Length and Area of Circles

ACTIVITY 32

Pi in the Sky

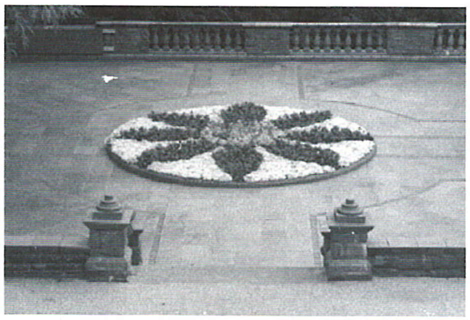
Lesson 32-1 Circumference and Area of a Circle

Learning Targets:

- Develop and apply a formula for the circumference of a circle.
- Develop and apply a formula for the area of a circle.

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Identify a Subtask

Lance owns The Flyright Company. His company specializes in making parachutes and skydiving equipment. After returning from a tour of Timberlake Gardens, he was inspired to create a garden in the circular drive in front of his office building. Lance plans to hire a landscape architect to design his garden. The architect needs Lance to provide the area and circumference of the proposed garden so the architect can estimate a budget.



Lance relies on the measures of some familiar objects to help him understand the measures of a circle.

Object	Distance Around the Object, *C*	Diameter, *d*	$\frac{C}{d}$
Coin	7.85 cm	2.5 cm	3.14
Clock	40.84 mm	13 mm	3.14
Frisbee	62.83 in.	20 in.	3.14

1. Complete the table to find the ratio of $C : d$.
2. Make a conjecture about the relationship between the circumference and the diameter of circular objects. $C : d = 3.14$

The ***circumference*** of a circle is the distance around the circle. Circumference is measured in linear units. The ratio of the circumference to the diameter of any circle is designated by the Greek letter π (pi).

3. Write a formula, in terms of diameter, *d*, that can be used to determine the circumference of a circle, *C*. $C = \pi d$
4. Write a formula, in terms of the radius, *r*, that can be used to determine the circumference of a circle, *C*. $C = 2\pi r$
5. What information does the circumference provide about the garden Lance wants to create?

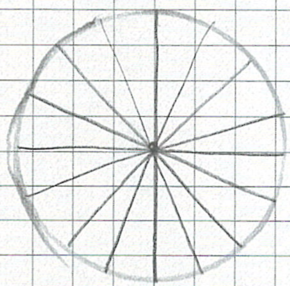
circumference is the distance around the garden

My Notes

CONNECT TO HISTORY

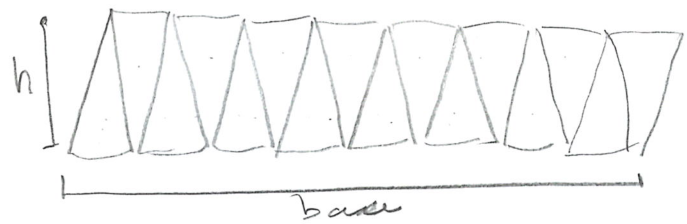
The first known calculation of π (pi) was done by Archimedes of Syracuse (287–212 BCE), one of the greatest mathematicians of the ancient world. Archimedes approximated the area of a circle using the Pythagorean Theorem to find the areas of two regular polygons: the polygon *inscribed within the circle* and the polygon *within which the circle was circumscribed*. Since the actual area of the circle lies between the areas of the inscribed and circumscribed polygons, the areas of the polygons gave upper and lower bounds for the area of the circle. Archimedes knew that he had not found the value of pi but only an approximation within those limits. In this way, Archimedes showed that pi is between $3\frac{10}{71}$ and $3\frac{1}{7}$.

My Notes



Lance uses a compass and paper folding to develop the formula for the area of a circle.

6. **Use appropriate tools strategically.** Use a compass to draw a large circle. Use scissors to cut the circle out.



7. Divide the circle into 16 equal ***sectors***.
8. Cut the sectors out and arrange the 16 pieces to form a parallelogram.
9. Write the formula for the area of a parallelogram, in terms of base b and height h.

$A = bh$

MATH TERMS

A **sector** is a pie-shaped part of a circle. A sector is formed by two radii and the arc determined by the radii.

10. Explain how the height of the parallelogram is related to the radius of the circle.

height ▱ = radius ⊙

$h = r$

11. Explain how the base of the parallelogram is related to the circumference of the circle.

base ▱ = 1/2 circumference ⊙

$b = \frac{1}{2}C = \frac{1}{2}(2\pi r) = \pi r$

12. How does the area of the parallelogram compare to the area of the original circle?

area ▱ = area ⊙

13. **Construct viable arguments.** Use your knowledge of area of parallelograms to prove the area formula of a circle.

$A_{▱} = b \cdot h = \pi r \cdot r = \pi r^2$ Area circle

14. What information does the area provide about the garden Lance wants to create?

the amount of ground the garden covers.

Additional Example:

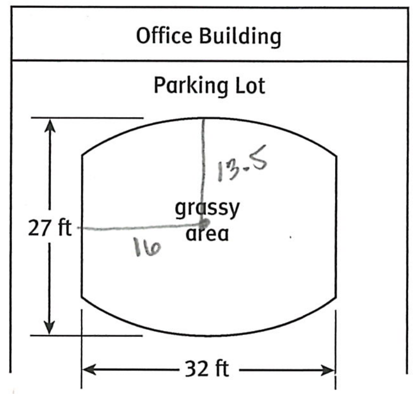
C of a ⊙ = 32π in

Find the area of the ⊙

$256\pi \text{ in}^2$

My Notes

The layout of The Flyright Company's building and parking lot is shown below.



The dimensions of the grassy area of the parking lot are 32 ft by 27 ft.

15. What are the radius, circumference, and area of the largest circle that will fit in the grassy area? Justify your answer.

$r = 27/2 = 13.5$ ft. $A = 13.5^2\pi$

$C = 27\pi$ ft. ≈ 84.8ft ≈ 572.6 ft^2

Even though larger circular gardens will fit in the grassy area, Lance decides that he would like the garden to have a diameter of 20 feet.

16. The landscape architect recommends surrounding the circular garden with decorative edging. The edging is sold in 12-foot sections that can bend into curves. How many sections of the edging will need to be purchased to surround the garden? Justify your answer.

6 sections. $C = 20\pi \approx 62.83$ ft.

$62.83 \div 12 = 5.236$ sections

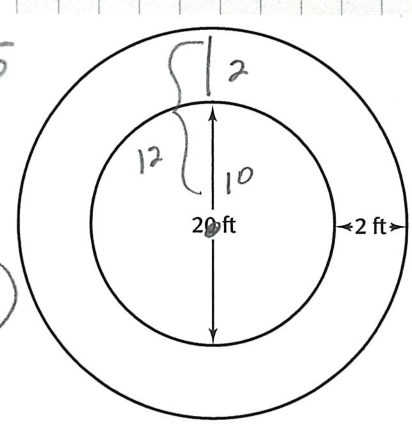
DISCUSSION GROUP TIP

As you share ideas with your group, be sure to explain your thoughts using precise language and specific details to help group members understand your ideas and reasoning.

17. To begin building the garden, the landscape architect needs to purchase soil. To maintain a depth of one foot throughout the garden, each bag can cover 3.5 square feet of the circle. How many bags of soil need to be purchased? Show the calculations that lead to your answer.

90 bags $A = \pi 10^2 = 314$ ft^2 $\div 3.5$
≈ 89.7

18. Lance and the architect are discussing the possibility of installing a sidewalk around the outside of the garden, as shown in the diagram. Determine the area of the sidewalk.



$\pi 12^2 - \pi 10^2 = 44\pi$ ft^2
≈ 138.2 ft^2

19. Critique the reasoning of others. Darnell claims that the circumference of a circular garden with radius 12 meters is greater than the perimeter of a square garden with side 18 meters. Do you agree? Explain your reasoning.

$C = \pi(24) = 75.36$m   yes

$P = 18(4) = 72$m

My Notes

(20) area of ⊙ is # of units to cover surface, circumference is distance.

(21) $C = 2\pi r$

$\frac{C}{2\pi} = r$

(22) $A = \pi r^2$

$A = \pi\left(\frac{C}{2\pi}\right)^2$

$A = \frac{\pi}{1}\left(\frac{C^2}{4\pi^2}\right)$

$A = \frac{C^2}{4\pi}$

(23) area will decrease

(24) $A = \pi 9^2$

$81\pi \approx 254.5\ ft^2$

Check Your Understanding

20. Explain why the units of measure for the circumference of a circle are linear, and those of the area of a circle are in square units.

21. Express the radius of a circle in terms of its circumference.

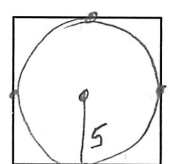
22. Express the area of a circle in terms of its circumference.

23. If Lance decides to increase the width of the walkway to 3 feet and keep the outer radius at 12 feet, what will happen to the area of the garden?

LESSON 32-1 PRACTICE

24. A circular rotating sprinkler sprays water over a distance of 9 feet. What is the area of the circular region covered by the sprinkler?

25. Reason quantitatively. What is the greatest circumference of the circle that can be inscribed in the square below?



10 cm

$d = 10$

$C = 10\pi$

≈ 31.42 cm

26. Suppose a landscaper pushes a wheelbarrow so that the wheel averages 25 revolutions per minute. If the wheel has a diameter of 18 inches, how many feet does the wheelbarrow travel each minute?

27. Construct viable arguments. Can the area of a circle ever equal the circumference of a circle? Support your answer with proof.

(26.) $1d = 18in = 1.5ft.$

$25 \times (1.5\pi)$

$37.5\pi \approx 117.75\ ft$

(27.) $A = C$

$\pi r^2 = 2\pi r$

$r^2 = 2r$

$r = 2$

yes, when radius = 2