Chapter 9

Power and Refrigeration Gas Cycles

9.1 INTRODUCTION

Several cycles utilize a gas as the working substance, the most common being the Otto cycle and the diesel cycle used in internal combustion engines. The word "cycle" used in reference to an internal combustion engine is technically incorrect since the working fluid does not undergo a thermodynamic cycle; air enters the engine, mixes with a fuel, undergoes combustion, and exits the engine as exhaust gases. This is often referred to as an *open cycle*, but we should keep in mind that a thermodynamic cycle does not really occur; the engine itself operates in what we could call a *mechanical cycle*. We do, however, analyze an internal combustion engine as though the working fluid operated on a cycle; it is an approximation that allows us to predict influences of engine design on such quantities as efficiency and fuel consumption.

9.2 GAS COMPRESSORS

We have already utilized the gas compressor in the refrigeration cycles discussed earlier and have noted that the control volume energy equation relates the power input to the enthalpy change as follows:

$$\dot{W}_{\text{comp}} = \dot{m}(h_e - h_i) \qquad (9.1)$$

where h_e and h_i are the exit and inlet enthalpies, respectively. In this form we model the compressor as a fixed volume into which and from which a gas flows; we assume that negligible heat transfer occurs from the compressor and ignore the difference between inlet and outlet kinetic and potential energy changes.

There are three general types of compressors: reciprocating, centrifugal, and axial-flow. Reciprocating compressors are especially useful for producing high pressures, but are limited to relatively low flow rates; upper limits of about 200 MPa with inlet flow rates of 160 m^3/min are achievable with a two-stage unit. For high flow rates with relatively low pressure rise, a centrifugal or axial-flow compressor would be selected; a pressure rise of several MPa for an inlet flow rate of over 10 000 m^3/min is possible.

The Reciprocating Compressor

A sketch of the cylinder of a reciprocating compressor is shown in Fig. 9-1. The intake and exhaust valves are closed when state 1 is reached, as shown on the *P-v* diagram of Fig. 9-2*a*. An isentropic compression follows as the piston travels inward until the maximum pressure at state 2 is reached. The exhaust valve then opens and the piston continues its inward motion while the air is exhausted until state 3 is reached at top dead center. The exhaust valve then closes and the piston begins its outward motion with an isentropic expansion process until state 4 is reached. At this point the intake value opens and the piston moves outward during the intake process until the cycle is completed.

During actual operation the *P-v* diagram would more likely resemble that of Fig. 9-2*b*. Intake and exhaust valves do not open and close instantaneously, the airflow around the valves results in pressure gradients during the intake and exhaust strokes, losses occur due to the valves, and some heat transfer may take place. The ideal cycle does, however, allow us to predict the influence of proposed design changes on work requirements, maximum pressure, flow rate, and other quantities of interest.

The effectiveness of a compressor is partially measured by the *volumetric efficiency*, which is defined as the volume of gas drawn into the cylinder divided by the displacement volume. That is,

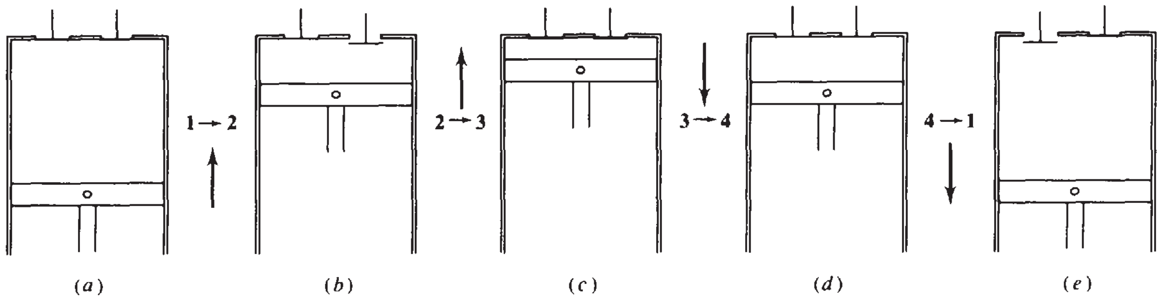


Fig. 9-1

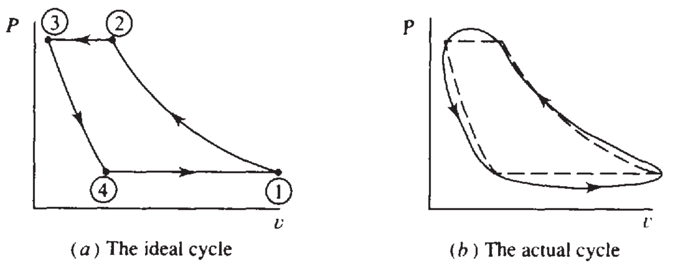


Fig. 9-2

referring to Fig. 9-2,

$$\eta_{\text{vol}} = \frac{V_1 - V_4}{V_1 - V_3} \tag{9.2}$$

The higher the volumetric efficiency the greater the volume of air drawn in as a percentage of the displacement volume. This can be increased if the clearance volume V_3 is decreased.

To improve the performance of the reciprocating compressor, we can remove heat from the compressor during the compression process $1 \rightarrow 2$. The effect of this is displayed in Fig. 9-3, where a polytropic process is shown. The temperature of state 2′ would be significantly lower than that of state

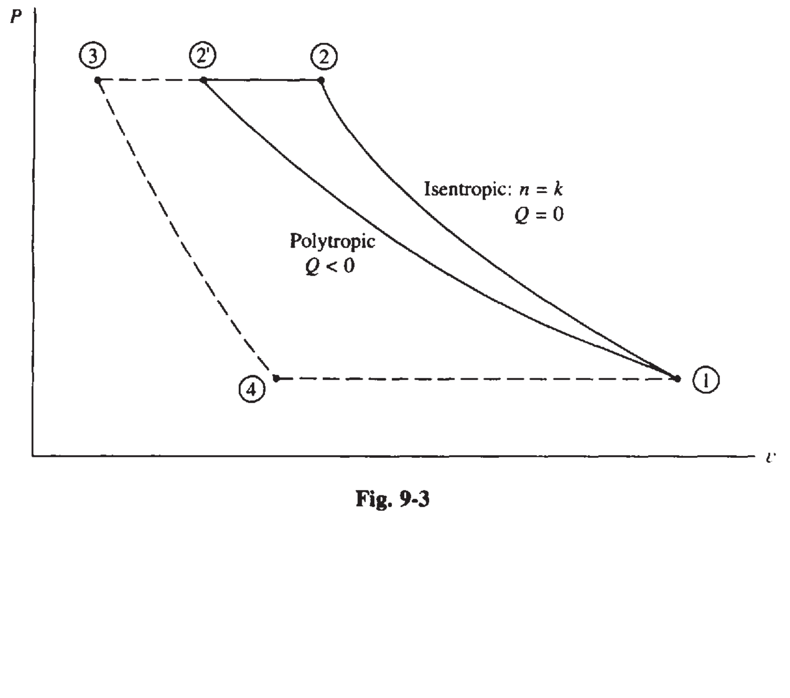


Fig. 9-3

2 and the work requirement for the complete cycle would be less since the area under the P-v diagram would decrease. To analyze this situation let us return to the control volume inlet-outlet description, as used with (*9.1*). The required work is, for an adiabatic compressor,

$$w_{\text{comp}} = h_2 - h_1 = c_p(T_2 - T_1) \tag{9.3}$$

assuming an ideal gas with constant specific heat. For an isentropic compression between inlet and outlet we know that

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} \tag{9.4}$$

This allows the work to be expressed as, using c_p given in (*4.30*),

$$w_{\text{comp}} = \frac{kR}{k-1}T_1\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right] \tag{9.5}$$

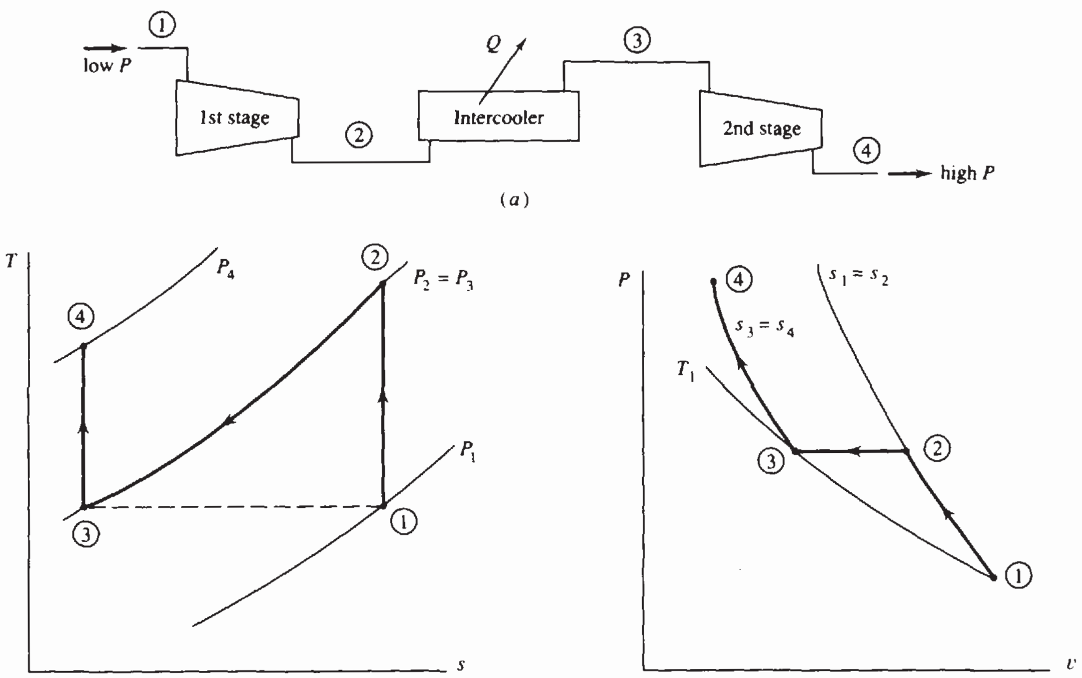
For a polytropic process we simply replace k with n and obtain

$$w_{\text{comp}} = \frac{nR}{n-1}T_1\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] \tag{9.6}$$

The heat transfer is then found from the first law.

By external cooling, with a water jacket surrounding the compressor, the value of n for air can be reduced to about 1.35. This reduction from 1.4 is difficult since heat transfer must occur from the rapidly moving air through the compressor casing to the cooling water, or from fins. This is an ineffective process, and multistage compressors with interstage cooling are often a desirable alternative. With a single stage and with a high P_2 the outlet temperature T_2 would be too high even if n could be reduced to, say, 1.3.

Consider a two-stage compressor with a single intercooler, as shown in Fig. 9-4*a*. The compression processes are assumed to be isentropic and are shown in the T-s and P-v diagrams of Fig. 9-4*b*.



(*b*)

Fig. 9-4

Referring to (9.5), the work is written as

$$w_{\text{comp}} = c_p T_1\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right] + c_p T_3\left[\left(\frac{P_4}{P_3}\right)^{(k-1)/k} - 1\right] = c_p T_1\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} + \left(\frac{P_4}{P_2}\right)^{(k-1)/k} - 2\right] \tag{9.7}$$

where we have used $P_2 = P_3$ and $T_1 = T_3$, for an ideal intercooler. To determine the intercooler pressure P_2 that minimizes the work, we let $\partial w_{\text{comp}}/\partial P_2 = 0$. This gives

$$P_2 = (P_1 P_4)^{1/2} \qquad \text{or} \qquad \frac{P_2}{P_1} = \frac{P_4}{P_3} \tag{9.8}$$

That is, the pressure ratio is the same across each stage. If three stages were used, the same analysis would lead to a low-pressure intercooler pressure of

$$P_2 = \left(P_1^2 P_6\right)^{1/3} \tag{9.9}$$

and a high-pressure intercooler pressure of

$$P_4 = \left(P_1 P_6^2\right)^{1/3} \tag{9.10}$$

where P_6 is the highest pressure. This is also equivalent to equal pressure ratios across each stage. Additional stages may be necessary for extremely high outlet pressures; an equal pressure ratio across each stage would yield the minimum work for the ideal compressor.

Centrifugal and Axial-Flow Compressors

A centrifugal compressor is sketched in Fig. 9-5. Air enters along the axis of the compressor and is forced to move outward along the rotating impeller vanes due to the effects of centrifugal forces. This results in an increased pressure from the axis to the edge of the rotating impeller. The diffuser section results in a further increase in the pressure as the velocity is reduced due to the increasing area in each subsection of the diffuser. Depending on the desired pressure-speed characteristics, the rotating impeller can be fitted with radial impeller vanes, as shown; with backward-curved vanes; or with forward-curved vanes.

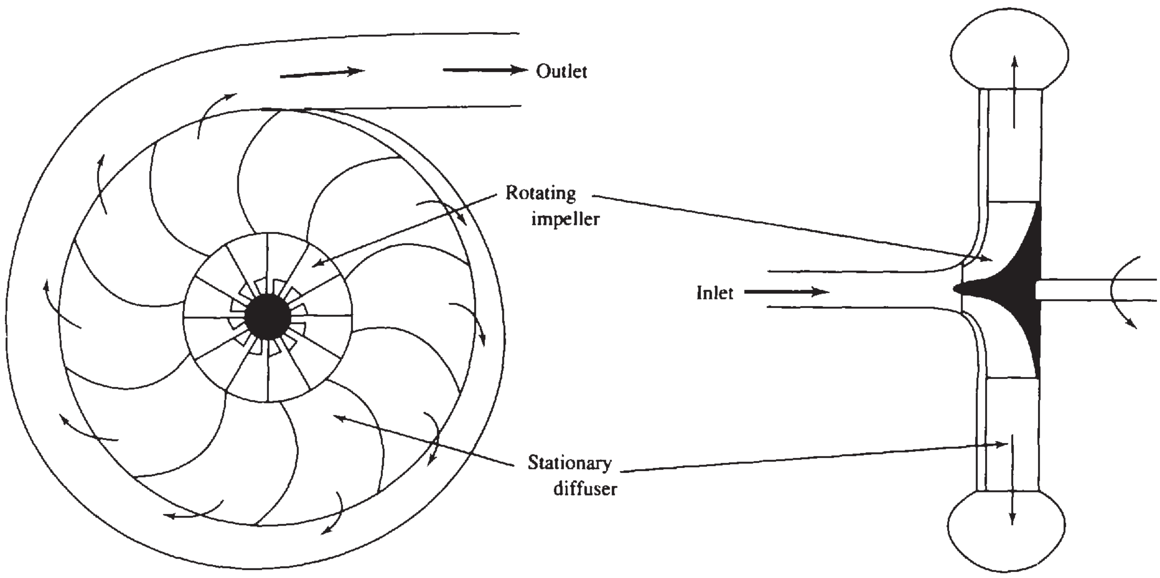


Fig. 9-5

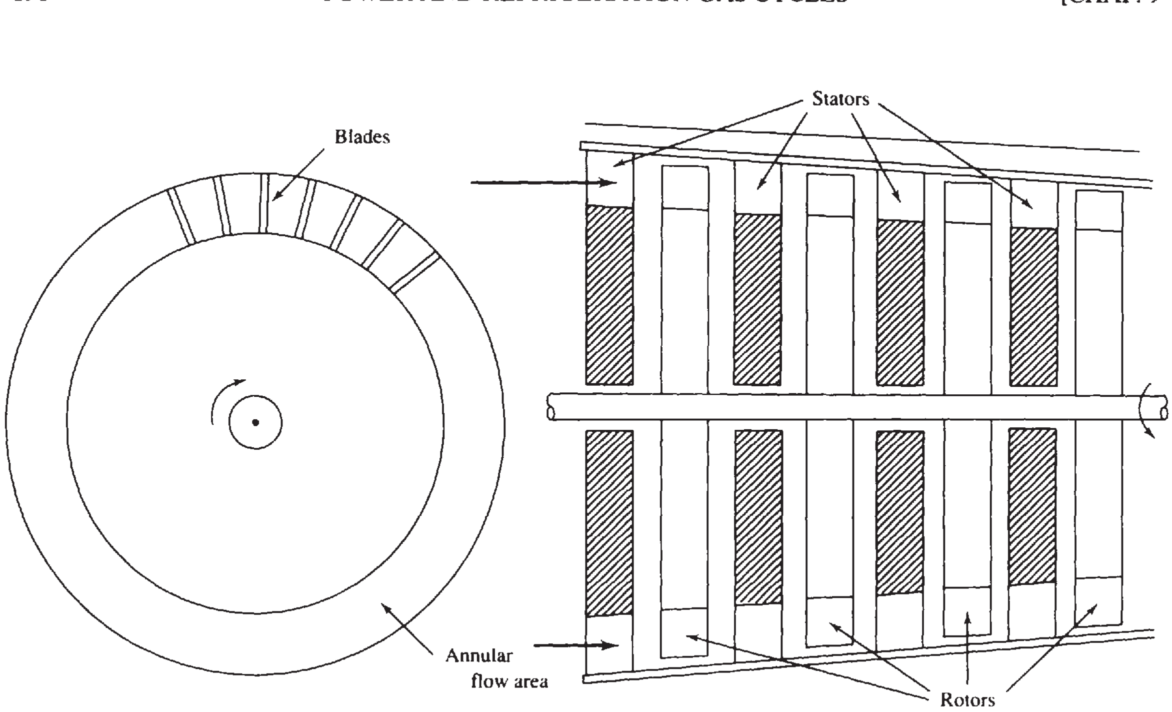


Fig. 9-6

An axial-flow compressor is illustrated in Fig. 9-6. It is similar in appearance to the steam turbine used in the Rankine power cycle. Several stages of blades are needed to provide the desired pressure rise, with a relatively small rise occurring over each stage. Each stage has a *stator*, a series of blades that are attached to the stationary housing, and a *rotor*. All the rotors are attached to a common rotating shaft which utilizes the power input to the compressor. The specially designed airfoil-type blades require extreme precision in manufacturing and installation to yield the maximum possible pressure rise while avoiding flow separation. The area through which the air passes decreases slightly as the pressure rises due to the increased density in the higher-pressure air. In fluid mechanics the velocity and pressure at each stage can be analyzed; in thermodynamics we are concerned only with inlet and outlet conditions.

EXAMPLE 9.1 A reciprocating compressor is to deliver 20 kg/min of air at 1600 kPa. It receives atmospheric air at 20 °C. Calculate the required power if the compressor is assumed to be 90 percent efficient. No cooling is assumed.

The efficiency of the compressor is defined as

$$\eta = \frac{\text{isentropic work}}{\text{actual work}} = \frac{h_{2'} - h_1}{h_2 - h_1}$$

where state 2 identifies the actual state reached and state 2′ is the ideal state that could be reached with no losses. Let us find the temperature $T_{2'}$ first. It is

$$T_{2'} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (293)\left(\frac{1600}{100}\right)^{(1.4-1)/1.4} = 647\text{ K}$$

Using the efficiency, we have

$$\eta = \frac{c_p(T_{2'} - T_1)}{c_p(T_2 - T_1)} \qquad \text{or} \qquad T_2 = T_1 + \frac{1}{\eta}(T_{2'} - T_1) = 293 + \left(\frac{1}{0.9}\right)(647 - 293) = 686\text{ K}$$

The power required to drive the adiabatic compressor (no cooling) is then

$$\dot{W}_{\text{comp}} = \dot{m}(h_2 - h_1) = \dot{m}c_p(T_2 - T_1) = \left(\frac{20}{60}\right)(1.006)(686 - 293) = 131.9\text{ kW}$$

EXAMPLE 9.2 Suppose that, for the compressor of Example 9.1, it is decided that because T_2 is too high, two stages with an intercooler are necessary. Determine the power requirement for the proposed two-stage adiabatic compressor. Assume 90 percent efficiency for each stage.

The intercooler pressure for minimum power input is given by (9.8) as $P_2 = \sqrt{P_1 P_4} = \sqrt{(100)(1600)} = 400$ kPa. This results in a temperature entering the intercooler of

$$T_{2'} = T_1\left(\frac{P_2}{P_1}\right)^{(1.4-1)/1.4} = 293\left(\frac{400}{100}\right)^{0.2857} = 435 \text{ K}$$

Since $T_3 = T_1$ and $P_4/P_3 = P_2/P_1$, we also have $T_{4'} = (293)(400/100)^{0.2857} = 435$ K. Considering the efficiency of each stage allows us to find

$$T_2 = T_1 + \frac{1}{\eta}(T_{2'} - T_1) = 293 + \left(\frac{1}{0.9}\right)(435 - 293) = 451 \text{ K}$$

This will also be the exiting temperature $T_{4'}$. Note the large reduction from the single-stage temperature of 686 K. Assuming no heat transfer in the compressor stages, the power necessary to drive the compressor is

$$\dot{W}_{\text{comp}} = \dot{m}c_p(T_2 - T_1) + \dot{m}c_p(T_4 - T_3) = \left(\frac{20}{60}\right) \times (1.00)(451 - 293) + \left(\frac{20}{60}\right)(1.00)(451 - 293) = 105 \text{ kW}$$

This is a 20 percent reduction in the power requirement.

9.3 THE AIR-STANDARD CYCLE

In this section we introduce engines that utilize a gas as the working fluid. Spark-ignition engines that burn gasoline and compression-ignition (diesel) engines that burn fuel oil are the two most common engines of this type.

The operation of a gas engine can be analyzed by assuming that the working fluid does indeed go through a complete thermodynamic cycle. The cycle is often called an *air-standard cycle*. All the air-standard cycles we will consider have certain features in common:

Air is the working fluid throughout the entire cycle. The mass of the small quantity of injected fuel is negligible.

There is no inlet process or exhaust process.

The combustion process is replaced by a heat transfer process with energy transferred from an external source.

The exhaust process, used to restore the air to its original state, is replaced with a constant-volume process transferring heat to the surroundings; no work is accomplished with a constant-volume process.

All processes are assumed to be in quasiequilibrium.

The air is assumed to be an ideal gas with constant specific heats.

A number of the engines we will consider make use of a closed system with a piston-cylinder arrangement, as shown in Fig. 9-7. The cycle shown on the *P-v* and *T-s* diagrams in the figure is representative. The diameter of the piston is called the *bore*, and the distance the piston travels in one direction is the *stroke*. When the piston is at top dead center (TDC), the volume occupied by the air in the cylinder is at a minimum; this volume is the *clearance volume*. When the piston moves to bottom dead center (BDC), the air occupies the maximum volume. The difference between the maximum volume and the clearance volume is the *displacement volume*. The clearance volume is often implicitly presented as the *percent clearance c*, the ratio of the clearance volume to the displacement volume. The *compression ratio r* is defined to be the ratio of the volume occupied by the air at BDC to the volume occupied by the air at TDC, that is, referring to Fig. 9-7,

$$r = \frac{V_1}{V_2} \tag{9.11}$$

The *mean effective pressure* (MEP) is another quantity that is often used when rating piston-cylinder engines; it is the pressure that, if acting on the piston during the power stroke, would produce an amount of work equal to that actually done during the entire cycle. Thus,

$$W_{\text{cycle}} = (\text{MEP})(V_{\text{BDC}} - V_{\text{TDC}}) \tag{9.12}$$

In Fig. 9-7 this means that the enclosed area of the actual cycle is equal to the area under the MEP dotted line.

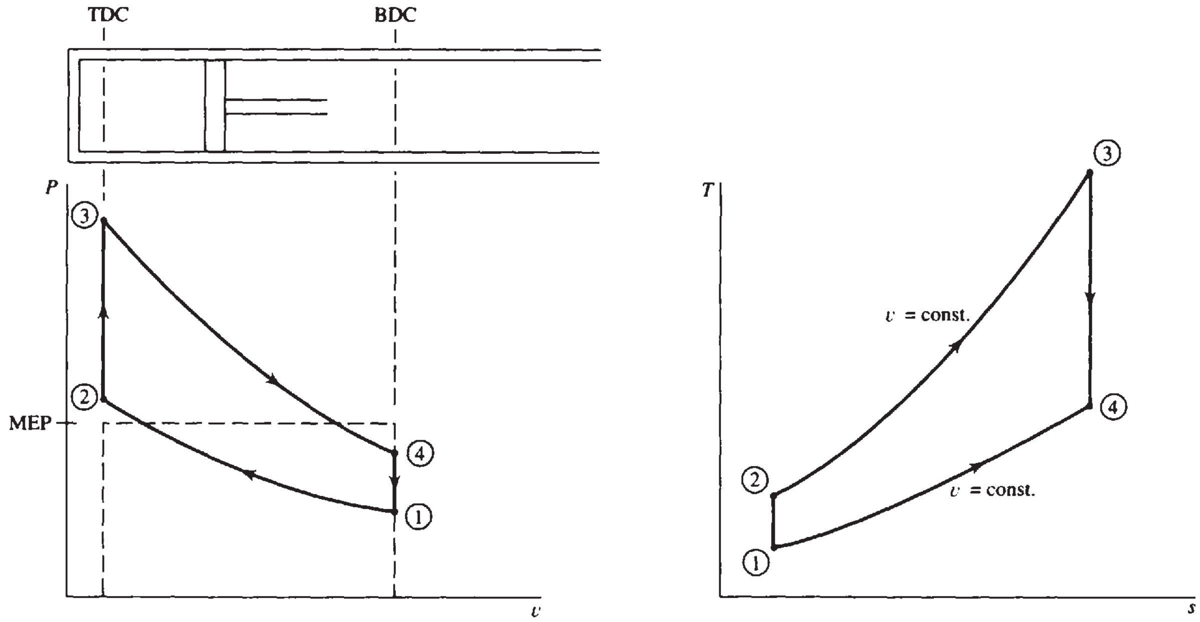


Fig. 9-7

EXAMPLE 9.3 An engine operates with air on the cycle shown in Fig. 9-7 with isentropic processes 1 → 2 and 3 → 4. If the compression ratio is 12, the minimum pressure is 200 kPa, and the maximum pressure is 10 MPa determine (*a*) the percent clearance and (*b*) the MEP.

(*a*) The percent clearance is given by

$$c = \frac{V_2}{V_1 - V_2}(100)$$

But the compression ratio is $r = V_1/V_2 = 12$. Thus,

$$c = \frac{V_2}{12V_2 - V_2}(100) = \frac{100}{11} = 9.09\%$$

(*b*) To determine the MEP we must calculate the area under the *P-V* diagram; this is equivalent to calculating the work. The work from 3 → 4 is, using $PV^k = C$,

$$W_{3\text{-}4} = \int P\,dV = C\int \frac{dV}{V^k} = \frac{C}{1-k}\left(V_4^{1-k} - V_3^{1-k}\right) = \frac{P_4V_4 - P_3V_3}{1-k}$$

where $C = P_4V_4^k = P_3V_3^k$. But we know that $V_4/V_3 = 12$, so

$$W_{3\text{-}4} = \frac{V_3}{1-k}(12P_4 - P_3)$$

Likewise, the work from 1 → 2 is

$$W_{1-2} = \frac{V_2}{1-k}(P_2 - 12P_1)$$

Since no work occurs in the two constant-volume processes, we find, using $V_2 = V_3$,

$$W_{\text{cycle}} = \frac{V_2}{1-k}(12P_4 - P_3 + P_2 - 12P_1)$$

The pressures P_2 and P_4 are found as follows:

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^k = (200)(12)^{1.4} = 1665\text{ kPa} \qquad P_4 = P_3\left(\frac{V_3}{V_4}\right)^k = (10\,000)\left(\frac{1}{12}\right)^{1.4} = 308\text{ kPa}$$

whence

$$W_{\text{cycle}} = \frac{V_2}{-0.4}[(12)(308) - 10\,000 + 1665 - (12)(200)] = 20\,070V_2$$

But $W_{\text{cycle}} = (\text{MEP})(V_1 - V_2) = (\text{MEP})(12V_2 - V_2)$; equating the two expressions yields

$$\text{MEP} = \frac{20\,070}{11} = 1824\text{ kPa}$$

9.4 THE CARNOT CYCLE

This ideal cycle was treated in detail in Chapter 5. Recall that the thermal efficiency of a Carnot engine,

$$\eta_{\text{carnot}} = 1 - \frac{T_L}{T_H} \tag{9.13}$$

exceeds that of any real engine operating between the given temperatures.

9.5 THE OTTO CYCLE

The four processes that form the cycle are displayed in the *T-s* and *P-V* diagrams of Fig. 9-8. The piston starts at state 1 at BDC and compresses the air until it reaches TDC at state 2. Combustion then occurs, resulting in a sudden jump in pressure to state 3 while the volume remains constant (this

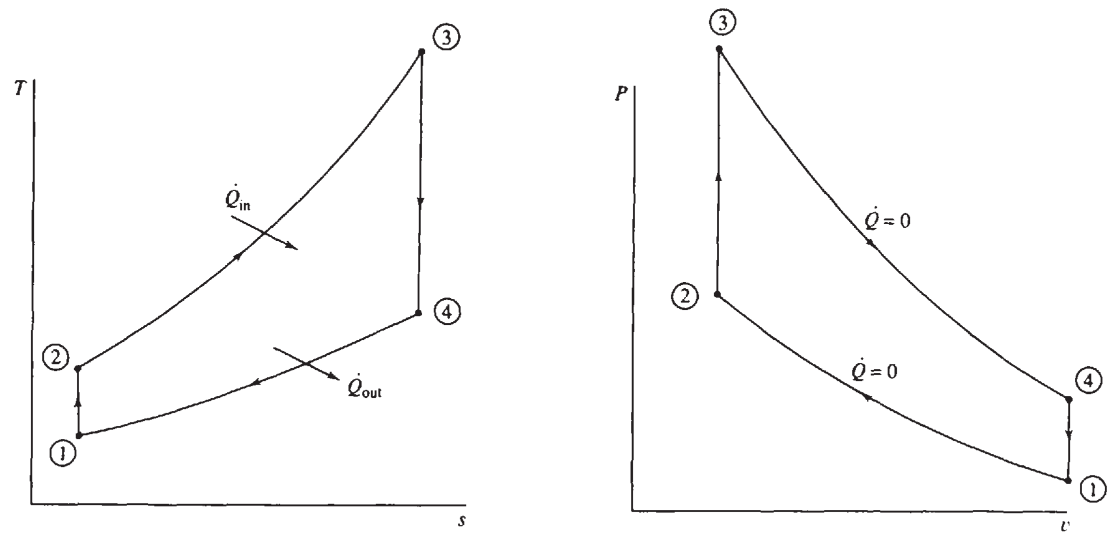


Fig. 9-8

combustion process is simulated with a quasiequilibrium heat addition process). The process that follows is the power stroke as the air (simulating the combustion products) expands isentropically to state 4. In the final process heat transfer to the surroundings occurs and the cycle is completed.

The thermal efficiency of the Otto cycle is found from

$$\eta = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} \tag{9.14}$$

Noting that the two heat transfer processes occur during constant-volume processes, for which the work is zero, there results

$$\dot{Q}_{\text{in}} = \dot{m}c_v(T_3 - T_2) \qquad \dot{Q}_{\text{out}} = \dot{m}c_v(T_4 - T_1) \tag{9.15}$$

where we have assumed each quantity to be positive. Then

$$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} \tag{9.16}$$

This can be written as

$$\eta = 1 - \frac{T_1}{T_2}\frac{T_4/T_1 - 1}{T_3/T_2 - 1} \tag{9.17}$$

For the isentropic processes we have

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \qquad \text{and} \qquad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{k-1} \tag{9.18}$$

But, using $V_1 = V_4$ and $V_3 = V_2$, we see that

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \tag{9.19}$$

Thus, (*9.17*) gives the thermal efficiency as

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{V_2}{V_1}\right)^{k-1} = 1 - \frac{1}{r^{k-1}} \tag{9.20}$$

We see, then, that the thermal efficiency in this idealized cycle is dependent only on the compression ratio r: the higher the compression ratio, the higher the thermal efficiency.

EXAMPLE 9.4 A spark-ignition engine is proposed to have a compression ratio of 10 while operating with a low temperature of 200 °C and a low pressure of 200 kPa. If the work output is to be 1000 kJ/kg, calculate the maximum possible thermal efficiency and compare with that of a Carnot cycle. Also calculate the MEP.

The Otto cycle provides the model for this engine. The maximum possible thermal efficiency for the engine would be

$$\eta = 1 - \frac{1}{r^{k-1}} = 1 - \frac{1}{(10)^{0.4}} = 0.602 \quad \text{or } 60.2\%$$

Since process 1 → 2 is isentropic, we find that

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (473)(10)^{0.4} = 1188\text{ K}$$

The net work for the cycle is given by

$$w_{\text{net}} = w_{1-2} + \overset{0}{\cancel{w_{2-3}}} + w_{3-4} + \overset{0}{\cancel{w_{4-1}}} = c_v(T_1 - T_2) + c_v(T_3 - T_4) \qquad \text{or}$$

$$1000 = (0.717)(473 - 1188 + T_3 - T_4)$$

But, for the isentropic process 3 → 4,

$$T_3 = T_4\left(\frac{v_4}{v_3}\right)^{k-1} = (T_4)(10)^{0.4} = 2.512T_4$$

Solving the last two equations simultaneously, we find $T_3 = 3508$ K and $T_4 = 1397$ K, so that

$$\eta_{\text{carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{473}{3508} = 0.865 \quad \text{or } 86.5\%$$

The Otto cycle efficiency is less than that of a Carnot cycle operating between the limiting temperatures because the heat transfer processes in the Otto cycle are not isothermal.

The MEP is found by using the equation

$$w_{\text{net}} = (\text{MEP})(v_1 - v_2)$$

We have

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(473)}{200} = 0.6788 \text{ m}^3/\text{kg} \qquad \text{and} \qquad v_2 = \frac{v_1}{10}$$

Thus

$$\text{MEP} = \frac{w_{\text{net}}}{v_1 - v_2} = \frac{1000}{(0.9)(0.6788)} = 1640 \text{ kPa}$$

9.6 THE DIESEL CYCLE

If the compression ratio is large enough, the temperature of the air in the cylinder when the piston approaches TDC will exceed the ignition temperature of diesel fuel. This will occur if the compression ratio is about 14 or greater. No external spark is needed; the diesel fuel is simply injected into the cylinder and combustion occurs because of the high temperature of the compressed air. This type of engine is referred to as a *compression-ignition engine*. The ideal cycle used to model the compression-ignition engine is the diesel cycle, shown in Fig. 9-9. The difference between this cycle and the Otto cycle is that, in the diesel cycle, the heat is added during a constant-pressure process.

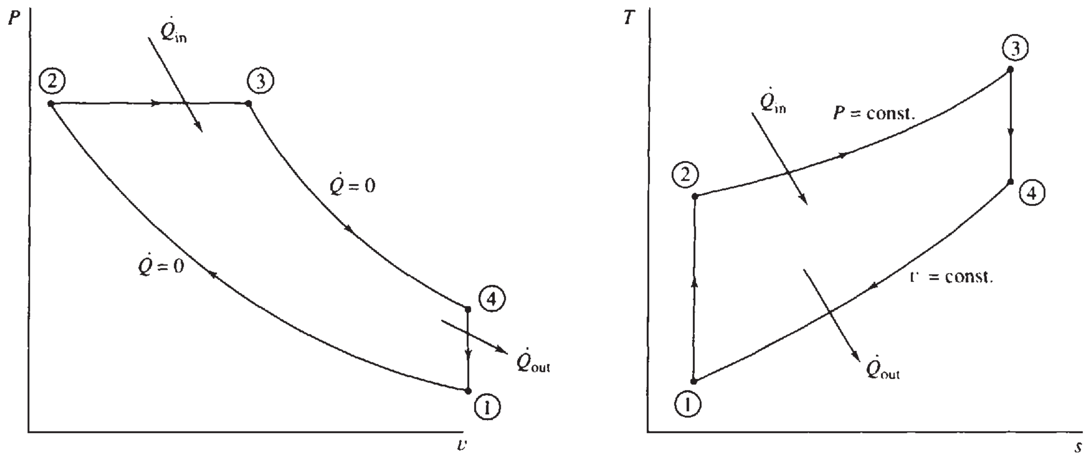


Fig. 9-9

The cycle begins with the piston at BDC, state 1; compression of the air occurs isentropically to state 2 at TDC; heat addition takes place (this models the injection and combustion of fuel) at constant pressure until state 3 is reached; expansion occurs isentropically to state 4 at BDC; constant volume heat rejection completes the cycle and returns the air to the original state. Note that the power stroke includes the heat addition process and the expansion process.

The thermal efficiency of the diesel cycle is expressed as

$$\eta = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} \tag{9.21}$$

For the constant-volume process and the constant-pressure process

$$\dot{Q}_{\text{out}} = \dot{m}c_v(T_4 - T_1) \qquad \dot{Q}_{\text{in}} = \dot{m}c_p(T_3 - T_2) \tag{9.22}$$

The efficiency is then

$$\eta = 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} \tag{9.23}$$

This can be put in the form

$$\eta = 1 - \frac{T_1}{kT_2}\frac{T_4/T_1 - 1}{T_3/T_2 - 1} \tag{9.24}$$

This expression for the thermal efficiency is often written in terms of the compression ratio r and the *cutoff ratio* r_c which is defined as V_3/V_2; there results

$$\eta = 1 - \frac{1}{r^{k-1}}\frac{r_c^k - 1}{k(r_c - 1)} \tag{9.25}$$

From this expression we see that, for a given compression ratio r, the efficiency of the diesel cycle is less than that of an Otto cycle. For example, if $r = 10$ and $r_c = 2$, the Otto cycle efficiency is 60.2 percent and the diesel cycle efficiency is 53.4 percent. As r_c increases, the diesel cycle efficiency decreases. In practice, however, a compression ratio of 20 or so can be achieved in a diesel engine; using $r = 20$ and $r_c = 2$, we would find $\eta = 64.7$ percent. Thus, because of the higher compression ratios, a diesel engine typically operates at a higher efficiency than a gasoline engine.

The decrease in diesel cycle efficiency with an increase in r_c can also be observed by considering the T-s diagram shown in Fig. 9-10. If we increase r_c, the end of the heat input process moves to state 3′. The increased work output is then represented by area 3–3′–4′–4–3. The heat input increases considerably, as represented by area 3–3′–a–b–3. The net effect is a decrease in cycle efficiency, caused

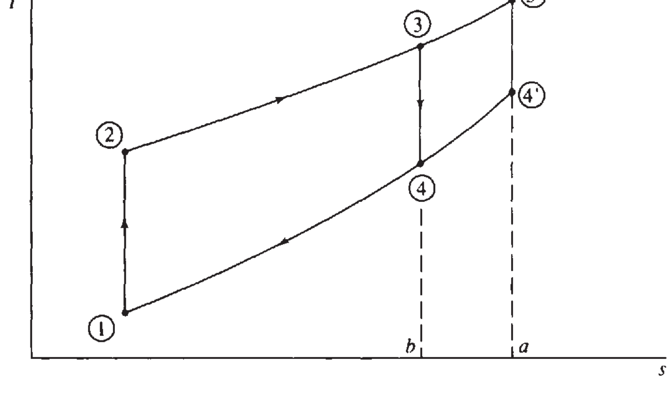


Fig. 9-10

obviously by the convergence of the constant-pressure and constant-volume lines on the T-s diagram. For the Otto cycle note that two constant-volume lines diverge, thereby giving an increase in cycle efficiency with increasing T_3.

EXAMPLE 9.5 A diesel cycle, with a compression ratio of 18 operates on air with a low pressure of 200 kPa and a low temperature 200 °C. If the work output is 1000 kJ/kg, determine the thermal efficiency and the MEP. Also, compare with the efficiency of an Otto cycle operating with the same maximum pressure.

The cutoff ratio r_c is found first. We have

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(473)}{200} = 0.6788 \text{ m}^3/\text{kg} \qquad \text{and} \qquad v_2 = v_1/18 = 0.03771 \text{ m}^3/\text{kg}$$

Since process 1 → 2 is isentropic, we find

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (473)(18)^{0.4} = 1503 \text{ K} \qquad \text{and} \qquad P_2 = P_1\left(\frac{v_1}{v_2}\right)^k = (200)(18)^{1.4} = 11.44 \text{ MPa}$$

The work for the cycle is given by

$$w_{\text{net}} = q_{\text{net}} = q_{2\text{-}3} + q_{4\text{-}1} = c_p(T_3 - T_2) + c_v(T_1 - T_4)$$

$$1000 = (1.00)(T_3 - 1503) + (0.717)(473 - T_4)$$

For the isentropic process 3 → 4 and the constant-pressure process 2 → 3, we have

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = T_3\left(\frac{v_3}{0.6788}\right)^{0.4} \qquad \frac{T_3}{v_3} = \frac{T_2}{v_2} = \frac{1503}{0.03771} = 39\,860$$

The last three equations can be combined to yield

$$1000 = (1.00)(39\,860v_3 - 1503) + (0.717)(473 - 46\,540v_3^{1.4})$$

This equation is solved by trial and error to give

$$v_3 = 0.0773 \text{ m}^3/\text{kg} \qquad \therefore T_3 = 3080 \text{ K} \qquad T_4 = 1290 \text{ K}$$

This gives the cutoff ratio as $r_c = v_3/v_2 = 2.05$. The thermal efficiency is now calculated as

$$\eta = 1 - \frac{1}{r^{k-1}}\frac{r_c^k - 1}{k(r_c - 1)} = 1 - \frac{1}{(18)^{0.4}}\frac{(2.05)^{1.4} - 1}{(1.4)(2.05 - 1)} = 0.629 \quad \text{or} \quad 62.9\%$$

Also, MEP $= w_{\text{net}}/(v_1 - v_2) = 1000/(0.6788 - 0.0377) = 641$ kPa.

For the comparison Otto cycle,

$$r_{\text{otto}} = v_1/v_3 = \frac{0.6788}{0.0773} = 8.78 \qquad \eta_{\text{otto}} = 1 - \frac{1}{r^{k-1}} = 0.581 \quad \text{or } 58.1\%$$

9.7. THE DUAL CYCLE

An ideal cycle that better approximates the actual performance of a compression-ignition engine is the *dual cycle*, in which the combustion process is modeled by two heat-addition processes: a constant-volume process and a constant-pressure process, as shown in Fig. 9-11. The thermal efficiency is found from

$$\eta = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} \tag{9.26}$$

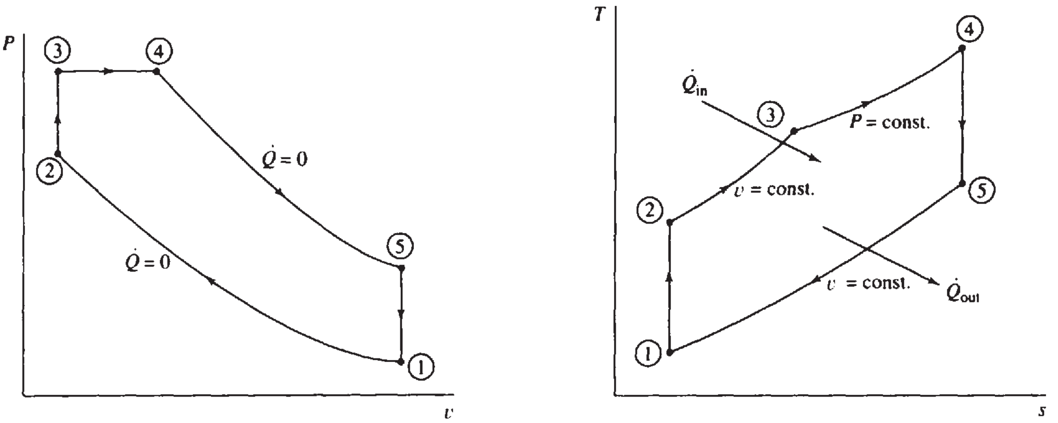


Fig. 9-11

where

$$\dot{Q}_{out} = \dot{m}c_v(T_5 - T_1) \qquad \dot{Q}_{in} = \dot{m}c_v(T_3 - T_2) + \dot{m}c_p(T_4 - T_3) \tag{9.27}$$

Hence, we have

$$\eta = 1 - \frac{T_5 - T_1}{T_3 - T_2 + k(T_4 - T_3)} \tag{9.28}$$

If we define the *pressure ratio* $r_p = P_3/P_2$, the thermal efficiency can be expressed as

$$\eta = 1 - \frac{1}{r^{k-1}} \frac{r_p r_c^k - 1}{kr_p(r_c - 1) + r_p - 1} \tag{9.29}$$

If we let $r_p = 1$, the diesel cycle efficiency results; if we let $r_c = 1$, the Otto cycle efficiency results. If $r_p > 1$, the thermal efficiency will be less than the Otto cycle efficiency but greater than the diesel cycle efficiency.

EXAMPLE 9.6 A dual cycle, which operates on air with a compression ratio of 16, has a low pressure of 200 kPa and a low temperature of 200 °C. If the cutoff ratio is 2 and the pressure ratio is 1.3, calculate the thermal efficiency, the heat input, the work output, and the MEP.

By (*9.29*),

$$\eta = 1 - \frac{1}{(16)^{0.4}} \frac{(1.3)(2)^{1.4} - 1}{(1.4)(1.3)(2-1) + 1.3 - 1} = 0.622 \quad \text{or} \quad 62.2\%$$

The heat input is found from $q_{in} = c_v(T_3 - T_2) + c_p(T_4 - T_3)$, where

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (473)(16)^{0.4} = 1434\,\text{K} \qquad T_3 = T_2\frac{P_3}{P_2} = (1434)(1.3) = 1864\,\text{K}$$

$$T_4 = T_3\frac{v_4}{v_3} = (1864)(2) = 3728\,\text{K}$$

Thus, $q_{in} = (0.717)(1864 - 1434) + (1.00)(3728 - 1864) = 2172$ kJ/kg. The work output is found from

$$w_{out} = \eta q_{in} = (0.622)(2172) = 1350\,\text{kJ/kg}$$

Finally, since

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(473)}{200} = 0.6788\,\text{m}^3/\text{kg}$$

we have

$$\text{MEP} = \frac{w_{out}}{v_1(1 - v_2/v_1)} = \frac{1350}{(0.6788)(15/16)} = 2120\,\text{kPa}$$

9.8 THE STIRLING AND ERICSSON CYCLES

The Stirling and Ericsson cycles, although not extensively used to model actual engines, are presented to illustrate the effective use of a *regenerator*, a heat exchanger which utilizes waste heat. A schematic diagram is shown in Fig. 9-12. Note that for both the constant-volume processes of the Stirling cycle (Fig. 9-13) and the constant-pressure processes of the Ericsson cycle (Fig. 9-14) the heat transfer q_{2-3} required by the gas is equal in magnitude to the heat transfer q_{4-1} discharged by the gas.

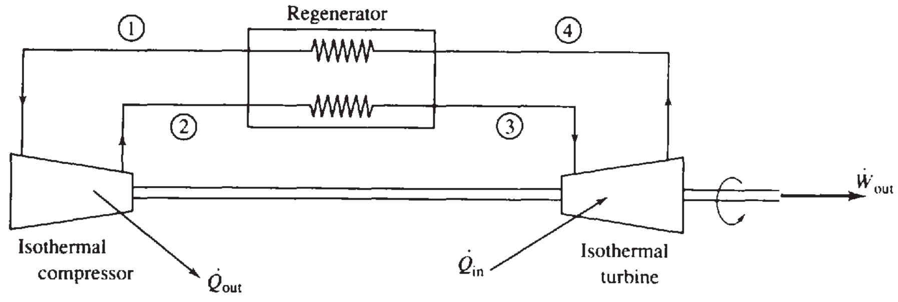


Fig. 9-12

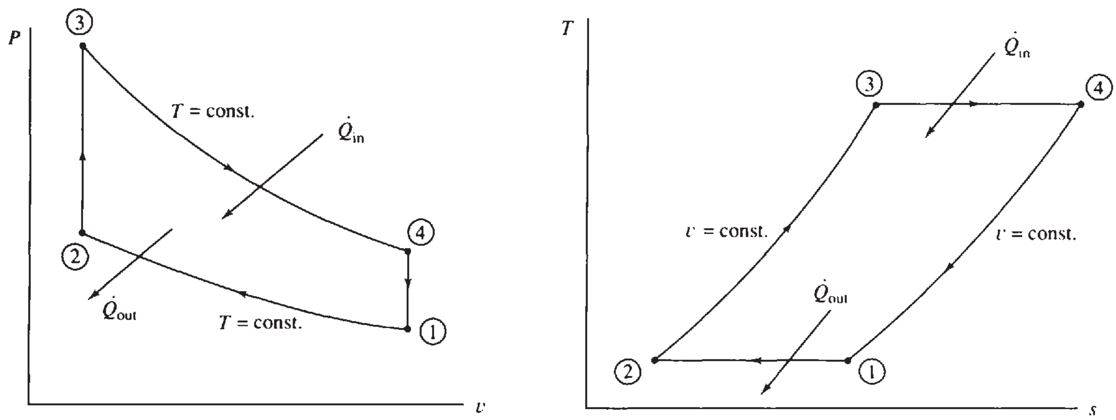


Fig. 9-13 Stirling cycle

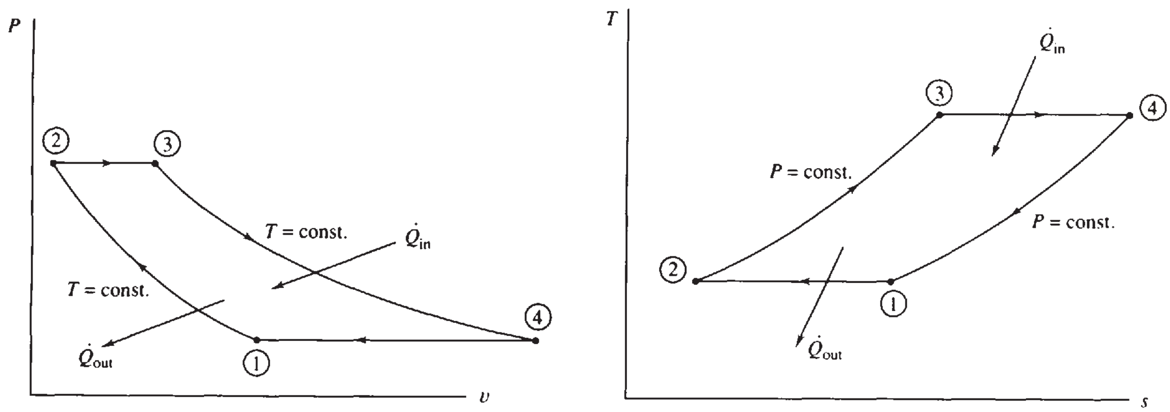


Fig. 9-14 Ericsson cycle

This suggests the use of a regenerator that will, internally to the cycle, transfer the otherwise wasted heat from the air during the process $4 \to 1$ to the air during the process $2 \to 3$. The net result of this is that the thermal efficiency of each of the two ideal cycles shown equals that of a Carnot cycle operating between the same two temperatures. This is obvious because the heat transfer in and out of each cycle occurs at constant temperature. Thus, the thermal efficiency is

$$\eta = 1 - \frac{T_L}{T_H} \tag{9.30}$$

Note that the heat transfer (the purchased energy) needed for the turbine can be supplied from outside an actual engine, that is, external combustion. Such external combustion engines have lower emissions but have not proved to be competitive with the Otto and diesel cycle engines because of problems inherent in the regenerator design and the isothermal compressor and turbine.

EXAMPLE 9.7 A Stirling cycle operates on air with a compression ratio of 10. If the low pressure is 30 psia, the low temperature is 200 °F, and the high temperature is 1000 °F, calculate the work output and the heat input.

For the Stirling cycle the work output is

$$w_{\text{out}} = w_{3\text{-}4} + w_{1\text{-}2} = RT_3 \ln \frac{v_4}{v_3} + RT_1 \ln \frac{v_2}{v_1} = (53.3)(1460 \ln 10 + 660 \ln 0.1) = 98{,}180 \text{ ft-lbf/lbm}$$

where we have used (*4.36*) for the isothermal process. Consequently,

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{660}{1460} = 0.548 \qquad q_{\text{in}} = \frac{w_{\text{out}}}{\eta} = \frac{98{,}180/778}{0.548} = 230 \text{ Btu/lbm}$$

EXAMPLE 9.8 An Ericsson cycle operates on air with a compression ratio of 10. For a low pressure of 200 kPa, a low 100 °C, and a high temperature of 600 °C, calculate the work output and the heat input.

For the Ericsson cycle the work output is

$$w_{\text{out}} = w_{1\text{-}2} + w_{2\text{-}3} + w_{3\text{-}4} + w_{4\text{-}1} = RT_1 \ln \frac{v_2}{v_1} + P_2(v_3 - v_2) + RT_3 \ln \frac{v_4}{v_3} + P_1(v_1 - v_4)$$

We must calculate P_2, v_1, v_2, and v_4. We know

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(373)}{200} = 0.5353 \text{ m}^3/\text{kg}$$

For the constant-pressure process $4 \to 1$,

$$\frac{T_4}{v_4} = \frac{T_1}{v_1} \qquad \frac{873}{v_4} = \frac{373}{0.5353} \qquad v_4 = 1.253 \text{ m}^3/\text{kg}$$

From the definition of the compression ratio, $v_4/v_2 = 10$, giving $v_2 = 0.1253$ m^3/kg. Using the ideal-gas law, we have

$$P_3 = P_2 = \frac{RT_2}{v_2} = \frac{(0.287)(373)}{0.1253} = 854.4 \text{ kPa}$$

The final necessary property is $v_3 = RT_3/P_3 = (0.287)(873)/854.4 = 0.2932$ m^3/kg. The expression for work output gives

$$w_{\text{out}} = (0.287)(373) \ln \frac{0.1253}{0.5353} + (854.4)(0.2932 - 0.1253)$$

$$+ 0.287 \times 873 \ln \frac{1.253}{0.2932} + (200)(0.5353 - 1.253) = 208 \text{ kJ/kg}$$

Finally,

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{378}{873} = 0.573 \qquad q_{\text{in}} = \frac{w_{\text{out}}}{\eta} = \frac{208}{0.573} = 364 \text{ kJ/kg}$$

9.9 THE BRAYTON CYCLE

The gas turbine is another mechanical system that produces power. It may operate on an open cycle when used as a tank engine or truck engine, or on a closed cycle when used in a nuclear power plant. In open cycle operation, air enters the compressor, passes through a constant-pressure combustion chamber, passes through a turbine, and then exits as products of combustion to the atmosphere, as shown in Fig. 9-15*a*. In closed cycle operation the combustion chamber is replaced with a heat exchanger in which energy enters the cycle from some exterior source; an additional heat exchanger transfers heat from the cycle so that the air is returned to its initial state, as shown in Fig. 9-15*b*.

The ideal cycle used to model the gas turbine is the Brayton cycle. It utilizes isentropic compression and expansion, as indicated in Fig. 9-16. The efficiency of such a cycle is given by

$$\eta = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_1}{T_2}\frac{T_4/T_1 - 1}{T_3/T_2 - 1} \tag{9.31}$$

Using the isentropic relations

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} \qquad \frac{P_3}{P_4} = \left(\frac{T_3}{T_4}\right)^{k/(k-1)} \tag{9.32}$$

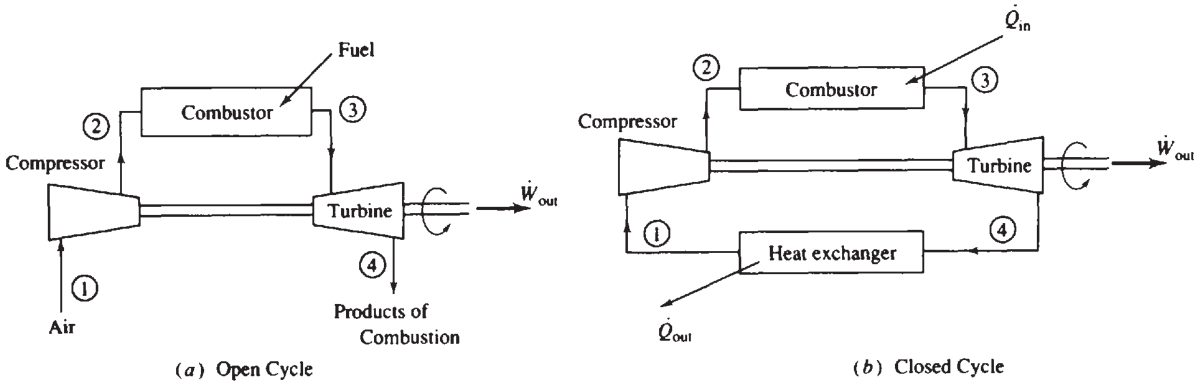


Fig. 9-15

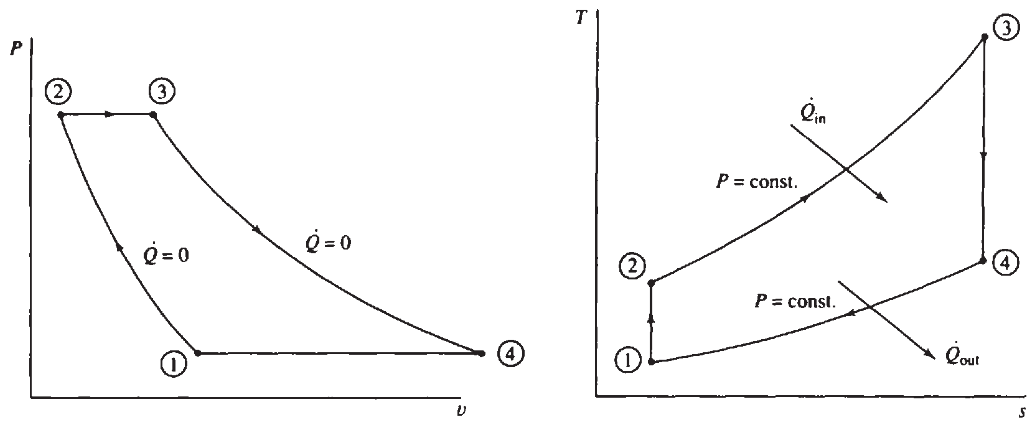


Fig. 9-16

and observing that $P_2 = P_3$ and $P_1 = P_4$, we see that

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2} \tag{9.33}$$

Hence, the thermal efficiency can be written as

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{P_1}{P_2}\right)^{(k-1)/k} \tag{9.34}$$

In terms of the pressure ratio $r_p = P_2/P_1$ the thermal efficiency is

$$\eta = 1 - r_p^{(1-k)/k} \tag{9.35}$$

Of course, this expression for thermal efficiency was obtained using constant specific heats. For more accurate calculations the gas tables should be used.

In an actual gas turbine the compressor and the turbine are not isentropic; some losses do occur. These losses, usually in the neighborhood of 15 percent, significantly reduce the efficiency of the gas turbine engine.

Another important feature of the gas turbine that seriously limits thermal efficiency is the high work requirement of the compressor, measured by the *back work ratio* $\dot{W}_{comp}/\dot{W}_{turb}$. The compressor may require up to 80 percent of the turbine's output (a back work ratio of 0.8), leaving only 20 percent for net work output. This relatively high limit is experienced when the efficiencies of the compressor and turbine are too low. Solved problems illustrate this point.

EXAMPLE 9.9 Air enters the compressor of a gas turbine at 100 kPa and 25 °C. For a pressure ratio of 5 and a maximum temperature of 850 °C determine the back work ratio and the thermal efficiency using the Brayton cycle.

To find the back work ratio we observe that

$$\frac{w_{comp}}{w_{turb}} = \frac{c_p(T_2 - T_1)}{c_p(T_3 - T_4)} = \frac{T_2 - T_1}{T_3 - T_4}$$

The temperatures are $T_1 = 298$ K, $T_3 = 1123$ K, and

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (298)(5)^{0.2857} = 472.0\text{ K} \qquad T_4 = T_3\left(\frac{P_4}{P_5}\right)^{(k-1)/k} = (1123)\left(\frac{1}{5}\right)^{0.2857} = 709.1\text{ K}$$

The back work ratio is then

$$\frac{w_{comp}}{w_{turb}} = \frac{472.0 - 298}{1123 - 709} = 0.420 \quad \text{or } 42.0\%$$

The thermal efficiency is $\eta = 1 - r^{(1-k)/k} = 1 - (5)^{-0.2857} = 0.369$ (36.9%).

EXAMPLE 9.10 Assume the compressor and the gas turbine in Example 9.9 both have an efficiency of 80 percent. Using the Brayton cycle determine the back work ratio and the thermal efficiency.

We can calculate the quantities asked for if we determine w_{comp}, w_{turb}, and q_{in}. The compressor work is

$$w_{comp} = \frac{w_{comp,s}}{\eta_{comp}} = \frac{c_p}{\eta_{comp}}(T_{2'} - T_1)$$

where $w_{comp,s}$ is the isentropic work. $T_{2'}$ is the temperature of state 2′ assuming an isentropic process; state 2 is the actual state. We then have, using $T_{2'} = T_2$ from Example 9.9,

$$w_{comp} = \left(\frac{1.00}{0.8}\right)(472 - 298) = 217.5\text{ kJ/kg}$$

Likewise, there results $w_{turb} = \eta_{turb} w_{turb,s} = \eta_{turb} c_p(T_3 - T_{4'}) = (0.8)(1.00)(1123 - 709.1) = 331.1$ kJ/kg, where $T_{4'}$ is T_4 as calculated in Example 9.9. State 4 is the actual state and state 4′ is the isentropic state. The back work ratio is then

$$\frac{w_{comp}}{w_{turb}} = \frac{217.5}{331.1} = 0.657 \quad \text{or } 65.7\%$$

The heat transfer input necessary in this cycle is $q_{in} = h_3 - h_2 = c_p(T_3 - T_2)$, where T_2 is the actual temperature of the air leaving the compressor. It is found by returning to the compressor:

$$w_{comp} = c_p(T_2 - T_1) \qquad 217.5 = (1.00)(T_2 - 298) \qquad \therefore T_2 = 515.5 \text{ K}$$

Thus, $q_{in} = (1.00)(1123 - 515.5) = 607.5$ kJ/kg. The thermal efficiency of the cycle can then be written as

$$\eta = \frac{w_{net}}{q_{in}} = \frac{w_{turb} - w_{comp}}{q_{in}} = \frac{331.1 - 217.5}{607.5} = 0.187 \quad \text{or } 18.7\%$$

9.10 THE REGENERATIVE GAS-TURBINE CYCLE

The heat transfer from the simple gas-turbine cycle of the previous section is simply lost to the surroundings—either directly, with the products of combustion, or from a heat exchanger. Some of this exit energy can be utilized since the temperature of the flow exiting the turbine is greater than the temperature of the flow entering the compressor. A counterflow heat exchanger, a regenerator, is used to transfer some of this energy to the air leaving the compressor, as shown in Fig. 9-17. For an ideal regenerator the exit temperature T_3 would equal the entering temperature T_5; and, similarly, T_2 would equal T_6. Since less energy is rejected from the cycle, the thermal efficiency is expected to increase. It is given by

$$\eta = \frac{w_{turb} - w_{comp}}{q_{in}} \tag{9.36}$$

Using the first law, expressions for q_{in} and w_{turb} are found to be

$$q_{in} = c_p(T_4 - T_3) \qquad w_{turb} = c_p(T_4 - T_5) \tag{9.37}$$

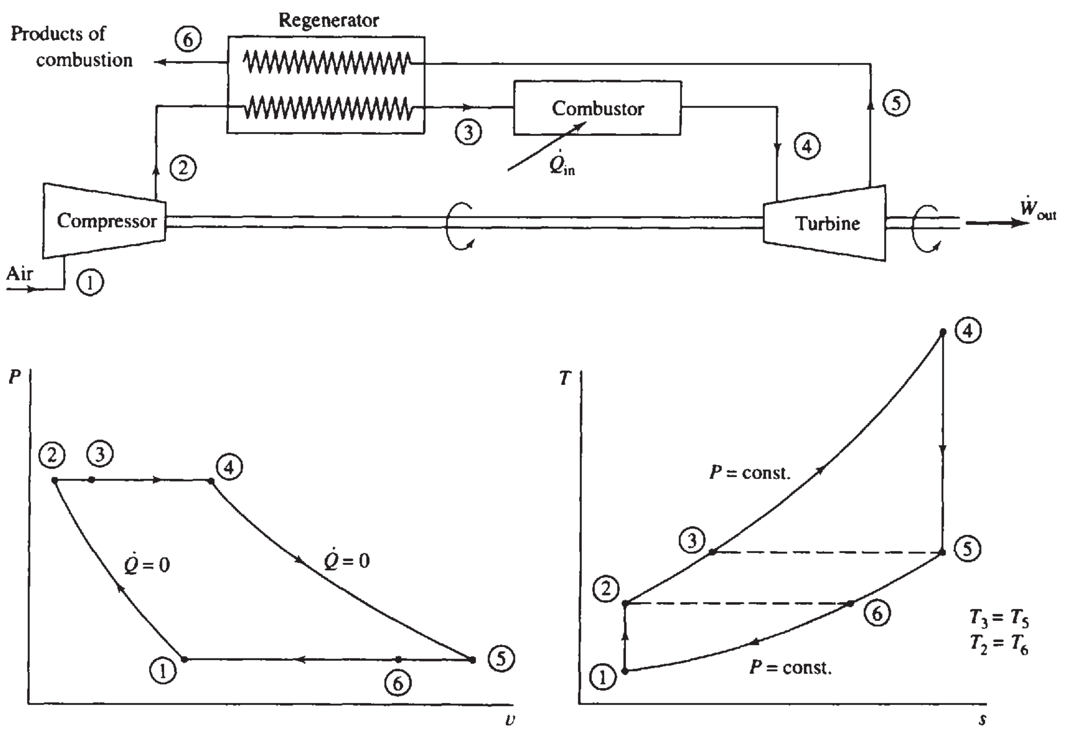


Fig. 9-17

Hence, for the ideal regenerator in which $T_3 = T_5$, $q_{in} = w_{turb}$ and the thermal efficiency can be written as

$$\eta = 1 - \frac{w_{comp}}{w_{turb}} = 1 - \frac{T_2 - T_1}{T_4 - T_5} = 1 - \frac{T_1}{T_4}\frac{T_2/T_1 - 1}{1 - T_5/T_4} \tag{9.38}$$

Using the appropriate isentropic relation, this can be written in the form

$$\eta = 1 - \frac{T_1}{T_4}\frac{(P_2/P_1)^{(k-1)/k} - 1}{1 - (P_1/P_2)^{(k-1)/k}} = 1 - \frac{T_1}{T_4} r_p^{(k-1)/k} \tag{9.39}$$

Note that this expression for thermal efficiency is quite different from that for the Brayton cycle. For a given pressure ratio, the efficiency increases as the ratio of minimum to maximum temperature decreases. But, perhaps more surprisingly, as the pressure ratio increases the efficiency decreases, an effect opposite to that of the Brayton cycle. Hence it is not surprising that for a given regenerative cycle temperature ratio, there is a particular pressure ratio for which the efficiency of the Brayton cycle will equal the efficiency of the regenerative cycle. This is shown for a temperature ratio of 0.25 in Fig. 9-18.

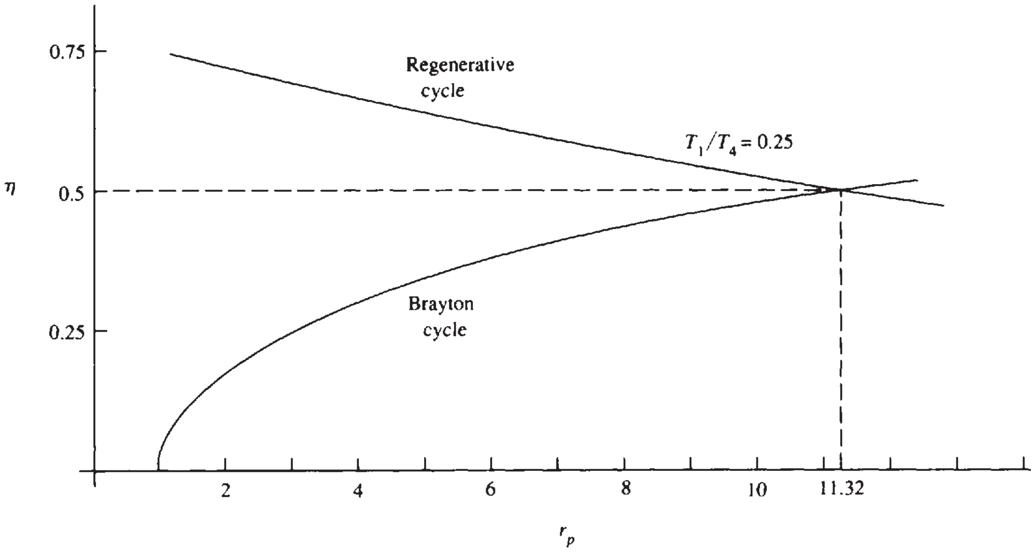


Fig. 9-18

In practice the temperature of the air leaving the regenerator at state 3 must be less than the temperature of the air entering at state 5. Also, $T_6 > T_2$. The effectiveness, or efficiency, of a regenerator is measured by

$$\eta_{reg} = \frac{h_3 - h_2}{h_5 - h_2} \tag{9.40}$$

This is equivalent to

$$\eta_{reg} = \frac{T_3 - T_2}{T_5 - T_2} \tag{9.41}$$

if we assume an ideal gas with constant specific heats. Obviously, for the ideal regenerator $T_3 = T_5$ and $\eta_{reg} = 1$. Regenerator efficiencies exceeding 80 percent are common.

EXAMPLE 9.11 Add an ideal regenerator to the gas-turbine cycle of Example 9.9 and calculate the thermal efficiency and the back work ratio.

The thermal efficiency is found using (*9.39*):

$$\eta = 1 - \frac{T_1}{T_4}\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = 1 - \left(\frac{298}{1123}\right)(5)^{0.2857} = 0.580 \quad \text{or } 58.0\%$$

This represents a 57 percent increase in efficiency, a rather large effect. Note that, for the information given, the back work ratio does not change; hence, $w_{comp}/w_{turb} = 0.420$.

9.11 THE INTERCOOLING, REHEAT, REGENERATIVE GAS-TURBINE CYCLE

In addition to the regenerator of the previous section there are two other common techniques for increasing the thermal efficiency of the gas turbine cycle. First, an intercooler can be inserted into the compression process; air is compressed to an intermediate pressure, cooled in an intercooler, and then compressed to the final pressure. This reduces the work required for the compressor, as was discussed in Sec. 9.2, and it reduces the maximum temperature reached in the cycle. The intermediate pressure is determined by equating the pressure ratio for each stage of compression; that is, referring to Fig. 9.19 [see (*9.8*)],

$$\frac{P_2}{P_1} = \frac{P_4}{P_3} \tag{9.42}$$

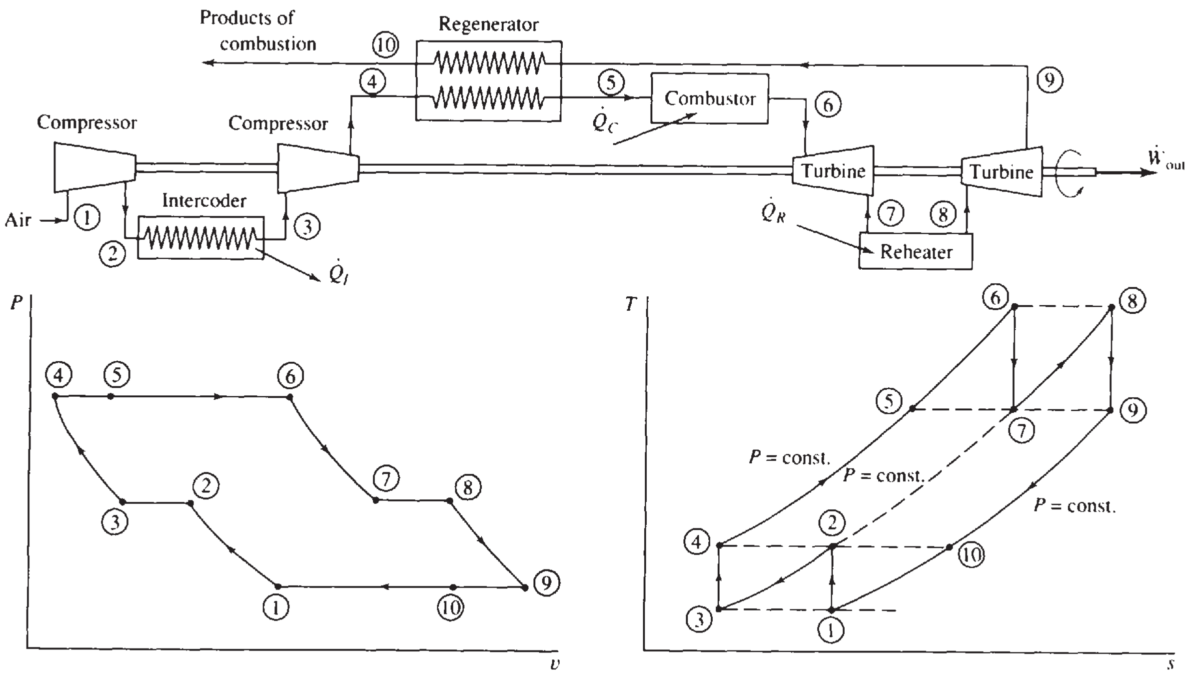


Fig. 9-19

The second technique for increasing thermal efficiency is to use a second combustor, called a *reheater*. The intermediate pressure is determined as in the compressor; we again require that the ratios be equal; that is,

$$\frac{P_6}{P_7} = \frac{P_8}{P_9} \tag{9.43}$$

Since $P_9 = P_1$ and $P_6 = P_4$, we see that the intermediate turbine pressure is equal to the intermediate compressor pressure for our ideal-gas turbine.

Finally, we should note that intercooling and reheating are never used without regeneration. In fact, if regeneration is not employed, intercooling and reheating *reduce* the efficiency of a gas-turbine cycle.

EXAMPLE 9.12 Add an ideal intercooler, reheater, and regenerator to the gas-turbine cycle of Example 9.9 and calculate the thermal efficiency. Keep all given quantities the same.

The intermediate pressure is found to be $P_2 = \sqrt{P_1 P_4} = \sqrt{(100)(500)} = 223.6$ kPa. Hence, for the ideal isentropic process,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (298)\left(\frac{223.6}{100}\right)^{0.2857} = 375.0 \text{ K}$$

The maximum temperature $T_6 = T_8 = 1123$ K. Using $P_7 = P_2$ and $P_6 = P_4$, we have

$$T_7 = T_6\left(\frac{P_7}{P_6}\right)^{(k-1)/k} = (1123)\left(\frac{223.6}{500}\right)^{0.2857} = 892.3 \text{ K}$$

Now all the temperatures in the cycle are known and the thermal efficiency can be calculated as

$$\eta = \frac{w_{out}}{q_{in}} = \frac{w_{turb} - w_{comp}}{q_C + q_R} = \frac{c_p(T_6 - T_7) + c_p(T_8 - T_9) - c_p(T_2 - T_1) - c_p(T_4 - T_3)}{c_p(T_6 - T_5) + c_p(T_8 - T_7)}$$

$$= \frac{230.7 + 230.7 - 77.0 - 77.0}{230.7 + 230.7} = 0.666 \quad \text{or } 66.6\%$$

This represents a 14.9 percent increase over the cycle of Example 9.11 with only a regenerator, and an 80.5 percent increase over the simple gas-turbine cycle. Obviously, losses in the additional components must be considered for any actual situation.

9.12 THE TURBOJET ENGINE

The turbojet engines of modern commercial aircraft utilize gas-turbine cycles as the basis for their operation. Rather than producing power, however, the turbine is sized to provide just enough power to drive the compressor. The energy that remains is used to increase the kinetic energy of the exiting exhaust gases by passing the gases through an exhaust nozzle thereby providing thrust to the aircraft. Assuming that all of the air entering the engine passes through the turbine and out the exhaust nozzle, as shown in Fig. 9-20, the net thrust on the aircraft due to one engine is

$$\text{thrust} = \dot{m}(V_5 - V_1) \qquad (9.44)$$

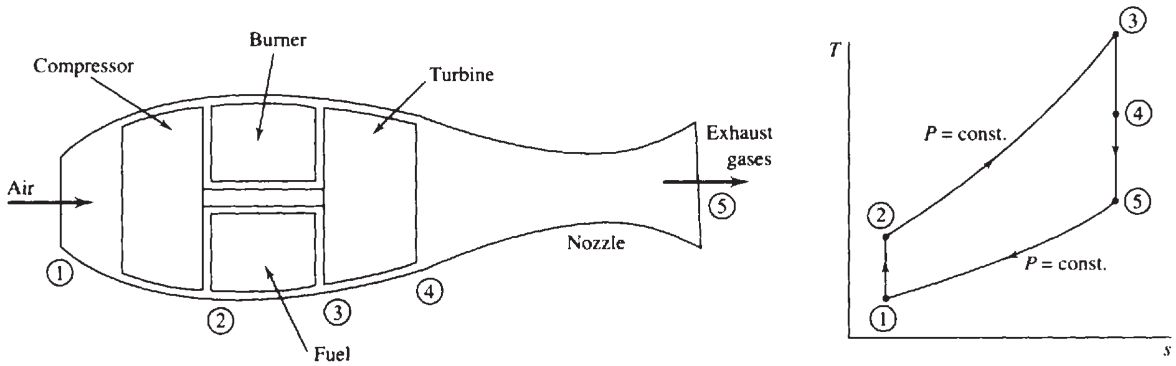


Fig. 9-20

where $\dot{m}$ is the mass flux of air passing through the engine. The mass flux of fuel is assumed to be negligibly small. In our ideal engine we assume that the pressures at section 1 and section 5 are equal to atmospheric pressure and that the velocity at section 1 is equal to the aircraft speed. A solved problem will illustrate the calculations for this application.

EXAMPLE 9.13 A turbojet engine inlets 100 lbm/sec of air at 5 psia and $-50\,°F$ with a velocity of 600 ft/sec. The compressor discharge pressure is 50 psia and the turbine inlet temperature is 2000 °F. Calculate the thrust and the horsepower developed by the engine.

To calculate the thrust we must first calculate the exit velocity. To do this we must know the temperatures T_4 and T_5 exiting the turbine and the nozzle, respectively. Then the energy equation can be applied across the nozzle as

$$\overbrace{\frac{V_4^2}{2}}^{\text{neglect}} + h_4 = \frac{V_5^2}{2} + h_5 \quad \text{or} \quad V_5^2 = 2c_p(T_4 - T_5)$$

Let us find the temperatures T_4 and T_5. The temperature T_2 is found to be (using $T_1 = 410\,°R$)

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (410)\left(\frac{50}{5}\right)^{0.2857} = 791.6\,°R$$

Since the work from the turbine equals the work required by the compressor, we have $h_2 - h_1 = h_3 - h_4$ or $T_3 - T_4 = T_2 - T_1$. Thus, $T_4 = 2460 - (791.6 - 410) = 2078\,°R$. The isentropic expansion through the turbine yields

$$P_4 = P_3\left(\frac{T_4}{T_3}\right)^{k/(k-1)} = (50)\left(\frac{2078}{2460}\right)^{3.5} = 27.70\,\text{psia}$$

The temperature T_5 at the nozzle exit where $P_5 = 5$ psia is found, assuming isentropic nozzle expansion, to be

$$T_5 = T_4\left(\frac{P_5}{P_4}\right)^{(k-1)/k} = (2078)\left(\frac{5}{27.7}\right)^{0.2857} = 1274\,°R$$

The energy equation then gives

$$V_5 = \left[2c_p(T_4 - T_5)\right]^{1/2} = \left[(2)(0.24)(778)(32.2)(2078 - 1274)\right]^{1/2} = 3109\,\text{ft/sec}$$

[Note: We use $c_p = (0.24\,\text{Btu/lbm-°R}) \times (778\,\text{ft-lbf/Btu}) \times (32.2\,\text{lbm-ft/lbf-sec}^2)$. This provides the appropriate units for c_p.]

The thrust is: thrust $= \dot{m}(V_5 - V_1) = (100/32.2)(3109 - 600) = 7790$ lbf. The horsepower is

$$\text{hp} = \frac{(\text{thrust})(\text{velocity})}{550} = \frac{(7790)(600)}{550} = 8500\,\text{hp}$$

where we have used the conversion 550 ft-lbf/sec = 1 hp.

9.13 THE COMBINED BRAYTON-RANKINE CYCLE

The Brayton cycle efficiency is quite low primarily because a substantial amount of the energy input is exhausted to the surroundings. This exhausted energy is usually at a relatively high temperature and thus it can be used effectively to produce power. One possible application is the combined Brayton-Rankine cycle in which the high-temperature exhaust gases exiting the gas turbine are used to supply energy to the boiler of the Rankine cycle, as illustrated in Fig. 9-21. Note that the temperature T_9 of the Brayton cycle gases exiting the boiler is less than the temperature T_3 of the Rankine cycle steam exiting the boiler; this is possible in the counterflow heat exchanger, the boiler.

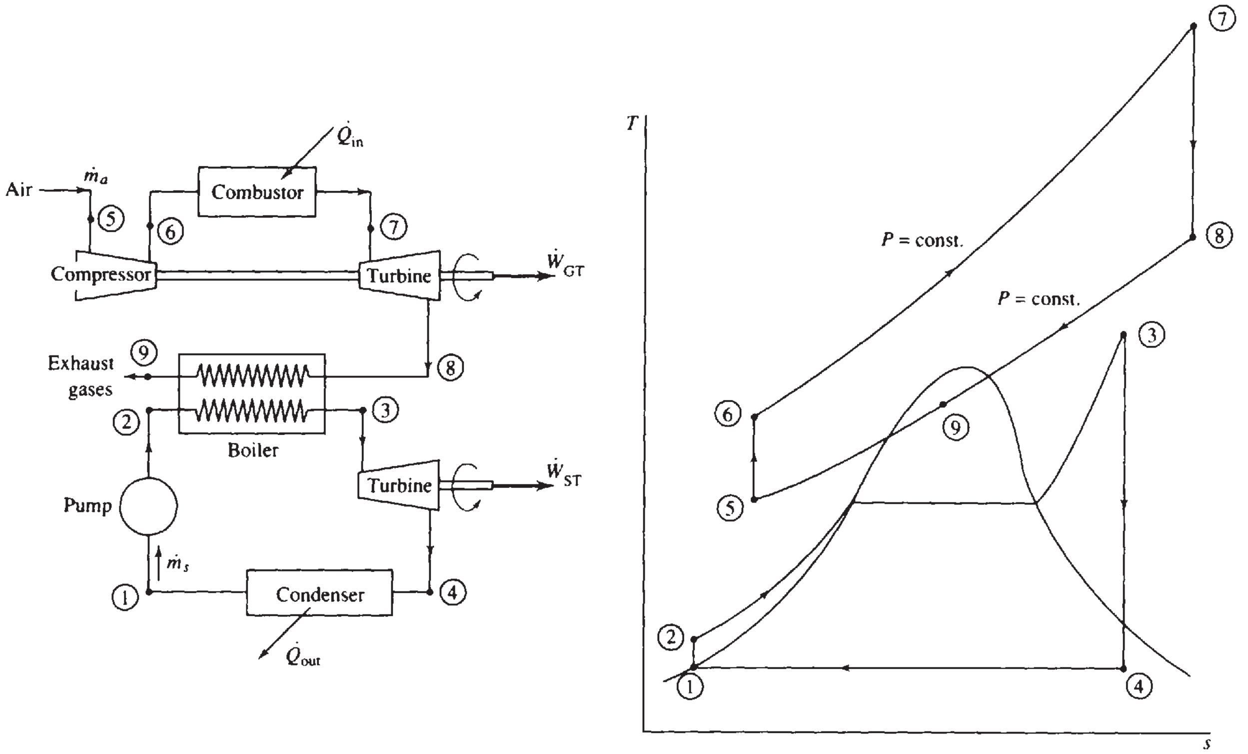


Fig. 9-21

To relate the air mass flux $\dot{m}_a$ of the Brayton cycle to the steam mass flux $\dot{m}_s$ of the Rankine cycle, we use an energy balance in the boiler; it gives (see Fig. 9-21),

$$\dot{m}_a(h_8 - h_9) = \dot{m}_s(h_3 - h_2) \tag{9.45}$$

assuming no additional energy addition in the boiler, which would be possible with an oil burner, for example.

The cycle efficiency would be found by considering the purchased energy as $\dot{Q}_{in}$, the energy input in the combustor. The output is the sum of the net output $\dot{W}_{GT}$ from the gas turbine and the output $\dot{W}_{ST}$ from the steam turbine. The combined cycle efficiency is thus given by

$$\eta = \frac{\dot{W}_{GT} + \dot{W}_{ST}}{\dot{Q}_{in}} \tag{9.46}$$

An example will illustrate the increase in efficiency of such a combined cycle.

EXAMPLE 9.14 A simple steam power plant operates between pressures of 10 kPa and 4 MPa with a maximum temperature of 400 °C. The power output from the steam turbine is 100 MW. A gas turbine provides the energy to the boiler; it accepts air at 100 kPa and 25 °C, has a pressure ratio of 5, and a maximum temperature of 850 °C. The exhaust gases exit the boiler at 350 K. Determine the thermal efficiency of the combined Brayton-Rankine cycle.

If we neglect the work of the pump, the enthalpy remains unchanged across the pump. Hence, $h_2 = h_1 = 192$ kJ/kg. At 400 °C and 4 MPa we have $h_3 = 3214$ kJ/kg and $s_3 = 6.7698$ kJ/kg · K. State 4 is located by noting that $s_4 = s_3$ so that the quality is

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.798 - 0.6491}{7.5019} = 0.8159$$

Thus, $h_4 = h_f + x_4 h_{fg} = 192 + (0.8159)(2393) = 2144$ kJ/kg. The steam mass flux is found using the turbine output as follows:

$$\dot{W}_{ST} = \dot{m}_s(h_3 - h_4) \qquad 100\,000 = \dot{m}_s(3214 - 2144) \qquad \dot{m}_s = 93.46 \text{ kg/s}$$

Considering the gas-turbine cycle,

$$T_6 = T_5\left(\frac{P_6}{P_5}\right)^{(k-1)/k} = (298)(5)^{0.2857} = 472.0 \text{ K}$$

Also,

$$T_8 = T_7\left(\frac{P_8}{P_7}\right)^{(k-1)/k} = (1123)\left(\frac{1}{5}\right)^{0.2857} = 709.1 \text{ K}$$

Thus we have, for the boiler,

$$\dot{m}_s(h_3 - h_2) = \dot{m}_a c_p(T_8 - T_9) \qquad (93.46)(3214 - 192) = (\dot{m}_a)(1.00)(709.1 - 350)$$

$$\dot{m}_a = 786.5 \text{ kg/s}$$

The output of the gas turbine is (note that this is not $\dot{W}_{GT}$)

$$\dot{W}_{turb} = \dot{m}_a c_p(T_7 - T_8) = (786.5)(1.00)(1123 - 709.1) = 325.5 \text{ MW}$$

The energy needed by the compressor is

$$\dot{W}_{comp} = \dot{m}_a c_p(T_6 - T_5) = (786.5)(1.00)\,(472 - 298) = 136.9 \text{ MW}$$

Hence, the net gas turbine output is $\dot{W}_{GT} = \dot{W}_{turb} - \dot{W}_{comp} = 325.5 - 136.9 = 188.6$ MW. The energy input by the combustor is

$$\dot{Q}_{in} = \dot{m}_a c_p(T_7 - T_6) = (786.5)(1.00)(1123 - 472) = 512 \text{ MW}$$

The above calculations allow us to determine the combined cycle efficiency as

$$\eta = \frac{\dot{W}_{ST} + \dot{W}_{GT}}{\dot{Q}_{in}} = \frac{100 + 188.6}{512} = 0.564 \quad \text{or } 56.4\%$$

Note that this efficiency is 59.3 percent higher than the Rankine cycle (see Example 8.2) and 52.8 percent higher than the Brayton cycle (see Example 9.9). Cycle efficiency could be increased even more by using steam reheaters, steam regenerators, gas intercoolers, and gas reheaters.

9.14. THE GAS REFRIGERATION CYCLE

If the flow of the gas is reversed in the Brayton cycle of Sec. 9.9, the gas undergoes an isentropic expansion process as it flows through the turbine, resulting in a substantial reduction in temperature, as shown in Fig. 9-22. The gas with low turbine exit temperature can be used to refrigerate a space to temperature T_2 by extracting heat at rate $\dot{Q}_{in}$ from the refrigerated space.

Figure 9-22 illustrates a *closed* refrigeration cycle. (An *open* cycle system is used in aircraft; air is extracted from the atmosphere at state 2 and inserted into the passenger compartment at state 1. This provides both fresh air and cooling.) An additional heat exchanger may be used, like the regenerator of the Brayton power cycle, to provide an even lower turbine exit temperature, as illustrated in Fig. 9-23. The gas does not enter the expansion process (the turbine) at state 5; rather, it passes through an *internal* heat exchanger (it does not exchange heat with the surroundings). This allows the temperature of the gas entering the turbine to be much lower than that of Fig. 9-22. The temperature T_1 after the expansion is so low that gas liquefication is possible. It should be noted, however, that the coefficient of performance is actually reduced by the inclusion of an internal heat exchanger.

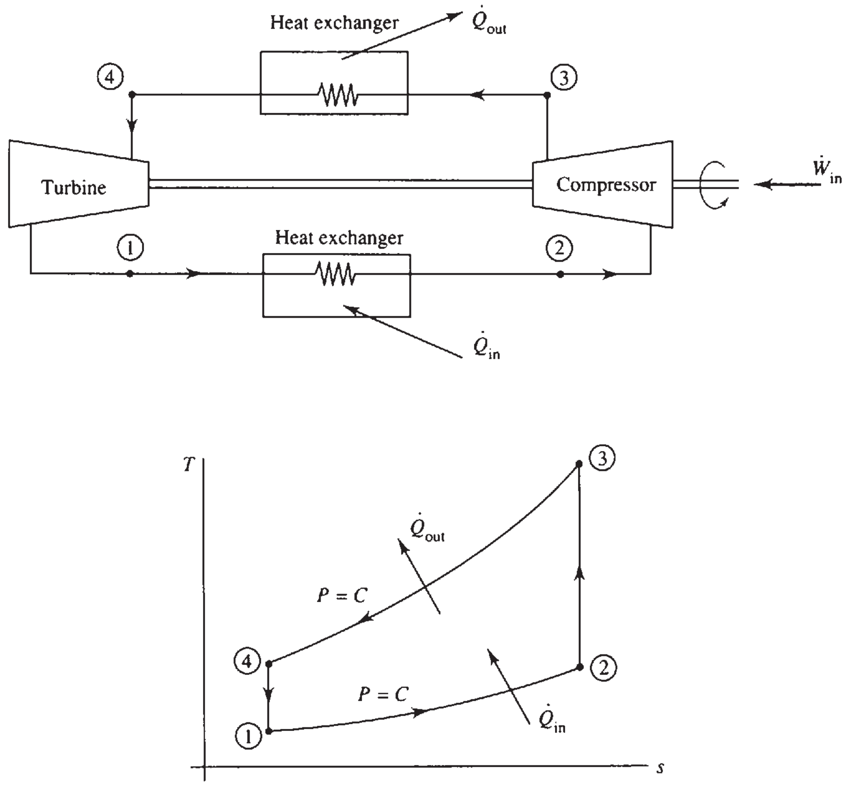


Fig. 9-22

A reminder: when the purpose of a thermodynamic cycle is to cool a space, we do not define a cycle's efficiency; rather, we define its *coefficient of performance*:

$$\text{COP} = \frac{\text{desired effect}}{\text{energy that costs}} = \frac{\dot{Q}_{in}}{\dot{W}_{in}} \tag{9.47}$$

where $\dot{W}_{in} = \dot{m}(w_{comp} - w_{turb})$.

EXAMPLE 9.15 Air enters the compressor of a simple gas refrigeration cycle at $-10\,°\text{C}$ and 100 kPa. For a compression ratio of 10 and a turbine inlet temperature of $30\,°\text{C}$ calculate the minimum cycle temperature and the coefficient of performance.

Assuming isentropic compression and expansion processes we find

$$T_3 = T_2\left(\frac{P_3}{P_2}\right)^{(k-1)/k} = (263)(10)^{0.2857} = 508\ \text{K}$$

$$T_1 = T_4\left(\frac{P_1}{P_4}\right)^{(k-1)/k} = (303)\left(\frac{1}{10}\right)^{0.2857} = 157\ \text{K} = -116\,°\text{C}$$

The COP is now calculated as follows:

$$q_{in} = c_p(T_2 - T_1) = (1.00)(263 - 157) = 106\ \text{kJ/kg}$$

$$w_{comp} = c_p(T_3 - T_2) = (1.00)(508 - 263) = 245\ \text{kJ/kg}$$

$$w_{turb} = c_p(T_4 - T_1) = (1.00)(303 - 157) = 146\ \text{kJ/kg}$$

$$\therefore \text{COP} = \frac{q_{in}}{w_{comp} - w_{turb}} = \frac{106}{245 - 146} = 1.07$$

This coefficient of performance is quite low when compared with that of a vapor refrigeration cycle. Thus gas refrigeration cycles are usual only for special applications.

EXAMPLE 9.16 Use the given information for the compressor of the refrigeration cycle of Example 9.15 but add an ideal internal heat exchanger, a regenerator, as illustrated in Fig. 9-23, so that the air temperature entering the turbine is $-40\,^\circ\text{C}$. Calculate the minimum cycle temperature and the coefficient of performance.

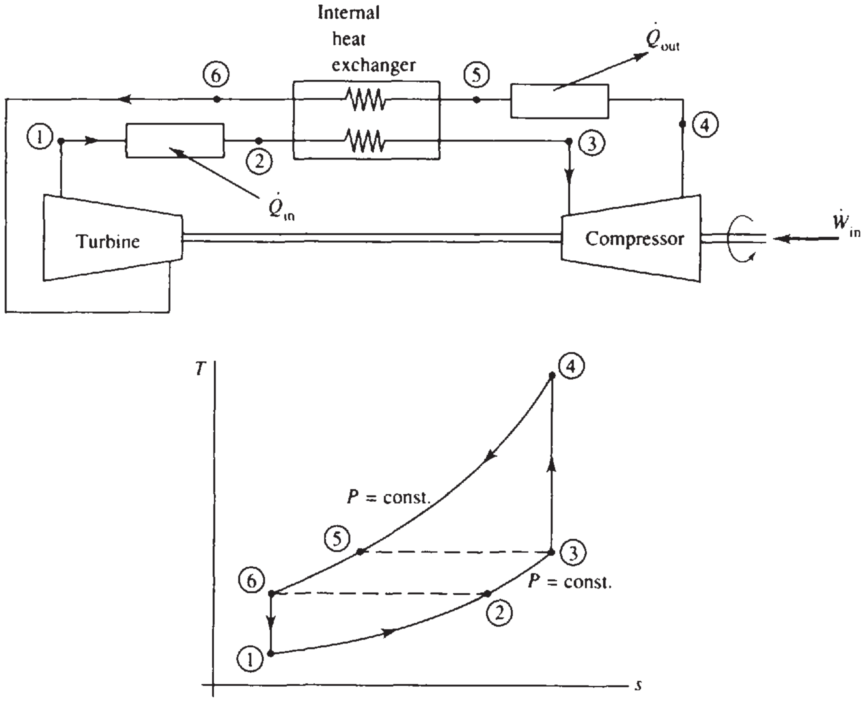


Fig. 9-23

Assuming isentropic compression we again have $T_4 = T_3(P_4/P_3)^{(k-1)/k} = (263)(10)^{0.2857} = 508$ K. For an ideal internal heat exchanger we would have $T_5 = T_3 = 263$ K and $T_6 = T_2 = 233$ K. The minimum cycle temperature is

$$T_1 = T_6\left(\frac{P_1}{P_6}\right)^{(k-1)/k} = (233)\left(\frac{1}{10}\right)^{0.2857} = 121\text{ K} = -152\,^\circ\text{C}$$

For the COP:

$$q_{in} = c_p(T_2 - T_1) = (1.00)(233 - 121) = 112\text{ kJ/kg}$$

$$w_{comp} = c_p(T_4 - T_3) = (1.00)(508 - 263) = 245\text{ kJ/kg}$$

$$w_{turb} = c_p(T_6 - T_1) = (1.00)(233 - 121) = 112\text{ kJ/kg}$$

$$\therefore \text{COP} = \frac{q_{in}}{w_{comp} - w_{turb}} = \frac{112}{245 - 112} = 0.842$$

Obviously, the COP is lower than that of the cycle with no internal heat exchanger. The objective is not to increase the COP but to provide extremely low refrigeration temperatures.

Solved Problems

9.1 An adiabatic compressor receives 20 m^3/min of air from the atmosphere at 20 °C and compresses it to 10 MPa. Calculate the minimum power requirement.

An isentropic compression requires the minimum power input for an adiabatic compressor. The outlet temperature for such a process is

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (293)\left(\frac{10\,000}{100}\right)^{0.2857} = 1092 \text{ K}$$

To find the mass flux, we must know the density. It is $\rho = P/RT = 100/(0.287)(293) = 1.189$ kg/m^3. The mass flux is then (the flow rate is given) $\dot{m} = \rho(AV) = (1.189)(20/60) = 0.3963$ kg/s. The minimum power requirement is now calculated to be

$$\dot{W}_{\text{comp}} = \dot{m}(h_2 - h_1) = \dot{m}c_p(T_2 - T_1) = (0.3963)(1.00)(1092 - 293) = 317 \text{ kW}$$

9.2 A compressor receives 4 kg/s of 20 °C air from the atmosphere and delivers it at a pressure of 18 MPa. If the compression process can be approximated by a polytropic process with $n = 1.3$, calculate the power requirement and the rate of heat transfer.

The power requirement is [see (*9.6*)]

$$\dot{W}_{\text{comp}} = \dot{m}\frac{nR}{n-1}T_1\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] = (4)\frac{(1.3)(0.287)}{1.3-1}(293)\left[\left(\frac{18\,000}{100}\right)^{0.3/1.3} - 1\right] = 3374 \text{ kW}$$

The first law for the control volume [see (*4.66*)] surrounding the compressor provides us with

$$\dot{Q} = \dot{m}\,\Delta h + \dot{W}_{\text{comp}} = \dot{m}c_p(T_2 - T_1) + \dot{W}_{\text{comp}} = \dot{m}c_pT_1\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] + \dot{W}_{\text{comp}}$$

$$= (4)(1.00)(293)\left[\left(\frac{18\,000}{100}\right)^{0.3/1.3} - 1\right] - 3374 = -661 \text{ kW}$$

In the above, we have used the compressor power as negative since it is a power input. The expression of (*9.6*) is the magnitude of the power with the minus sign suppressed, but when the first law is used we must be careful with the signs. The negative sign on the heat transfer means that heat is leaving the control volume.

9.3 An adiabatic compressor is supplied with 2 kg/s of atmospheric air at 15 °C and delivers it at 5 MPa. Calculate the efficiency and power input if the exiting temperature is 700 °C.

Assuming an isentropic process and an inlet temperature of 15 °C, the exit temperature, would be

$$T_{2'} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (288)\left(\frac{5000}{100}\right)^{0.2857} = 880.6 \text{ K}$$

The efficiency is then

$$\eta = \frac{w_s}{w_a} = \frac{c_p(T_{2'} - T_1)}{c_p(T_2 - T_1)} = \frac{880.6 - 288}{973 - 288} = 0.865 \quad \text{or } 86.5\%$$

The power input is $\dot{W}_{\text{comp}} = \dot{m}c_p(T_2 - T_1) = (2)(1.00)(973 - 288) = 1370$ kW.

9.4 An ideal compressor is to compress 20 lbm/min of atmospheric air at 70 °F at 1500 psia. Calculate the power requirement for (*a*) one stage, (*b*) two stages, and (*c*) three stages.

(*a*) For a single stage, the exit temperature is

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (530)\left(\frac{1500}{14.7}\right)^{0.2857} = 1987\,^\circ\text{R}$$

The required power is

$$\dot{W}_{\text{comp}} = \dot{m}c_p(T_2 - T_1) = \left(\frac{20}{60}\right)[(0.24)(778)](1987 - 530) = 90{,}680 \text{ ft-lbf/sec} \quad \text{or } 164.9 \text{ hp}$$

(*b*) With two stages, the intercooler pressure is $P_2 = (P_1P_4)^{1/2} = [(14.7)(1500)]^{1/2} = 148.5$ psia. The intercooler inlet and exit temperatures are (see Fig. 9-4)

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = 530\left(\frac{148.5}{14.7}\right)^{0.2857} = 1026\,^\circ\text{R}$$

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = 530\left(\frac{1500}{148.5}\right)^{0.2857} = 1026\,^\circ\text{R}$$

The power required for this two-stage compressor is

$$\dot{W}_{\text{comp}} = \dot{m}c_p(T_2 - T_1) + \dot{m}c_p(T_4 - T_3)$$
$$= \left(\frac{20}{60}\right)[(0.24)(778)](1026 - 530 + 1026 - 530) = 61{,}740 \text{ ft-lbf/sec}$$

or 112.3 hp. This represents a 31.9 percent reduction compared to the single-stage compressor.

(*c*) For three stages, we have, using (*9.9*) and (*9.10*),

$$P_2 = \left(P_1^2P_6\right)^{1/3} = \left[(14.7)^2(1500)\right]^{1/3} = 68.69 \text{ psia}$$

$$P_4 = \left(P_1P_6^2\right)^{1/3} = \left[(14.7)(1500)^2\right]^{1/3} = 321.0 \text{ psia}$$

The high temperature and power requirement are then

$$T_2 = T_4 = T_6 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (530)\left(\frac{68.69}{14.7}\right)^{0.2857} = 823.3\,^\circ\text{R}$$

$$\dot{W}_{\text{comp}} = 3\dot{m}c_p(T_2 - T_1) = (3)\left(\frac{20}{60}\right)[(0.24)(778)](823.3 - 530) = 54{,}770 \text{ ft-lbf/sec}$$

or 99.6 hp. This represents a 39.6 percent reduction compared to the single-stage compressor.

9.5 The calculations in Prob. 9.4 were made assuming constant specific heats. Recalculate the power requirements for (*a*) and (*b*) using the more accurate air tables (Appendix F).

(*a*) For one stage, the exit temperature is found using P_r. At stage $T_1 = 530\,^\circ$R: $h_1 = 126.7$ Btu/lbm, $(P_r)_1 = 1.300$. Then,

$$(P_r)_2 = (P_r)_1\frac{P_2}{P_1} = (1.300)\left(\frac{1500}{14.7}\right) = 132.7$$

This provides us with $T_2 = 1870\,^\circ$R and $h_2 = 469.0$ Btu/lbm. The power requirement is

$$\dot{W}_{\text{comp}} = \dot{m}(h_2 - h_1) = \left(\frac{20}{60}\right)(469 - 126.7)(778) = 88{,}760 \text{ ft-lbf/sec or } 161.4 \text{ hp}$$

(*b*) With two stages, the intercooler pressure remains at 148.5 psia. The intercooler inlet condition is found as follows:

$$(P_r)_2 = (P_r)_1\frac{P_2}{P_1} = (1.300)\left(\frac{148.5}{14.7}\right) = 13.13$$

whence $T_2 = 1018\,°\text{R}$ and $h_2 = 245.5$ Btu/lbm. These also represent the compressor exit (see Fig. 9-4), so that

$$\dot{W}_{comp} = \dot{m}(h_2 - h_1) + \dot{m}(h_4 - h_3)$$
$$= \left(\frac{20}{60}\right)(245.5 - 126.7 + 245.5 - 126.7)(778) = 61{,}620\ \text{ft-lbf/sec}$$

or 112.0 hp. Obviously, the assumption of constant specific heats is quite acceptable. The single-stage calculation represents an error of only 2 percent.

9.6 A Carnot engine operates on air between high and low pressures of 3 MPa and 100 kPa with a low temperature of 20 °C. For a compression ratio of 15, calculate the thermal efficiency, the MEP, and the work output.

The specific volume at TDC (see Fig. 6-1) is $v_1 = RT_1/P_1 = (0.287)(293)/100 = 0.8409\ \text{m}^3/\text{kg}$. For a compression ratio of 15 (we imagine the Carnot engine to have a piston-cylinder arrangement), the specific volume at BDC is

$$v_3 = \frac{v_1}{15} = \frac{0.8409}{15} = 0.05606\ \text{m}^3/\text{kg}$$

The high temperature is then $T_3 = P_3 v_3/R = (3000)(0.05606)/0.287 = 586.0\ \text{K}$.

The cycle efficiency is calculated to be $\eta = 1 - T_L/T_H = 1 - 293/586 = 0.500$. To find the work output, we must calculate the specific volume of state 2 as follows:

$$P_2 v_2 = P_1 v_1 = (100)(0.8409) = 84.09 \qquad P_2 v_2^{1.4} = P_3 v_3^{1.4} = (3000)(0.05606)^{1.4} = 53.12$$
$$\therefore\ v_2 = 0.3171\ \text{m}^3/\text{kg}$$

The entropy change $(s_2 - s_1)$ is then

$$\Delta s = c_v \ln 1 + R \ln \frac{v_2}{v_1} = 0 + 0.287 \ln \frac{0.3171}{0.8409} = -0.2799\ \text{kJ/kg}\cdot\text{K}$$

The work output is now found to be $w_{net} = \Delta T\,|\Delta s| = (586 - 293)(0.2799) = 82.0\ \text{kJ/kg}$. Finally,

$$w_{net} = (\text{MEP})(v_1 - v_2) \qquad 82.0 = (\text{MEP})(0.8409 - 0.3171) \qquad \text{MEP} = 156.5\ \text{kPa}$$

9.7 An inventor proposes a reciprocating engine with a compression ratio of 10, operating on 1.6 kg/s of atmospheric air at 20 °C, that produces 50 hp. After combustion the temperature is 400 °C. Is the proposed engine feasible?

We will consider a Carnot engine operating between the same pressure and temperature limits; this will establish the ideal situation without reference to the details of the proposed engine. The specific volume at state 1 (see Fig. 6-1) is

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(293)}{100} = 0.8409\ \text{m}^3/\text{kg}$$

For a compression ratio of 10, the minimum specific volume must be $v_3 = v_1/10 = 0.8409/10 = 0.08409$. The specific heat at state 2 is now found by considering the isothermal process from 1 to 2 and the isentropic process from 2 to 3:

$$P_2 v_2 = P_1 v_1 = 100 \times 0.8409 = 84.09 \qquad P_2 v_2^k = \frac{0.287(673)}{0.08409}(0.08409)^{1.4} = 71.75$$
$$\therefore\ v_2 = 0.6725\ \text{m}^3/\text{kg}$$

The change in entropy is

$$\Delta s = R \ln \frac{v_2}{v_1} = 0.287 \ln \frac{0.6725}{0.8409} = -0.0641\ \text{kJ/kg}\cdot\text{K}$$

The work output is then $w_{net} = \Delta T\,|\Delta s| = (400 - 20)(0.0641) = 24.4$ kJ/kg. The power output is

$$\dot{W} = \dot{m}w_{net} = (1.6)(24.4) = 39.0\text{ kW} \quad \text{or } 52.2\text{ hp}$$

The maximum possible power output is 52.2 hp; the inventor's claims of 50 hp is highly unlikely, though not impossible.

9.8 A six-cylinder engine with a compression ratio of 8 and a total volume at TDC of 600 mL intakes atmospheric air at 20 °C. The maximum temperature during a cycle is 1500 °C. Assuming an Otto cycle, calculate (*a*) the heat supplied per cycle, (*b*) the thermal efficiency, and (*c*) the power output for 4000 rpm.

(*a*) The compression ratio of 8 allows us to calculate T_2 (see Fig. 9-8):

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = (293)(8)^{0.4} = 673.1\text{ K}$$

The heat supplied is then $q_{in} = c_v(T_3 - T_2) = (0.717)(1773 - 673.1) = 788.6$ kJ/kg. The mass of air in the six cylinders is

$$m = \frac{P_1V_1}{RT_1} = \frac{(100)(600 \times 10^{-6})}{(0.287)(293)} = 0.004281\text{ kg}$$

The heat supplied per cycle is $Q_{in} = mq_{in} = (0.004281)(788.6) = 3.376$ kJ.

(*b*) $\eta = 1 - r^{1-k} = 1 - 8^{-0.4} = 0.5647$ or 56.5%.

(*c*) $W_{out} = \eta Q_{in} = (0.5647)(3.376) = 1.906$ kJ.

For the idealized Otto cycle, we assume that one cycle occurs each revolution. Consequently,

$$\dot{W}_{out} = (W_{out})\text{ (cycles per second)} = (1.906)(4000/60) = 127\text{ kW} \quad \text{or } 170\text{ hp}$$

9.9 A diesel engine intakes atmospheric air at 60 °F and adds 800 Btu/lbm of energy. If the maximum pressure is 1200 psia calculate (*a*) the cutoff ratio, (*b*) the thermal efficiency, and (*c*) the power output for an airflow of 0.2 lbm/sec.

(*a*) The compression process is isentropic. The temperature at state 2 (see Fig. 9-9) is calculated to be

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (520)\left(\frac{1200}{14.7}\right)^{0.2857} = 1829\,°\text{R}$$

The temperature at state 3 is found from the first law as follows:

$$q_{in} = c_p(T_3 - T_2) \qquad 800 = (0.24)(T_3 - 1829) \qquad \therefore T_3 = 5162\,°\text{R}$$

The specific volumes of the three states are

$$v_1 = \frac{RT_1}{P_1} = \frac{(53.3)(520)}{(14.7)(144)} = 13.09\text{ ft}^3/\text{lbm} \qquad v_2 = \frac{RT_2}{P_2} = \frac{(53.3)(1829)}{(1200)(144)} = 0.5642\text{ ft}^3/\text{lbm}$$

$$v_3 = \frac{RT_3}{P_3} = \frac{(53.3)(5162)}{(1200)(144)} = 1.592\text{ ft}^3/\text{lbm}$$

The cutoff ratio is then $r_c = v_3/v_2 = 1.592/0.5642 = 2.822$.

(*b*) The compression ratio is $r = v_1/v_2 = 13.09/0.5642 = 23.20$. The thermal efficiency can now be calculated, using (*9.25*):

$$\eta = 1 - \frac{1}{r^{k-1}}\frac{r_c^k - 1}{k(r_c - 1)} = 1 - \frac{1}{(23.2)^{0.4}}\frac{(2.822)^{1.4} - 1}{(1.4)(2.822 - 1)} = 0.6351 \quad \text{or } 63.51\%$$

(*c*) $\dot{W}_{out} = \eta\dot{Q}_{in} = \eta\dot{m}q_{in} = [(0.6351)(0.2)(800)](778) = 79{,}060$ ft-lbf/sec or 143.7 hp.

9.10 A dual cycle is used to model a piston engine. The engine intakes atmospheric air at 20 °C, compresses it to 10 MPa, and then combustion increases the pressure to 20 MPa. For a cutoff ratio of 2, calculate the cycle efficiency and the power output for an airflow of 0.1 kg/s.

The pressure ratio (refer to Fig. 9-11) is $r_p = P_3/P_2 = 20/10 = 2$. The temperature after the isentropic compression is

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (293)\left(\frac{10\,000}{100}\right)^{0.2857} = 1092\ \text{K}$$

The specific volumes are

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(293)}{100} = 0.8409\ \text{m}^3/\text{kg} \qquad v_2 = \frac{RT_2}{P_2} = \frac{(0.287)(1092)}{10\,000} = 0.03134\ \text{m}^3/\text{kg}$$

The compression ratio is then $r = v_1/v_2 = 0.8409/0.03134 = 26.83$. This allows us to calculate the thermal efficiency:

$$\eta = 1 - \frac{1}{r^{k-1}}\frac{r_p r_c^{k-1} - 1}{kr_p(r_c - 1) + r_p - 1} = 1 - \frac{1}{(26.83)^{0.4}}\frac{(2)(2)^{0.4} - 1}{(1.4)(2)(2-1) + 2 - 1} = 0.8843$$

To find the heat input, the temperatures of states 3 and 4 must be known. For the constant-volume heat addition,

$$\frac{T_3}{P_3} = \frac{T_2}{P_2} \qquad \therefore T_3 = T_2\frac{P_3}{P_2} = (1092)(2) = 2184\ \text{K}$$

For the constant-pressure heat addition,

$$\frac{T_3}{v_3} = \frac{T_4}{v_4} \qquad \therefore T_4 = T_3\frac{v_4}{v_3} = (2184)(2) = 4368\ \text{K}$$

The heat input is then

$$q_{in} = c_v(T_3 - T_2) + c_p(T_4 - T_3) = (0.717)(2184 - 1092) + (1.00)(4368 - 2184) = 2967\ \text{kJ/kg}$$

so that

$$w_{out} = \eta q_{in} = (0.8843)(2967) = 2624\ \text{kJ/kg}$$

The power output is $\dot{W}_{out} = \dot{m}w_{out} = (0.1)(2624) = 262.4$ kW.

9.11 Air at 90 kPa and 15 °C is supplied to an ideal cycle at intake. If the compression ratio is 10 and the heat supplied is 300 kJ/kg, calculate the efficiency and the maximum temperature for (*a*) a Stirling cycle, and (*b*) an Ericsson cycle.

(*a*) For the constant-temperature process, the heat transfer equals the work. Referring to Fig. 9-13, the first law gives

$$q_{out} = w_{1\text{-}2} = RT_1 \ln\frac{v_1}{v_2} = (0.287)(288)\ln 10 = 190.3\ \text{kJ/kg}$$

The work output for the cycle is then $w_{out} = q_{in} - q_{out} = 300 - 190.3 = 109.7$ kJ/kg. The efficiency is

$$\eta = \frac{w_{out}}{q_{in}} = \frac{109.7}{300} = 0.366$$

The high temperature is found from

$$\eta = 1 - \frac{T_L}{T_H} \qquad \therefore T_H = \frac{T_L}{1-\eta} = \frac{288}{1 - 0.366} = 454\ \text{K}$$

(*b*) For the Ericsson cycle of Fig. 9-14, the compression ratio is v_4/v_2. The constant-temperature heat addition $3 \to 4$ provides

$$q_{in} = w_{3\text{-}4} = RT_4 \ln\frac{v_4}{v_3} \qquad \therefore 300 = (0.287)T_4 \ln\frac{v_4}{v_3}$$

The constant-pressure process 2 → 3 allows

$$\frac{T_3}{v_3} = \frac{T_2}{v_2} = \frac{288}{v_4/10}$$

The constant-pressure process 4 → 1 demands

$$\frac{T_4}{v_4} = \frac{T_1}{v_1} = \frac{P_1}{R} = \frac{90}{0.287} = 313.6$$

Recognizing that $T_3 = T_4$, the above can be combined to give

$$300 = (0.287)(313.6v_4)\ln\frac{v_4}{v_3} \qquad v_3 = 0.1089v_4^2$$

The above two equations are solved simultaneously by trial and error to give

$$v_4 = 3.94\ \text{m}^3/\text{kg} \qquad v_3 = 1.69\ \text{m}^3/\text{kg}$$

Thus, from the compression ratio, $v_2 = v_4/10 = 0.394\ \text{m}^3/\text{kg}$. The specific volume of state 1 is

$$v_1 = \frac{RT}{P_1} = \frac{(0.287)(288)}{90} = 0.9184\ \text{m}^3/\text{kg}$$

The heat rejected is then

$$q_{\text{out}} = RT_1 \ln\frac{v_1}{v_2} = (0.287)(288)\ln\frac{0.9184}{0.394} = 70.0\ \text{kJ/kg}$$

The net work for the cycle is $w_{\text{out}} = q_{\text{in}} - q_{\text{out}} = 300 - 70.0 = 230$ kJ/kg. The efficiency is then $\eta = w_{\text{out}}/q_{\text{in}} = 230/300 = 0.767$. This allows us to calculate the high temperature:

$$\eta = 1 - \frac{T_L}{T_H} \qquad 0.767 = 1 - \frac{288}{T_H} \qquad \therefore T_H = 1240\ \text{K}$$

9.12 A gas-turbine power plant is to produce 800 kW of power by compressing atmospheric air at 20 °C to 800 kPa. If the maximum temperature is 800 °C, calculate the minimum mass flux of the air.

The cycle is modeled as an ideal Brayton cycle. The cycle efficiency is given by (*9.35*):

$$\eta = 1 - r_p^{(1-k)/k} = 1 - \left(\frac{800}{100}\right)^{-0.4/1.4} = 0.4479$$

The energy added in the combustor is (see Fig. 9-15) $\dot{Q}_{\text{in}} = \dot{W}_{\text{out}}/\eta = 800/0.4479 = 1786$ kW. The temperature into the combustor is

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (293)\left(\frac{800}{100}\right)^{0.2857} = 530.7\ \text{K}$$

With a combustor outlet temperature of 1073 K, the mass flux follows from a combustor energy balance:

$$\dot{Q}_{\text{in}} = \dot{m}c_p(T_3 - T_2) \qquad 1786 = (\dot{m})(1.00)(1073 - 530.7) \qquad \therefore \dot{m} = 3.293\ \text{kg/s}$$

This represents a minimum, since losses have not been included.

9.13 If the efficiency of the turbine of Prob. 9.12 is 85 percent and that of the compressor is 80 percent, calculate the mass flux of air needed, keeping the other quantities unchanged. Also calculate the cycle efficiency.

The compressor work, using $T_{2'} = 530.7$ from Prob. 9.12, is

$$w_{\text{comp}} = \frac{w_{\text{comp},s}}{\eta_{\text{comp}}} = \frac{1}{\eta_{\text{comp}}}c_p(T_{2'} - T_1) = \left(\frac{1}{0.8}\right)(1.00)(530.7 - 293) = 297.1\ \text{kJ/kg}$$

The temperature of state 4′, assuming an isentropic process, is

$$T_{4'} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1073)\left(\frac{100}{800}\right)^{0.2857} = 592.4\ \text{K}$$

The turbine work is then

$$w_{\text{turb}} = \eta_{\text{turb}} w_{\text{turb},s} = \eta_{\text{turb}} c_p(T_{4'} - T_3) = (0.85)(1.00)(592.4 - 1073) = 408.5\ \text{kJ/kg}$$

The work output is then $w_{\text{out}} = w_{\text{turb}} - w_{\text{comp}} = 408.5 - 297.1 = 111.4$ kJ/kg. This allows us to determine the mass flux:

$$\dot{W}_{\text{out}} = \dot{m} w_{\text{out}} \qquad 800 = (\dot{m})(111.4) \qquad \therefore \dot{m} = 7.18\ \text{kg/s}$$

To calculate the cycle efficiency, we find the actual temperature T_2. It follows from an energy balance on the actual compressor:

$$w_{\text{comp}} = c_p(T_2 - T_1) \qquad 297.1 = (1.00)(T_2 - 293) \qquad \therefore T_2 = 590.1\ \text{K}$$

The combustor rate of heat input is thus $\dot{Q}_{\text{in}} = \dot{m}(T_3 - T_2) = (7.18)(1073 - 590.1) = 3467$ kW. The efficiency follows as

$$\eta = \frac{\dot{W}_{\text{out}}}{\dot{Q}_{\text{in}}} = \frac{800}{3467} = 0.2307$$

Note the sensitivity of the mass flux and the cycle efficiency to the compressor and turbine efficiency.

9.14 Assuming the ideal-gas turbine and regenerator shown in Fig. 9-24, find $\dot{Q}_{\text{in}}$ and the back work ratio.

Mathcad

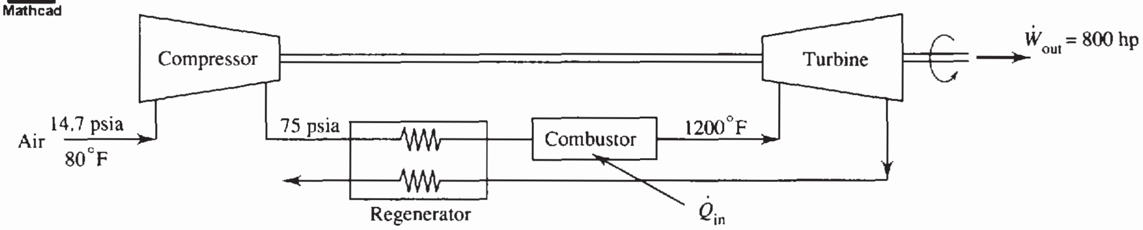


Fig. 9-24

The cycle efficiency is (see Fig. 9-17)

$$\eta = 1 - \frac{T_1}{T_4} r_p^{(k-1)/k} = 1 - \left(\frac{540}{1660}\right)\left(\frac{75}{14.7}\right)^{0.2857} = 0.4818$$

The rate of energy input to the combustor is

$$\dot{Q}_{\text{in}} = \frac{\dot{W}_{\text{out}}}{\eta} = \frac{(800)(550/778)}{0.4818} = 1174\ \text{Btu/sec}$$

The compressor outlet temperature is

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (540)\left(\frac{75}{14.7}\right)^{0.2857} = 860.2\,^\circ\text{R}$$

The turbine outlet temperature is

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1660)\left(\frac{14.7}{75}\right)^{0.2857} = 1042\,^\circ\text{R}$$

The turbine and compressor work are then

$$w_{\text{comp}} = c_p(T_2 - T_1) = (1.00)(860.2 - 540) = 320.2\ \text{Btu/lbm}$$

$$w_{\text{turb}} = c_p(T_3 - T_4) = (1.00)(1660 - 1042) = 618\ \text{Btu/lbm}$$

The back work ratio is then $w_{\text{comp}}/w_{\text{turb}} = 320.2/618 = 0.518$.

9.15 To Prob. 9.14 add an intercooler and a reheater. Calculate the ideal cycle efficiency and the back work ratio.

The intercooler pressure is (see Fig. 9-19), $P_2 = \sqrt{P_1 P_4} = \sqrt{(14.7)(75)} = 33.2$ psia. The temperatures T_2 and T_4 are

$$T_4 = T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (540)\left(\frac{33.2}{14.7}\right)^{0.2857} = 681.5\,^\circ\text{R}$$

Using $P_7 = P_2$ and $P_6 = P_4$, there results

$$T_9 = T_7 = T_6\left(\frac{P_7}{P_6}\right)^{(k-1)/k} = (1660)\left(\frac{33.2}{75}\right)^{0.2857} = 1315\,^\circ\text{R}$$

The work output of the turbine and input to the compressor are

$$w_{\text{turb}} = c_p(T_8 - T_9) + c_p(T_6 - T_7) = (0.24)(778)(1660 - 1315)(2) = 128{,}800\ \text{ft-lbf/lbm}$$

$$w_{\text{comp}} = c_p(T_4 - T_3) + c_p(T_2 - T_1) = (0.24)(778)(681.5 - 540)(2) = 52{,}840\ \text{ft-lbf/lbm}$$

The heat inputs to the combustor and the reheater are

$$q_{\text{comb}} = c_p(T_6 - T_5) = (0.24)(1660 - 1315) = 82.8\ \text{Btu/lbm}$$

$$q_{\text{reheater}} = c_p(T_8 - T_7) = (0.24)(1660 - 1315) = 82.8\ \text{Btu/lbm}$$

The cycle efficiency is now calculated to be

$$\eta = \frac{w_{\text{out}}}{q_{\text{in}}} = \frac{w_{\text{turb}} - w_{\text{comp}}}{q_{\text{comb}} + q_{\text{reheater}}} = \frac{(128{,}800 - 52{,}840)/778}{82.8 + 82.8} = 0.590$$

The back work ratio is $w_{\text{comp}}/w_{\text{turb}} = 52{,}840/128{,}800 = 0.410$

9.16 A turbojet aircraft flies at a speed of 300 m/s at an elevation of 10 000 m. If the compression ratio is 10, the turbine inlet temperature is 1000 °C, and the mass flux of air is 30 kg/s, calculate the maximum thrust possible from this engine. Also, calculate the rate of fuel consumption if the heating value of the fuel is 8400 kJ/kg.

The inlet temperature and pressure are found from Table B-1 to be (see Fig. 9-20)

$$T_1 = 223.3\ \text{K} \qquad P_1 = 0.2615 \qquad P_0 = 26.15\ \text{kPa}$$

The temperature exiting the compressor is

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (223.3)(10)^{0.2857} = 431.1\ \text{K}$$

Since the turbine drives the compressor, the two works are equal so that

$$c_p(T_2 - T_1) = c_p(T_3 - T_4) \qquad \therefore\ T_3 - T_4 = T_2 - T_1$$

Since $T_3 = 1273$, we can find T_4 as $T_4 = T_3 + T_1 - T_2 = 1273 + 223.3 - 431.1 = 1065.2$ K. We can now calculate the pressure at the turbine exit to be, using $P_3 = P_2 = 261.5$ kPa,

$$P_4 = P_3\left(\frac{T_4}{T_3}\right)^{k/(k-1)} = (261.5)\left(\frac{1065.2}{1273}\right)^{3.5} = 140.1\ \text{kPa}$$

The temperature at the nozzle exit, assuming an isentropic expansion, is

$$T_5 = T_4\left(\frac{P_5}{P_4}\right)^{(k-1)/k} = (1065.2)\left(\frac{26.15}{140.1}\right)^{0.2857} = 659.4 \text{ K}$$

The energy equation provides us with the exit velocity $V_5 = [2c_p(T_4 - T_5)]^{1/2} = [(2)(1000)(1065.2 - 659.4)]^{1/2} = 901$ m/s, where $c_p = 1000$ J/kg · K must be used in the expression. The thrust can now be calculated as

$$\text{thrust} = \dot{m}(V_5 - V_1) = (30)(901 - 300) = 18\,030 \text{ N}$$

This represents a maximum since a cycle composed of ideal processes was used.

The heat transfer rate in the burner is $\dot{Q} = \dot{m}c_p(T_3 - T_2) = (30)(1.00)(1273 - 431.1) = 25.26$ MW. This requires that the mass flux of fuel $\dot{m}_f$ be

$$8400\dot{m}_f = 25\,260 \qquad \therefore \dot{m}_f = 3.01 \text{ kg/s}$$

9.17 A gas-turbine cycle inlets 20 kg/s of atmospheric air at 15 °C, compresses it to 1200 kPa, and heats it to 1200 °C in a combustor. The gases leaving the turbine heat the steam of a Rankine cycle to 350 °C and exit the heat exchanger (boiler) at 100 °C. The pump of the Rankine cycle operates between 10 kPa and 6 MPa. Calculate the maximum power output of the combined cycle and the combined cycle efficiency.

The temperature of gases leaving the gas turbine is (see Fig. 9-21)

$$T_8 = T_7\left(\frac{P_8}{P_7}\right)^{(k-1)/k} = (1473)\left(\frac{100}{1200}\right)^{0.2857} = 724.2 \text{ K}$$

This temperature of the air exiting the compressor is

$$T_6 = T_5\left(\frac{P_6}{P_5}\right)^{(k-1)/k} = (288)\left(\frac{1200}{100}\right)^{0.2857} = 585.8 \text{ K}$$

The net power output of the gas turbine is then

$$\dot{W}_{GT} = \dot{W}_{turb} - \dot{W}_{comp} = \dot{m}c_p(T_7 - T_8) - \dot{m}c_p(T_6 - T_5)$$
$$= (20)(1.00)(1473 - 724.2 - 585.8 + 288) = 9018 \text{ kW}$$

The temperature exiting the condenser of the Rankine cycle is 45.8 °C. An energy balance on the boiler heat exchanger allows us to find the mass flux $\dot{m}_s$ of the steam:

$$\dot{m}_a c_p(T_8 - T_9) = \dot{m}_s(h_3 - h_2) \qquad (20)(1.00)(724.2 - 100) = \dot{m}_s(3043 - 191.8)$$
$$\dot{m}_s = 3.379 \text{ kg/s}$$

The isentropic process 3 → 4 allows h_4 to be found:

$$s_4 = s_3 = 6.3342 = 0.6491 + 7.5019x_4 \qquad \therefore x_4 = 0.7578$$
$$\therefore h_4 = 191.8 + (0.7578)(2392.8) = 2005 \text{ kJ/kg}$$

The steam turbine output is $\dot{W}_{ST} = \dot{m}(h_3 - h_4) = (3.379)(3043 - 2005) = 3507$ kW. The maximum power output (we have assumed ideal processes in the cycles) is, finally,

$$\dot{W}_{out} = \dot{W}_{GT} + \dot{W}_{ST} = 9018 + 3507 = 12\,525 \text{ kW} \quad \text{or } 12.5 \text{ MW}$$

The energy input to this combined cycle is $\dot{Q}_{in} = \dot{m}_a c_p(T_7 - T_6) = (20)(1.00)(1473 - 585.8) = 17.74$ MW. The cycle efficiency is then

$$\eta = \frac{\dot{W}_{out}}{\dot{Q}_{in}} = \frac{12.5}{17.74} = 0.70$$

9.18 A simple gas cycle produces 10 tons of refrigeration by compressing air from 200 kPa to 2 MPa. If the maximum and minimum temperatures are 300 °C and −90 °C, respectively, find the compressor power and the cycle COP. The compressor is 82 percent efficient and the turbine is 87 percent efficient.

The ideal compressor inlet temperature (see Fig. 9-22) is $T_2 = T_3(P_2/P_3)^{(k-1)/k} = (573)(200/2000)^{0.2857} = 296.8$ K. Because the compressor is 82 percent efficient, the actual inlet temperature T_2 is found as follows:

$$\eta_{\text{comp}} = \frac{w_s}{w_a} = \frac{c_p(T_3 - T_{2'})}{c_p(T_3 - T_2)} \qquad \therefore T_2 = \left(\frac{1}{0.82}\right)[(0.82)(573) - 573 + 296.8] = 236.2 \text{ K}$$

The low-temperature heat exchanger produces 10 tons = 35.2 kW of refrigeration:

$$\dot{Q}_{\text{in}} = \dot{m}c_p(T_2 - T_1) \qquad 35.2 = \dot{m}(1.00)(236.2 - 183) \qquad \therefore \dot{m} = 0.662 \text{ kg/s}$$

The compressor power is then $\dot{W}_{\text{comp}} = \dot{m}c_p(T_3 - T_2) = (0.662)(1.00)(573 - 236.2) = 223$ kW. The turbine produces power to help drive the compressor. The ideal turbine inlet temperature is

$$T_{4'} = T_1\left(\frac{P_4}{P_1}\right)^{(k-1)/k} = (183)\left(\frac{2000}{200}\right)^{0.2857} = 353.3 \text{ K}$$

The turbine power output is $\dot{W}_{\text{turb}} = \dot{m}\eta_{\text{turb}}c_p(T_{4'} - T_1) = (0.662)(0.87)(1.00)(353.3 - 183) = 98.1$ kW. The cycle COP is now calculated to be

$$\text{COP} = \frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{net}}} = \frac{(10)(3.52)}{223 - 98.1} = 0.282$$

9.19 Air enters the compressor of a gas refrigeration cycle at −10 °C and is compressed from 200 kPa to 800 kPa. The high-pressure air is then cooled to 0 °C by transferring energy to the surroundings and then to −30 °C with an internal heat exchanger before it enters the turbine. Calculate the minimum possible temperature of the air leaving the turbine, the coefficient of performance, and the mass flux for 8 tons of refrigeration. Assume ideal components.

Refer to Fig. 9-23 for designation of states. The temperature at the compressor outlet is

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (283)\left(\frac{800}{200}\right)^{0.2857} = 420.5 \text{ K}$$

The minimum temperature at the turbine outlet follows from an isentropic process:

$$T_1 = T_6\left(\frac{P_1}{P_6}\right)^{(k-1)/k} = (243)\left(\frac{200}{800}\right)^{0.2857} = 163.5 \text{ K}$$

The coefficient of performance is calculated as follows:

$$q_{\text{in}} = c_p(T_2 - T_1) = (1.00)(243 - 163.5) = 79.5 \text{ kJ/kg}$$

$$w_{\text{comp}} = c_p(T_4 - T_3) = (1.00)(420.5 - 283) = 137.5 \text{ kJ/kg}$$

$$w_{\text{turb}} = c_p(T_6 - T_1) = (1.00)(243 - 163.5) = 79.5 \text{ kJ/kg}$$

$$\therefore \text{COP} = \frac{q_{\text{in}}}{w_{\text{comp}} - w_{\text{turb}}} = \frac{79.5}{137.5 - 79.5} = 1.37$$

We find the mass flux as follows:

$$\dot{Q}_{\text{in}} = \dot{m}q_{\text{in}} \qquad (8)(3.52) = (\dot{m})(79.5) \qquad \dot{m} = 0.354 \text{ kg/s}$$