6.5 Trapezoids

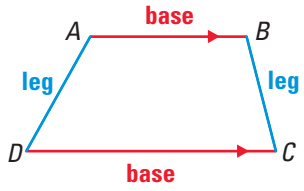
Goal

Use properties of trapezoids.

Key Words

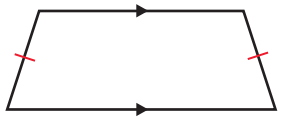
- trapezoid
- bases, legs, and base angles of a trapezoid
- isosceles trapezoid
- midsegment of a trapezoid

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases**. The nonparallel sides are the **legs**.



A trapezoid has two pairs of **base angles**. In trapezoid $ABCD$, $\angle C$ and $\angle D$ are one pair of base angles. $\angle A$ and $\angle B$ are the other pair.

If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.

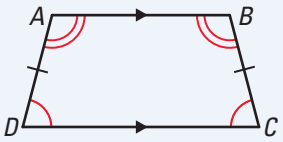

isosceles trapezoid

THEOREMS 6.12 and 6.13

Theorem 6.12

Words If a trapezoid is isosceles, then each pair of base angles are congruent.

Symbols In the isosceles trapezoid $ABCD$, $\angle A \cong \angle B$ and $\angle C \cong \angle D$.



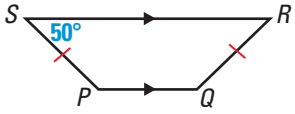
Theorem 6.13

Words If a trapezoid has a pair of congruent base angles, then it is isosceles.

Symbols In trapezoid $ABCD$, if $\angle C \cong \angle D$ then $ABCD$ is isosceles.

EXAMPLE 1 Find Angle Measures of Trapezoids

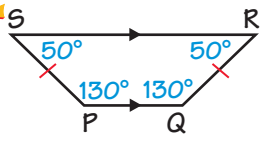
$PQRS$ is an isosceles trapezoid. Find the missing angle measures.



Solution

1. $PQRS$ is an isosceles trapezoid and $\angle R$ and $\angle S$ are a pair of base angles. So, $m\angle R = m\angle S = 50°$.
2. Because $\angle S$ and $\angle P$ are same-side interior angles formed by parallel lines, they are supplementary. So, $m\angle P = 180° - 50° = 130°$.
3. Because $\angle Q$ and $\angle P$ are a pair of base angles of an isosceles trapezoid, $m\angle Q = m\angle P = 130°$.

Visualize It!

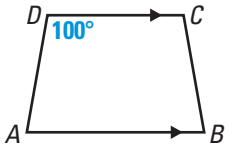


In Example 1, fill in the missing angle measures as you find them.

Checkpoint ✓ Find Angle Measures of Trapezoids

ABCD **is an isosceles trapezoid. Find the missing angle measures.**

1.



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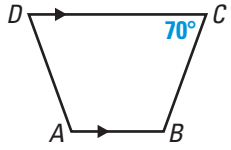

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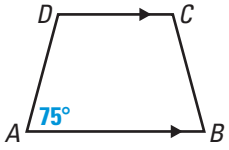
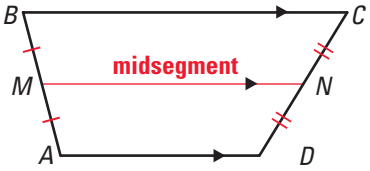


Student Help

VOCABULARY TIP
The midsegment of a trapezoid is sometimes called the *median* of a trapezoid.

Midsegments The **midsegment of a trapezoid** is the segment that connects the midpoints of its legs. The midsegment of a trapezoid is parallel to the bases.

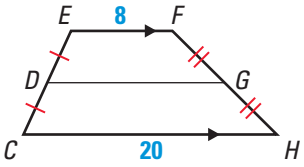


The length of the midsegment of a trapezoid is half the sum of the lengths of the bases.

$$MN = \frac{1}{2}(AD + BC)$$

EXAMPLE 2 Midsegment of a Trapezoid

Find the length of the midsegment $\overline{DG}$ of trapezoid *CEFH*.



Solution

Use the formula for the midsegment of a trapezoid.

$DG = \frac{1}{2}(EF + CH)$ — Formula for midsegment of a trapezoid

$= \frac{1}{2}(8 + 20)$ — Substitute 8 for *EF* and 20 for *CH*.

$= \frac{1}{2}(28)$ — Add.

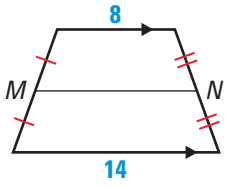
$= 14$ — Multiply.

ANSWER ▶ The length of the midsegment $\overline{DG}$ is 14.

Checkpoint ✓ Midsegment of a Trapezoid

Find the length of the midsegment $\overline{MN}$ of the trapezoid.

4.



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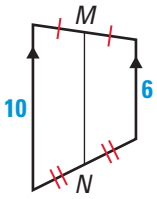

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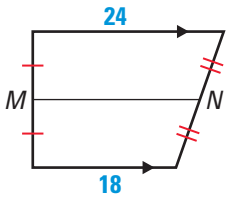


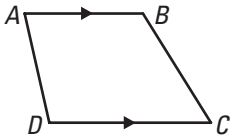
6.5 Exercises

Guided Practice

Vocabulary Check

1. Name the *bases* of trapezoid $ABCD$.

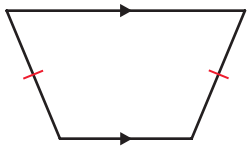
2. Name the *legs* of trapezoid $ABCD$.



Skill Check

Decide whether the quadrilateral is a *trapezoid*, an *isosceles trapezoid*, or *neither*.

3.



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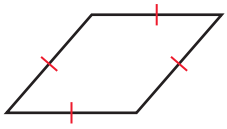

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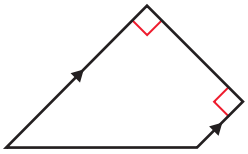
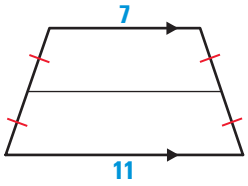


Find the length of the midsegment.

6.



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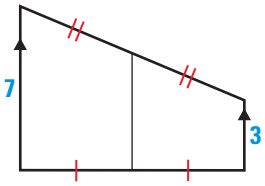

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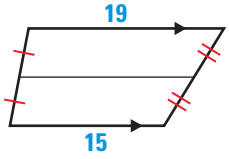


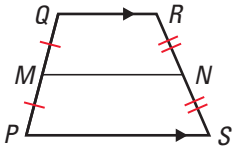
Practice and Applications

Extra Practice

See p. 686.

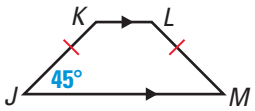
Parts of a Trapezoid **Match the parts of trapezoid *PQRS* with the correct description.**

9. $\overline{QR}$ and $\overline{PS}$	**A.** legs
10. $\angle Q$ and $\angle S$	**B.** base angles
11. $\angle R$ and $\angle Q$	**C.** opposite angles
12. $\overline{MN}$	**D.** bases
13. $\overline{PQ}$ and $\overline{RS}$	**E.** midsegment



Finding Angle Measures ***JKLM*** **is an isosceles trapezoid. Find the missing angle measures.**

14.



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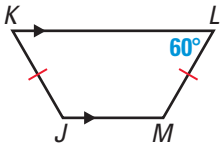

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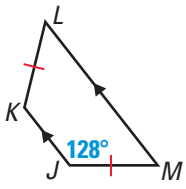
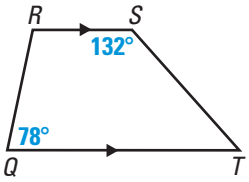


Homework Help

Example 1: Exs. 14–19
Example 2: Exs. 20–26

Finding Angle Measure *QRST* is a trapezoid. Find the missing angle measures.

17.



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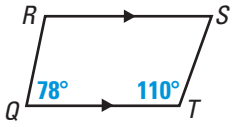

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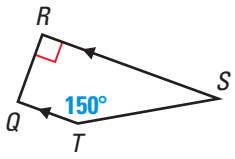
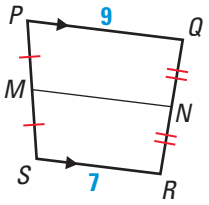


Finding Midsegments Find the length of the midsegment $\overline{MN}$ of the trapezoid.

20.



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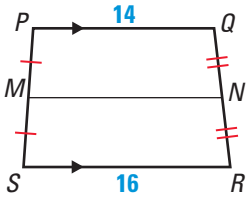

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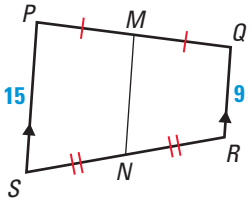
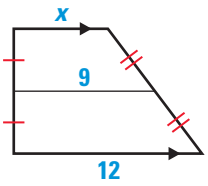


Using Algebra Find the value of x.

23.



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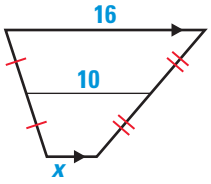

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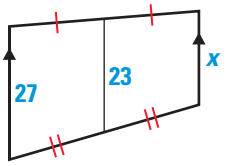
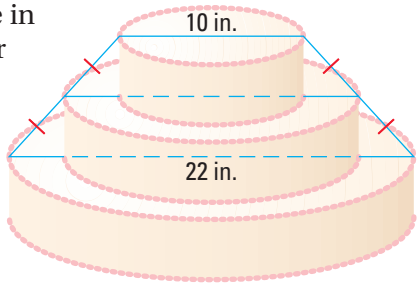


Link to Cake Design

CAKE DESIGNERS form and sculpt shapes and figures onto cakes by using tools such as icing bags, handmade paper cones, or cutters.

26. Cake Design The top layer of the cake in the diagram at the right has a diameter of 10 inches. The bottom layer has a diameter of 22 inches. What is the diameter of the middle layer?



Coordinate Geometry The vertices of a trapezoid are $A(2, 6)$, $B(8, 6)$, $C(8, 2)$, and $D(4, 2)$.

27. Plot the vertices on a coordinate plane. Connect them to form trapezoid *ABCD*.

28. Name the bases of trapezoid *ABCD*.

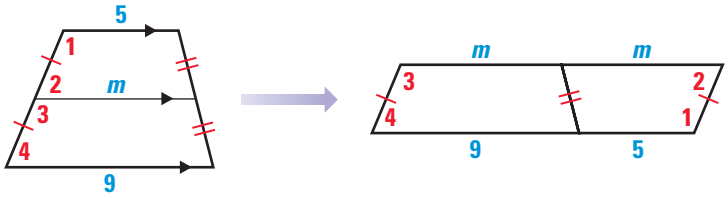
29. Name the legs of trapezoid *ABCD*.

30. Find the coordinates of the midpoint of each leg. Then plot these points on the coordinate plane you drew in Exercise 27. What is the line segment that connects these two points called?

Student Help
CLASSZONE.COM

HOMEWORK HELP
Extra help with problem solving in Exs. 31–33 is at classzone.com

Visualize It! **In Exercises 31–33, use the figures shown below.** The figure on the left is a trapezoid with midsegment of length m. The figure on the right is formed by cutting the trapezoid along its midsegment and rearranging the two pieces.



31. Which theorem or postulate from Chapter 3 can you use to show that $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$ in the figure on the left?

32. What kind of quadrilateral is on the right? Explain your answer.

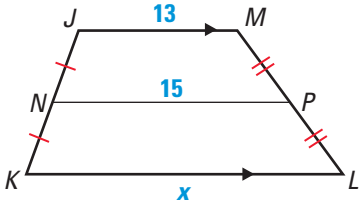
33. Challenge How does the diagram help you see that the length of the midsegment is half the sum of the lengths of the bases?

Standardized Test Practice

34. Multiple Choice In the trapezoid at the right, what is the value of x?

Ⓐ 13 Ⓑ 15

Ⓒ 17 Ⓓ 28



35. Multiple Choice Which of the following must a trapezoid have?

Ⓕ congruent bases

Ⓖ diagonals that bisect each other

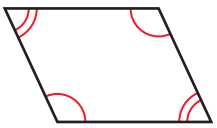
Ⓗ exactly one pair of parallel sides

Ⓙ a pair of congruent opposite angles

Mixed Review

Logical Reasoning **Tell whether the quadrilateral is a parallelogram. Explain your reasoning.** *(Lesson 6.3)*

36.



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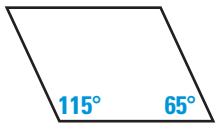

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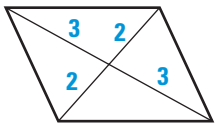

Algebra Skills

Multiplying **Multiply. Write the answer in simplest form.** *(Skills Review, p. 659)*

39. $\frac{1}{2} \times 20$ **40.** $52 \times \frac{1}{4}$ **41.** $136 \times \frac{1}{8}$ **42.** $\frac{3}{4} \times 60$

43. $\frac{2}{3} \times \frac{3}{7}$ **44.** $\frac{7}{8} \times \frac{2}{14}$ **45.** $\frac{5}{6} \times \frac{1}{3}$ **46.** $\frac{4}{21} \times \frac{7}{16}$