these figures to find the area of each trapezoid.

***Show your work.***

Trapezoid A




Trapezoid B




Possible work: See trapezoids A and B above.

Area of A = $(4 \times 3.5) + \left(\frac{1}{2} \times 3.5 \times 3\right) = 14 + 5.25 = 19.25$ cm$^2$

Area of B = $\left(\frac{1}{2} \times 4 \times 3.5\right) + \left(\frac{1}{2} \times 7 \times 3.5\right) = 7 + 12.25 = 19.25$ cm$^2$

*Solution:* The area of the trapezoid is 19.25 square centimeters.

M 9 Use the results of problem 8. Does the area of a figure depend on how it is separated into smaller figures?

No; the area of a figure is the same no matter what smaller figures are used to determine the area.





**Key**

**B** Basic **M** Medium **C** Challenge

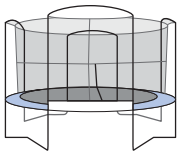
Name: _____

## Find Circumference Using a Formula

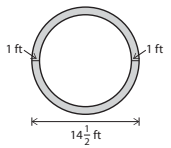
**Study the example showing how to find the circumference of a circle using a formula. Then solve problems 1–9.**

**Example**

The diameter of the outer edge of a trampoline is 14 feet. There is 1 foot between the outer edge of the trampoline and a safety enclosure. What is the circumference of the enclosure?



To find the circumference of the enclosure, first find its diameter. The diagram shows that the diameter of the enclosure is $14\frac{1}{2} - (2 \times 1)$, or $12\frac{1}{2}$ feet.




The formula for the circumference of a circle is $C = \pi d$, where $C$ is the circumference and $d$ is the diameter. Use 3.14 for $\pi$.

$C = \pi d = 3.14d = 3.14 \times 12\frac{1}{2}$ ft $= 39.25$ ft

B 1 What is the circumference of the trampoline including its outer edge? Use 3.14 for $\pi$.

$C = 3.14 \times 14\frac{1}{2}$ ft $= 45.53$ ft

B 2 What is the difference between the circumference of the trampoline and the circumference of the enclosure?

$45.53 - 39.25 = 6.28$ ft

M 3 Explain how to find the circumference of a circle if you know the radius of the circle.

Multiply the radius of the circle by 2 to find the diameter. Then multiply the diameter of the circle by 3.14 to find the circumference.

M 4 The United States Capitol Rotunda is a circular room located in the Capitol Building in Washington, D.C. The radius of the room is 48 feet. What is its circumference?

The diameter is $2 \times 48$, or 96 ft; $C = \pi d = 3.14 \times 96 = 301.44$ ft

**Vocabulary**

**circumference** the distance around a circle.

**diameter** the distance across a circle through the center.

**radius** the distance from the center to any point on a circle.





**Solve. Use 3.14 for $\pi$. Show your work.**

M 5 Pavit has a clock with a minute hand that extends to the clock's edge. The minute hand is 5.25 inches long. What is the circumference of the clock?

The diameter is $2 \times 5.25$, or 10.5 in.; $C = \pi d = 3.14 \times 10.5 = 32.97$ in.

M 6 The high school soccer field is decorated with the logo of the school's soccer team in a circular design. The center of the design is placed in the middle of the field. The edge of the design is $9\frac{3}{4}$ yards from its center. What is the circumference of the design?

The diameter is $9\frac{3}{4}$ yd + $9\frac{3}{4}$ yd, or 19.5 yd.

$C = \pi d = 3.14 \times 19.5$ yd $= 61.23$ yd

The circumference of the design is 61.23 yd.

M 7 Andrea built a circular barbecue pit with a circumference of 9 feet. She wants a rod to go across the pit to use for a rotisserie. The rod must go from one edge of the pit to the other, passing through its center. To the nearest foot, how long must the rod be?

$C = \pi d$; $9 = 3.14d$; $d = \frac{9}{3.14} \approx 2.87$

To the nearest foot, the rod must be 3 feet long.

C 8 Is the relationship between the diameter of a circle and its circumference a proportional relationship? Explain why or why not.

Yes; Possible explanation: For any circle, the ratio $\frac{C}{d}$ is $\pi$, so the relationship is proportional. The constant of proportionality is $\pi$.

M 9 The radius of Earth is about 6,371 kilometers and the radius of the moon is about 1,737 kilometers. Which ratio is closest to the ratio of the circumference of the moon to the circumference of Earth?

A $\frac{13}{50}$

B $\frac{21}{80}$

(C) $\frac{28}{101}$

D $\frac{27}{96}$

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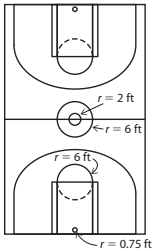
## Find the Area of a Circle

**Study the example problem showing how to find the area of a circle. Then solve problems 1–9.**

**Example**

Basketball courts contain several different circles. What is the area of the large center circle in the diagram? Use 3.14 for $\pi$.




The formula for the area of a circle is $A = \pi r^2$.

The radius of the large circle is 6 feet.

$A = \pi r^2 = 3.14 \times 6^2$
$= 3.14 \times 6 \times 6$
$= 3.14 \times 36 = 113.04$

The area is 113.04 square feet.

B 1 What is the area of the smaller circle at the center of the court? Use 3.14 for $\pi$.

$A = \pi r^2 = 3.14 \times 2^2 = 3.14 \times 2 \times 2 = 12.56$

The area is 12.56 square feet.

B 2 To the nearest tenth, what is the area of the circle formed by the hoop at the end of the court? Use 3.14 for $\pi$.

$A = \pi r^2 = 3.14 \times 0.75^2 = 3.14 \times 0.5625 = 1.76625$

To the nearest tenth, the area is 1.8 square feet.

B 3 Describe how to find the area of a circle if you know the diameter of the circle.

Divide the diameter by 2 to find the radius of the circle. Then use the formula $A = \pi r^2$.

M 4 A pie is baked in a circular pan with a diameter of $10\frac{1}{2}$ inches. What is the approximate area of the pie that is left after half of the pie has been eaten?

Possible solution: The radius of the pie is $10\frac{1}{2} \div 2$, or 5.25 inches.

The area of half the pie is $\frac{1}{2}\pi r^2 = \frac{1}{2} \times 3.14 \times 5.25^2 = 43.273125$.

The area of the remaining pie is about 43 square inches.





**Solve. Use 3.14 for $\pi$.**

B 5 Ms. Kwan's class is playing games using a circular parachute during recess. The parachute has a radius of 8 feet. What is the area of the parachute?

$A = \pi r^2 = 3.14 \times 8^2 = 3.14 \times 64 = 200.96$ square feet

M 6 Find the areas of circles with radii of 1, 2, and 4 centimeters. Then predict how the area of a circle changes when the radius is doubled.

1 cm: 3.14 $cm^2$ 2 cm: 12.56 $cm^2$ 4 cm: 50.24 $cm^2$

*Prediction:* When the radius is doubled, the area is multiplied by 4.

M 7 How is finding the area of a circle with a given radius like finding the circumference of the circle?

Possible answer: Both involve the product of $r$ and $\pi$. The circumference is the product of 2, $\pi$, and $r$, while the area is the product of $r$, $\pi$, and $r$.

C 8 The exact area of a circle is $81\pi$ square inches. What are the radius and diameter of the circle? Show your equation and explain your answers.

Because $\pi r^2 = 81\pi$, $r^2 = 81$. I know that $9 \times 9 = 81$, so $r = 9$.

Therefore, the radius is 9 inches and the diameter is $2 \times 9 = 18$ inches.

C 9 Simon has 18.5 feet of fencing. He makes a circular garden with the fencing. What is the area of Simon's garden to the nearest square foot?

***Show your work.***

The circumference of the garden is 18 ft, so $3.14d = 18.5$.

$d = 18.5 \div 3.14$, or about 5.89 feet.

The radius is about $5.89 \div 2$, or about 2.95 feet.

The area of the garden is about $3.14 \times 2.95^2$, or about 27.33 square feet.

*Solution:* To the nearest square foot, the area of the garden is 27 square feet.

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## Use Areas of Circles

**Study the example problem showing how to solve problems by using the areas of circles. Then solve problems 1–6.**

**Example**

Marcella and Carlos play a game in which they drop pennies on a piece of paper to see who can cover the most area with pennies. They each have 25 pennies to drop. If a penny lands on another one, the player has to remove it and it doesn't count. A penny is about 2 centimeters in diameter.

Marcella can count 17 pennies that she dropped. To find the area Marcella's pennies cover, start by finding the area 1 penny covers.

The area 1 penny covers can be found using the formula for the area of a circle. The radius of each penny is half its diameter, or 1 centimeter.

$A = \pi r^2$

$A = 3.14 \times 1^2$

$A = 3.14 \times 1 \times 1$

$A = 3.14$ square centimeters

Multiply the area one penny covers by 17 to find the area covered by 17 pennies. So 17 pennies cover $17 \times 3.14$, or 53.38 square centimeters.

B 1 Estimate the area that would be covered by all 25 pennies.

Sample estimate: $25 \times 3 = 75$ square centimeters

B 2 Use 3.14 for $\pi$ to find the area that would be covered by 25 pennies. Compare your answer to your estimate in problem 1.

78.5 square centimeters; This result is close to my estimate in problem 1.

M 3 Marcella and Carlos repeat the game using dimes instead of pennies. Each dime has a diameter of 0.7 inches. Carlos can count 15 dimes. To the nearest hundredth of a square inch, what area do his dimes cover on the paper?

One dime: $A = \pi r^2 = \pi \times 0.35^2 = 3.14 \times 0.1225 = 0.38465$ square inches.

So 15 dimes cover $15 \times 0.38465$ square inches, or about 5.77 square inches.





**Solve. Use 3.14 for $\pi$.**

M 4 Donna bakes biscuits on a rectangular cookie sheet that is 30 centimeters by 50 centimeters. Each biscuit has a diameter of 8 centimeters.

a. What is the maximum number of biscuits that Donna can bake at once on the cookie sheet?

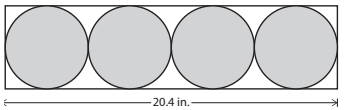
She can place 3 biscuits in each of 6 rows for a total of 18 biscuits.

b. If Donna bakes the maximum number of biscuits, what area do the biscuits take up on the cookie sheet? Explain.

One biscuit: $A = \pi r^2 = \pi \times 4^2 = 3.14 \times 16 = 50.24$ square centimeters

So 18 biscuits cover $18 \times 50.24$ square centimeters, or 904.32 square centimeters.

M 5 Find the area of the unshaded part of the figure.




***Show your work.***

The diameter of each circle is $20.4 \div 4 = 5.1$ in., so the radius is 2.55 in.

The area of one circle is: $A = \pi r^2 = \pi \times 2.55^2 = 20.41785$ square inches.

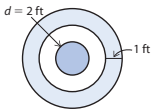
The area of all four circles is: $4 \times 20.41785 = 81.6714$ square inches.

The area of the rectangle is: $A = bh = 20.4 \times 5.1 = 104.04$ square inches.

The area of the unshaded part of the figure is $104.04 - 81.6714 = 22.3686$ square inches.

*Solution:* The area of the unshaded part is about 22.4 square inches.

C 6 A circular garden has a diameter of 6 feet. There are 2 sections of blue flowers and 1 section of white flowers as shown. What is the area of the section with the white flowers?




***Show your work.***

The section with the white flowers is a circle with diameter of 4 feet minus a circle with a diameter of 2 feet.
A circle with a diameter of 4 feet has a radius of 2 feet.
$A = \pi \times 2^2 = 3.14 \times 4 = 12.56$ square feet.
A circle with a diameter of 2 feet has a radius of 1 foot.
$A = \pi \times 1^2 = 3.14 \times 1 = 3.14$ square feet.
12.56 square feet − 3.14 square feet = 9.42 square feet

*Solution:* The area of the section with white flowers is 9.42 square feet.

Name: _____

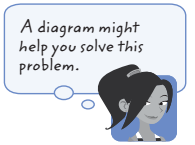
## Area and Circumference of a Circle

**Solve the problems. Use 3.14 for $\pi$.**

**M**

**1** The diameter of a DVD is 12 centimeters. The hole in the center of a DVD has a diameter of 1.5 centimeters.

Write and solve an equation to find the area of the DVD. Round to the nearest hundredth if necessary.




***Show your work.***

Let $M$ = area of outer circle and let $N$ = area of hole.

$A = M - N$

$A = \pi \cdot 6^2 - \pi \cdot 0.75^2$

$A = 36\pi - 0.5635\pi$

$A = 111.27$

*Solution:* 111.27 square centimeters

**C**

**2** Keb wants to put a swimming pool in his backyard. He buys a square plastic tarp to go under it. The circumference of the pool is 48 feet. The tarp is 16 feet on each side.


**a.** Show that the tarp is large enough for the pool to fit entirely on the tarp.

**b.** To the nearest percent, what percent of the tarp will be covered by the pool?

***Show your work.***

**a.** Find the diameter of the pool: $C = \pi d$; $48 = 3.14d$; $d = 48 \div 3.14 \approx 15.29$ ft. The diameter of the pool is less than the length of each side of the tarp, so the tarp is large enough for the pool to fit entirely on the tarp.

**b.** The radius of the pool is about $15.29 \div 2 \approx 7.65$ ft. The area of the pool is $3.14 \times 7.65^2 \approx 184$ sq ft. The area of the tarp is $16 \times 16 = 256$ sq ft. $184 \div 256 \approx 0.72$, so about 72% of the tarp will be covered by the pool.

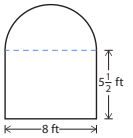




**Solve.**

**M**

**3** A gate is made up of a rectangle and a semicircle, as shown. Find the area of the gate.





***Show your work.***

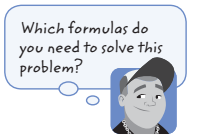
Total area = area of rectangle + area of semicircle =

$\left(8 \cdot 5\frac{1}{2}\right) + \left(\frac{1}{2}\pi \cdot 4^2\right) = 44 + \left(\frac{1}{2} \cdot 3.14 \cdot 16\right) = 44 + 25.12 = 69.12$

*Solution:* 69.12 square feet

**C**

**4** Tuan wants to add trim only around the edges of the gate in problem 3. How much trim does he need?

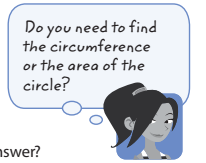



**A** 19.28 feet **C** 44.12 feet

**(B)** 31.56 feet **D** 45.56 feet

**M**

**5** Chas builds a circular pen for his dog. The radius of the pen is 9.2 feet. Rounded to the nearest tenth, much fencing did Chas use for the pen?




**A** 28.9 feet **C** 115.6 feet

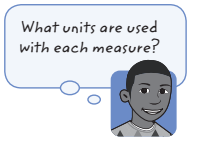
**(B)** 57.8 feet **D** 266 feet

Skye chose **A** as the correct answer. How did she get that answer?

Skye multiplied $\pi$ by the radius. She should have multiplied by the diameter.

**M**

**6** A circular plate has a radius of 12 inches. Complete the table. Include a number and a unit in the last column.




| Property of Circle | Formula | Measure |
|---|---|---|
| Area | $\pi r^2$ | 452.16 square inches |
| Diameter | $2r$ | 24 inches |
| Circumference | $\pi d$ | 75.36 inches |