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## **Shadow Prices In EMIX**

This is in response to an action item assigned to me after I made a comment in an EMIX meeting that there are prices not determined by a tariff, negotiation or a market process. One example is a "shadow price".

The definition of a shadow price from Wikipedia is below. Wikipedia further explains the concept beyond what I have attached below.

A power example is the case of a micro grid disconnected from the main grid. In this case supply and demand must balance within the microgrid. There is no market price. If the microgrid has an optimization method to minimize the cost of serving the load then the constraint "load - generation = 0" must be satisfied. At the optimal solution, the shadow price is the incremental cost of one additional unit of generation. The is also the price that could be used internally to communicate scarcity to any price-responsive loads.

## **Shadow price**

From Wikipedia, the free encyclopedia

In <u>constrained optimization</u> in <u>economics</u>, the **shadow price** is the change in the objective value of the optimal solution of an <u>optimization</u> problem obtained by relaxing the <u>constraint</u> by one unit – it is the <u>marginal utility</u> of relaxing the constraint, or equivalently the <u>marginal cost</u> of strengthening the constraint.

In a business application, a shadow price is the maximum price that management is willing to pay for an extra unit of a given limited resource. For example, if a production line is already operating at its maximum 40 hour limit, the shadow price would be the maximum price the manager would be willing to pay for operating it for an additional hour, based on the benefits he would get from this change.

More formally, the shadow price is the value of the <u>Lagrange multiplier</u> at the optimal solution, which means that it is the infinitesimal change in the objective function arising from an infinitesimal change in the constraint. This follows from the fact that at the optimal solution the gradient of the objective function is a linear combination of the constraint function gradients with the weights equal to the <u>Lagrange multipliers</u>. Each <u>constraint</u> in an <u>optimization</u> problem has a shadow price or <u>dual</u> variable.

The value of the shadow price can provide decision makers powerful insight into problems. For instance if you have a constraint that limits the amount of labor available to 40 hours per week, the shadow price will tell you how much you would be willing to pay for an additional hour of labor. If your shadow price is \$10 for the labor constraint, for instance, you should pay no more than \$10 an hour for additional labor. Labor costs of less than \$10/hour will increase the objective value; labor costs of more than \$10/hour will decrease the objective value. Labor costs of exactly \$10 will cause the objective function value to remain the same.