HOW TO WRITE MATHEMATICAL PAPERS

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1. The Title

The title of your paper should be informative. A title such as "On a conjecture of Daisy Dud" conveys no information, unless the reader knows Daisy Dud and she has made only one conjecture in her lifetime. Generally, titles should have no more than ten words, although, admittedly, I have not followed this advice on several occasions.

2. The Introduction

The Introduction is the most important part of your paper. Although some mathematicians advise that the Introduction be written last, I advocate that the Introduction be written first. I find that writing the Introduction first helps me to organize my thoughts. However, I return to the Introduction many times while writing the paper, and after I finish the paper, I will read and revise the Introduction several times.

Get to the purpose of your paper as soon as possible. Don't begin with a pile of notation. Even at the risk of being less technical, inform readers of the purpose of your paper as soon as you can. Readers want to know as soon as possible if they are interested in reading your paper or not. If you don't immediately bring readers to the objective of your paper, you will lose readers who might be interested in your work but, being pressed for time, will move on to other papers or matters because they do not want to read further in your paper.

To state your main results precisely, considerable notation and terminology may need to be introduced. At this point, you do not want the reader to be bogged down with technical definitions and notation, and so it is therefore preferable to informally describe your results in such instances. Try to be as informative and precise as possible without drifting off into too much technical jargon.

Why are you writing this paper? The logic in climbing a mountain, "because it is there," does not apply to writing and publishing a paper. Just because you can prove a theorem does not mean that you should publish it and its proof. For example, the theorem may be of interest to no one else, the proof may involve no new ideas, or, despite a proof not being in the literature, the theorem can be easily proved by many, in particular, students.

Put your paper in an historical context. Indicate what you have done in relation to what others have done. Briefly survey the pertinent results of others to your work.

On the other hand, as you place your results in an historical perspective, do not name drop. Ramanujan, J.–P. Serre, and P. Deligne are common names that writers like to drop to enhance their own particular work. Readers will recognize that you are referring to these famous mathematicians in an attempt to bring attention to your work, which likely may not receive any notice otherwise.

3. Displaying and Referring to Mathematical Formulas; Notation; Definitions

One of the most common shortcomings of both inexperienced and experienced writers is the failure to inform readers of their reasoning. Often, such failures arise when one writes down a long series of equalities without explaining those previous theorems, lemmas, equations, etc. that were used in constructing this long string. Tag equations! Refer to the tagged equations. Tell readers your reasoning at every step. You know all the steps; all is logical to you. But how long did it take you to develop this proof? You cannot expect readers to come to the same reasoning in a much shorter period of time without being aided along the way. Every step (except trivial ones) should be documented.

The steps that you undergo should be related in sentences. Don't just add a parenthetical remark, indicating that a certain equation was used. This is lazy writing. It is also bad grammar. Moreover, aesthetically, such writing is ugly. Parenthetical remarks after equations often cause texing problems. Such lazy composition causes difficulties for copy editors and compositors.

Display complicated expressions and equations, for if you put them in the text, they will look like a huge mess. Your paper should be attractive, neat, and beautiful. An attractive paper puts the editor and referee in a good mood. You want the editor to exclaim, "Ah, this paper looks nice! We should publish it." Putting the editor or referee in a positive psychological mood is important in convincing them of the worth of your paper.

Do not use LHS, RHS, etc.; you are not at the blackboard. If you need an abbreviation, it is much better to write: Let $S(x, m, n)$ denote the left-hand side of (3.2.1729). If you use an abbreviation, always explain the abbreviation at its first appearance.

Number equations, as well as theorems, corollaries, lemmas, etc. by section. Subdividing your paper into sections enables readers to more easily locate equations and theorems. For example, if an author writes, "Recall that in Theorem 31, we proved ...," it might take the reader nearly a half or full minute to find the theorem, but if the author writes, "Recall that in Theorem 4.3, we proved ...," it is likely that only a few seconds are needed for a search.

Use standard notation. Although you may think that your notation is better than existing notation, swallow your pride and avoid your new, spiffy, thoughtful notation. For example, in the mid-twentieth century, a book entitled, *The Elliptic Functions as They Should Be*, was published. The author, Albert Eagle, desired to introduce a more organized, descriptive, and useful notation to replace the current notation in the theory of elliptic functions. His book was a flop; no one has ever used his notation. Notation evolves over a long period, and it is very difficult to "change horses in midstream." Many mathematicians have usually contributed to the notation in any particular field. Use as little notation as possible. Think about which subscripts, superscripts, additional arguments, and fonts would be best. Try to avoid subscripts of subscripts. Clarity and familiarity of notation aid the readers in her or his understanding of what you are trying to communicate.

As an editor, I have read many reports from referees, and one of the most common complaints of referees is that authors forget to give definitions. It is very easy to use concepts and terminology that you have not defined. Have you defined everything that you need to define? Have you explained all the notation that you have used?

4. WRITE IN THE KING'S ENGLISH

Be precise. You are not talking to your research collaborator, and you are not at the blackboard. Sloppy descriptions are not acceptable. Maybe you know what you are writing about, but this does not translate into the reader knowing what you are writing about. Your writing will be recorded for decades and centuries. Remember that your paper will be read by many whose native language is not English. Thus, colloquialisms, slang, and current jargon may not be understood by your readers.

Everything in your paper must be written in sentences. You are not at a blackboard, where shorthand, sentence fragments, and poor grammar are accepted. All equations must be parts of complete sentences. Punctuate with commas and periods after equations. Making a list of notation is acceptable at a blackboard, but not in a paper. You should aspire to the same literary levels as William Shakespeare, J. K. Rowling, and Leo Tolstoy.

Theorems, corollaries, etc. are in their own environments and are not to be included as parts of text sentences. Thus, "We now prove the following

Theorem 4.1.

is incorrect. One should write, "We now prove the following theorem." Then begin to use the theorem environment. The aforementioned misusage is very common and is one of my personal pet peeves. So, if I am reading your paper or thesis, beware!

Do not use nouns as adjectives. Some misuses have become so common that they are now standard and accepted. For example, "weight 2 modular form" and "half-integer weight modular form" should be, respectively, "modular form of weight 2" and "a modular form of half-integral weight." We cannot change the course of history, and such terminology will continue, but let us show the world that we know the difference between a noun and an adjective and not promulgate any further nouns as adjectives.

Run-on sentences are among the most common and flagrant misuses of the King's English found in mathematical writing. Here is a simple example: This theorem is due to Ramanujan, see [1729]. Here, a replacement of the comma by a semicolon would correct the run-on sentence. Often run-on sentences can be corrected with the use of a dependent clause. Here is an example: Let $f(x)$ be a continuous function, the Wiggleworm summation formula holds. This run-on sentence can be corrected by writing: If $f(x)$ is a continuous function, then the Wiggleworm summation formula holds.

Never begin a sentence with a mathematical symbol. Confusion or ambiguity may result in such an instance, especially if the previous sentence ended in a mathematical symbol.

My mathematical stepfather, Marvin Knopp, admonished me about my careless use of dangling participles while writing drafts of my thesis. I have never forgotten this advice, and neither have any of Knopp's doctoral students. Knopp is almost as famous for eschewing dangling participles as he is for his theorems on modular forms. So, if you would like me to read your paper or thesis and you want to avoid receiving a returned copy with lots of red marks on it, refrain from using dangling participles. Here is a common example: Employing equations (3.2) and (3.3), the proof is complete. Who is doing the "employing?" Answer: the writer, or the reader. But the writer and reader are nowhere to be found in the sentence; the participle "employing" is dangling without a modifier.

Section, Chapter, Theorem, etc. are proper nouns when referenced as Section 3.4, Chapter 6, Theorem 1729, etc., for example. They should be capitalized. However, "the following section," "this chapter is devoted to the theory of dunking apples," "the theorem about basket weaving," etc. are instances where improper nouns are used, and therefore should not be capitalized.

Mathematicians are fond of "we have," which can be erased in at least 80% of instances. They are usually superfluous. Mathematicians toss a frequent "we have" into the discourse because they feel that they should write something before an equation. Such usage is another example of lazy writing.

Mathematicians are also very fond of colons. They are inserted in most cases because the author does not know what else to do. The writer thinks that there probably should be some punctuation here, so a colon is tossed in to satisfy the need for punctuation. In most cases, they can be deleted. In some cases, a period should replace a colon.

Do not use the word "get." Definition (Webster): "To come into the state of having; become the owner or receiver of; obtain." We arrive at conclusions, equations, etc. by the process of thinking, deducing, reasoning, seeing, etc.; we do not "get" them. We also never use the word, "receive." For example, "Using (3.2), we receive the equation ..." is never used.

Very few authors adhere to the next advice, and, honestly, it is not a huge crime if one does not. Generally, one can delete most "shall's" and "will's" from a manuscript. For example, one might write, "We will describe this method in Chapter 6." Well, Chapter 6 has already been written; the method has been described, and so the "future" has already arrived. It would be better to write, "This method is described in Chapter 6."

5. Citations in the Text of Your Paper

In citing books, always give page numbers. No one is going to search through a 400 page book to find the theorem, equation, etc. you are quoting.

Early in my career, I read three or four papers on how to write mathematics. These writers advised that in citing authors, one should give initials the first time an author is mentioned. Afterward, the initials can be dropped. Unfortunately, many journals do not follow this practice; they eliminate all initials. Suppose that two authors have the same last name? For example, 25% of Koreans have the last name Kim. Quite recently, two authors sent me a preprint entitled, "On a conjecture of Berndt and Kim." Since Byungchan Kim and Sun Kim are two of my recent students, and since there are many mathematicians named Kim, I asked the authors to supply initials. The authors, Kathrin Bringmann and Amanda Folsom, were happy to comply with my request, and their paper was published in a recent issue of the *Ramanujan Journal*. D. H. Lehmer and Emma Lehmer were well-known twentieth century number theorists who are still frequently cited. If an author wrote, "By a theorem of Lehmer," the reader's first response would be: "Which one?"

Give references for all results that you quote, except for those theorems that would be known to everyone, e.g., Taylor's Theorem. As an editor, I find that another of the most frequent complaints from referees is that authors do not divulge sources for the theorems that they use. Not only is this information important and necessary, the lack of it does not inspire confidence in the reader about the correctness of the author's results.

6. List of References

Be consistent in recording references. For the names of authors, you should either use initials for all authors, or you should provide the given names for all authors. For most journals, only the initials of authors are printed. In the preparation of your paper, if you know the journal to which you will be submitting your paper, check the journal's style for references. Use punctuation as prescribed by the journal. Some journals italicize the title of papers, while some italicize the names of journals. Attempt to use accepted abbreviations for journals; if in doubt, consult MathSciNet for standard abbreviations. Sloppiness in preparing references may convince readers that the author was also sloppy in proving theorems in the paper. I have seen many papers with multiple mistakes and inconsistencies in almost every reference. Such inattention to accuracy and consistency is inexcusable. Moreover, if a reader finds a reference to be incorrect or has trouble locating it, she or he will not be happy. It is not in your best interest to make readers unhappy.

I offer a few further remarks about style in references. For most journals, especially in the U.S., space is put between the initials of authors. For instance, we would write C. F. Gauss. However, other publishers, especially in Asia and Europe, do not put space between initials, and so would type C.F. Gauss. As a personal example, Springer has always required me to not put space between the initials of authors in my books.

In writing p. or pp., either in the text of in the references, always insert space before page numbers. Page numbers should always be separated by en-dashes. For example, one should write pp. 17–29, and not pp. 17-29.

7. Epilogue

I have undoubtedly made several grammatical errors in preparing this scribe about how to write mathematical papers. Please inform me of such mistakes.

Edward Dunne, an editor for the American Mathematical Society, recently told me of a lecture given by J.–P. Serre on how not to write mathematical papers. He sent me two links, and I quote from his email to me. "The first is YouTube: http://youtu.be/tJZpdXWm4Gg. The YouTube video's quality seems a bit poor. So here is another link (from William Stein's site at Washington): http://modular.math.washington.edu/edu/basic/serre/ Then click on the "AVI" file to get the pure video file. You need an appropriate player to watch it. I think Microsoft Media player works, but I'm not sure." I actually have not seen this lecture, for I did not want to be accused of "cribbing" from Serre while preparing my own advice on writing papers.