Unit S Student Success Sheet (SSS)

Trigonometric Identities Part 3 (section 5.5)

Standards: Trig 11.0

Segerstrom High School -- Math Analysis Honors

Name: ____

Period: ___________

Thinkbinder Study Group: www.bit.ly/ChatUnitS

Reminders:

- Practice Problems (PQ & PT) are completed in **spiral bound notebook only**.
- **All pages** in spiral notebook should be labeled accordingly:

Unit ______ Concept ______ - (title of assignment)

Examples:

Unit **S** Concept **1** – **Practice Quiz**

Unit **S** Concept **1-4** – **Practice Test**

Need Help? Support is available!

- Website with all video links and resources:
  kirchmathanalysis.blogspot.com
- Edmodo Group Codes for class communication:
  http://bit.ly/edmodo2013

"If you want to achieve excellence, you can get there today. As of this second, quit doing less-than-excellent work."
Thomas J Watson

Concept #	What we will be learning…	Mandatory Practice	Optional Extra practice from textbook
1	Writing products as sums	Practice quiz 1	
2	Writing sums as products	Practice quiz 2	
3	Using power-reducing formulas	Practice quiz 3	
4	using half-angle formulas	Practice quiz 4	
5	Finding function values with double angles and half angles (right triangles)	Practice quiz 5	
6	Solving multiple angle equations (using multiple-angle identities)	Practice quiz 6	
7	solving equations with half-angle formulas	Practice quiz 8	

IN THIS UNIT…

We will conclude our study of trigonometric identities by looking at five more types of formulas: double-angle, half-angle, product to sum, sum to product, and power-reducing.

Like last unit, these formulas expand our usage of the unit circle from the main angles of 0, 30, 45, 60, and 90 to many more.

Double-Angle Formulas (See the proofs on page 405.)

$$\sin 2u = 2 \sin u \cos u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

Half-Angle Formulas

$$\sin \frac{u}{2} = \pm\sqrt{\frac{1 - \cos u}{2}} \qquad \cos \frac{u}{2} = \pm\sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

The signs of $\sin \frac{u}{2}$ and $\cos \frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

$$\sin^2 u = \frac{1 - \cos(2u)}{2}$$

$$\cos^2 u = \frac{1 + \cos(2u)}{2}$$

$$\tan^2 u = \frac{1 - \cos(2u)}{1 + \cos(2u)}$$

Sum-to-Product Formulas (See the proof on page 406.)

$$\sin u + \sin v = 2 \sin\left(\frac{u + v}{2}\right) \cos\left(\frac{u - v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u + v}{2}\right) \sin\left(\frac{u - v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u + v}{2}\right) \cos\left(\frac{u - v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u + v}{2}\right) \sin\left(\frac{u - v}{2}\right)$$

#1 Writing products as sums

1. $6\sin\frac{\pi}{3}\cos\frac{\pi}{3}$; u= _________ v= __________

2. $5\sin 3\alpha \sin 4\alpha$; u= _________ v= __________

Additional Problems on "extra videos"
Extra video #10 covers the following problems. Use these as extra practice and the videos as a guide if you need help.

3. $4\sin\frac{\pi}{3}\cos\frac{5\pi}{6}$ 4. $10\cos 75^\circ \cos 15^\circ$

PQ problems:

5. $\sin 5\theta \cos 3\theta$ 6. $6\sin 45^\circ \cos 15^\circ$

PT problems:

7. $5\cos(-5\beta)\cos 3\beta$ 8. $\cos 2\theta \cos 4\theta$ 9. $\sin(x+y)\sin(x-y)$

8. 10. $\sin(x+y)\cos(x-y)$ 11. $\cos(\theta-\pi)\sin(\theta+\pi)$ 12. $\sin(\theta+\pi)\sin(\theta-\pi)$

#2 Writing sums as products

1. $\sin 5\theta - \sin\theta$; u= _________ v= __________

2. $\sin 195^\circ + \sin 105^\circ$; u= _________ v= __________

Additional Problems on "extra videos"
Extra video #11 covers the following problems. Use these as extra practice and the videos as a guide if you need help.

3. $\sin 3\theta + \sin\theta$ 4. $\cos 165^\circ - \cos 75^\circ$

PQ problems:

5. $\cos 6x + \cos 2x$ 6. $\cos\frac{5\pi}{12} + \cos\frac{\pi}{12}$ 7. $\sin\frac{11\pi}{12} - \sin\frac{7\pi}{12}$

PT problems:

8. $\sin x + \sin 7x$ 9. $\sin(\alpha+\beta) - \sin(\alpha-\beta)$ 10. $\cos(\phi+2\pi) + \cos\phi$

11. $\cos\left(\theta+\frac{\pi}{2}\right) - \cos\left(\theta-\frac{\pi}{2}\right)$ 12. $\sin\left(x+\frac{\pi}{2}\right) + \sin\left(x-\frac{\pi}{2}\right)$

#3 Using power-reducing formulas

1. $\boldsymbol{cos^4 x} =$ $cos^2 x cos^2 x$

using power reducing formulas = $\frac{1+\cos(2x)}{2}\frac{1+\cos(2x)}{2}$ =

FOILing the numerator= $\frac{1+2\cos(2x)+\boldsymbol{cos^2(2x)}}{4}$

Still not in the first power!

Let m=2x

$$\frac{1+2\cos(2x)+cos^2(m)}{4}$$

$cos^2 m = \frac{1+\cos(2m)}{2}$; therefore = $\frac{\mathbf{1+cos(4x)}}{\mathbf{2}}$

Our whole equation of:

$\frac{1+2\cos(2x)+\boldsymbol{cos^2(2x)}}{4}$ now becomes: $\frac{1+2\cos(2x)+\frac{\mathbf{1+cos(4x)}}{\mathbf{2}}}{4}$

Now, we must make it look nicer… make all of the numerator have a denominator of 2

$$\frac{\frac{2+4\cos(2x)+1+\cos(4x)}{2}}{4} =$$

Add like terms in the numerator and multiply top and bottom by ¼ to get rid of the 4 in the denominator

$$\frac{3+4\cos(2x)+\cos(4x)}{8}$$

2. Try it with $\boldsymbol{sin^4 x} =$

Additional Problems on "extra videos"
Extra video #8 covers the following problems. Use these as extra practice and the videos as a guide if you need help.

3. $\sin^4 x$ **4.** $\sin^2 x \cos^4 x$

PQ problems:

5. $\sin^2 x \cos^2 x$ **6.** $\sin^4 x \cos^2 x$

PT problems:

7. $\sin^2 2x$ **8.** $\cos^2 2x$ **9.** $\sin^2 2x \cos^2 2x$

#4 using half-angle formulas

1. 15^{o} ; u= ________; u/2 is in quadrant _______________, so sine is ___, cosine is ___

2. $\frac{\pi}{8}$; u= ________; u/2 is in quadrant _______________, so sine is ___, cosine is ___

3. 112.5^{o} ; u= ________; u/2 is in quadrant _______________, so sine is ___, cosine is ___

4. $\frac{3\pi}{8}$; u= ________; u/2 is in quadrant _______________, so sine is ___, cosine is ___

Additional Problems on "extra videos"
Extra video #9 covers the following problems. Use these as extra practice and the videos as a guide if you need help.

5. $\frac{\pi}{12}$

PQ problems:

6. $165°$ 7. $\frac{7\pi}{8}$

PT problems:

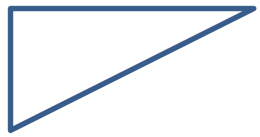
8. $157° \, 30'$ 9. $\frac{7\pi}{12}$

#5 Finding function values with double angles and half angles (right triangles)

0 to 90 -	90 to 180	180 to 270	270 to 360
Half angle - 0 to 45 still 1st quadrant!	Half angle - 45 to 90 1st quadrant!	Half angle - 90 to 135 2nd quadrant!	Half angle - 135 to 180 - 2nd quadrant!

1. $\sin\theta = -\dfrac{5}{13}$ and $\pi < \theta < \dfrac{3\pi}{2}$

Find $\sin\dfrac{\theta}{2}$



$$+\sqrt{\frac{1-\cos u}{2}} = +\sqrt{\frac{1--\frac{12}{13}}{2}}$$

$\dfrac{\theta}{2}$ is in Quadrant __II__; sin is ____positive___

$$= +\sqrt{\frac{25}{26}} = +\frac{5}{\sqrt{26}} = +\frac{5\sqrt{26}}{26}$$

You are identifying the quadrant for the HALF ANGLE $\frac{\theta}{2}$ from the information they give you about the REAL ANGLE θ

2. $\tan\theta = -\dfrac{7}{24}$ and $\dfrac{3\pi}{2} < \theta < 2\pi$

Find $\sin\dfrac{\theta}{2}$

Quadrant ______________; sin is ___________

3. $\cot\theta = \dfrac{\sqrt{21}}{2}$ and $0 < \theta < \dfrac{\pi}{2}$

Find $\cos\dfrac{\theta}{2}$

Quadrant ______________; cos is ___________

4. $\cos\theta = -\dfrac{\sqrt{105}}{19}$ and $\dfrac{\pi}{2} < \theta < \pi$

Find $\sin\dfrac{\theta}{2}$

Quadrant ______________; sin is ___________

5. $\sec\theta = -\dfrac{5}{4}$ and $\pi < \theta < \dfrac{3\pi}{2}$

Find $\tan\dfrac{\theta}{2}$

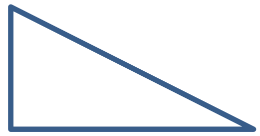
Quadrant ______________; tan is ___________

0 to 90 -	**90 to 180**	**180 to 270**	**270 to 360**
Double angle - 0 to 180 - in 1st or 2nd quadrant	Double angle -180 to 360 (3rd or 4th quadrant)	Double angle - 360 to 540 - 1st or 2nd quadrant	Double angle - 540 to 720 - 3rd or 4th quadrant

6.

$\csc\ \theta = \frac{5}{3}$ and $\frac{\pi}{2} < \theta < \pi$

Find $\cos 2\theta$



$cos^2u - sin^2u$

$(\frac{4}{5})^2 - (\frac{3}{5})^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

7.

$\tan\ \theta = \frac{5}{12}$ and $\pi < \theta < \frac{3\pi}{2}$

Find $\sin 2\theta$

Quadrant ______________;

8.

$\sin\ \theta = \frac{7}{25}$ and $\frac{\pi}{2} < \theta < \pi$

Find $\tan 2\theta$

Quadrant ______________;

9.

$\tan\ \theta = \frac{3}{4}$ and $0 < \theta < \frac{\pi}{2}$

Find $\cos 2\theta$

Quadrant ______________;

10.

$\csc\ \theta = -\frac{25}{7}$ and $\frac{3\pi}{2} < \theta < 2\pi$

Find $\tan 2\theta$

Quadrant ______________;

Additional Problems on "extra videos" **Extra video #12 covers the following problems. Use these as extra practice and the videos as a guide if you need help.**	
11. $\cos u = -\frac{2}{7}, \quad \pi/2 < u < \pi$	**12.** $\sec u = -\frac{5}{2}, \quad \pi/2 < u < \pi$
PQ problems: *Find the half angle and double angle values for all three trig functions (6 parts per problem). In addition, once you find the original three trig functions, find the reciprocal trig functions as well.*	
13. $\tan u = \frac{1}{2}, \quad \pi < u < 3\pi/2$	**14.** $\csc u = 3, \quad \pi/2 < u < \pi$
PT problems: *Find the half angle and double angle values for all three trig functions (6 parts per problem). In addition, once you find the original three trig functions, find the reciprocal trig functions as well.*	
15. $\sin u = \frac{3}{5}, \quad 0 < u < \pi/2$	**16.** $\cot u = -6, \quad 3\pi/2 < u < 2\pi$

#6 Solving multiple angle equations (using multiple-angle identities)

***Please note to graph, mode must be in radians!**

1. $\sin 2x - \sin x = 0$ algebraically

2. $\sin 2x \sin x = \cos x$ algebraically

3. $(\sin 2x + \cos 2x)^2 = 1$ graphically

4. $\sin 6x + \sin 2x = 0$ graphically

Additional Problems on "extra videos"
Extra video #13 covers the following problems. Use these as extra practice and the videos as a guide if you need help.

5. $\sin 2x + \cos x = 0$ **6.** $\cos 2x - \cos x = 0$ **7.** $\cos 2x + \sin x = 0$

PQ problems:

8. $4 \sin x \cos x = 1$ 9. $\tan 2x - \cot x = 0$ 10. $\tan 2x - 2 \cos x = 0$

PT problems:

11. $\sin 4x = -2 \sin 2x$ 12. $\dfrac{\cos 2x}{\sin 3x - \sin x} - 1 = 0$ 13. $\sin^2 3x - \sin^2 x = 0$

#7 solving equations with half-angle formulas

1. $\sin \dfrac{x}{2} - \cos x = 0$

2. $\dfrac{1-cosx}{2} = cos^2 x$

DON'T FORGET TO CHECK FOR EXTRANEOUS SOLUTIONS ANYTIME YOU "square both sides" IN THE PROCESS OF SOLVING!

Additional Problems on "extra videos"
Extra video #14 covers the following problems. Use these as extra practice and the videos as a guide if you need help.

3. $\sin \dfrac{x}{2} + \cos x - 1 = 0$

PQ problems:

4. $\cos \dfrac{x}{2} - \sin x = 0$

PT problems:

5. $\tan \dfrac{x}{2} - \sin x = 0$

Answer key

See kirchmathanalysis.blogspot.com for answer key to most problems in this packet!