Massive Gravitons Stability , and a Review of How Many Gravitons Make Up a Gravity Wave detectable / congruent with B.P. Abbott, et.al., Nature 460, 991 (2009).

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Abstract: The following questions are raised. First, can there be a stable (massive) graviton? Secondly, what is the relationship between a Gravity Wave, and Gravitons? The inter relationship between a graviton, and a gravity wave is raised, with an idea of eventually making sense of a numerical count presented by Claus Kieffer, in his book on Quantum gravity, in conjunction with how a gravity wave/ graviton count could have some over lap.

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INTRODUCTION

As presented in the introduction the article asks first for criteria for massive graviton stability, and then applies stable massive (4 D) gravitons in terms of a KK DM model, with small 4 D graviton mass to obtain re acceleration of the universe a billion years ago. The problem, next, of identification of how GW and gravitons mesh together will be part of a formalistic description of how to confirm a role for gravitons in re acceleration of the universe a billion years ago.

Identification of Graviton Stability Requirements from Graviton Frequency

If the graviton is, with small mass, stable, then its macro effects show up in the deceleration parameter behavior, indicating re acceleration a billion years ago. We look at work presented by Maggiorie,[7] which specifically delineated for non zero graviton mass, where we write $h \equiv \eta^{uv} h_{uv} = Trace \cdot (h_{uv})$ and $T = Trace \cdot \left(T^{uv}\right)$ that

$$-3m_{graviton}^{2} h = \frac{\kappa}{2} \cdot T \tag{1}$$

Our work uses Visser's[4] 1998 analysis of non zero graviton mass for both T and h. We will use the above equation with a use of particle count n_f for a way to present initial

GW relic inflation density using the definition given by Maggiore [1,2,7] as a way to state that a particle count

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f)\cdot\Omega_{gw}(f) \Rightarrow h_0^2\Omega_{gw}(f) \cong 3.6\cdot\left[\frac{n_f}{10^{37}}\right]\cdot\left(\frac{f}{1kHz}\right)^4 \qquad (2)$$

where n_f is the frequency-based numerical count of gravitons per unit phase space. To do so, let us give the reasons for using Visser's [4] values for T and h above, in Eqn. (1).

We begin our inquiry by initially looking at a modification of what was presented by R. Maartens [3] , as done by Beckwith [1,2]

$$m_n(Graviton) = \frac{n}{L} + 10^{-65} \text{ grams} \qquad (3)$$

On the face of it, assignment of a mass of about 10^{-65} grams for a 4 dimensional graviton, allowing for $m_0(Graviton - 4D) \sim 10^{-65}$ grams[1,2] violates all known quantum mechanics, and is to be avoided. Numerous authors, including Maggiore[7] have demonstrated how adding a term to the Fiertz Lagrangian for gravitons, and assuming massive gravitons leads to results which appear to violate field theory

Visser's Treatment of a Stress Energy Tensor for Massive Gravitons

Visser[4] in 1998, stated a stress energy treatment of gravitons along the lines of

$$T_{uv}\Big|_{m\neq 0} = \left[\left(\frac{\hbar}{l_P^2\lambda_g^2}\right)\cdot\left(\frac{GM}{r}\right)\cdot\exp\left(\frac{r}{\lambda_g}\right) + \left(\frac{GM}{r}\right)^2\right]\times\begin{bmatrix}4&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} \qquad (4)$$

Furthermore, his version of $g_{uv} = \eta_{uv} + h_{uv}$ can be written as setting

$$h_{uv} \equiv 2\frac{GM}{r}\cdot\left[\exp\left(\frac{-m_g r}{\hbar}\right)\right]\cdot\left(2\cdot V_\mu V_\nu + \eta_{uv}\right) \qquad (5)$$

If one adds velocity 'reduction' put in with regards to speed propagation of gravitons[5]

$$v_g = c\cdot\sqrt{1 - \frac{m_g^2\cdot c^4}{\hbar^2\omega_g^2}} \qquad (6)$$

One can insert all this into Eqn. (1) to obtain a real value for the square of frequency > 0, i.e.

$$\hbar^2\omega^2 \cong m_g^2 c^4 \cdot \left[1/\left(1-\breve{A}\right)\right] > 0 \tag{7}$$

$$\breve{A} = \left\{1 - \frac{1}{6m_g c^2}\left(\frac{\hbar^2}{l_P^2\lambda_g^2}\cdot\exp\left[-\frac{r}{\lambda_g}+\frac{m_g\cdot r}{\hbar}\right]+\left(\frac{MG}{r}\right)\cdot\exp\left(\frac{m_g r}{\hbar}\right)\right)\right\}^2 \tag{8}$$

According to Kim [8], if the square of the frequency of a graviton, with mass, is >0, and real valued, it is likely that the graviton is stable, with regards to perturbations. Kim's article [11] is with regards to Gravitons in brane / string theory, but it is likely that the same dynamic for semi classical representations of a graviton with mass. Conditions for a positive value of the square of frequency in Eqn. (7) is the same as analyzing conditions for when the following inequality holds, in terms of parameter space values for the variables in Eqn. (9) below. Eqn. (9) allows for stable giant gravitons.

$$0 < \frac{1}{6m_g c^2}\left(\frac{\hbar^2}{l_P^2\lambda_g^2}\cdot\exp\left[-\frac{r}{\lambda_g}+\frac{m_g\cdot r}{\hbar}\right]+\left(\frac{MG}{r}\right)\cdot\exp\left(\frac{m_g r}{\hbar}\right)\right) < 1 \tag{9}$$

Review of what can be said about Gravitons and GW, i.e. Inter relationship and Overlap

We will start off with a review of what Kip Thorne has to say about the number of nodes of GW in a post inflationary regime of space time, and compare that with Claus Kieffer's effective gravitational mass, m_g. From there, we will attempt to make an estimate of how to relate coherent states of gravitons, with GW, partly in terms of Kieffer's number of Gravitons with a specified momentum value $p^\mu = \hbar k^\mu$.

Thorne makes the prediction that a maximum allowed frequency per individual Gravity WAVE may be obtained via

$$f_{\max} \approx \left[2\times10^{-7}\,Hz\right]\cdot\left(kT/1GeV\right) \tag{10}$$

Arguably, some linkage between Gravity waves, and gravitons could be obtained via comparing Eqn. (10) and Eq. (11) with a prediction made by Kieffer as to the number of gravitons per cubic centimeter , as a function of frequency, i.e.

$$N_{gravitons} = \frac{\omega\cdot c^2}{16\pi\hbar G}\cdot\left[\left(e^{\alpha\beta *}e_{\alpha\beta} - .5\left|e_\alpha^\alpha\right|^2\right) \equiv \left|Amp\right|^2\right] \tag{11}$$

Kieffer's figures for frequency versus amplitude lead to the following results, i.e.for frequency ~ 1000 Hertz, $\left|Amp\right| \sim 10^{-21} \Rightarrow N_{gravitons} \sim 3\times10^{14}/cm^3$, whereas for frequency ~ 10^{-3} Hertz, $\left|Amp\right| \sim 10^{-22} \Rightarrow N_{gravitons} \sim 10^{7}/cm^3$. One could estimate, roughly that if frequencies climbed, say to frequency ~ 100,000 Hertz, $\left|Amp\right| \sim 10^{-23} \Rightarrow N_{gravitons} \sim 3\times10^{15}/cm^3$. If so, then if one looked at what the number of ‘gravitons’ **at the end of inflation**, about at a time specified by $t \sim 10^{-30} s$, and a radii of ~ 1 meter, then if a graviton wave length were approximately $\lambda_{graviton} \sim c / f_{graviton} \approx 1 \cdot meter$, leading to

$$f_{graviton} \approx c / \lambda_{graviton} \sim 2.99 \times 10^{8} / \sec \cong 3 \times 10^{8}$$
$$\Rightarrow \omega_{graviton} \equiv 2\pi f_{graviton} \sim 10^{9} Hertz \qquad (13)$$

If gravitons have mass, then the relative speed is reduced according to the equation $v_g = c \cdot \sqrt{1 - \frac{m_g^2 \cdot c^4}{\hbar^2 \omega_g^2}} \leq c$ depending upon the mass of the graviton assumed. . Note though, that values of $\omega_{graviton} \equiv 2\pi f_{graviton} \sim 10^{9} Hertz$, and in some cases 10 GHz have been postulated by L. Grishkuk, which leads to the conundrum of determining if graviton production can be made in sync with GW node estimates, as given by Eqn. 10, above. Note that for a temperature of T~ 10^{30} Kelvin ~ 10^{26} eV=10^{17} GeV which is about what the value of the temperature in the middle to the end of inflation, (from a peak value of about T~ 10^{32} Kelvin, initially) when put into Eqn. (10) above would lead to $2\pi \cdot f_{\max}\big|_{GW} \sim 2 \times 10^{-7} Hz \times \left[10^{17}\right] \sim 10 \cdot GHz$. Putting this into Eqn. (11) by rough estimation would lead to, $|Amp| \sim 10^{-24} \Rightarrow N_{gravitons} \sim 3 \times 10^{18} / cm^{3}$. Beckwith, via his own estimation has $N_{gravitons} \sim 3 \times 10^{6} / cm^{3}$, which would lead to $|Amp| \sim 10^{-30} \Rightarrow N_{gravitons} \sim 3 \times 10^{6} / cm^{3}$ as escaping graviton/ information at the end of the big bang, presumably being information from a prior universe as transferred to a present universe.

The question this leads to is what to make of $|Amp| \sim 10^{-30}$. I.e. Maggiore has that $|Amp| \sim 10^{-22}$ is in amplitude ~ 1/1000 the size of a typical atom, and R. Weiss stated in conversation with the author at ADM 50, that he sees no way that $|Amp| \sim 10^{-24}$ could be obtained within the next ten years.

The question remains, then, what would the implications of setting $|Amp| \sim 10^{-30} \Leftrightarrow N_{relic-graviton-End-of-Inflation} \sim 10^{6}$ as produced at the end of inflation, i.e. at looking at, say

$$0 < \frac{1}{6 m_g c^2} \left(\frac{\hbar^2}{\lambda_g^2} \cdot \exp\left[-\frac{r}{\lambda_g} \right] \right) < 1 \qquad (14)$$

One obtains, if looking at this, then that $0 \leq \exp\left[-\frac{r}{\lambda_g} \right]$, where one has that for when r >> λ_g , but also $m_g \neq 0$, that

$$h_0^2 \Omega_{gw}(f) \cong 3.6 \cdot \left[\frac{n_f \sim 10^7}{10^{37}} \right] \cdot \left(\frac{f \approx 10^{10} \cdot Hz}{10^3 Hz} \right) \sim 3.6 \cdot 10^{17} \cdot 10^{37} \sim 10^{-20} \qquad (15)$$

I.e. Houston, we have a problem!

For the record, one can look at

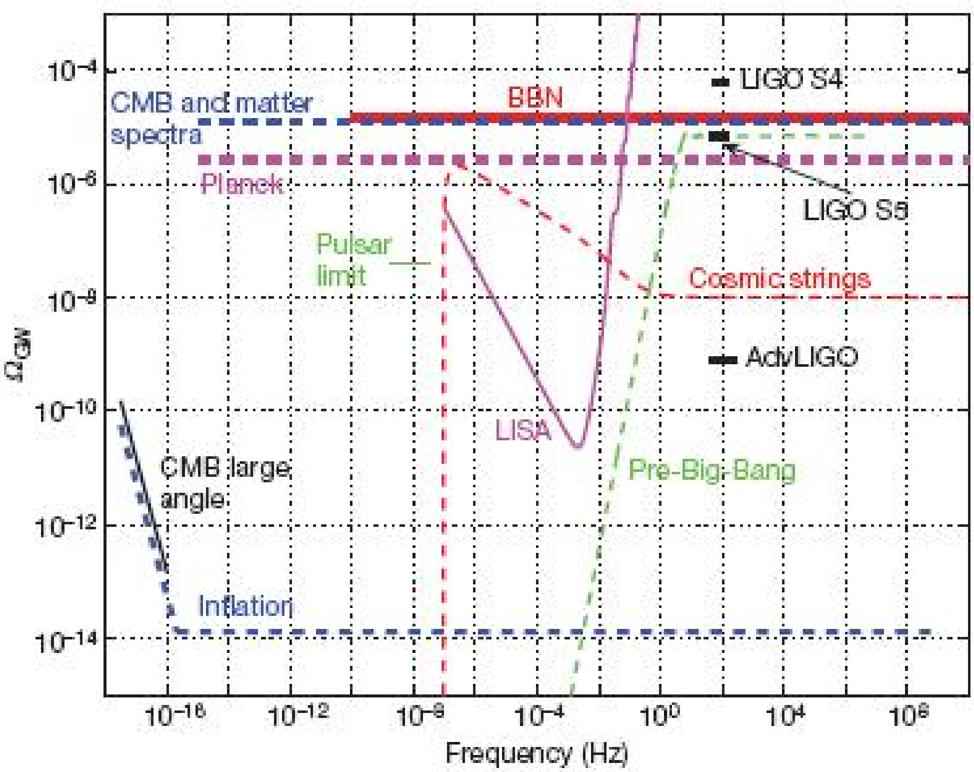

Figure 1 : This figure from B.P. Abbott, et.al.,, Nature 460, 991 (2009).

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In order to obtain GW which could be commensurate, with Inflation, the values for n_f would have to be changed to , say as follows

If graviton frequency ~ 100,000 Hertz, and $|Amp| \sim 10^{-24} \Rightarrow N_{gravitons} \sim 3 \times 10^{18} / cm^3$, then

$$h_0^2\Omega_{gw}(f) \cong 3.6 \cdot \left[\frac{n_f \sim 10^{18}}{10^{37}}\right] \cdot \left(\frac{f \approx 10^5 \cdot Hz}{10^3\,Hz}\right) \sim 10^{-17} \qquad \Rightarrow\Leftarrow \tag{17}$$

<u>In order to fix this, we need either of the FOLLOWING,</u>

One would need to up by an **<u>order of magnitude</u>** of three, either $n_f \sim 10^{21}$, for a frequency of about $f \approx 10^5 \cdot Hz$, or else $n_f \sim 10^{18}$ for $f \approx 10^8 \cdot Hz \Leftrightarrow \omega \sim 10^9\,Hz$

Having $n_f \sim 10^{18}$ at the end of inflation would probably do it. Determining if $n_f \sim 10^{18}$ gravitons is due to a single early universe, stochastic background DETECTED GW is another matter entirely.