Quantum Screening Effects on the Entanglement Fidelity for Elastic Collisions in Electron-Ion Quantum Plasmas

Dae-Han Ki[a] and Young-Dae Jung[a,b]

[a] Department of Applied Physics, Hanyang University, Ansan, Kyunggi-Do 426-791, South Korea
[b] Department of Electrical and Computer Engineering, MC 0407, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0407, U. S. A.

Reprint requests to Y.-D. J.; E-mail: ydjung@hanyang.ac.kr

Z. Naturforsch. **65a,** 1143 – 1146 (2010); received February 17, 2010 / revised April 26, 2010

The quantum screening effects on the entanglement fidelity for the elastic collision are investigated in electron-ion quantum plasmas. The partial wave analysis and modified Debye-Hückel interaction potential are employed to obtain the entanglement fidelity function in electron-ion quantum plasmas as a function of the collision energy, charge of the ion, and quantum wave number. It is found that the quantum screening effects significantly enhance the entanglement fidelity in electron-ion quantum plasmas. It is also found that the entanglement fidelity increases with an increase of the ion charge. The quantum screening effects on the entanglement fidelity is also found to be increased with increasing plasma density. In addition, it is found that the quantum screening effects decreases with an increase of the collision energy.

Key words: Quantum Screening; Entanglement Fidelity; Electron-Ion Quantum Plasmas.

The elastic atomic collision [1 – 3] has received much attention since this process is one of the most important atomic processes in many areas of physics such as astrophysics, atomic physics, and plasma physics. Recently, the elastic collision in plasmas have been extensively investigated since the elastic electron-ion process is known as a plasma diagnostic tool for providing useful information on various plasma parameters [4 – 6]. Moreover, the entanglement fidelity for the collision process has received notable attention since it has been shown that the quantum correlation plays a key role in understanding the quantum measurement and information processing [7]. Very recently, the physical properties of quantum plasmas have been explored in nano-scale objects such as nanowires, quantum dot, semiconductor devices, and also in astrophysical compact objects such as neutron stars and white dwarfs [8, 9]. In addition, a recent excellent investigation by Shukla and Eliasson [10] using the linear dielectric response formalism in electron-ion quantum plasmas showed the existence of the oscillatory behaviour of the modified Debye-Hückel interaction potential which is quite different from the standard Debye-Hückel model. Hence, it would be expected that the entanglement fidelity for the elastic collision in electron-ion quantum plasmas would be different from that in ordinary Debye-Hückel classical plasmas. It would be also expected that the investigation on the behaviour of the entanglement fidelity for elastic collisions in electron-ion quantum plasmas provides a useful information on the quantum correlation and information. Thus, in this paper we investigate the quantum screening effects on the entanglement fidelity for the elastic collision in electron-ion quantum plasmas. The oscillatory modified Debye-Hückel interaction [10] and partial wave analysis are employed to obtain the entanglement fidelity for the elastic collision in electron-ion quantum plasmas as a function of the collision energy, charge of the ion, and quantum wave number.

In quantum collision processes, the stationary-state Schrödinger equation [1] would be expressed by

$$(\nabla^2 + k^2)\varphi(k;\mathbf{r}) = \frac{2\mu}{\hbar^2} V(\mathbf{r})\varphi(k;\mathbf{r}), \quad (1)$$

where $\varphi(k;\mathbf{r})$ stands for the scattered wave function, $k[= (2\mu E/\hbar^2)^{1/2}]$ is the wave number, μ is the reduced mass of the collision system, $E(= \mu v^2/2)$ is the collision energy, v is the collision velocity, and $V(\mathbf{r})$ is the interaction potential. Using the partial wave expansion technique [1], the scattered wave function function $\varphi(k;\mathbf{r})$ would be represented by the following form:

$$\varphi(k;\mathbf{r}) = \sum_{l=0}^{\infty} \mathrm{i}^l (2l+1) C_l(k) P_l(\cos\theta) R_l(k;r), \quad (2)$$

where $C_l(k)$ is the expansion coefficient for the angular momentum quantum number l, $P_l(\cos\theta)$ is the Legendre polynomial, $R_l(k;r)$ is the solution of the radial part of the Schrödinger equation:

$$\left[\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}}{\mathrm{d}r}\right) - \frac{l(l+1)}{r^2} - \frac{2\mu}{\hbar^2}V(r) + k^2\right] \cdot R_l(k;r) = 0, \quad (3)$$

where the asymptotic form of the radial wave function would be given by $R_l(k;r) \propto \sin(kr - \pi l/2 + \eta_l)/(kr)$ [11], and η_l is the phase-shift for the partial wave l. According to the boundary condition at the origin, the radial wave function $R_l(k;r)$ and expansion coefficient $C_l(k)$ for the central field $V(r)$ would be, respectively, expressed in the following forms [1]:

$$\begin{aligned} R_l(k;r) &= j_l(kr) \\ &+ \frac{2\mu k}{\hbar^2}\left[n_l(kr)\int_0^r \mathrm{d}r' r'^2 V(r') R_l(k;r') j_l(kr')\right. \\ &\left. + j_l(kr)\int_r^\infty \mathrm{d}r' r'^2 V(r') R_l(k;r') n_l(kr')\right], \end{aligned} \quad (4)$$

$$C_l(k) = \frac{1}{(2\pi)^{3/2}}\left[1 + \frac{2\mathrm{i}\mu k}{\hbar^2}\int_0^\infty \mathrm{d}r r^2 V(r) \cdot R_l(k;r) j_l(kr)\right]^{-1}, \quad (5)$$

where $j_l(kr)$ and $n_l(kr)$ are the spherical Bessel and Neumann functions. From a recent excellent work by Mishima et al. [7], the entanglement fidelity $F(k)$ for the elastic collision can be obtained by the absolute square of the scattered wave function. Since the s-state provides the main contribution to the low energy projectiles, the entanglement fidelity for the case of low-energy collisions would be then

$$F(k) \propto \left|\int \mathrm{d}^3\mathbf{r}\varphi(k;\mathbf{r})\right|^2 = \frac{\left|\int_0^\infty \mathrm{d}r\, r^2 j_0(kr)\right|^2}{1 + \left|\frac{2\mu k}{\hbar^2}\int_0^\infty \mathrm{d}r\, r^2 j_0(kr) V(r)\right|^2}. \quad (6)$$

In a recent pioneering work of Shukla and Eliasson [10], the useful analytic form of the modified Debye-Hückel potential of a test charge in electron-ion quantum plasmas has been obtained by the linear dielectric response function. From this effective potential model [7], the modified Debye-Hückel interaction potential $V_\mathrm{q}(r)$ between the electron and ion with charge Ze in quantum plasmas would be obtained as

$$V_\mathrm{q}(r) = -\frac{Ze^2}{r}\exp\left(-\frac{k_\mathrm{q} r}{\sqrt{2}}\right)\cos\left(\frac{k_\mathrm{q} r}{\sqrt{2}}\right), \quad (7)$$

where k_q $[\equiv (4m_\mathrm{e}^2\omega_\mathrm{pe}^2/\hbar^2)^{1/2}]$ is the quantum wave number, m_e is the electron mass, ω_pe is the electron plasma frequency, and $\hbar$ is the rationalized Planck constant. Thus, the oscillatory effective quantum interaction potential (7) is shown to be quite different from the conventional Debye-Hückel model [8] due to the extra oscillating term $\cos(k_\mathrm{q} r/\sqrt{2})$. Hence, the quantum screening effects on the entanglement fidelity for the elastic collision in electron-ion quantum plasmas would be investigated by the fidelity ratio function R_q $(\equiv f_\mathrm{q}/f_\mathrm{C})$ denoted by the ratio of the entanglement fidelity $f_\mathrm{q}(k)$ for the electron-ion quantum plasma (V_q) to the entanglement fidelity $f_\mathrm{C}(k)$ for the case of the pure Coulomb interaction (V_C):

$$\begin{aligned} R_\mathrm{q}(k) &= \frac{1 + \left|\frac{2Zm_\mathrm{e}e^2k}{\hbar^2}\int_0^\infty \mathrm{d}r\, r\frac{\sin(kr)}{kr}\right|^2}{1 + \left|\frac{2Zm_\mathrm{e}e^2k}{\hbar^2}\int_0^\infty \mathrm{d}r\, r\exp\left(-\frac{k_\mathrm{q}r}{\sqrt{2}}\right)\cos\left(\frac{k_\mathrm{q}r}{\sqrt{2}}\right)\frac{\sin(kr)}{kr}\right|^2} \\ &= \frac{1 + \left(\frac{2k}{a_\mathrm{Z}}\right)^2\left|\int_0^\infty \mathrm{d}r\, r\frac{\sin(kr)}{kr}\right|^2}{1 + \left(\frac{2k}{a_\mathrm{Z}}\right)^2\left|\mathrm{Re}\int_0^\infty \mathrm{d}r\, r\exp\left(-\frac{k_\mathrm{q}r}{\sqrt{2}}(1-\mathrm{i})\right)\frac{\sin(kr)}{kr}\right|^2}, \end{aligned} \quad (8)$$

where a_Z $(= a_0/Z)$ is the Bohr radius of the hydrogenic ion with nuclear charge Ze, a_0 $(= \hbar^2/m_\mathrm{e}e^2)$ is the Bohr radius of the hydrogen atom, and 'Re' stands for the real part.

As shown in the second line of (8), the fidelity ratio function has the strong dependence on the quantum correlation through the parameter k_q. After some mathematical manipulations, the fidelity ratio R_q, i. e., the quantum correlation effects on the entanglement fidelity for the elastic collision in electron-ion quantum plasmas is then found to be

$$R_\mathrm{q}(\bar{E}, \bar{k}_\mathrm{q}, Z) = \frac{(\bar{E} + 4Z^2)(\bar{E}^2 + \bar{k}_\mathrm{q}^4)^2}{4\bar{E}^4 Z^2 + \bar{E}(\bar{E}^2 + \bar{k}_\mathrm{q}^4)^2}, \quad (9)$$

where $\bar{E}$ $(\equiv m_\mathrm{e}v^2/2Ry)$ is the scaled collision energy, Ry $(= m_\mathrm{e}e^4/2\hbar^2 \approx 13.6\ \mathrm{eV})$ is the Rydberg constant, and $\bar{k}_q$ ($\equiv k_q a_0$) is the scaled quantum wave number. An excellent discussion on the additional far-field oscillatory wake potential associated with a moving test charge in an unmagnetized quantum plasma was also given by Shukla and Eliasson [10]. In addition, an excellent investigation [12] on important nonlinear aspects of wave-wave and wave-electron interactions and influence of the external magnetic field and the electron angular momentum spin on the electromagnetic wave dynamics in dense quantum plasmas was given by Shukla and Eliasson. In the nonrelativisitc limit, the electron-ion bremsstrahlung cross section $d\sigma_b$ [13] is would be determined by $d\sigma_b = \int d\sigma_{sc} dW_\omega$, where $d\sigma_{sc}$ is the scattering cross section and dW_ω represents the probability for photon emission within the frequency $d\omega$. Hence, the fidelity ratio R_q would be related to the bremsstrahlung emission spectrum. Thus, it would be expected that the bremsstrahlung emission spectrum due to the electron-ion encounters in quantum plasmas provides the useful information on the entanglement fidelity for the electron-ion scattering in quantum plasmas.

Figure 1 shows the fidelity ratio R_q as a function of the scaled quantum wave number $\bar{k}_q$ for various values of the ion charge number Z. As it is seen, it is found that the quantum screening effects significantly enhance the entanglement fidelity in electron-ion quantum plasmas. Hence, it is understand that the entanglement fidelity increases with increasing the plasma density. It is also found that the entanglement fidelity increases with an increase of the ion charge. Therefore, we have found that the entanglement fidelity in the quantum hydrogen plasma ($Z = 1$) would be always smaller than that in any quantum plasmas with the ion charge number $Z > 1$. Figure 2 represents the surface plot of the fidelity ratio R_q as a function of the scaled collision energy $\bar{E}$ and scaled quantum wave number $\bar{k}_q$. It is shown that the quantum screening effects on the entanglement fidelity is found to be increased with increasing quantum wave number. Figure 3 shows the fidelity ratio R_q as a function of the ion charge number Z for various values of the collision energy. As shown in Figure 3, the fidelity ratio decreases with an increase of the collision energy. Thus, we understood that the quantum screening effects are found to be decreased with increasing collision energy. Hence, we have found that the quantum shielding effect plays a significant role in the entanglement fidelity

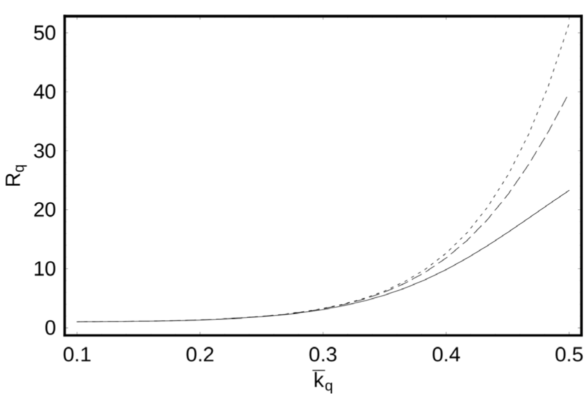

Fig. 1. Fidelity ratio R_q as a function of the scaled quantum wave number $\bar{k}_q$ for $\bar{E} = 0.1$. The solid line represents the case of $Z = 1$. The dashed line represents the case of $Z = 2$. The dotted line represents the case of $Z = 8$.

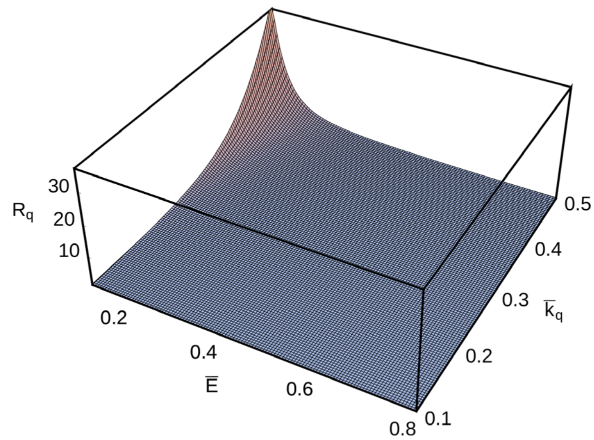

Fig. 2 (colour online). Surface plot of the fidelity ratio R_q as a function of the scaled collision energy $\bar{E}$ and scaled quantum wave number $\bar{k}_q$ for $Z = 2$.

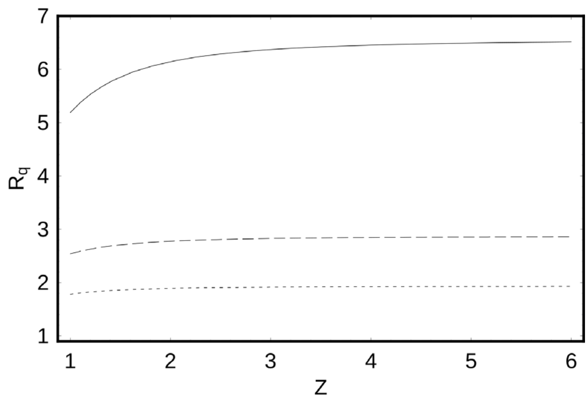

Fig. 3. Fidelity ratio R_q as a function of the ion charge number Z for $\bar{k}_q = 0.5$. The solid line represents the case of $\bar{E} = 0.2$. The dashed line represents the case of $\bar{E} = 0.3$. The dotted line represents the case of $\bar{E} = 0.4$.

for the elastic collision in electron-ion quantum plasmas. These results would provide useful information on the transfer of the quantum information in electron-ion quantum plasmas.

Acknowledgements

One of the authors (Y.-D. J.) gratefully acknowledges Dr. M. Rosenberg for useful discussions and warm hospitality while visiting the Department of Electrical and Computer Engineering at the University of California, San Diego. This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (Grant No. 2010-0015910).

[1] A. G. Sitenko, Lectures in Scattering Theory, Pergamon Press, Oxford 1971.
[2] V. P. Shevelko, Atoms and Their Spectroscopic Properties, Springer-Verlag, Berlin 1997.
[3] V. P. Shevelko and H. Tawara, Atomic Multielectron Processes, Springer-Verlag, Berlin 1998.
[4] V. P. Shevelko and L. A. Vainshtein, Atomic Physics for Hot Plasmas, Institute of Physics, Bristol 1993.
[5] T. S. Ramazanov and K. N. Dzhumagulova, Phys. Plasmas **9**, 3758 (2002).
[6] H. F. Beyer and V. P. Shevelko, Introduction to the Physics of Highly Charged Ions, Institute of Physics, Bristol 2003.
[7] K. Mishima, M. Hayashi, and S. H. Lin, Phys. Lett. A **333**, 371 (2004).
[8] M. Marklund and P. K. Shukla, Rev. Mod. Phys. **78**, 591 (2006).
[9] P. K. Shukla, L. Stenflo, and R. Bingham, Phys. Lett. A **359**, 218 (2006).
[10] P. K. Shukla and B. Eliasson, Phys. Lett. A **372**, 2897 (2008).
[11] H. A. Bethe and E. E. Salpeter, Quantum Mechanics of One- and Two-Electron Atoms, Springer-Verlag, Berlin 1957.
[12] P. K. Shukla and B. Eliasson, Phys. Usp. **53**, 51 (2010).
[13] R. J. Gould, Electromagnetic Processes, Princeton University Press, Princeton 2006.