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RELIABILITY OF ELECTRICAL SUBSTATION EQUIPMENT CONNECTED BY RIGID BUS

Junho SONG[1] and Armen DER KIUREGHIAN[2]

SUMMARY

Electrical substation equipment such as transformers, circuit breakers, switches and surge arresters are often connected through rigid buses. In practice, various types of flexible connectors are inserted between one equipment and the rigid bus to help reduce an adverse interaction effect between the equipment items under dynamic loading. Seismic reliability assessment of such a connected system is a challenging task because: 1) the behavior of the connector is highly nonlinear, 2) the ground motion is stochastic in nature, and 3) the substation is a complex system, whose reliability cannot be directly deduced from marginal component reliability estimates. This paper describes a newly developed methodology for estimating the seismic reliability of connected equipment items and the substation system by use of nonlinear random vibration analysis employing the equivalent linearization method, computation of first-passage probabilities for scalar and vector processes, and the computation of bounds on system reliability by linear programming. Numerical examples show equipment fragilities for various types of connectors, including a newly designed connector with superior properties, and fragility estimates for individual equipment items and system fragility for an example electrical substation system.

INTRODUCTION

Lifelines, such as transportation networks, gas- and water-distribution systems, and power transmission and communication networks, operate as critical backbones of urban communities. Recent earthquakes in Loma Prieta (1989), Northridge (1995) and Kobe (1996) have demonstrated that damage to critical lifelines can cause severe losses to an urban society and economy. Moreover, the failure of lifeline systems may delay delivery of first aid and post-earthquake recovery. Therefore, it is important to reinforce critical lifeline systems so as to assure their functionality during future earthquakes.

An important element within the power transmission network is the electrical substation, which consists of a complex set of interconnected equipment items, such as transformers, circuit breakers, switches and surge arresters. Many of these equipment items are connected to each other through assemblies of rigid bus and various types of flexible connectors, including conventional flexible strap connectors (FSC),

[1] Doctoral Candidate, Department of Civil & Environmental Engineering, University of California, Berkeley, USA. Email: jhsong@ce.berkeley.edu
[2] Taisei Professor of Civil Engineering, University of California, Berkeley, USA. Email: adk@ce.berkeley.edu

slider connectors (SC), and a newly designed S-shape FSC named *S*-FSC. Figure 1 shows the mechanical model of two interconnected equipment items with a rigid bus and a flexible connector. It is known (Der Kiureghian [1]) that the interaction between the connected equipment items can result in a significant amplification of the demand on the equipment item with higher frequency. To assure a desired level of their functionality during future earthquakes, it is essential to have a methodology for assessing the reliability of interconnected electrical substation equipment items and system. This problem is not straightforward because: (1) the equipment items cannot be analyzed individually due to the dynamic interaction between them, (2) the flexible connectors behave nonlinearly, (3) ground motions are stochastic in nature, and (4) the substation is a complex system whose reliability cannot be directly deduced from marginal component reliability estimates.

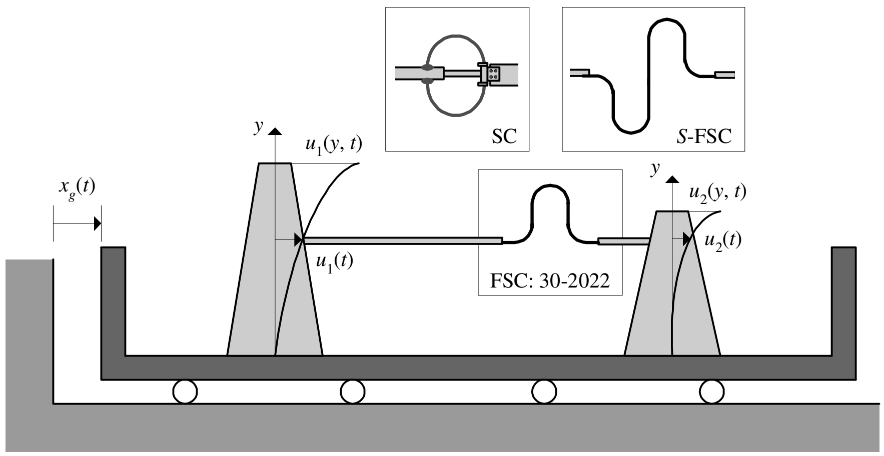

Figure 1. Mechanical model of two equipment items connected by a rigid bus conductor

This paper develops a methodology for estimating the reliability of an interconnected equipment system or its components. Briefly stated, the methodology involves the following steps: (1) the connected equipment items are represented by single-degree-of-freedom (SDOF) linear models; (2) the nonlinear hysteretic behavior of the rigid bus connector is described by differential-equation-type analytical models; (3) the second moments of the responses of the connected system are computed by nonlinear random vibration analysis employing the equivalent linearization method (ELM); (4) marginal and joint fragilities of equipment items are obtained by use of approximate first-excursion probability functions; and (5) the fragility of the entire substation system is approximated by system reliability bounds employing marginal and joint component fragilities together with a linear programming algorithm. The proposed methodology is demonstrated through a numerical example.

MODELING OF EQUIPMENT ITEMS

Due to the plethora of configurations of equipment types and connections, and lack of detailed information on equipment configurations, it is expedient to idealize each equipment item as simply as possible. Hence, we model each equipment item as a SDOF oscillator characterized by its effective mass m, stiffness k, damping ratio ζ, and an effective external inertia force l (Der Kiureghian [1]). The effective coefficients, capturing the essential dynamic features of the equipment, are obtained employing an appropriate "shape" function $\psi(y)$ such that the displacement $u(y,t)$ at the coordinate y and time t can be represented as $u(y,t)=\psi(y)z(t)$, where $z(t)$ is the generalized coordinate describing the variation of the displacement amplitude in time (see Figure 1). Using this shape function, we obtain for each equipment item

$$m_i = \int_0^{L_i} \rho_i(y)[\psi_i(y)]^2 dy \tag{1a}$$

$$k_i = \int_0^{L_i} EI_i(y)[\psi_i''(y)]^2 dy \tag{1b}$$

$$c_i = 2\zeta_i \sqrt{m_i k_i} \tag{1c}$$

$$l_i = \psi_i(y_i)\int_0^{L_i} \rho_i(y)\psi_i(y) dy \tag{1d}$$

where the subscript i is the equipment index, $\rho(\cdot)$ is the distributed mass density, $EI(\cdot)$ is the flexural stiffness, L is the total length of the equipment and c is the damping coefficient. For SDOF idealization of more complex structures and discussion on the accuracy of the SDOF modeling in interaction studies, see Song [2].

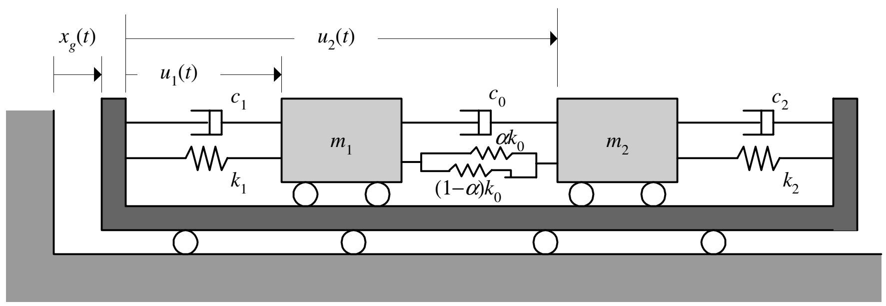

Figure 2. Idealized model of two equipment items connected by a rigid bus

Based on the above idealizations, the connected equipment items shown in Figure 1 are represented by the two-degree-of-freedom system shown in Figure 2. The corresponding equations of motion are

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{R}(\mathbf{u}, \dot{\mathbf{u}}, z) = -\mathbf{L}\ddot{x}_g \tag{2}$$

where

$$\mathbf{u} = \begin{Bmatrix} u_1(t) \\ u_2(t) \end{Bmatrix}, \quad \mathbf{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} c_1 + c_0 & -c_0 \\ -c_0 & c_2 + c_0 \end{bmatrix} \tag{3a}$$

$$\mathbf{R}(\mathbf{u}, \dot{\mathbf{u}}, z) = \begin{Bmatrix} k_1 u_1(t) - q(\Delta u(t), \Delta \dot{u}(t), z(t)) \\ k_2 u_2(t) + q(\Delta u(t), \Delta \dot{u}(t), z(t)) \end{Bmatrix}, \quad \mathbf{L} = \begin{Bmatrix} l_1 \\ l_2 \end{Bmatrix} \tag{3b}$$

$\ddot{x}_g$ is the ground acceleration, $u_i(t)$ is the displacement of the i-th equipment item at its attachment point relative to the base, $\Delta u(t) = u_2(t) - u_1(t)$ is the relative displacement between the two equipment items, $z(t)$ is an auxiliary variable representing the inelasticity of the connector, and c_0 denotes the effective viscous damping of the rigid bus connector assembly. The function $q(\Delta u(t), \Delta \dot{u}(t), z(t))$ denotes the resisting force of the inelastic rigid bus connector based on an assumed hysteretic model. The equations of motion can be easily extended to interconnected systems involving more than two equipment items.

MODELING OF RIGID BUS CONNECTORS

The hysteretic behavior of the rigid bus connectors was investigated by finite element (FE) analysis (Der Kiureghian [3]) and quasi-static tests (Filiatrault [4], Stearns [5]). The results exhibit a nonlinear hysteretic behavior of the connectors, which can mitigate the adverse interaction effects with energy dissipation and softening. For the purpose of the reliability analysis, FE analysis is not convenient since it would require a fairly large nonlinear dynamic model. On the other hand, differential-equation type hysteresis models, such as the Bouc-Wen model, have the advantage of computational simplicity. Furthermore, such models allow nonlinear random vibration analysis by use of ELM, thus providing a means to account for the stochasticity of ground motions.

For the differential-equation-type hysteresis model, the resisting force $q\left(\Delta u(t), \Delta \dot{u}(t), z(t)\right)$ in (3b) is written as

$$q(\Delta u, \Delta \dot{u}, z) = \alpha k_0 \Delta u + (1-\alpha) k_0 z \tag{4}$$

where α is the post- to pre-yield stiffness ratio and k_0 is the initial stiffness. The evolution of z for a given hysteretic model is described by an auxiliary ordinary differential equation involving z, $\dot{z}$, Δu and $\Delta \dot{u}$.

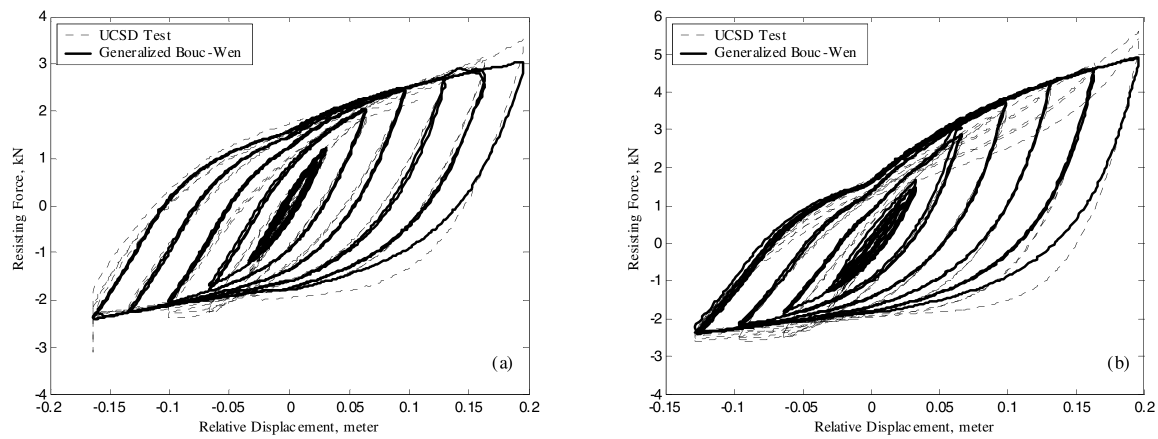

Figure 3. Comparison of test and analysis hysteresis loops by use of generalized Bouc-Wen model: (a) FSC PG&E: 30-2022 and (b) FSC PG&E: 30-2021

The conventional FSC's, including PG&E: 30-2022 shown in Figure 1, have highly asymmetric hysteresis loops due to their geometric nonlinearity. In order to achieve a good agreement with the asymmetric loops, Der Kiureghian [3] developed a modified Bouc-Wen model with parameters, which were functions of the response. Despite its close agreement with test results, this model cannot be used for nonlinear random vibration analysis. Song [6] developed a generalized Bouc-Wen model that has response-invariant parameters, but is capable of modeling asymmetric hysteresis loops with reasonable accuracy. The auxiliary differential equation of this model is given as

$$g = \dot{z} - \Delta \dot{u}[A - |z|^n \psi(\Delta u, \Delta \dot{u}, z)] = 0 \tag{5a}$$

$$\psi = \beta_1 \operatorname{sgn}(\Delta \dot{u} z) + \beta_2 \operatorname{sgn}(\Delta u \Delta \dot{u}) + \beta_3 \operatorname{sgn}(\Delta u z) + \beta_4 \operatorname{sgn}(\Delta \dot{u}) + \beta_5 \operatorname{sgn}(z) + \beta_6 \operatorname{sgn}(\Delta u) \tag{5b}$$

where A is a scale parameter of the hysteresis loop, n is a parameter that controls the sharpness of the hysteresis loop, β_i, $i = 1,2,...,6$, are parameters controlling the shape of the hysteresis loop, and $\mathrm{sgn}(\cdot)$ is the signum function. Figure 3 compares this model with hysteresis loops obtained in experiments by Filiatrault [4]. Fairly good agreement with experimental results is observed.

For describing the slider connector, the analytical bi-linear model by Kaul [7] is adopted (Song [8]). The corresponding auxiliary equation is

$$g = \dot{z} - \Delta\dot{u}\,[U(z + \Delta u_y) - U(z - \Delta u_y) + U(z - \Delta u_y)U(-\Delta\dot{u}) + U(-z - \Delta u_y)U(\Delta\dot{u})] = 0 \tag{6}$$

where $U(\cdot)$ denotes the unit step function and Δu_y is the yield displacement. The model is fitted to the experimental hysteresis loops with fairly close agreement, as shown in Figure 4a.

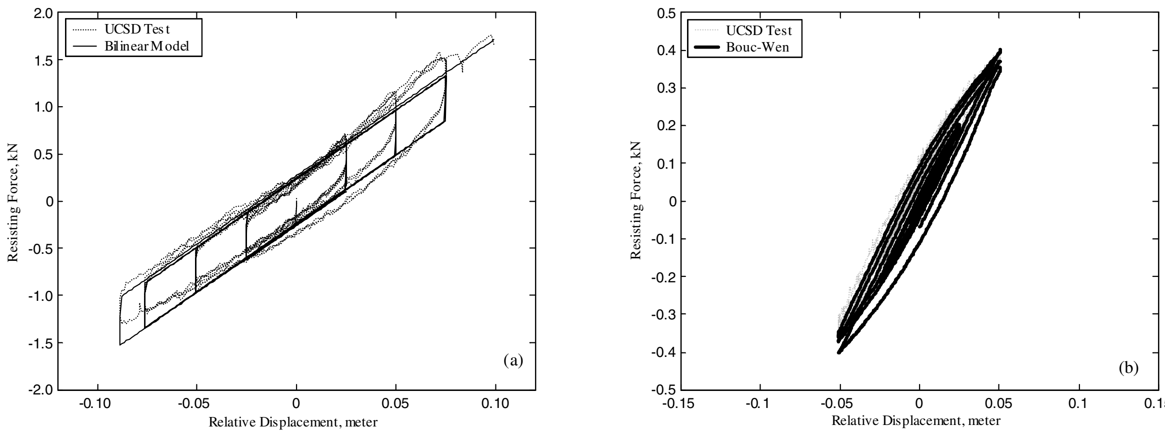

Figure 4. Comparison of test and analysis hysteresis loops by use of (a) bi-linear model for slider connector and (b) Bouc-Wen model for *S*-FSC

The parametric studies (Der Kiureghian [1]) on linearly connected equipment systems showed that increasing the flexibility of the connector can significantly reduce the adverse interaction effect on the higher frequency equipment item. Inelastic behavior (softening) and energy dissipation by the connector also reduce the interaction effect. However, the amount of the reduction depends on the intensity of ground motion, which is intrinsically random. Furthermore, inelastic deformation in a FSC may require retooling or replacement after an earthquake event, which may cause significant restoration cost or delay of service. Therefore, a highly flexible FSC, which essentially behaves linearly, is desirable because it guarantees a significant amount of reduction in interaction, independently of the intensity of ground motion. For this purpose, Song [2] & [6] developed a new design of FSC, *S*-FSC, which achieves these goals by its new anti-symmetric geometry. Its hysteresis loops were predicted by FE models developed for the conventional FSC's (Der Kiureghian [3]). Based on the predicted loops, the original Bouc-Wen model (Wen [9]) is adopted. The model achieves a close agreement with the experimental hysteresis loops by Stearns [5], as shown in Figure 4b. The auxiliary differential equation is

$$g = \dot{z} - \Delta\dot{u}\left\{A - |z|^n\left[\beta + \gamma\,\mathrm{sgn}(\Delta\dot{u}z)\right]\right\} = 0 \tag{7}$$

where β and γ are the parameters controlling the shape of the hysteresis loop.

The accuracy of the hysteresis models in dynamic analyses was confirmed through comparison with shake-table test results or analysis results by use of the modified Bouc-Wen model (Song [2]). For example, Figure 5 compares the displacement time histories of two equipment items connected by the slider connector and subjected to the Newhall (1994, Northridge) ground motion, as obtained in tests conducted by Filiatrault [4] at UCSD and by analysis based on the bi-linear model fitted to the experimental loops (Song [8]). Excellent agreement between experimental and analytical results is evident.

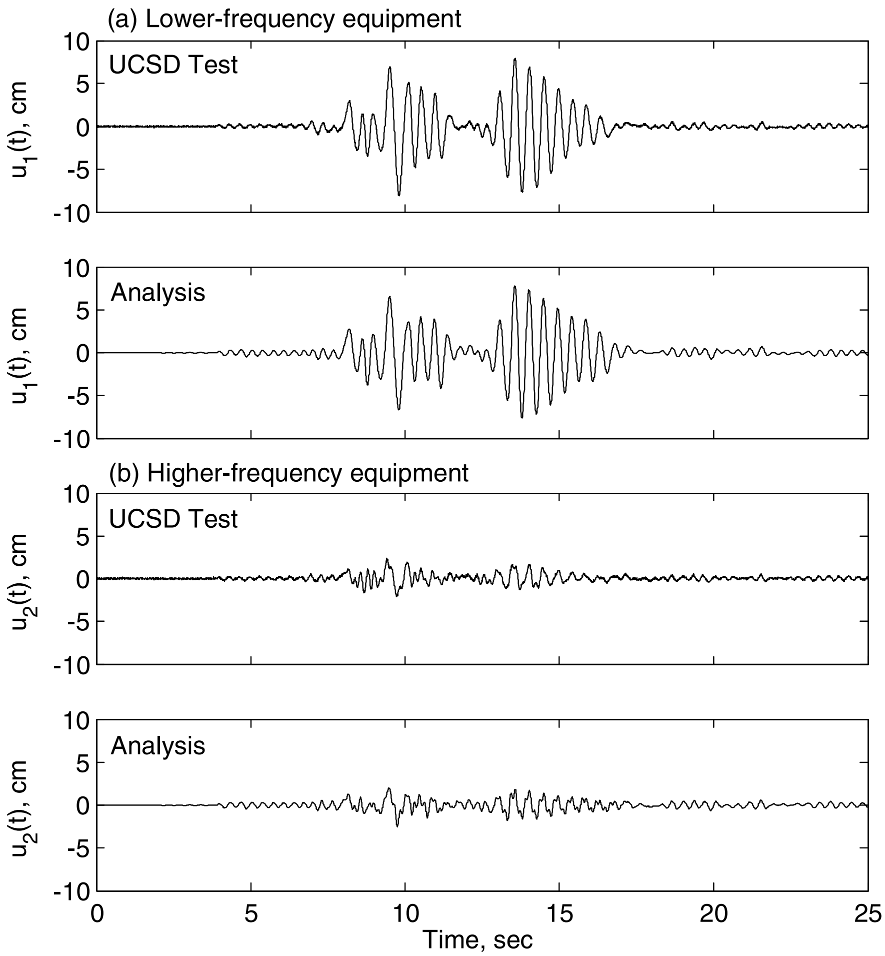

Figure 5. Displacement time histories of equipment items connected by slider connector

NONLINEAR RANDOM VIBRATION ANALYSIS BY ELM

Since ground motions are random, it is necessary to develop a stochastic approach so as to evaluate average responses for a class of ground motions, rather than for arbitrarily selected time histories. The ELM is applicable to general nonlinear, multi-degree-of-freedom structures subjected to stationary or nonstationary excitations. The required computational effort in ELM is significantly less than that required in simulation methods.

In the case of a zero mean, stationary Gaussian acceleration process $\ddot{x}_g(t)$, using the method described by Atalik [10], the auxiliary differential equation of the hysteretic model is linearized in the form

$$\dot{z} + C_1 \Delta \dot{u} + C_2 \Delta u + C_3 z = 0 \tag{8}$$

where the equivalent linear coefficients C_1, C_2 and C_3 are determined by minimizing the mean-square error, subject to the response having the Gaussian distribution. The result is

$$C_1 = \mathrm{E}\left[\frac{\partial g}{\partial \Delta \dot{u}}\right],\ C_2 = \mathrm{E}\left[\frac{\partial g}{\partial \Delta u}\right],\ C_3 = \mathrm{E}\left[\frac{\partial g}{\partial z}\right] \tag{9}$$

where g defines the hysteresis law. The solutions for C_1, C_2 and C_3 are obtained as algebraic functions of the second moments of Δu, $\Delta \dot{u}$ and z with the aid of the following well known property for a zero-mean Gaussian vector $\mathbf{x}$:

$$\mathrm{E}[\mathbf{x}h(\mathbf{x})] = \mathrm{E}[\mathbf{x}\mathbf{x}^{\mathrm{T}}]\mathrm{E}[\nabla h(\mathbf{x})] \tag{10}$$

where $h(\cdot)$ is a general nonlinear function. The expressions for C_1, C_2 and C_3 are given by Wen [11] for the original Bouc-Wen model, by Song [6] for the generalized Bouc-Wen model, and by Kaul [7] for the bi-linear model.

By use of the SDOF models for the equipment items and the linearized equation of the connector, an equivalent linear system of equations for the connected equipment system is developed. The system of equations is reduced to a first-order system

$$\dot{\mathbf{y}} = \mathbf{G}\mathbf{y} + \mathbf{f} \tag{11}$$

where $\mathbf{y}$ is the state-space vector including the equipment displacements and velocities, and the auxiliary variable z, $\mathbf{G}$ is the equivalent linear coefficient matrix, and $\mathbf{f}$ is the vector including the ground acceleration scaled by l_i/m_i. For example, for the two interconnected equipment items in (2), $\mathbf{y}$, $\mathbf{G}$ and $\mathbf{f}$ are

$$\mathbf{y} = \{u_1 \quad \dot{u}_1 \quad u_2 \quad \dot{u}_2 \quad z\}^{\mathrm{T}} \tag{12a}$$

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\left(\dfrac{k_1 + \alpha k_o}{m_1}\right) & -\left(\dfrac{c_1 + c_o}{m_1}\right) & \dfrac{\alpha k_o}{m_1} & \dfrac{c_o}{m_1} & \dfrac{(1-\alpha)k_o}{m_1} \\ 0 & 0 & 0 & 1 & 0 \\ \dfrac{\alpha k_o}{m_2} & \dfrac{c_o}{m_2} & -\left(\dfrac{k_2 + \alpha k_o}{m_2}\right) & -\left(\dfrac{c_2 + c_o}{m_2}\right) & -\dfrac{(1-\alpha)k_o}{m_2} \\ C_2 & C_1 & -C_2 & -C_1 & -C_3 \end{bmatrix} \tag{12b}$$

$$\mathbf{f} = \left\{0 \quad -\frac{l_1}{m_1}\ddot{x}_g \quad 0 \quad -\frac{l_2}{m_2}\ddot{x}_g \quad 0\right\}^{\mathrm{T}} \tag{12c}$$

When the excitation $\mathbf{f}$ is a delta-correlated vector process, the covariance matrix $\mathbf{S}$ of the zero-mean vector $\mathbf{y}$, i.e., $\mathbf{S} = \mathrm{E}[\mathbf{y}\mathbf{y}^T]$, satisfies the equations (Lin [12])

$$\frac{d}{dt}\mathbf{S} = \mathbf{G}\mathbf{S} + \mathbf{S}\mathbf{G}^{\mathrm{T}} + \mathbf{B} \tag{13}$$

in the case of nonstationary processes, where $B_{ij} = 0$ except $B_{ii} = 2\pi\Phi_o^i(t)$, where $\Phi_o^i(t)$ is the evolutionary power spectral density of the time-modulated non-stationary delta-correlated process $f_i(t)$. The corresponding equations in the case of stationary processes are

$$\mathbf{G}\mathbf{S}+\mathbf{S}\mathbf{G}^{\mathrm{T}}+\mathbf{B}=\mathbf{0} \tag{14}$$

where B_{ij} = 0 except $B_{ii}=2\pi\Phi_o^i$. Suppose the ground acceleration is assumed to be a Gaussian, filtered white-noise having the Kanai-Tajimi power spectral density (Clough [13])

$$\Phi_{\ddot{x}_g\ddot{x}_g}(\omega)=\frac{\omega_g^4+4\zeta_g^2\omega_g^2\omega^2}{(\omega_g^2-\omega^2)^2+4\zeta_g^2\omega_g^2\omega^2}\Phi_o \tag{15}$$

where ω_g, ζ_g and Φ_0 are the parameters defining the dominant frequency, bandwidth and intensity of the process, respectively. In this case, the ground displacement relative to the bedrock, $x_g^r(t)$, is the solution of the differential equation

$$\ddot{x}_g^r+2\zeta_g\omega_g\dot{x}_g^r+\omega_g^2x_g^r=w(t) \tag{16}$$

where $w(t)$ is a modulated white noise with evolutionary power spectral density $\Phi_0(t)$. The equivalent linear system (11) can be used for this case by adding two new variables x_g^r and $\dot{x}_g^r$ to the state space vector $\mathbf{y}$ and augmenting (16) into the $\mathbf{G}$ matrix. All the components of $\mathbf{f}$ are zero, except the component corresponding to $\dot{x}_g$ in vector $\mathbf{y}$, which is $w(t)$. The corresponding $\mathbf{B}$ matrix has only one non-zero term, B_{ii} $=2\pi\Phi_o^i(t)$, where i is the component index indicating $\dot{x}_g^r$ in $\mathbf{y}$ (Wen [11]).

The covariance matrix equation, (13) or (14), is solved by transforming the matrices $\mathbf{G}$ and $\mathbf{G}^{\mathrm{T}}$ into a complex Schur form and computing the solution of the resulting system (Bartels [14]). It should be noted that the solution of these equations requires an iterative scheme, since matrix $\mathbf{G}$ involves the second moments of the response in terms of the coefficients in (9).

The solution of matrix $\mathbf{S}$ provides the second moments of the responses, i.e., the standard deviations and covariances of the equipment displacements and velocities. The standard deviations of the equipment displacements are employed for estimating the effect of interaction by use of the response ratios defined by Der Kiureghian [1]

$$R_i=\frac{\mathrm{rms}[u_i(t)]}{\mathrm{rms}[u_{i0}(t)]},\, i=1,2 \tag{17}$$

where $u_i(t)$ and $u_{i0}(t)$ respectively denote the displacements of equipment i in its connected and stand-alone configurations at time t, and "rms" denotes the root-mean-square, which for the zero-mean processes is identical to the standard deviation. It should be obvious that a response ratio with a value greater (resp. smaller) than unity indicates that the interaction effect amplifies (resp. de-amplifies) the response of the corresponding equipment item in the connected system relative to its response in its stand-alone configuration.

As an example, consider two equipment items connected by an *S*-FSC. The parameters have the values m_1 = 250 kg, m_2 = 125 kg, k_1 = 9.88 kN/m, k_2 = 123 kN/m, k_0 = 8.58 kN/m, ζ_i = 0.02 for $i=1,2$, c_0 = 0, and l_1/m_1 = l_2/m_2 = 1.0. The analytical model for the *S*-FSC is fitted to the second *S*-FSC specimen tested by Stearns [5]. The parameters used for the Bouc-Wen model are $\alpha=0.206$, A = 1, $n=1$, $\beta=0.175$ and γ = 0.176. The ground acceleration is considered as a stationary, filtered white-noise process defined by the Kanai-Tajimi power spectral density of (15) with ω_g = 5π rad/sec and ζ_g = 0.6. The amplitude of the process Φ_0 is varied to examine the variation in the nonlinearity of the system with increasing intensity of the ground motion. The rms response ratios are evaluated by 1) the ELM, 2) linear random vibration analysis and 3) nonlinear time history analyses by use of the proposed FSC models subjected to five simulated ground motions based on the specified power spectral density. Figure 6 plots the response ratio of the lower- and higher-frequency equipment items versus the rms value of the ground acceleration. The ELM results show close agreement with the time history simulations. The results based on the ELM clearly show how the softening and energy dissipation of the *S*-FSC reduces the adverse interaction effect on the equipment items and how this reduction is a function of the intensity of the ground motion.

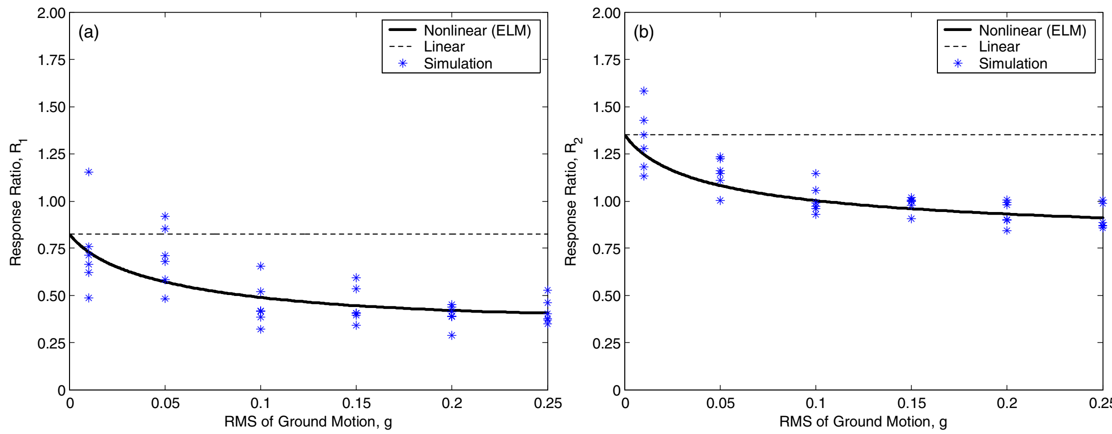

Figure 6. Response ratios for equipment items connected by *S*-FSC: (a) lower-frequency equipment and (b) higher-frequency equipment

MARGINAL FRAGILITY OF EIQUPMENT ITEMS

One of the quantities of central interest in assessing the reliability of a stochastic system is the probability that a random response $X(t)$ will remain within prescribed limits during a given period of time. The first-passage probability $L_{|X|}(\tau;a)$ is defined as the probability that $|X(t)|$ will not exceed the limit a during a time interval $t \in (0,\tau)$. A well-known approximation on the first-passage probability is given for a random process $X(t)$ having a symmetric distribution (Lutes [15])

$$L_{|X|}(\tau;a) \approx P(|X(0)| < a)\exp\left[-\int_0^\tau 2\nu^+(t,a)dt\right] \tag{18}$$

where $\nu^+(t,a)$ is the mean up-crossing rate of the process $X(t)$ over the limit $X(t)=a$. This result is based on a Poisson assumption, where the crossing events are assumed to be statistically independent of each other. When $X(t)$ is a zero-mean Gaussian process, the mean up-crossing rate is given by the well known formula

$$\nu^+(t,a) = (2\pi)^{-1}\sqrt{\lambda_2/\lambda_0}\exp(-a^2/2\lambda_0) \tag{19}$$

where

$$\lambda_i = \int_0^\infty \omega^i G_{XX}(\omega)d\omega \qquad i = 0,1,... \tag{20}$$

are the spectral moments and $G_{xx}(\omega)$ is the one-sided power spectral density of $X(t)$. Note that the zeroth and second spectral moments are the squares of the standard deviations of $X(t)$ and $\dot{X}(t)$, respectively. Therefore, the nonlinear random vibration analysis by use of the ELM allows us to compute the probability that an equipment item in the connected system will exceed a certain limit during a prescribed duration of excitation.

VanMarcke [16] improved the first-passage probability by accounting for the bandwidth of the process and the time spent in the unsafe region. The improved approximation is given as

$$L_{X1}(\tau;a) \approx P(|A(0)| < a)\exp\left[-\int_0^{\tau} \eta(t,a)dt\right] \tag{21}$$

where $A(t)$ denotes the envelope process of $X(t)$ and $\eta(t,a)$ is the conditional crossing rate given no crossings prior to time t. For a Gaussian process,

$$P(|A(0)| < a) = 1 - \exp(-r(0)^2/2) \tag{22a}$$

$$\eta(t,a) \approx \frac{\pi^{-1}\sqrt{\lambda_2(t)/\lambda_0(t)}}{1-\exp(-r(t)^2/2)}\left\{1-\exp[-\sqrt{\pi/2}\,\delta(t)r(t)]\right\} \tag{22b}$$

where $r(t) = a/\sigma_X(t)$ is the normalized barrier level and $\delta(t) = [1-\lambda_1(t)^2/\lambda_0(t)\lambda_2(t)]^{1/2}$ is a shape factor that characterizes the bandwidth of the process. Note that in order to compute the first-passage probability with the improved formula, one additionally needs to compute the first spectral moment, λ_1. This can be computed by numerical integration, using the frequency response function of the response of interest,

$$\lambda_1 = \int_0^{\infty} \omega |h(\omega)|^2 G_{\ddot{X}_g\ddot{X}_g}(\omega)d\omega \tag{23}$$

The frequency response function, $h(\omega)$, for a response quantity is derived from the equivalent linear equation of motion (11) with the $\mathbf{G}$ matrix computed by the converged solution of (13) or (14).

As an example, consider two equipment items having the parameter values m_1 = 400 kg, m_2 = 200 kg, k_1 = 15.8 kN/m, k_2 = 198 kN/m, l_i/m_i = 1.0 and ζ_i = 0.02 and connected by each of the connector types mentioned above. Assume the ground acceleration is a stationary process having the power spectral density in (15) with $\omega_g = 5\pi$ rad/sec and ζ_g = 0.6. The amplitude parameter Φ_0 is varied to examine the variation in the nonlinearity of the system with increasing intensity of the ground motion. Figure 7 shows the probability that the displacement of the higher-frequency equipment item will exceed the prescribed limit of 5.08 cm during 20 seconds of excitation. It is seen that the *S*-FSC and the slider connector significantly enhance the reliability of the higher-frequency equipment item by reducing the interaction effect. In this example, the exceedance probabilities are overestimated by Poisson assumption due to the narrow bandwidth of the response process (VanMarcke [16]).

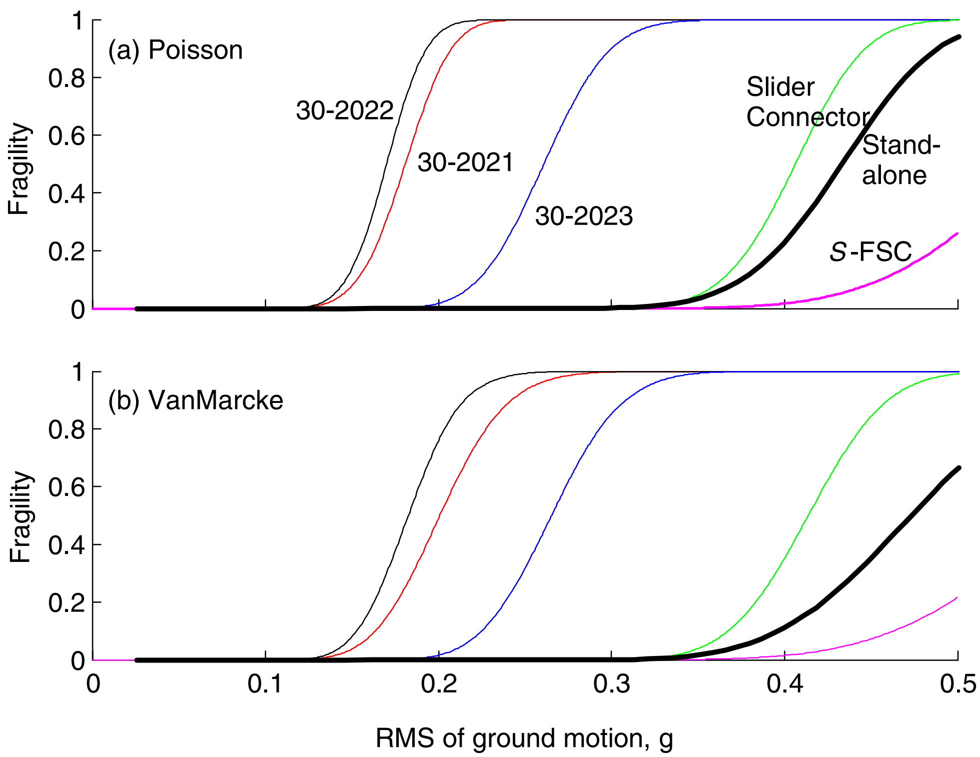

Figure 7. Fragilities of higher-frequency equipment item for different connectors

JOINT FRAGILITY OF EQUIPMENT ITEMS

An electrical substation is a complex system consisting of many interconnected equipment items. In most cases, the failure events of the constituent equipment items are statistically dependent, particularly in presence of dynamic interaction between them. Therefore, the reliability of the substation system cannot be deduced directly from marginal component fragilities. However, bounds on the system reliability can be obtained for any given set of marginal and joint component fragilities. Analytical bounding formulas (Ditlevsen [17], Zhang [18]) or linear programming (LP) (Song [19]) can be used for this purpose. Either analysis method requires the probabilities of joint component failure events to achieve narrow bounds.

The joint failure probability of a set of components under stochastic loading is defined as the probability that every component response exceeds its prescribed limit at least once during a given period of excitation. For example, the bi-component failure probability of the response processes $X_i(t)$ and $X_j(t)$ is defined as

$$P_{ij} \equiv P[(R_\tau^i > a_i) \cap (R_\tau^j > a_j)] \tag{24a}$$

$$R_\tau^i = \max_{0 \le t \le \tau} |X_i(t)| \tag{24b}$$

By applying the addition rule of probability, the joint failure probability in (24) is written as the sum of three probabilities:

$$P_{ij} = P(R_\tau^i > a_i) + P(R_\tau^j > a_j) - P[(R_\tau^i > a_i) \cup (R_\tau^j > a_j)] = P_i + P_j - \widetilde{P}_{ij} \tag{25}$$

The probabilities P_i and P_j are computed using the approximate first-passage time probability functions in (18) or (21). $\widetilde{P}_{ij}$ is the probability that the vector process $\mathbf{X} = \{X_i(t), X_j(t)\}^{\mathrm{T}}$ out-crosses the rectangular domain $|x_i| < a_i$ and $|x_j| < a_j$ at least once during the interval $t \in (0, \tau)$. Approximate solutions of $\widetilde{P}_{ij}$ in the form of an exponential function have been derived in Song [2]. The approximation based on a Poisson out-crossings assumption is given as

$$\widetilde{P}_{ij} = P[(R_\tau^i > a_i) \cup (R_\tau^j > a_j)] \approx 1 - A_{ij} \exp[-\tilde{\eta}_{ij}(a_i, a_j)\tau] \tag{26a}$$

$$A_{ij} = P[(|X_i(0)| < a_i) \cap (|X_j(0)| < a_j)] \tag{26b}$$

$$\tilde{\eta}_{ij}(a_i, a_j) \approx \tilde{\nu}_i + \tilde{\nu}_j \tag{26c}$$

where $\tilde{\nu}_i$ and $\tilde{\nu}_j$ are the mean out-crossing rates over the finite edges of the rectangular barrier. Specifically, $\tilde{\nu}_i$ is the mean out-crossing rate above the threshold $|x_i| = a_i$ with $|x_j| < a_j$, and $\tilde{\nu}_j$ is the mean out-crossing rate above the threshold $|x_j| = a_j$ with $|x_i| < a_i$. An improved approximation is obtained by using VanMarcke's approximations (22) for $\widetilde{P}_i$ and $\widetilde{P}_j$ and a similar approximation developed in Song [2] for $\widetilde{P}_{ij}$. The latter approximation employs a bi-variate Rayleigh distribution to compute A_{ij} and replaces each $\tilde{\nu}_i$ by $\tilde{\nu}_i[\eta_i(a_i)/\nu_i(a_i)]$, where the bracketed quotient is intended to account for the types of corrections that are inherent in VanMarcke's approximation. In Song [2], detailed derivations are presented for the cases of 2- and 3-dimensional Gaussian processes, and comparisons with Monte Carlos simulations are used to verify the accuracy of these approximations.

RELIABILITY OF ELECTRICAL SUBSTATION SYSTEMS

Song [19] has shown that linear programming (LP) can be used to obtain the narrowest possible bounds on the failure probability of a system for any given partial information on marginal and joint component failure probabilities. The lower (upper) bound on the system failure probability is obtained by solving the LP problem

$$\begin{array}{ll} \text{minimize (maximize)} & \mathbf{c}^{\mathrm{T}}\mathbf{p} \\ \text{subject to} & \mathbf{a}_1\mathbf{p} = \mathbf{b}_1 \\ & \mathbf{a}_2\mathbf{p} \geq \mathbf{b}_2 \end{array} \qquad \begin{array}{r} (27a) \\ (27b) \\ (27c) \end{array}$$

where $\mathbf{p} = \{p_1, p_2, \cdots, p_{2^n}\}$ is a vector containing the probabilities of all mutually exclusive and collectively exhaustive events in the sample space of the n components, $\mathbf{c}$ is a vector of binary constants making $\mathbf{c}^{\mathrm{T}}\mathbf{p}$ the probability of the system failure event, and (27b) and (27c) are equality and inequality constraints expressing known values and bounds, respectively, on marginal or joint component probabilities. This methodology is used in conjunction with the marginal and joint equipment fragility estimates described in the preceding sections to assess the reliability of an electrical substation system.

NUMERICAL EXAMPLE

Consider a substation system consisting of five equipment items, as shown in Figure 8. Equipment items 1 and 2 and equipment items 3 and 4 are connected to each other by three identical assemblies of a rigid bus and an S-FSC. Other connections are assumed to be sufficiently flexible not to cause dynamic interaction. The ground acceleration is defined as a stationary process having the power spectral density in (15) with $\omega_g = 5\pi$ rad/sec and $\zeta_g = 0.6$; the amplitude, Φ_0, is varied to compute the fragility of the system as a function of the root-mean-square of the ground acceleration. The duration of the stationary response is assumed to be 20 sec. The equipment items have the parameter values $m_1 = 438$ kg, $m_2 = 210$ kg, $m_3 = 403$

kg, m_4 = 193 kg, m_5 = 200 kg, l_i/m_i = 1.0 and ζ_i = 0.02 for $i = 1,...,5$, $k_1 = k_3$ = 158 kN/m, and $k_2 = k_4 = k_5$ = 198 kN/m. The *S*-FSC is described by a Bouc-Wen model having the parameters k_0 = 3×8.58 = 25.7 kN/m, $\alpha = 0.206$, $A = 1$, $n = 1$, $\beta = 0.175$ and $\gamma = 0.176$.

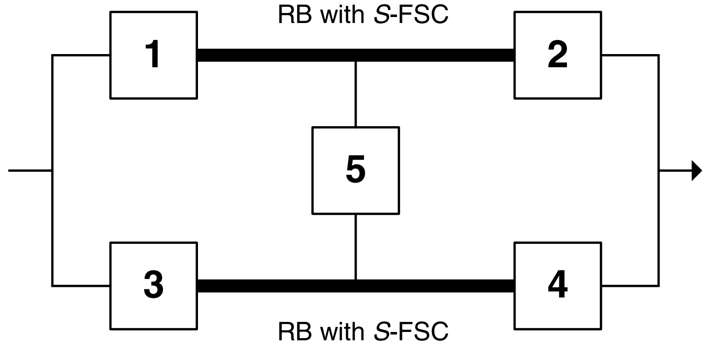

Figure 8. Substation system with five equipment items

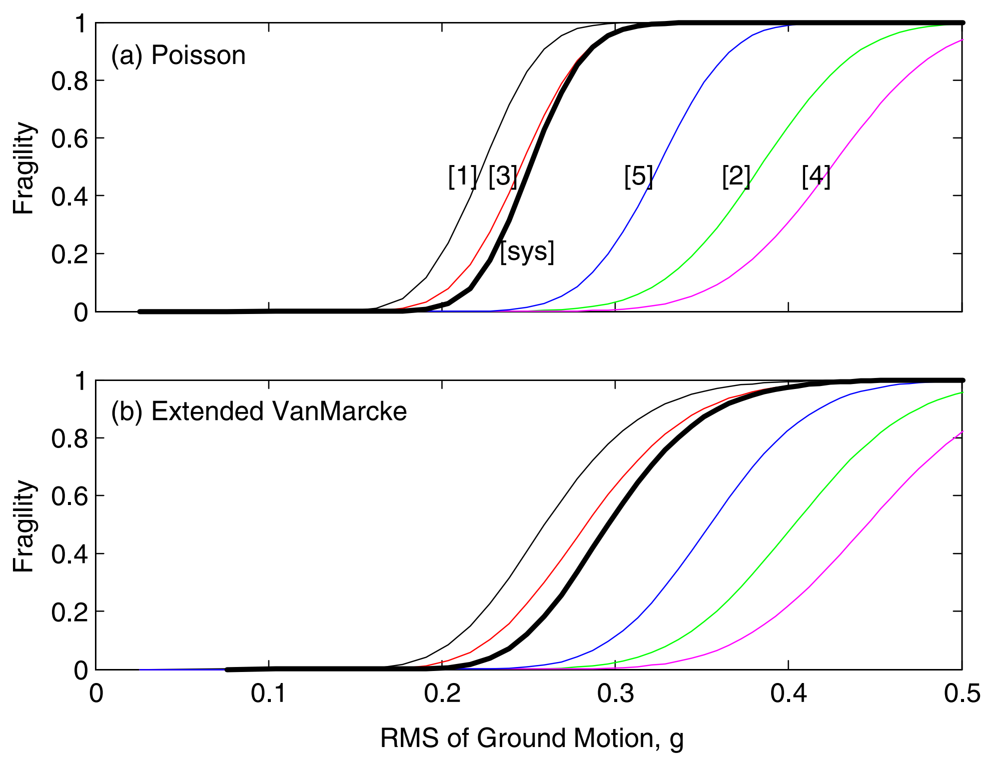

Figure 9. Equipment and system fragility estimates by (a) Poisson assumption and (b) based on extended VanMarcke formula

For each intensity level, the spectral moment λ_0, λ_1 and λ_2 are computed by nonlinear random vibration analysis using the ELM. The joint and marginal equipment failure probabilities, $P_1, \cdots, P_5$, P_{12}, $P_{13}, \cdots, P_{45}$ are computed by the Poisson-based or improved formulas. The prescribed failure limits of displacement responses are ± 7.62 cm for equipment 1 and 3, and ± 3.81 cm for equipment 2, 4 and 5. By use of the LP methodology, probability bounds on the system failure event

$$E_1E_3 \cup E_2E_4 \cup E_1E_4E_5 \cup E_2E_3E_5 \tag{28}$$

are estimated employing only marginal and bi-component fragilities. Figure 10 shows the fragility of each equipment item and the lower and upper bounds on the system fragility. For this example, the system probability bounds are practically coinciding.

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