MOTIVATION FOR LEARNING MATHEMATICS IN TERMS OF NEEDS AND GOALS

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This article suggests a framework for analysing students' motivation for learning mathematics. In the present paper, motivation is defined as a potential to direct behaviour. This potential is structured through needs and goals. The author examines students' motivation in terms of needs and goals, and the emphasis is on the psychological needs for competence and autonomy. The proposed theoretical framework as an analytical tool is useful in describing the students' goals and changes in goals in details. It could also contribute to increased insight into relations between different aspects of instructional designs and the students' motivation for learning mathematic. The usefulness of the theoretical framework will be illustrated with some findings from the study.

INTRODUCTION

In mathematics education there has not been done much work on people's motivation to date (Evans & Wedege, 2004; Hannula, 2006). Only a few researchers have distinguished between intrinsic and extrinsic motivation in mathematics (Goodchild, 2001; Holden, 2003; Middleton & Spanias, 1999), or between task orientation and ego orientation (Nicholls, Cobb, Wood, Yackel, & Patashnick, 1990; Yates, 2000). Some mathematics educators have discussed students' motivation under the terms of motivational beliefs (Kloosterman, 1996; Op't Eynde, De Corte, & Verschaffel, 2002) and interest (Köller, Baumert, & Schnabel, 2001; Schiefele & Csikszentmihalyi, 1995). Evans and Wedege (2004; , 2006) consider people's motivation and resistance to learn mathematics as interrelated phenomena.

Hannula (2006) points out that many of the above approaches fail to describe the quality of the individual's motivation for learning mathematics in sufficient detail. He suggests that the reason for this is that the authors' approaches aim to measure predefined aspects of motivation, not to describe it (p. 166). Hannula developed a theoretical foundation of motivation as a structure of needs and goals, and his study shows that the students' goals vary a lot from person to person. The aim of this article is to present (develop) a theoretical framework for analysing the students' motivation for learning mathematics, in terms of needs and goals. The article reports on a particular aspect of a study where the focus is the development of Norwegian upper secondary school students' motivation for learning mathematics when they experience an inquiry mathematics teaching approach. The study followed a design-research approach in that it involved both instructional design and classroom based research (Cobb, 2001). I collected a large and varied pool of data (participant observation, semi-structured interviews, videotapes of students working, conversations with the teacher, students' diaries, collection of material, assessment) on seven of the students. The focus of this article is the development of theory. Some findings from the study will be presented, mainly to illustrate the usefulness of the theoretical framework. Due to space constraint, the original data and analyses cannot be included. The interested reader should return to original papers.

MOTIVATION

Motivation is defined in different ways in the literature of (achievement) motivation, and I have chosen to use the following definition:

> Motivation is a potential to direct behaviour that is built into the system that controls emotion. This potential may be manifested in cognition, emotion and/or behaviour. (Hannula, 2004, p. 3)

Motivation is considered as a potential to direct behaviour, and therefore, my focus is on the orientation of motivation. According to the definition, students' motivation may be manifested in cognition, emotion and/or behaviour. For example, a student's motivation to get a good grade in mathematics may be manifested in happiness (emotion) if he or she scores high on a test. It may also be manifested in studying for a test (behaviour) and in new conceptual learning (cognition) when studying for the test. Needs are specified instances of the potential to direct behaviour (Hannula, 2004). Psychological needs that are often emphasised in educational settings are competence, relatedness (or social belonging) and autonomy (e.g. Boekaerts, 1999; Ryan & Deci, 2000). I have chosen to define motivation as a potential to direct behaviour and therefore the orientation of motivation becomes central. Thus it is necessary to add a more fine grained conceptualization of motivation focusing on needs and goals.

Self Determination Theory and needs

Self Determination Theory (SDT) is a general theory of motivation that focuses on psychological needs, and I have chosen to use Ryan and Deci's (2002) definition of needs. Before presenting the definition, I will give a short presentation of the theory. Most contemporary theories of motivation assume that people engage in activities to the extent that they believe the behaviours will lead to desired goals or outcomes (Deci & Ryan, 2000). Within Self determination theory one is concerned about the goals of the behaviour and what energizes this behaviour. SDT is founded on three assumptions. The first assumption is that human beings have an innate tendency to integrate. *Integrating* means to forge interconnections among aspects of one own psyches as well as with other individuals and groups in his or her social world:

> …all individuals have natural, innate and constructive tendencies to develop an even more elaborated and unified sense of self. (Ryan & Deci, 2002, p. 5)

This assumption of active, integrative tendencies in development is not unique to SDT. However, specific to this theory is that this evolved integrative tendency cannot be taken for granted. The second assumption in SDT is that social-contextual factors may facilitate and enable the integration tendency, or they may undermine this fundamental process of the human nature:

> …SDT posits that there are clear and specifiable social-contextual factors that support this innate tendency, and that there are other specifiable factors that thwart or hinder this fundamental process of human nature. (Ryan & Deci, 2002, p. 5)

In other words, there is a dialectic between an active organism and a dynamic environment (social context) such that the environment act on the individual, and is shaped by the individual. To describe and organize the environment as supporting versus thwarting the integrative process, the concepts of needs are used. *Needs* are defined through optimal functioning (growth and well-being), and I have chosen to use the following definition:

> There are necessary conditions for the growth and well-being of people's personalities and cognitive structures, just as there are for their physical development and functioning. These nutriments are referred to within SDT as basic psychological needs. (Ryan & Deci, 2002, p. 7)

Looking back at Hannula's definition, psychological needs are specified instances of the general potential to direct behaviour. The third assumption in SDT is that human beings have three basic psychological needs, the needs for competence, relatedness and autonomy (Deci & Ryan, 2000; Ryan & Deci, 2002). Within SDT, competence, relatedness and autonomy are defined in the following way:

> *Competence* refers to feeling effective in one's ongoing interactions with the social environment and experiencing opportunities to exercise and express one's capacities (Ryan & Deci, 2002, p. 7). *Relatedness* refers to feeling connected to others, to caring for and being cared for by others, to having a sense of belongingness both with other individuals and with one's community (Ryan & Deci, 2002, p. 7). *Autonomy* refers to being the perceived origin or source of one's own behaviour (Ryan & Deci, 2002, p. 8). (My italics in the three quotations)

According to the definition, competence is not an attained skill or capacity, but it is a felt sense of confidence and effectiveness in action. The individual feels and experiences competence in the specific situation, it is not a product that shall be used (Wæge, 2007). In that case it is different from the way it is used by Hannula (2002). Hannula defines competence as the individual's functional understanding and skills. He considers competence to be a product, something the individual could use. Relatedness, in the definition above, refers to the psychological feeling of being together with other persons in a secure community or unity. In a similar way as for the construct of competence, Hannula considers social belonging (or relatedness) to be a target to attain. It also includes a goal of social status in the group. Within SDT relatedness refers to the students' feelings of belongingness with others. When individuals are autonomous they experience themselves as volitional initiators of their own actions. Cobb and colleagues (Cobb, 2000; Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997; diSessa & Cobb, 2004) use the concept of intellectual autonomy as a characteristic of a student's way of participating in the practices of a classroom community. They speak of the students' awareness and willingness to draw on their own intellectually capabilities when making mathematical decisions and judgments as they participate in mathematics activities. Hannula define autonomy as "the need to have control over own actions and to feel self-determining" (Hannula, 2002, p. 74). His definition differs from Ryan and Deci's definition in that it adds an aspect of having control over own actions.

The concept of needs is useful because it allows the specification of the social-contextual conditions that will facilitate motivation. According to SDT, students' motivation will be maximized within social contexts that provide them with the opportunity to satisfy their basic psychological needs for competence, autonomy and relatedness. I have chosen to use Ryan and Deci's definitions of the three psychological needs [1]. The data in the study did not give a basis for detailed analyses of the student's needs for relatedness and the goals the students' have in relation to this need. Therefore, the need for relatedness was not a focus in my study. In my study I focused on the students' needs for competence and autonomy. In his study, Hannula focuses on the three psychological needs for competence, relatedness and autonomy, but as I pointed out above, his definitions of the constructs differ from Ryan and Deci's definitions, which are the ones I have chosen to use.

Needs and goals structures

Hannula's definition of motivation (above) purports the potential to direct behaviour is structured through needs and goals. Needs and goals are specified instances of the potential to direct behaviour. According to Hannula, goals are derived from needs, and the difference between needs and goals is their different level of specificity. A need may be directed toward a relatively large category of objects, while a goal is directed toward a specific object (Hannula, 2004). For example, in my study, Berit realised her need for competence as a more specific goal of gaining a good grade. She translated her need for autonomy into the more specific goal of developing her own ideas, independently of the teacher. Another student, David, realised his need for relatedness as a goal to gain the mathematics teacher's confidence and respect.

According to Boekaerts, the students' goal structures are complex, and they tend to pursue multiple goals. The goals are related to each other, and pursuing one goal might be necessary to attain another goal or different goals may be seen as contradictory (Boekaerts, 1999; Shah & Kruglanski, 2000). Learning goals and performance goals are usually considered as contradictory to each other (Lemos, 1999; Linnenbrink & Pintrich, 2000), but Hannula's (2004) and my own findings (Wæge, 2007) indicate that these goals should not be seen as mutually exclusive goals in mathematics education. To exemplify this I present an utterance of a student [2]:

Berit: […] I think it has been pretty enjoyable. In the beginning I thought it was a bit difficult (Interviewer: Mm) because I was not used to this kind of teaching approach. […] I think this mathematical approach is much better. The full-day test [3] was pretty special this time, because usually I didn't quite understand what I was doing {inaudible}. Do this, follow rules and things like that. This time I thought that I understood everything and I thought the test went very well. And then I get a 4[4] and when I didn't understand it I used to get a 5. But I almost think it's better to try to understand a little more and nevertheless get a lower grade. Anyhow, I think it is possible to increase the grade. It's only a new way of thinking. It's quite interesting, I think {laughing} strange, yes.

My analysis of Berit shows that she has a specific goal of relational understanding in mathematics (Skemp, 1976). Her sense of mastery and her feeling of succeeding in mathematics are higher when she experiences that she understands the mathematics problems, than when she uses rules without understanding. Another important goal for Berit is to get good grades on the mathematics tests. Her goals of relational understanding in mathematics and good grades in mathematics support each other mutually. Getting good grades are important to Berit, but relational understanding in mathematics is the most important goal for her.

FIVE MOTIVATION VARIABLES

There is a serious methodological problem with research on a mental construct like motivation. Students' motivation cannot directly be observed, and thus measured, and it needs to be reconstructed through interpretation of the observable. I have developed an instrument to assess students' motivation for learning mathematics in terms of cognition, emotion and behaviour. In doing this I focus on the five sets of motivational variables that Stipek, Salmon, Givvin & Kazemi (1998) used in their study entitled: "The value (and convergence) of practices suggested by motivation research and promoted by mathematics education reformers" [2]. These are the students'

1. focus on learning and understanding mathematics concepts as well as on getting right answers;
2. enjoyment in engaging in mathematics activities;
3. related positive (or negative) feelings about mathematics.
4. willingness to take risks and to approach challenging tasks;

5. self-confidence as mathematics learners;

All these motivation variables figure prominently in the achievement motivation literature and in the mathematics reform literature. The five motivation variables are closely related to the needs for competence and autonomy. The first and the fourth variable, students' focus on learning and their willingness to take risks and approach challenging tasks, are closely related to the students' need for competence. Deci and Ryan (2002) claim that the students' need for competence leads them to seek adequately challenging mathematics tasks and to attempt to maintain and develop their mathematical understanding and skills. In my analysis I distinguish between students' learning orientation and performance orientation (Nicholls, Cobb, Wood, Yackel, & Patashnick, 1990). In addition, I also make a distinction between relational understanding and instrumental understanding (Skemp, 1976). The fifth variable, students' self-confidence, is related to students' willingness to approach tasks (Stipek, Salmon, Givvin, & Kazemi, 1998). The second and the third variable, students' enjoyment and their feelings about mathematics, are related to the students' intrinsic motivation in mathematics. According to Deci and Ryan (2002), intrinsic motivation represents a prototype of self-determined activity. They suggest that there is a strong connection between people's intrinsic motivation and their need for autonomy and competence. Mathematics classrooms that support the students' needs for autonomy and competence will enhance their intrinsic motivation in mathematics. Contextual events that students experience as thwarting satisfaction of these needs will undermine their intrinsic motivation.

In analysing the data, I assess these five motivation variables and I analyse the needs and goals of the students in relation to these specific motivational orientations. More specifically, the analysis is divided into two parts. First I analyse the data according to the five motivation variables. Although the variables can be seen as interrelated they are analysed separately in order to provide detailed insight into the students' motivation for learning mathematics. In the second part, I analyse the student's needs and goals in relation to these five specific motivational orientations. Furthermore, my emphasis is on the students' need for autonomy and competence.

TEACHING APPROACH

The teaching approach in the study was intended to give more space for the students to satisfy their needs for competence and autonomy, than teacher-centred and teacher-controlled teaching approaches. In the study attention was given to the development of students' mathematical thinking and reasoning. Our (the teacher and I) task was to create instructional activities that supported the development of both collective mathematical meanings evolving in the classroom community and the mathematical understanding of the individual student. We tried to support

> …the collective learning of the classroom community, during which taken-as-shared mathematical meanings emerge as the teacher and students negotiate interpretations and solutions (Gravemeijer, Cobb, Bowers, & Whitenack, 2000, p. 226).

The teacher always asked the students "What did you think when you solved this problem? What strategies did you use?" In the written tasks we developed, the students were frequently asked to explain their solutions and strategies, and the students were invited to find several solution strategies to a problem. The teacher tried to promote a classroom microculture (Cobb, Boufi, McClain, & Whitenack, 1997) where active participation and encouragement to understand were emphasised. In some of the instructional activities the students had to develop their own ideas, apply the mathematics in realistic situations and draw their own conclusions. Collaboration was important in our teaching approach. When the student's were given problems they were not familiar with, we wanted the students to collaborate. The students had an opportunity to experience themselves and their peers as active participants in creating mathematical insight. Every student brought a personal contribution at his or her level. These elements of our design study were suitable for meeting the students need for competence, autonomy and relatedness.

THE THEORETICAL FRAMEWORK – SOME KEY POINTS

The proposed theoretical framework for analysing students' motivation is useful in describing students' goals and changes in goals in detail. The framework is useful in clarifying students' notion of what it might mean to understand in mathematics. For example, the analysis of Berit shows that for her, to understand means to know what to do and why. We may also understand the relations between different goals through the use of such a framework. The complete analysis of Berit shows that there was a strong connection between her goal of relational understanding and her goal of finding her own solutions. She believes that finding own strategies for solving problems helps her in learning and understanding mathematics. As I described above, her goal of getting a good grades in mathematics and mastery goal, in this case a goal of relational understanding in mathematics, mutually supported each other.

The study shows that students' motivation for learning mathematics, although it is considered relatively stable, can be influenced by changes in the teaching approach. The case of Berit shows that students' motivation for learning mathematics might change in a relatively short time. Within the first semester of the school year, Berit changed her goal of instrumental understanding (Skemp, 1976) to a goal of relational understanding in mathematics.

We may also understand the relations between different aspects of the instructional designs developed in the study and the students' motivation for learning mathematics in terms of needs and goals through this framework. The analysis of Berit indicate that a combination of working with mathematics problems and routine tasks from the textbook, and the fact that the students were given opportunities to find their own solutions and rules for solving the problems, in collaboration with peer students and with guidance from the teacher, contributed to a sense of understanding and mastery with Berit.

I perceive that the theoretical framework as an analytical tool captured the complexity and the richness of the students' motivation in detail, and the tool made it possible for me to present detailed descriptions of the students' motivation for learning mathematics.

NOTES

1. See Wæge (2007) for a detailed description of my interpretation of the definitions.

2. Key to transcripts: […] extracts edited out of transcript for sake of clarity; {inaudible} unclear words; {text} comments about context or emotional behaviour like laughing; {.} 1 sec pause, {..} 2 sec pause, and so on.

The interviews took place in Norwegian. I have tried to translate from colloquial Norwegian to colloquial English, but it does not give an exact word for word translation. My analysis took place without any translation, that is, I analysed the transcripts in the original language.

3. At the end of each semester, the students have an all-day test in mathematics.

4. 1 is the lowest grade and 6 is the highest.

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