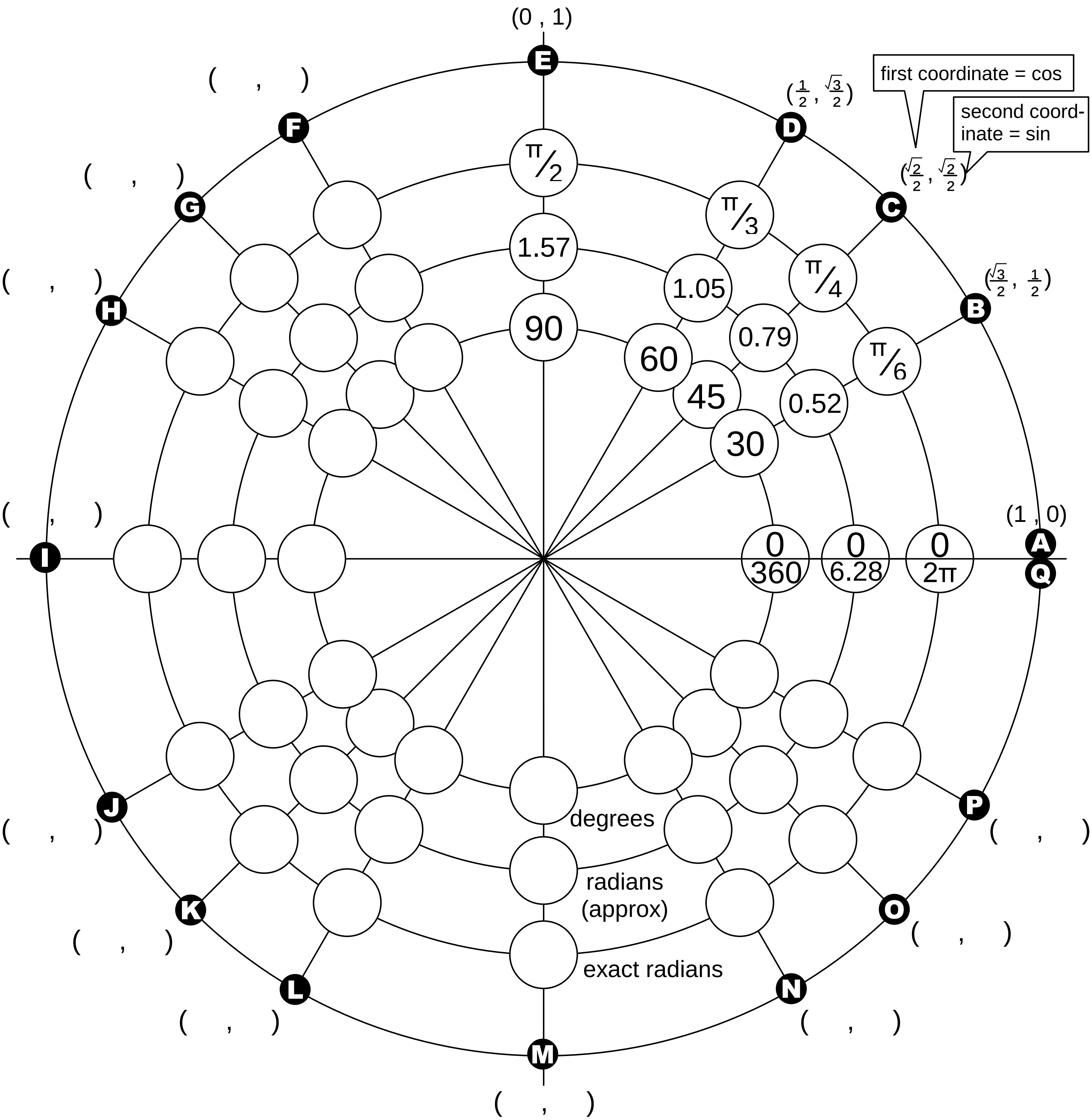
The Unit Circle Explorer's Guide

A working knowledge of the inner depths of the unit circle can make trigonometry much easier.

This worksheet aims to develop a better understanding of how angles are named and how the coordinates of these angles on the unit circle are determined.

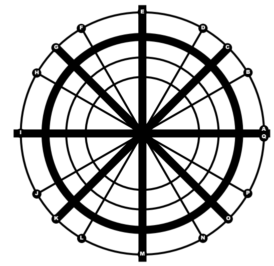
INSTRUCTIONS
Follow the step-by-step instructions on the third page. Fill in the information on the circle when requested. You may want to separate the two pages of this worksheet.

Authored by Gordon Wong


(0 , 1)
E
first coordinate = cos
second coord-inate = sin
D
$(\frac{1}{2}, \frac{\sqrt{3}}{2})$
C
$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
B
$(\frac{\sqrt{3}}{2}, \frac{1}{2})$
(1 , 0)
A
Q
F
G
H
I
J
K
L
M
N
O
P
$\pi/2$
1.57
90
$\pi/3$
1.05
60
$\pi/4$
0.79
45
$\pi/6$
0.52
30
0
360
0
6.28
0
2π
degrees
radians
(approx)
exact radians

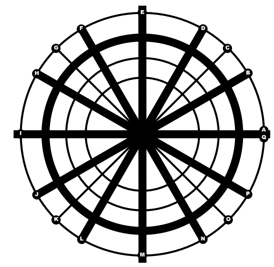
DETERMINING THE ANGLES IN RADIANS (EXACT)

The lines radiating from the origin are labelled in order from A to Q in the same direction that we measure angles (counterclockwise). Going around the circumference of the unit circle from A to Q is a distance of 2π, and represents 2π rad.



1. Positions A, C, E, G, I, K, M, O and Q are evenly spaced around the circle. Going from one position to the next means moving $\frac{1}{8}$ of the way around the circle. The whole circle is 2π, so C is $\frac{1}{8} \times 2\pi$, E is $\frac{2}{8} \times 2\pi$, and so on. These two are already done; they come out to $\frac{\pi}{4}$ and $\frac{\pi}{2}$.

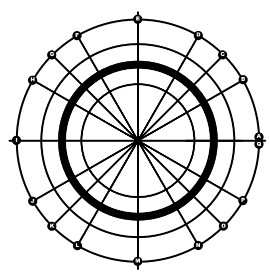
   Calculate the radian values for E, G, I, K, M, O, and Q and fill them into the diagram.



2. Positions A, B, D, E, F, H, I, J, L, M, N, P and Q are evenly spaced around the circle. Going from one position to the next means moving $\frac{1}{12}$ of the way around the circle. The whole circle is 2π, so B is $\frac{1}{12} \times 2\pi$, D is $\frac{2}{12} \times 2\pi$, and so on. These two are already done; they come out to $\frac{\pi}{6}$ and $\frac{\pi}{3}$. The positions on the x- and y-axes will be duplicated from step 1.

   Calculate the radian values for the remaining positions and fill them into the diagram.

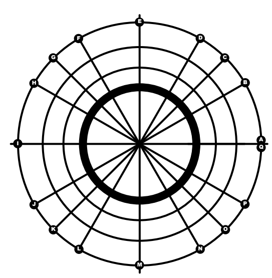
DETERMINING THE ANGLES IN RADIANS (DECIMAL APPROXIMATION)



3. Use the [π] button on your calculator (or $\pi = 3.14$) to convert all the angles into their numerical equivalents.

   Calculate these values to two decimal places and fill them into the middle ring on the diagram.

DETERMINING THE ANGLES IN DEGREES



4. Since π radians is equal to 180°, convert all of your exact radian measures from A–Q into degrees by multiplying by $\frac{180}{\pi}$.
   E.g. $\frac{\pi}{2} \times \frac{180}{\pi} = \frac{180}{2} = 90°$.

   Calculate these values and fill them into the inner ring on the diagram.

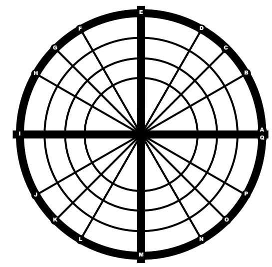
More on the next page….

DETERMINING THE COORDINATES OF THE ENDPOINTS ON THE UNIT CIRCLE

As it says in the upper-right corner of the diagram, the first coordinate of the endpoint on the unit circle is the cosine of the angle, and the second coordinate is the sine of the angle. Recall that the radius of the unit circle is 1, so

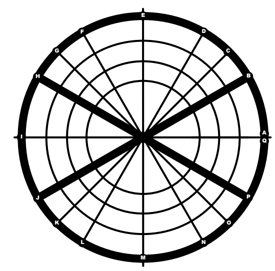
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} = \frac{x}{1} = x \qquad \sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} = \frac{y}{1} = y$$

5. Consider points A, E, I, M and Q. Points A and Q are both at (1, 0), as already marked on the circle, and E is (0, 1). I and M can be found by symmetry. Flip the points around the x- and y-axes.



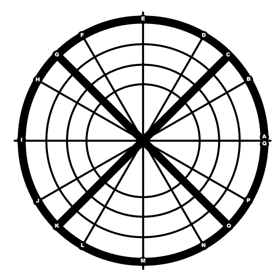
Fill the coordinates into the brackets at the remaining points.

6. Consider points B, H, J and P. The endpoint at position B is already marked, and it has coordinates of $(\frac{\sqrt{3}}{2}, \frac{1}{2})$. The other three points can be determined by symmetry. Flip the point around the x- and y-axes.



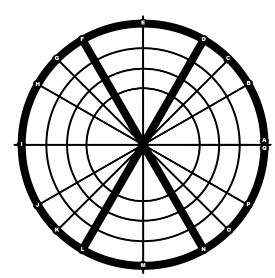
Fill the coordinates into the brackets at the other points.

7. Consider points C, G, K and O. The endpoint at position C is already marked, and it has coordinates of $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$. The other three points can be determined by symmetry. Flip the point around the x- and y-axes.



Fill the coordinates into the brackets at the other points.

8. Consider points D, F, L and N. The endpoint at position D is already marked, and it has coordinates of $(\frac{1}{2}, \frac{\sqrt{3}}{2})$. The other three points can be determined by symmetry. Flip the point around the x- and y-axes.



Fill the coordinates into the brackets at the other points.

Your diagram is now complete. You now have a handy guide to help you see the relationships between angles, cosines and sines.