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Vertex colouring using the adjacency matrix

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Abstract. Recently, graph theory is one of the most rapidly developing sciences. Graphs in its applications are generally used to represent discrete objects and relationships between these objects. The visual representation of a graph is to declare an object as a vertex, while the relationship between objects is expressed as an edge. One topic in graph theory is colouring. This graph colouring is divided into vertex colouring, edge colouring and area colouring. The problem of the vertex colouring is to determine the minimum number of colours to colour the vertex so that the interconnected vertex has different colours. The problem of edge colouring is to determine the minimum number of colours to colour the edge so that the interconnected edge has different colours. The problem with area colouring is to determine the minimum number of colours to colour the area so that the adjacent area has a different colour. In this article the discussion will focus on the problem of vertex colouring. Previously there have been several vertex colouring methods, such as the Welch Powell method and the backtracking method. In this paper we will discuss the method of vertex colouring using the adjacent matrix. Adjacent matrix (M) is a square matrix where the element M_{ij} is 1 if V_iV_j is connected and element M_{ij} is 0 if V_iV_j is not connected. In the discussion, this method is presented in the form of the pseducode and flowchart so that it can be computerized more easily. The novelty of this research is to detect the character of the adjacency matrix so that it can apply to vertex colouring through the matrix.

1. Introduction

Graph theory is rapidly moving into the mainstream of mathematics mainly because of its applications in diverse fields which include electrical engineering (communications networks and coding theory), computer science (algorithms and computations) and operations research (scheduling).Graph colouring is one of the most important concepts in graph theory and is used in many real time applications like Job scheduling [8] , Aircraft scheduling [8], computer network security[9], Map colouring and GSM mobile phone networks[1] Automatic channel allocation for small wireless local area networks[3].The proper colouring of a graph is the colouring of the vertices with minimal number of colours such that no two adjacent vertices should have the same colour. The minimum number of colours is called as the chromatic number and the graph is called properly coloured graph.

We know that graphs are simply model of relation. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. In many real world problem, we get partial information about that problem. So

there is vagueness in the description of the objects or in its relationships or in both. To describe this type of relation, we need to design fuzzy graph model. Fuzzy graph colouring is one of the most important problems of fuzzy graph theory. It uses in combinatorial optimization like traffic light control [8], exam-scheduling [3], register allocation etc. Two types of colouring namely vertex colouring and edge colouring are usually associated with any graph.

1.1 Adjacency matrix

To simplify calculations on computer programs, graphs can be represented using matrices[2]. One of them is the adjacency matrix.[1]Let $G = (V, E)$ is a simple graph where$|V| = n$,and$n > 1$.[8] Suppose the vertex of G is$v_1, v_2, \ldots, v_n$. So, the adjacency matrix of graph G is a matrix $n \times n$ where:

$$A = [a_{ij}], [a_{ij}] = \begin{cases} 1, & if\ i\ is\ adjacent\ to\ j \\ 0, & if\ i\ is\ not\ adjacent\ to\ j \end{cases}$$

[6]Adjacency matrix is a simple symmetrical graph, so$a_{ij} = a_{ji}$. This caused both have a value 1 when v_i and v_j are connected, and have a value 0 if both of them are not connected.Sometimes this is also called the zero-one matrix.[3]For directed graphs, the adjacency matrix is not necessarily symmetry (it will be symmetry if it is a complete directed graph). [7] In addition, a simple graph doesn't have a loop, so the element diagonal of matrix is always has value 0.[4]The adjacency matrix cannot be used to represent a graph that has double sides. To overcome this, the a_{ij} element on the adjacency matrix is equal to the number associated with(v_i, v_j). [5] For pseudo graphs, the loop in the v_i node is expressed as one in position (i, j) in the adjacency matrix. Figure 1 shows some examples of adjacency matrix.
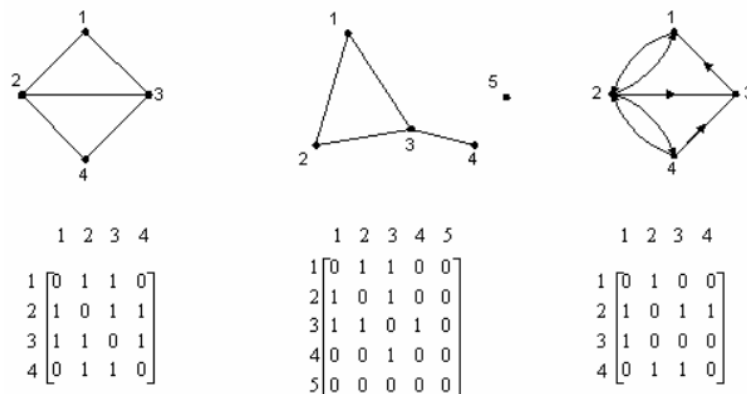

Figure 1. Example of graphs with it is adjacency matrix.

1.2Proposed algorithm

Now we will introduce a new colouring method, namely vertex colouring using an adjacency matrix. The following is a colouring method that is proposed to colour the vertex in a graph using its adjacency matrix:

1. Make an adjacency matrix of the graph which its vertex will be coloured
2. Sum the matrix elements in each row
3. Select the row matrix that has the biggest value
4. Strikethrough the selected matrix row and give the colour at its vertex
5. Strikethrough the row of matrix that corresponds to the column of the selected row that has a value 0
6. Give the colour at its vertex with the same colour of selected row
7. Select another row of matrix which has not been strikethrough and have the biggest row value (if the biggest row value more than one, please choose one)
   Repeat step 4, give the another colour at its vertex and so on until all the matrix row are strikethrough or all vertex have been coloured

To understand this colouring method, an example is given to colour the vertex of graph using the adjacency matrix.
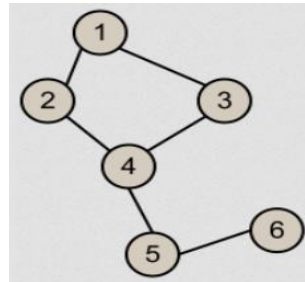
Figure 2. Graphs sample.

First step, make adjacency matrices for graph base on Figure 2. The results of the adjacency matrix can be seen in Figure 3.

Figure 3. Matrix adjacency.

The second step is sum the values of each matrix row. The results can be seen in Figure 4.

Figure 4. Sum of the matrix row.

The third step is choosing the biggest row value. In this graph the biggest value is 3, that is at vertex 4. The fourth step strikethrough the selected matrix row and put colour to the selected vertex (e.g. a).The results of this step can be seen in Figure 5.
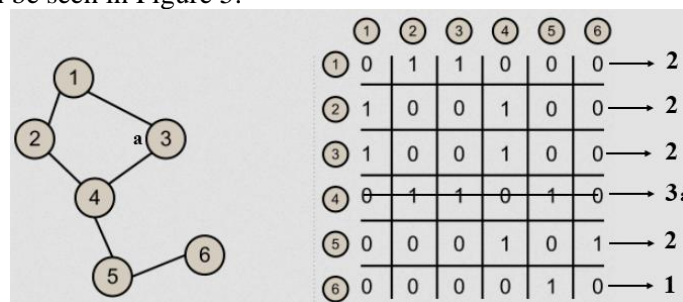

Figure 5. Selected matrix row colouring.

The fifth step 5, Strikethrough the row of matrix that corresponds to the column of the selected matrix row and has a value 0. In this case, the selected rows are vertex 1, vertex 4 and vertex 6. The sixth steps, put the colour at its vertex by the same colour like selected row. The colouring results can be seen in Figure 6.
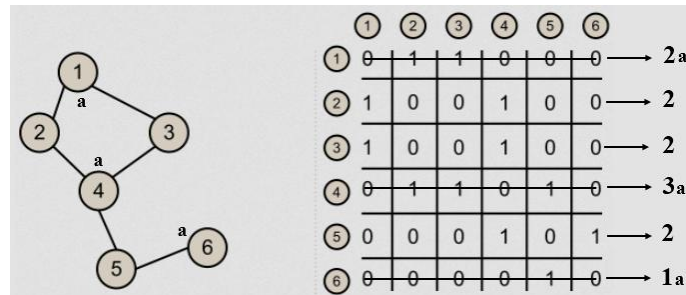

Figure 6. The colour of vertex that have same to the selected row.

The seven steps, select a matrix row which has not been strikethrough and has the biggest row value (if more than one row have the biggest values, please choose one).In this case there are three vertices that have the biggest row values, those are vertex 2, vertex 3, and vertex 5. Suppose vertex 2 is selected as the selected row. The next step strikethrough the selected row and put a different colour to the selected vertex (eg b).The results of this step can be seen in Figure 7.
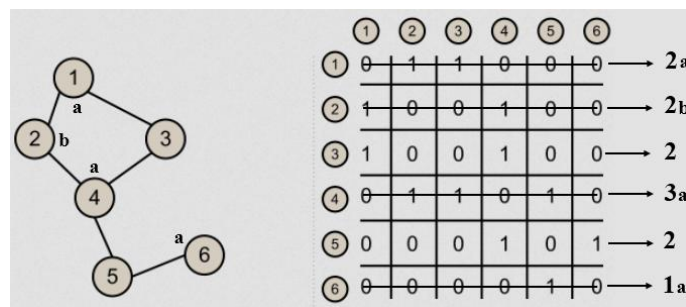
Figure 7. Another selected matrix row.

The eight steps, strikethrough the row of the matrix that matches with the column of the selected row which have 0 value, those are vertex 2, 3, 5 and vertex 6. Vertex 6 is not chosen because it has been strikethrough or has been coloured. The results can be seen in Figure 8.
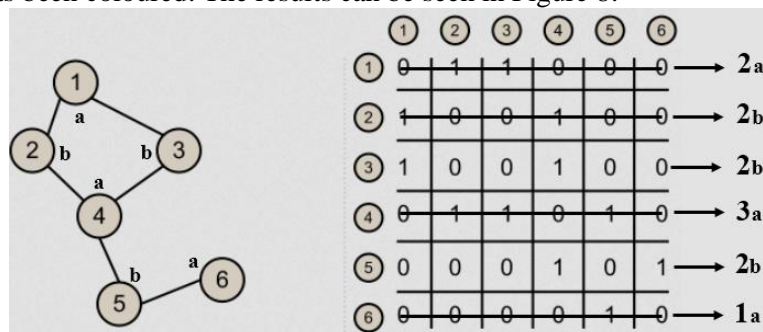
Figure 8. Vertex colouring according to the selected row.

Repeat step 7, give the other colour at vertex and so on until all the row of matrix are strikethrough and coloured. The result of vertex colouring using the adjacency matrix above is the minimum colour that used to colour the vertex (chromatic number) is 2.

2. Result

Furthermore, to simplify the computerization process is given a pseudo code of vertex colouring algorithm using an adjacency matrix input total vertex (n).

1. for $i = 1$ to n
   a. for $j = 1$ to n
      i. input $V(i, j)$
      ii. if $V(i, j) = 1$
         1. $row(i) = row(i) + 1$
2. $max = 0$
3. for $k = 1$ to n
   a. if $(max < row(k))$ and $(c(k) = 0)$
      i. $max = row(k)$
      ii. $best = k$
4. $color = color + 1$
5. for $j = 1$ to n
   a. if $(V(best, j) = 0)$ and $(c(j) = 0)$
      i. $c(j) = color$
6. for $i = 1$ to n
   a. if $c(i) = 0$
      i. go to (3)
7. output ($'chromatic\ number\ = ', color$)
8. for $k = 1$ to n
   a. output ($'color\ of\ vertex(', k, ') = ', c(k)$)

From the pseudo code above, it can be explained that:
Line (1) is a command to input the total vertex of the graph to be coloured.
Line (2) is a command to input adjacency matrix elements.
Line (2.a.ii) is a command to sum the value of each matrix row.
Line (4) is the command to find the biggest row value and the vertex has not been coloured.
Line (5) is a command to define vertex colour.
Line (6) is a command to colour vertices based on selected matrix row.
Line (7) is the command to test whether all vertices have been coloured. If not, then go back to line (3) and if all vertices have been coloured then go to the next command.
Line (8) is a command to print chromatic number.
Line (9) is a command to print colours from each vertex.

3. Conclusion

Based on the results of the study it can be concluded that the vertex colouring method uses an adjacency matrix can be applied to any graph without looking at the type or class of the graph.By using this method, vertex colouring of graphs can be done using a computer program easily so that the time needed will be faster. Based on this research, in future the algorithms can be developed to find the shortest path, minimum dominating set and rainbow graph.

References
[1]   Bondy J A and Murty U S R 2008 Graph theory *Graduate Texts in Mathematics* Springer, New York pp 244
[2]   Bhutani K R 1989 On automorphisms of fuzzy graphs *Pattern Recognition Lett. 9* pp 159- 162
[3]   Mordeson J N 1993 Fuzzy line graphs *Pattern Recognition Lett. 4* pp 381-384

[4]   Lavanya S and Sattanathan R 2009 Fuzzy total colouring of fuzzy graphs *International Journal of Information Technology and Knowledge Management* **2** pp 37-39

[5]   Lin C 2007 Simple proofs of results on paths representing all colours in proper vertex colourings *Graph and Combinatorics* **23** pp 201-203

[6]   Shariefuddin P and Ashay D 2007 A colourful path *Journal of the Korean Society for Industrial and applied Mathematics* **11** 4

[7]   Li H 2001 A generalization of the Gallai-Roy theorem *Graphs and Combinatorics* 17, PP: 681-685

[8]   Perri M, Janne R and Marina P2004 Automatic channel allocation for small wireless area networks using graph coloring algorithm approach *IEEE* pp 187-201