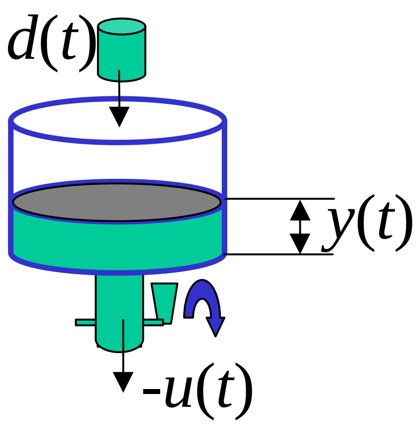
Lecture 4 - PID Control

- 90% (or more) of control loops in industry are PID
- Simple control design model $\rightarrow$ simple controller

P control

- Integrator plant:

$$\dot{y} = u + d$$



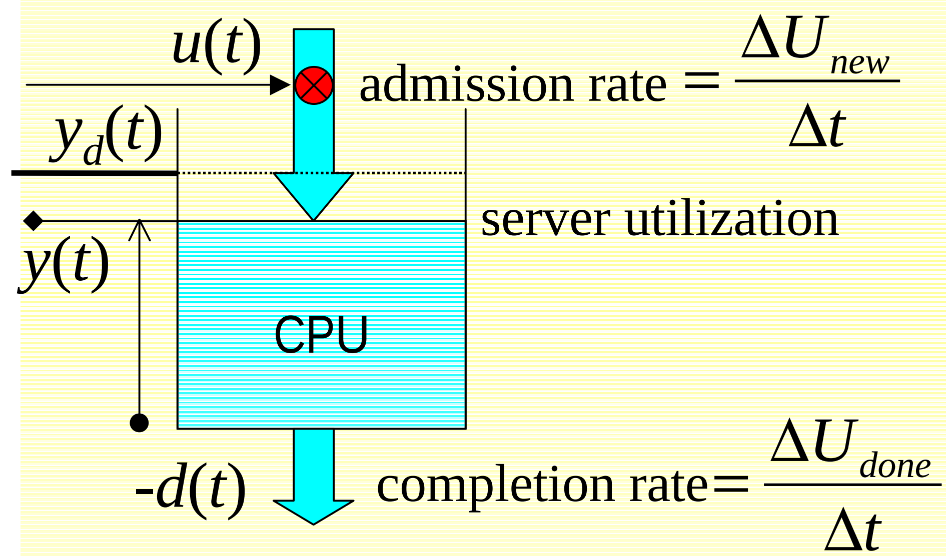
- P controller:

$$u = -k_P(y - y_d)$$

Example:
Utilization control in a video server

Video stream *i*

– processing time *c*[*i*], period *p*[*i*]
– CPU utilization: *U*[*i*]=c[*i*]/*p*[*i*]



P control

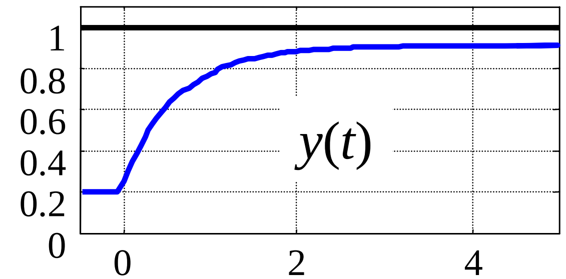
- Closed-loop dynamics

$$\dot{y} + k_P y = k_P y_d + d \qquad\qquad y = \frac{k_P}{s + k_P} y_d + \frac{1}{s + k_P} d$$

- Steady-state (s = 0) $y_{SS} = y_d + \frac{1}{k_P} d_{SS}$

- Transient $y(t) = y(0) e^{-t/T} + \left(y_d + \frac{1}{k_P} d_{SS} \right) \cdot \left(1 - e^{-t/T} \right)$

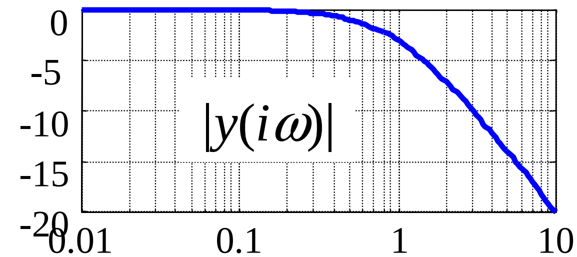
$$T = 1/k_P$$



- Frequency-domain (bandwidth)

$$y_d(t) = \hat{y}_d(i\omega) e^{i\omega t}$$
$$d(t) = \hat{d}(i\omega) e^{i\omega t}$$

$$|\hat{y}(i\omega)| = \frac{\left| \hat{y}_d(i\omega) + \hat{d}(i\omega)/k_P \right|}{\sqrt{(\omega/k_P)^2 + 1}}$$



I control

$$y = g \cdot u + d,$$

- Introduce integrator into control

$$\dot{u} = v,$$

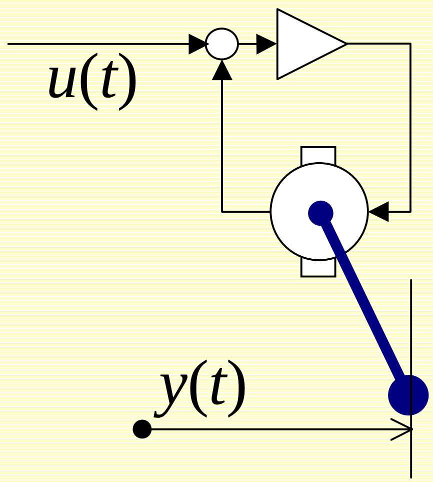
$$v = -k_I (y - y_d)$$

- Closed-loop dynamics

$$y = \frac{gk_I}{s + gk_I} y_d + \frac{s}{s + gk_I} d$$

Example:

- Servosystem command



- More:
  - stepper motor
  - flow through a valve
  - motor torque ...

Sampled time I control

- Step to step update:

$$y(t) = g \cdot u(t) + d(t)$$
$$u(t) = u(t-1) + v(t-1)$$
$$v(t) = k_I \left[y(t) - y_d \right]$$

sampled time integrator

- Closed-loop dynamics

$$y = g \cdot u + d$$
$$u = \frac{k_I}{z-1}\left[y - y_d \right]$$
$$\Longrightarrow \quad y = \frac{gk_I}{z - 1 + gk_I} y_d + \frac{z-1}{z-1+gk_I} d$$

- Deadbeat control: $gk_I = 1 \quad \Longrightarrow \quad y = z^{-1} y_d + \left(1 - z^{-1}\right) d$

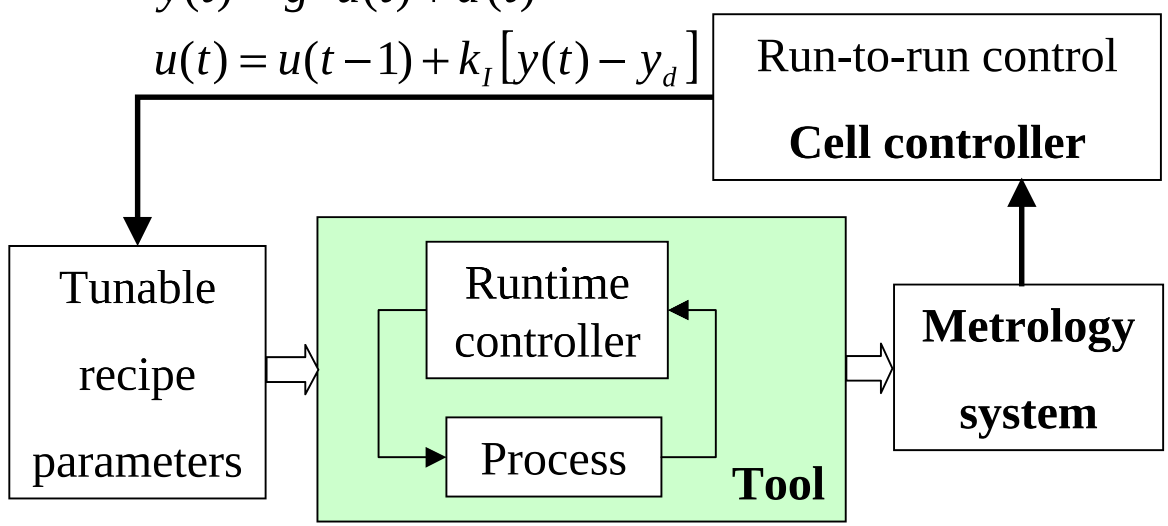
Run-to-run (R2R) control

- Main APC (Advanced Process Control) approach in semiconductor processes
- Modification of a product recipe between tool "runs"

- Processes:
  - vapor phase epitaxy
  - lithography
  - chemical mechanical planarization (CMP)
  - plasma etch

$$y(t) = g \cdot u(t) + d(t)$$

$$u(t) = u(t-1) + k_I\left[y(t) - y_d\right]$$



PI control

- First-order system:

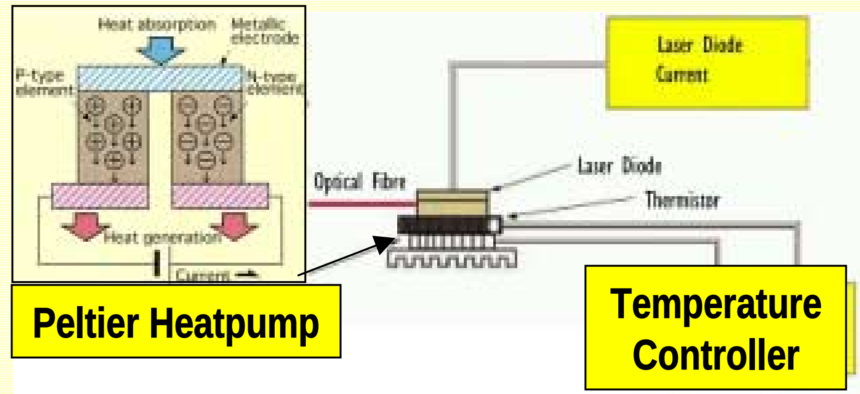
$$\tau\dot{y} = -y + u + d$$

- P control + integrator for cancelling steady state error

$$e = y - y_d;$$
$$\dot{v} = e$$
$$u = -k_I v - k_P e$$

Example:

- WDM laser-diode temperature control



$y(t)$ = temperature - ambient temperature

heat capacity; heat loss to environment; pumped heat; produced heat

$$\tau\dot{y} = -y + u + d$$

- Other applications
  - ATE
  - EDFA optical amplifiers
  - Fiber optic laser modules
  - Fiber optic network equipment

PI control

- P Control + Integrator for cancelling steady state error

$$e = y - y_d;$$

$$\dot{v} = e$$

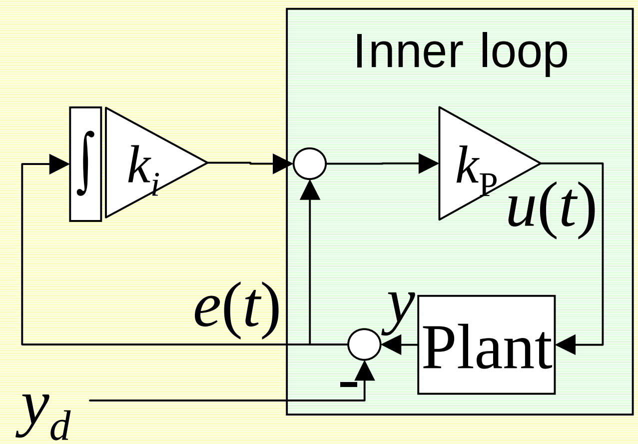
$$u = -k_I v - k_P e = k_P(e - \underbrace{k_i}_{\frac{k_I}{k_P}} v)$$

- Velocity form of the controller

$$\dot{u} = -k_I e - k_P \dot{e}$$

$$u(t+1) = u(t) - k_I e(t) - k_P\left[e(t) - e(t-1)\right]$$

Cascade loop interpretation:



$$k_I = k_i \cdot k_P$$

PI control

- Closed-loop dynamics

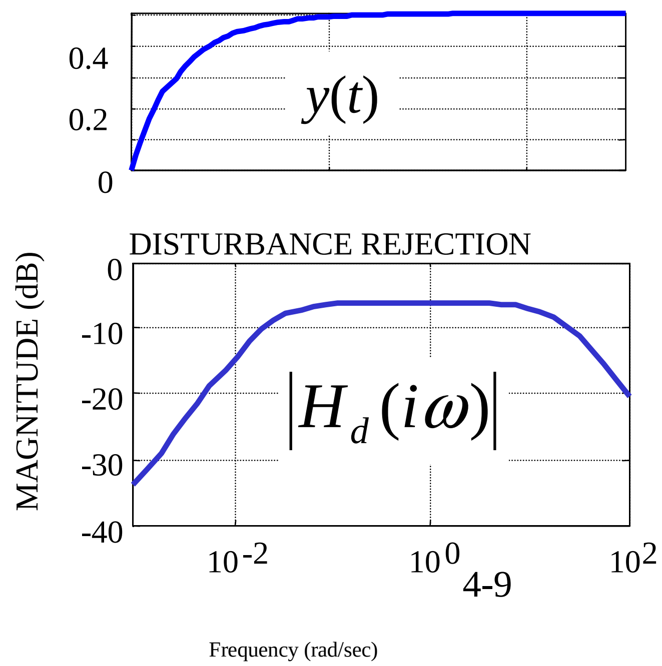
$$y = \frac{sk_P + k_I}{s(\tau s+1) + sk_P + k_I} y_d + \frac{s}{s(\tau s+1) + sk_P + k_I} d$$

- Steady state (s = 0): $y_{SS} = y_d$.
  No steady-state error!
- Transient dynamics: look at the characteristic equation

$$\tau\lambda^2 + (1+k_P)\lambda + k_I = 0$$

- Disturbance rejection

$$|\hat{y}(i\omega)| = |H_d(i\omega)| \cdot |\hat{d}(i\omega)|$$



PLL Example

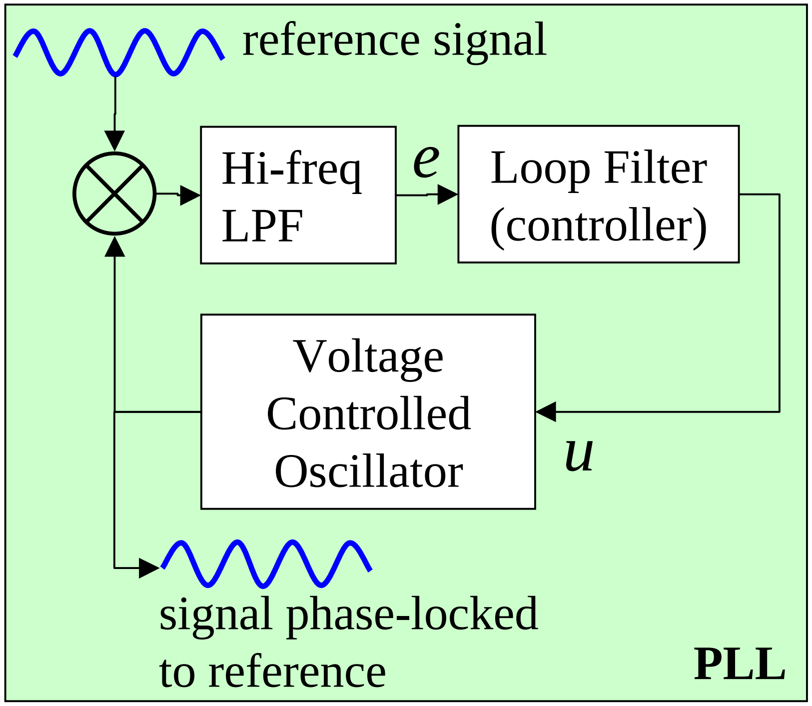
- Phase-locked loop is arguably a most prolific feedback system

$$e = 2K_m \mathrm{LPF}\langle r \times v \rangle$$

$$= 2K_m \mathrm{LPF}\langle A\sin(\omega t + \theta_d) \times \cos(\omega_o t + \theta_o) \rangle$$

$$e \approx AK_m \sin(\omega t - \omega_o t + \theta_d - \theta_o)$$

$$\dot{\theta}_o = \Delta\omega_o = K_o u$$



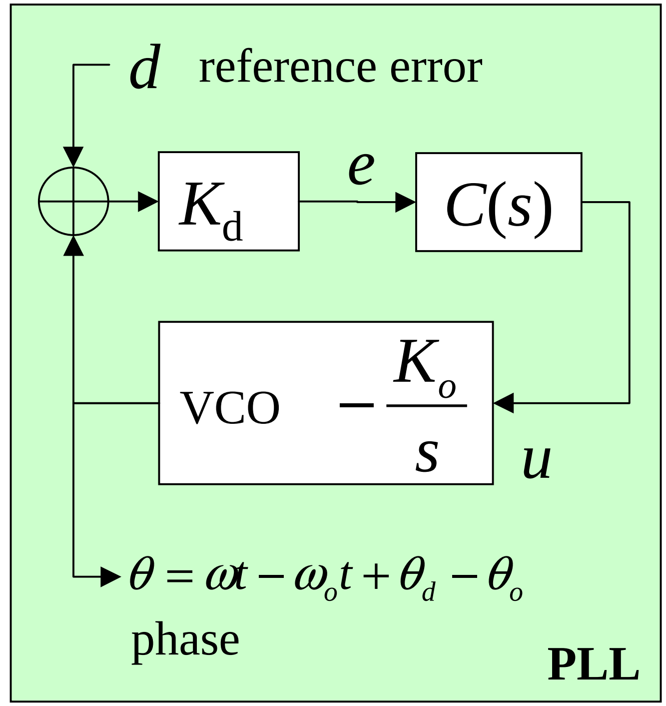
PLL Loop Model

- Small-signal model:

$$\theta = \omega t - \omega_o t + \theta_d - \theta_o << 1$$

$$e = K_d \sin(\theta) \approx K_d \theta$$

$$\dot{\theta} = \underbrace{\omega - \omega_o + \dot{\theta}_d}_{d} - \underbrace{K_o u}_{\theta_o}$$



- Loop dynamics:

$$\dot{\theta} = d - K_o u$$

$$e = K_d \theta$$

$$u = k_P e + k_I \int e \cdot dt$$

$$\Rightarrow \quad \theta = \frac{s}{s^2 + K_o K_d k_P s + k_I} d$$

PD control

- 2-nd order dynamics

$$\ddot{y} = u + d$$

- PD control

$$e = y - y_d$$

$$u = -k_D \dot{e} - k_P e$$

- Closed-loop dynamics

$$\ddot{e} + k_D \dot{e} + k_P e = d$$

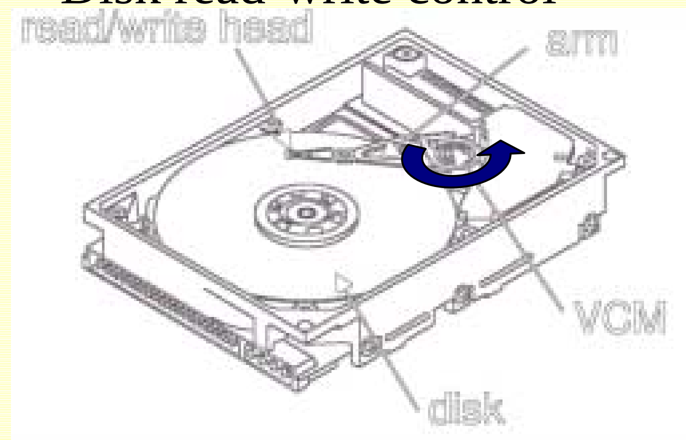
$$e = \frac{1}{s^2 + k_D s + k_P} d$$

- Optimal gains (critical damping)

$$k_D = 2\tau; \qquad k_P = \tau^2$$

Example:

- Disk read-write control



$$J\ddot{\varphi} = T_{VCM} + T_{DISTURB}$$

Voice
Coil
Motor

PD control

- Derivative (rate of *e*) can be obtained
  - speed sensor (tachometer)
  - low-level estimation logic
- Signal differentiation
  - is noncausal
  - amplifies high-frequency noise
- Causal (low-pass filtered) estimate of the derivative

$$\dot{e} \approx \frac{s}{\tau_D s + 1} e = \frac{1}{\tau_D} e + \frac{1/\tau_D}{\tau_D s + 1} e$$

- Modified PD controller:

$$u = -k_D \frac{s}{\tau_D s + 1} e - k_P e$$

PD control performance

- The performance seems to be infinitely improving for

$$k_D = 2\tau; k_P = \tau^2; \qquad \tau \to \infty$$

- This was a simple design model, remember?
- Performance is limited by
  - system being different from the model
    - flexible modes, friction, VCM inductance
  - sampling in a digital controller
  - rate estimation would amplify noise if too aggressive
  - actuator saturation
  - you might really find *after* you have tried to push the performance
- If high performance is really that important, careful application of more advanced control approaches might help

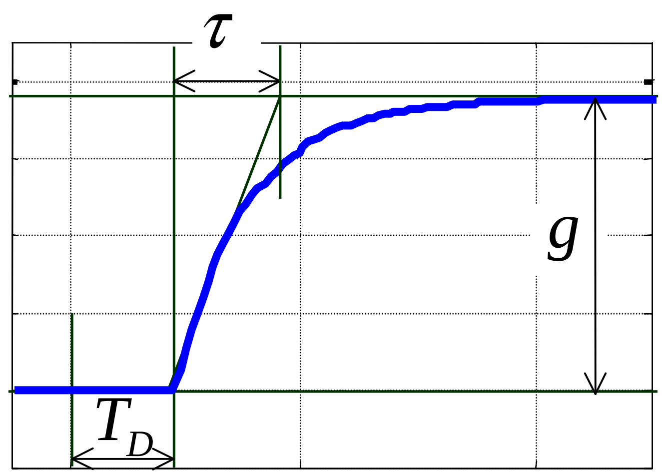
Plant Type

- Constant gain - I control
- Integrator - P control
- Double integrator - PD control
- Generic second order dynamics - PID control

PID Control

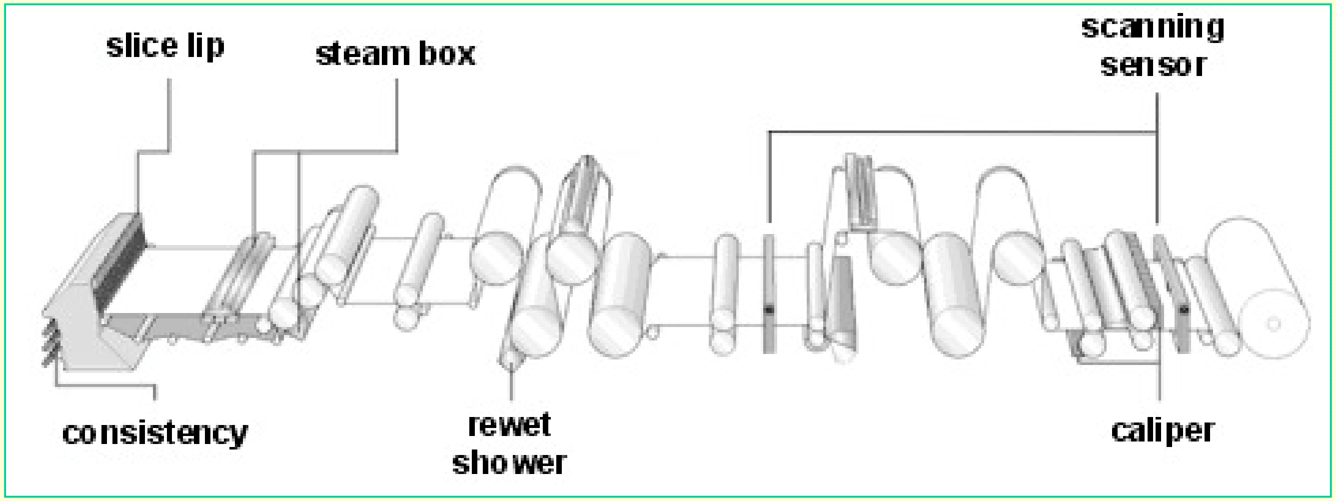
- Generalization of P, PI, PD
- Early motivation: control of first order processes with deadtime

$$y = \frac{g e^{-T_D s}}{\tau s + 1} u$$



Example:

- Paper machine control



PID Control

- PID: three-term control

$$e = y - y_d$$

$$u = -k_D \dot{e} - k_P e - k_I \int e \cdot dt$$

independent sensor or an estimate

- Sampled-time PID

$$u = -k_D\left(1 - z^{-1}\right)e + k_P e + k_I \frac{1}{1 - z^{-1}} e$$

future — present — past

- Velocity form
  – bumpless transfer between manual and automatic

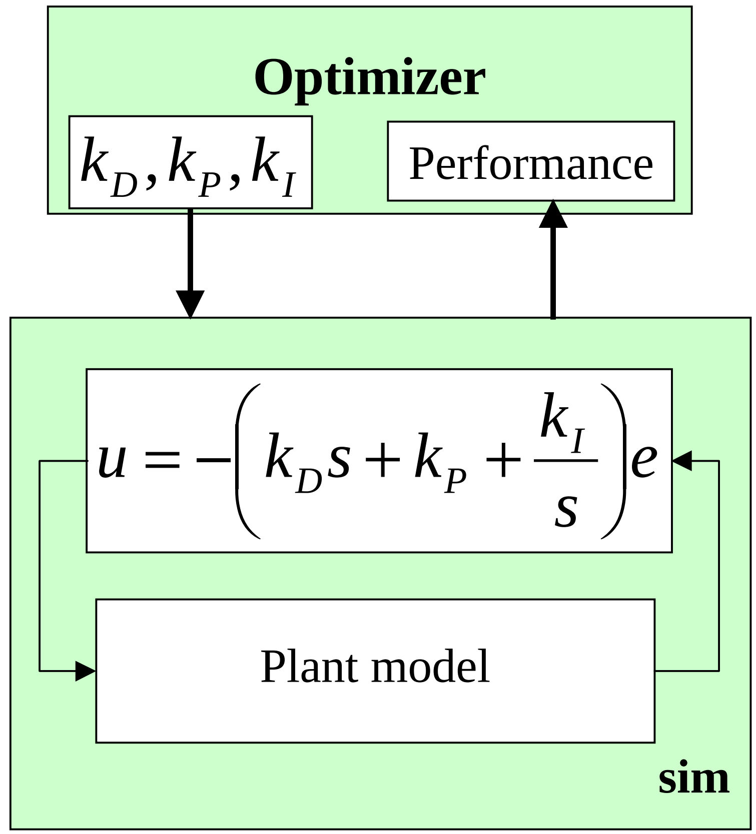
$$\Delta u = -k_D \Delta^2 e - k_P \Delta e - k_I e$$

$$\Delta = 1 - z^{-1}$$

$$u(t+1) = u(t) - k_I e(t) - k_P\left(e(t) - e(t-1)\right) - k_D\left(e(t) - 2e(t-1) + e(t-2)\right)$$

Tuning PID Control

- Model-based tuning
- Look at the closed-loop poles
- Numerical optimization
  - For given parameters run a sim, compute performance parameters and a performance index
  - Optimize the performance index over the three PID gains using grid search or Nelder method.



Zeigler-Nichols tuning rule

- Explore the plant:
  - set the plant under P control and start increasing the gain till the loop oscillates
  - note the critical gain k_C and oscillation period T_C
- Tune the controller:

	k_P	k_I	k_D
P	$0.5k_C$	——	——
PI	$0.45k_C$	$1.2k_P/T_C$	——
PID	$0.5k_C$	$2k_P/T_C$	$k_P T_C/8$

- Z and N used a Monte Carlo method to develop the rule
- Z-N rule enables tuning if a model and a computer are both unavailable, only the controller and the plant are.

Integrator anti wind-up

- In practice, control authority is always limited:
  - $u_{\text{MIN}} \leq u(t) \leq u_{\text{MAX}}$
- Wind up of the integrator:
  - if $|u_c| > u_{MAX}$ the integral v will keep growing while the control is constant. This results in a heavy overshoot later
- Anti wind-up:
  - switch the integrator off if the control has saturated

$$\dot{v} = e$$

$$u_c = -k_I v - k_P e$$

$$u = \begin{cases} u_{MAX}, & u_c > u_{MAX} \\ u_c, & u_{MIN} \leq u_c \leq u_{MAX} \\ u_{MIN}, & u_c < u_{MIN} \end{cases}$$

$$\dot{v} = \begin{cases} e, & \text{for } u_{MIN} \leq u_c \leq u_{MAX} \\ 0, & \text{if } u_c > u_{MAX} \text{ or } u_c < u_{MIN} \end{cases}$$

Industrial PID Controller

- A box, not an algorithm
- Auto-tuning functionality:
  - pre-tune
  - self-tune
- Manual/cascade mode switch
- Bumpless transfer between different modes, setpoint ramp
- Loop alarms
- Networked or serial port