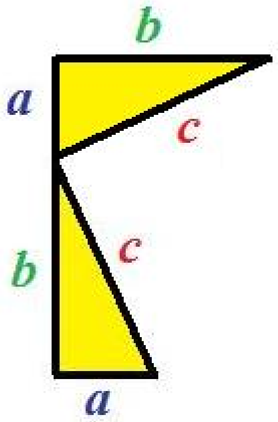
Why is Pythagoras' Theorem True?

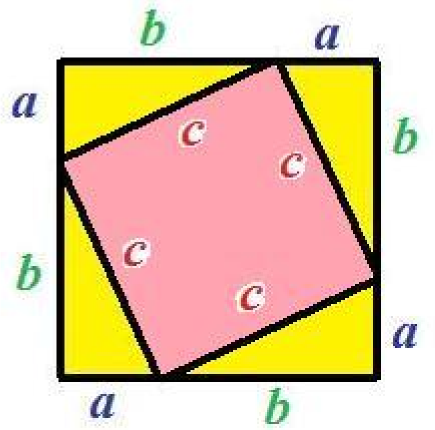
$$c^2 = a^2 + b^2$$

Pythagoras' Theorem is a famous fact from plane geomtery, familiar to nearly every student of math. The theorem is useful, elegant, but perhaps a bit odd: why squares? We can show that this theorem really is true (rather than an elaborate hoax, or an academic conspiracy), and perhaps get a sense of "why squares" in a proof by paint drawings.

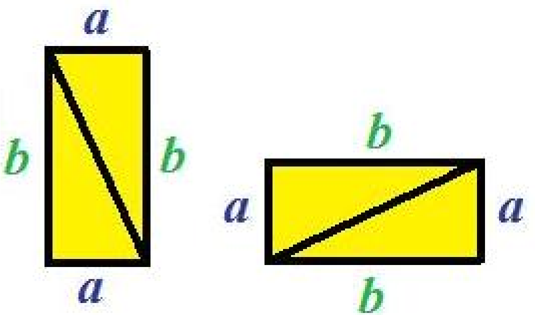
Here is a right triangle with legs a, b and hypotenuse c.



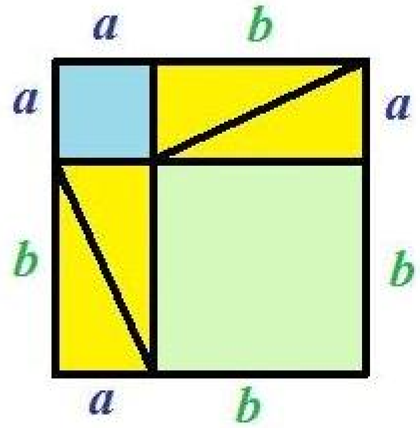
I took another copy of the yellow triangle, rotated it a quarter turn, and placed it on top of the first.



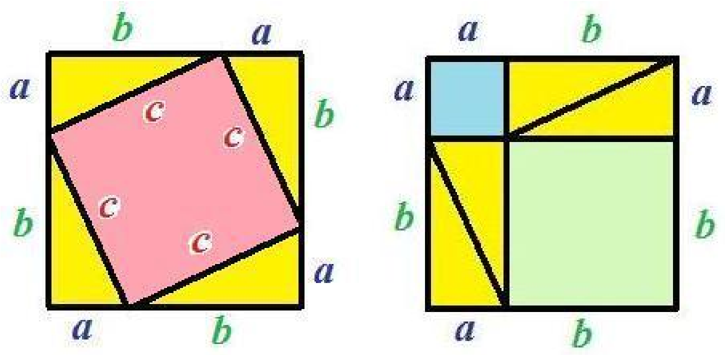
Two more copies of the triangle are placed similarly. Notice that this encloses a (pink) sqaure with side length c. Also notice that altogether the four yellow triangles and the pink square form a big square with side length $a + b$.



I'm going to make a big square with sides $a + b$ in another way. First, notice that if I put together two yellow triangles along the hypotenuse, they form a rectangle with sides a and b.

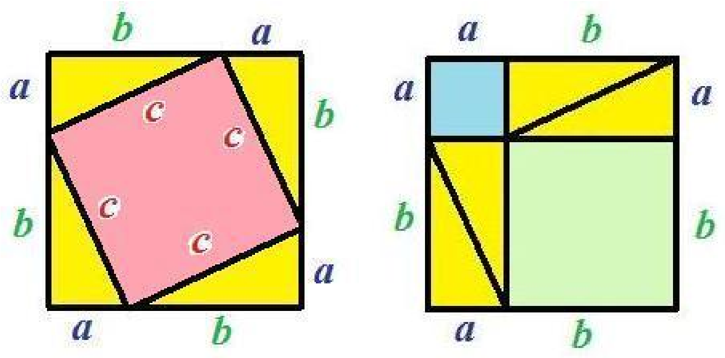


Take four yellow triangles put together as rectangles, a (blue) square of side length a, and a (green) square of side length b. We can piece them together as above to get a big square of side length $a + b$ again.



We can equate the areas for the two ways of breaking apart the big $a + b$ square:

pink square $+ 4 \cdot$ triangle $=$ blue square $+$ green square $+ 4 \cdot$ triangle



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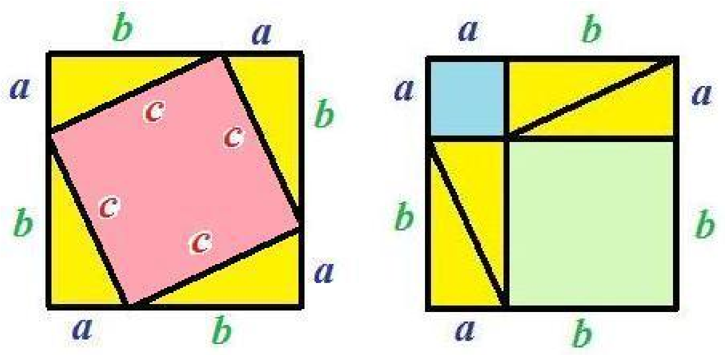

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