

5. FLOW OF WATER THROUGH SOIL

5.1 FLOW OF WATER IN A PIPE

The flow of water through a rough open pipe may be expressed by means of the Darcy-Weisbach resistance equation

$$\Delta h = f \frac{L}{D} \frac{v^2}{2g} \quad (5.1)$$

in which Δh is the head loss over a length L of pipe of diameter D . The average velocity of flow is v . f is a measure of pipe resistance.

In Fig. 4.1 standpipes or piezometers have been connected to the pipe at points P and Q. The heights to which the water rises in these piezometers indicate the heads at these points. The difference between the elevations for the water surfaces in the piezometers is the head loss (Δh). If the hydraulic gradient (i) is defined as

$$i = \frac{\Delta h}{L} \quad (5.2)$$

then it is clear from equation (4.1) that the velocity v is proportional to the square root of i . The expression for rate of discharge of water Q may be written as

$$Q = v \frac{\pi D^2}{4} = v A = \left(\frac{2gD}{f}\right)^{1/2} i^{1/2} A \quad (5.3)$$

If the pipe is filled with a pervious material such as sand the rate of discharge of water through the sand is no longer proportional to the square root of i . Darcy, in 1956, found that Q was proportional to the first power of i

$$Q = k i A \quad (5.4)$$

$$\text{or} \quad v = \frac{Q}{A} = k i \quad (5.5)$$

where k is the constant of proportionality which is called the coefficient of permeability or the hydraulic conductivity. Actually, k in equation (4.4) is not simply a material constant since it depends upon the characteristics of the fluid as well as the soil through which the fluid is seeping.

Equation (4.4) or (4.5) expresses what has come to be known as Darcy's Law. Several studies have found that the relationship between v and i is non linear but this has been largely discounted by Mitchell (1976) who concluded that if all other factors are held constant, Darcy's Law is valid. Because of the small value of v that applies to water seeping through soil, the flow is considered to be laminar. For coarse sands and gravels the flow may not be laminar so the validity of Darcy's Law in these cases may be in doubt.

5.2 THE COEFFICIENT OF PERMEABILITY

Typical values of k for various types of soil are shown in Table 5.1. This table illustrates the enormous range of values of permeability for soils.

TABLE 5.1

TYPICAL VALUES OF PERMEABILITY

Gravel	greater than 10^{-2} m/sec
Sand	10^{-6} m/sec to 10^{-2} m/sec
Silt	10^{-9} m/sec to 10^{-5} m/sec
Clay	10^{-11} m/sec to 10^{-8} m/sec

Several empirical equations have been proposed for the evaluation of the coefficient of permeability. One of the earliest is that proposed by Hazen for uniform sands

$$k \text{ (cm/sec)} = C_1 D_{10}^2 \quad (5.6)$$

where

D_{10} = effective size in mm

C_1 = constant, varying from 1.0 to 1.5

It was mentioned above that the coefficient of permeability (k) is not strictly a material constant. The viscosity and density of the permeant have been found to influence the value of k . The two characteristics may be eliminated by using "absolute permeability" (K) having dimensions of (length)² and defined as

$$K = k \left(\frac{\mu}{\rho_w g} \right) \quad (5.7)$$

where μ is the viscosity of the permeant.

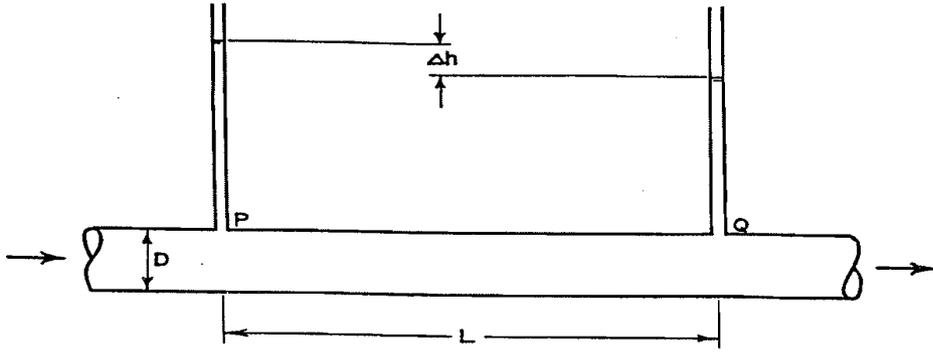


Fig. 5.1 Flow Through a Pipe

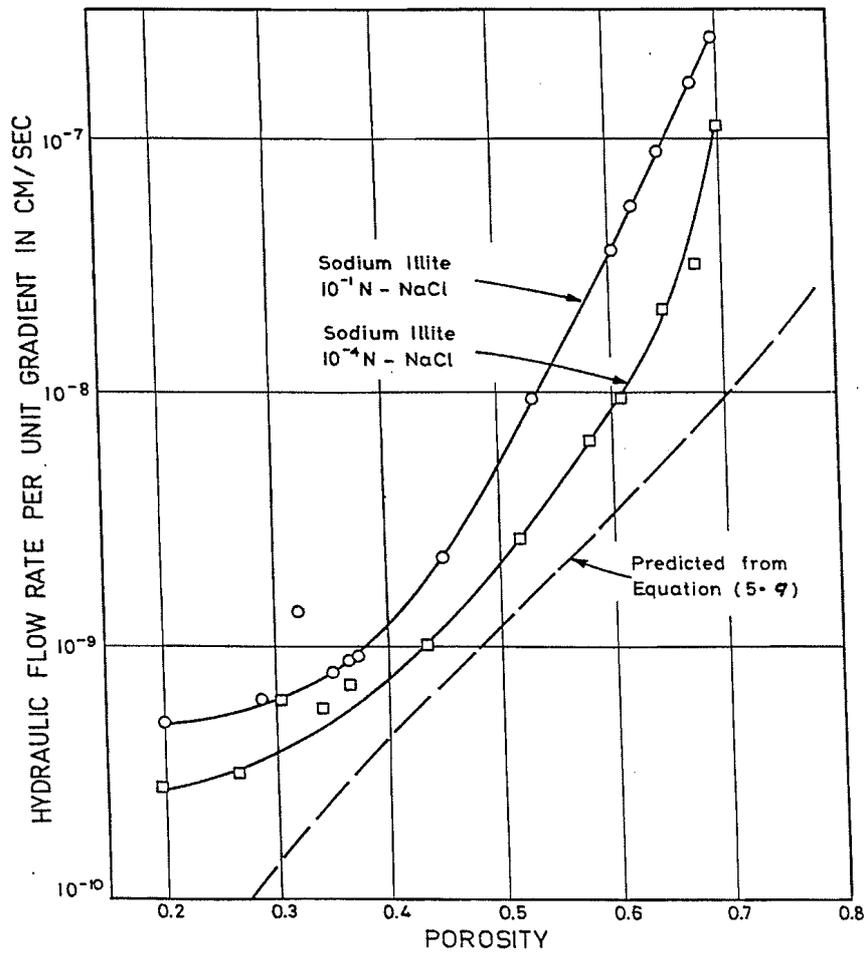


Fig. 5.2 Flow Rates versus Porosity
(after Olsen, 1962)

In carrying out permeability tests the viscosity is standardized by carrying out the tests at 20°C or by making a correction for tests carried out at other temperatures.

$$k_{20} = k_t (\mu_t / \mu_{20}) \quad (5.8)$$

where

$$k_{20} = \text{coefficient of permeability at } 20^\circ\text{C}$$

$$k_t = \text{coefficient of permeability at temperature } t$$

$$\mu_{20} = \text{viscosity at } 20^\circ\text{C}$$

$$\mu_t = \text{viscosity at temperature } t$$

An equation that has been proposed for absolute permeability (K) of sandy soils is the Kozeny-Carman equation.

$$K = \frac{1}{k_o T^2 S_o^2} \left(\frac{e^3}{1+e} \right) \quad (5.9)$$

where

$$k_o \text{ is a pore shape factor } (\approx 2.5)$$

$$T \text{ is the tortuosity factor } (\approx 2^{1/2})$$

$$S_o \text{ is the specific surface area per unit volume of particles}$$

$$e \text{ is the void ratio}$$

The Kozeny-Carman equation has been found to work well with sands but is inadequate with clays. This inadequacy with clays is illustrated in Figures 4.2 and 4.3. Olsen (1962) has shown that the major reason for these discrepancies with clay soils is the existence of unequal pore sizes.

The preceding comments indicate that several factors may influence the permeability of a soil, and these must be taken into account particularly when laboratory tests are used to assess the permeability of a soil stratum.

5.3 WHAT IS v ?

The v in equation (5.5) is known as the superficial or discharge velocity for the very good reason that it is not the actual velocity of flow of the water through the soil.

Consider a typical cross section through the soil in the pipe as illustrated in Fig. 5.4. The hatched portion represents the soil mineral particles of soil with an average cross sectional area equal to A_s . The remaining cross sectional area in the pipe $A_v (= \frac{V_v}{L}$ where L is the length of pipe and V_v is

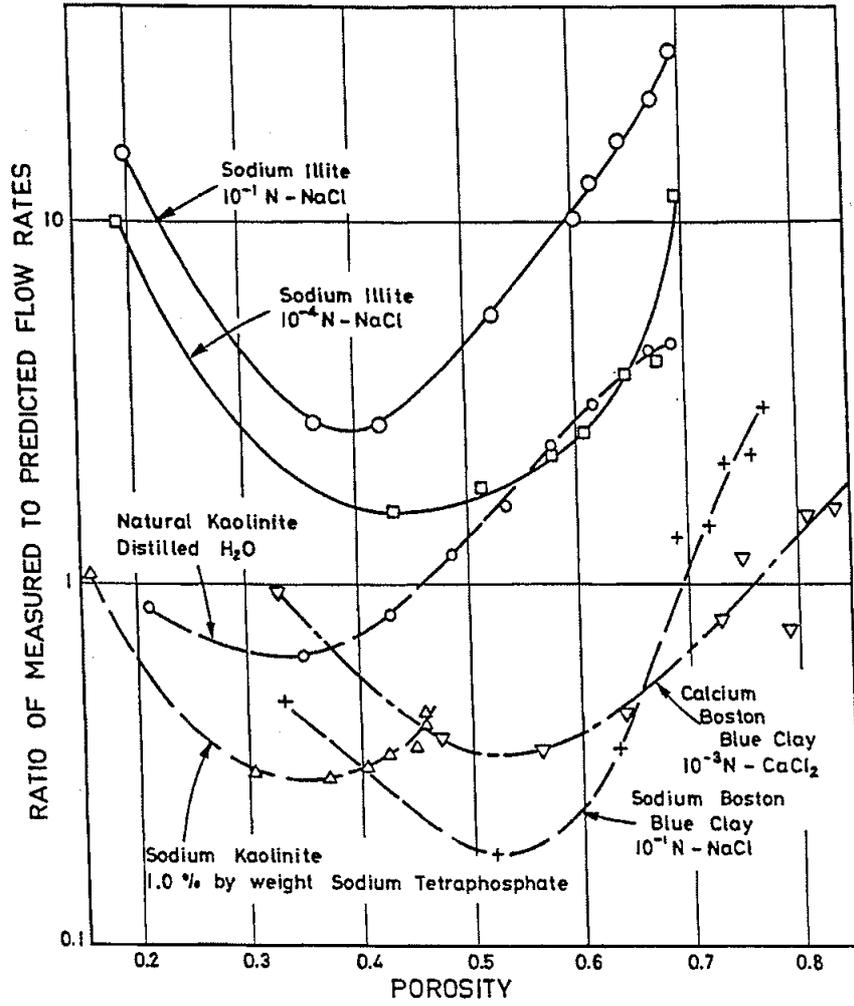


Fig. 5.3 Discrepancies Between Measured and Predicted Flow Rates (after Olsen, 1962)

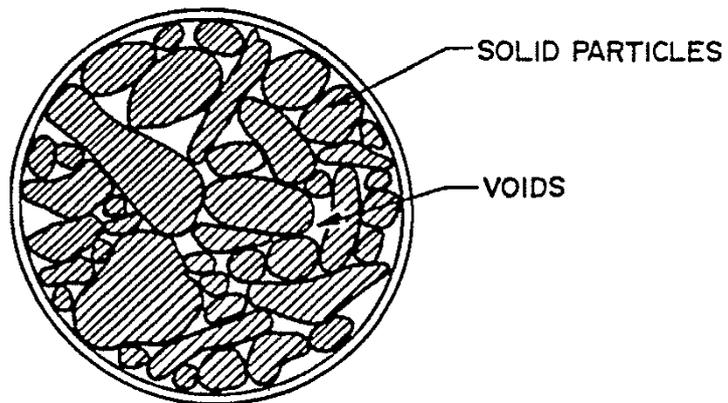


Fig. 4.4 Typical Cross-Section in a Soil Filled Pipe

the volume of voids) represents the void space through which the water flows. If A represents the total internal cross sectional area of the pipe then clearly

$$A = A_v + A_s$$

The rate of discharge of water Q is given by

$$Q = v A$$

Therefore $v = \frac{Q}{A} = k i$ from equation (5.5)

But the water is actually flowing through an area A_v with a velocity which will be indicated by v_s . The rate of discharge Q is

$$Q = v_s A_v \text{ which also } = v A$$

Therefore

$$v_s = v \frac{A}{A_v}$$

$$= v \frac{V}{V_v} \text{ where } V \text{ is the total internal volume of the pipe}$$

$$= \frac{v}{n} \tag{5.10}$$

So v (superficial or discharge velocity) and the actual velocity of flow v_s (effective or seepage velocity) are never equal.

5.4 WHAT IS i ?

As shown by equation (5.2) the hydraulic gradient (i) is obtained by dividing the head loss (Δh) by the length (L) over which this head is lost. In the case of horizontal seepage the head loss is entirely a loss in pressure head. In the case of seepage down a hill the Δh is derived from a loss of elevation. It is sometimes incorrectly assumed that the Δh in all cases is either a pressure head loss or an elevation head loss. In fact it is the loss of total head where, according to Bernoulli's equation

$$\begin{aligned} \text{total head } (h_t) &= p/\rho_w g + v^2/2g + h_e & (5.11) \\ \text{where } p/\rho_w g &= \text{pressure head } (h_p) \\ p &= \text{pressure} \end{aligned}$$

$$\begin{aligned} v^2/2g &= \text{velocity head } (h_v) \\ h_e &= \text{elevation head} \end{aligned}$$

In seepage problems the velocity of flow is very small and the velocity head is negligible in comparison with the pressure and elevation heads. Therefore in seepage problems

$$h_t = h_e + h_p \quad (5.12)$$

The interrelationship between elevation head, pressure head and total head for a one dimensional steady state seepage problem (seepage in one direction only) is illustrated in Fig. 4.5. Here water flows in a soil filled pipe of length L from point I to point O. In order to provide a base for measurement of elevation head it is necessary to define a datum. This horizontal datum line is chosen arbitrarily. The total head h_t is represented by the vertical distance between the datum and line ABCD. The elevation h_e for a particular point (P) is represented by the vertical distance between the datum and the point in question.

At the inlet I the total head is given by

$$h_{tI} = h_{eI} + h_{pI}$$

At the outlet O the total head is given by

$$h_{tO} = h_{eO} + h_{pO}$$

The loss in total head is

$$\Delta h = h_{tI} - h_{tO}$$

and this head is lost in the soil as indicated by line ABCD. The head losses in the two sections of pipe, one on the upstream side of the inlet (I) and the other on the downstream side of the outlet (O) are negligible compared with the head loss through the soil. This can easily be demonstrated by carrying out calculations using a pipe friction formula such as equation (4.1).

The total head gradient i is given by

$$i = \frac{h}{L} \quad (5.2)$$

which, it should be noted, is not equal to the slope of the line BC unless the pipe IO is in a horizontal position.

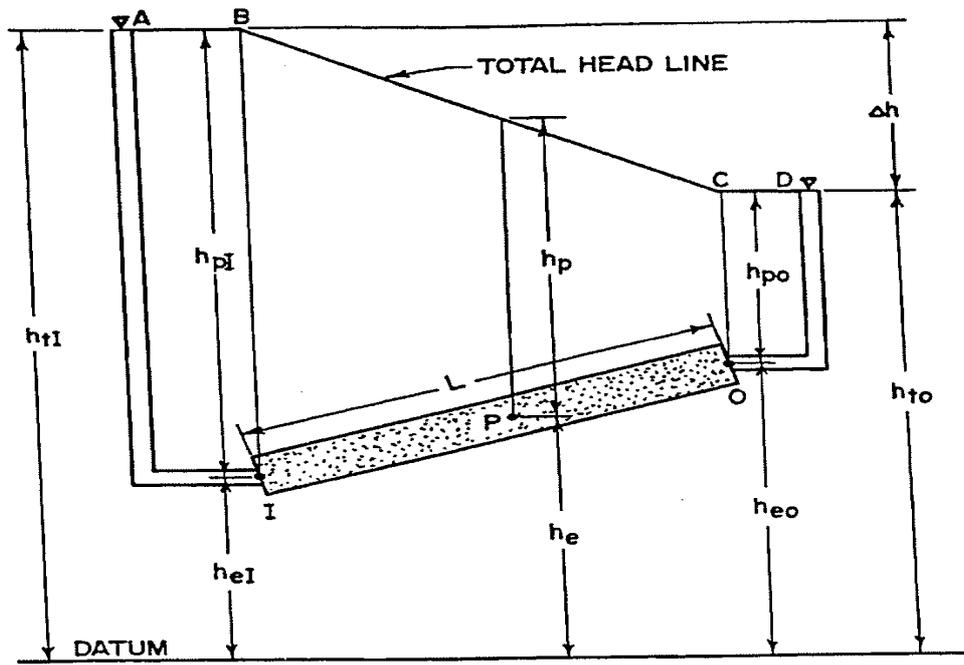


Fig. 5.5 One Dimensional Seepage

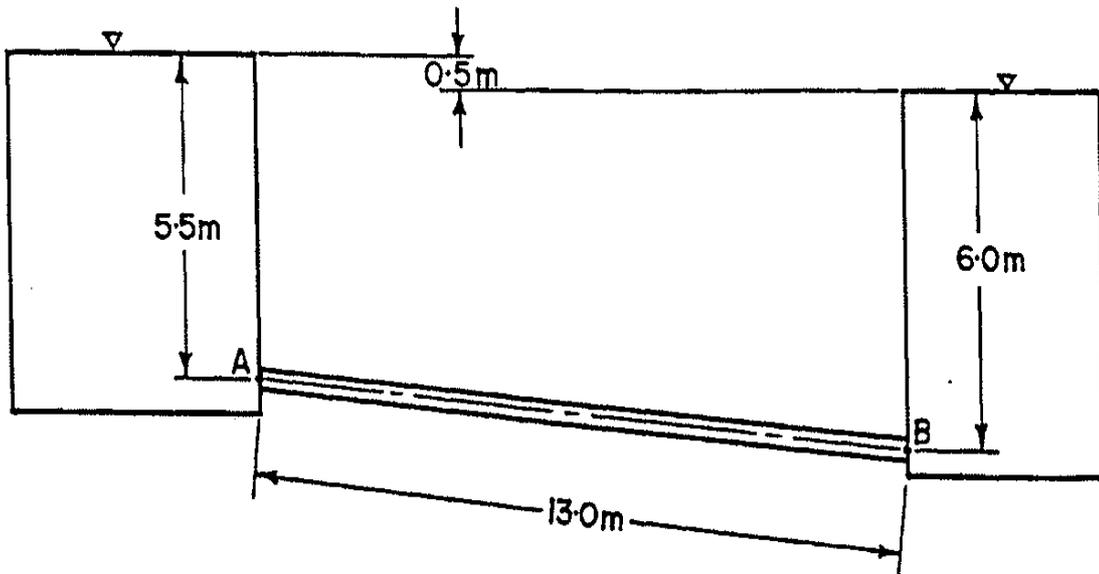


Fig. 5.6

5.5 PORE PRESSURE IN THE SOIL

The pressure head h_p is a measure (in units of length) of the pore pressure of the water in the soil. Again referring to Fig. 4.5 the pressure head at any point P may be found by measuring the vertical distance (h_p) between point P and the total head line ABCD. In other words if a piezometric tube was inserted into the soil at point P and suspended upwards with the end of the tube open to atmospheric pressure the water would rise in the tube a distance h_p above point P.

The pore pressure (u) may be calculated from the pressure head (h_p) by means of the expression

$$u = \rho_w g h_p \quad (5.13)$$

where ρ_w = density of water
 g = acceleration due to gravity

EXAMPLE

Referring to Fig. 5.6, a pipe (AB) 13.0m long connects two reservoirs. The pipe, which is filled with sand has a gross internal cross sectional area of 0.2m^2 . The permeability of the sand is 10^{-6}m/sec . Determine the amount of seepage that occurs under steady state flow conditions and calculate the pore pressure at the mid point of the pipe.

Firstly, an arbitrary datum must be selected. Suppose that this is set at a distance of 2.0m below the centreline of the pipe outlet (point B).

Then elevation head at B

$$h_{eB} = 2.0\text{m}$$

pressure head at B

$$h_{pB} = 6.0\text{m}$$

therefore total head at B

$$h_{tB} = 2.0 + 6.0 = 8.0\text{m}$$

Similarly,

$$h_{eA} = 3.0\text{m}$$

$$h_{pA} = 5.5\text{m}$$

$$h_{tA} = 3.0 + 5.5 = 8.5\text{m}$$

Loss in total head between points A and B

$$\Delta h = h_{tA} - h_{tB}$$

$$= 8.5 - 8.0 = 0.5\text{m}$$

Note that this loss of total head is equal to the difference in elevations of the reservoir water levels.

The gradient of total head

$$i = \frac{\Delta h}{L} = \frac{0.5}{13.0}$$

Using Darcy's law, the rate of seepage flow

$$\begin{aligned} Q &= k i A \\ &= 10^{-6} \times \frac{0.5}{13.0} \times 0.2 \\ &= 0.77 \times 10^{-8} \text{ m}^3/\text{sec} \end{aligned}$$

As the total heads at points A and B are 8.5m and 8.0m respectively the total head at the mid point of the pipe is 8.25m. The elevation head at the mid point of the pipe is 2.5m

$$\begin{aligned} \therefore \text{pressure head at mid point of pipe} &= 8.25 - 2.5 = 5.75\text{m} \\ \text{pore pressure} &= \rho_w g h_p \\ \text{where } \rho_w &= 1000 \text{ kg/m}^3 \\ g &= 9.81 \text{ m/sec}^2 \\ \therefore \text{pore pressure} &= 1000 \times 9.81 \times 5.75 \text{ N/m}^2 \\ &= 56.4 \text{ kN/m}^2 \end{aligned}$$

EXAMPLE

Fig. 5.7 (a) represents a steady state one dimensional seepage situation in which the upstream and down stream water levels are maintained constant. The container in which the soil (permeability $k = 1.2 \times 10^{-5}$ m/sec, and porosity $n = 0.2$) has been placed consists of two sections having different diameters. The areas of the upper (A_U) and lower (A_L) sections are 28cm^2 and 14cm^2 respectively.

- Determine the rate of seepage flow through the soil.
- Calculate the actual velocity of flow through the upper section of soil.
- Sketch the distribution of pore water pressure throughout the soil.

Firstly it is necessary to establish an arbitrary datum from which the elevation head may be measured. Let this datum be located a distance of 4cm below the lower boundary of the soil.

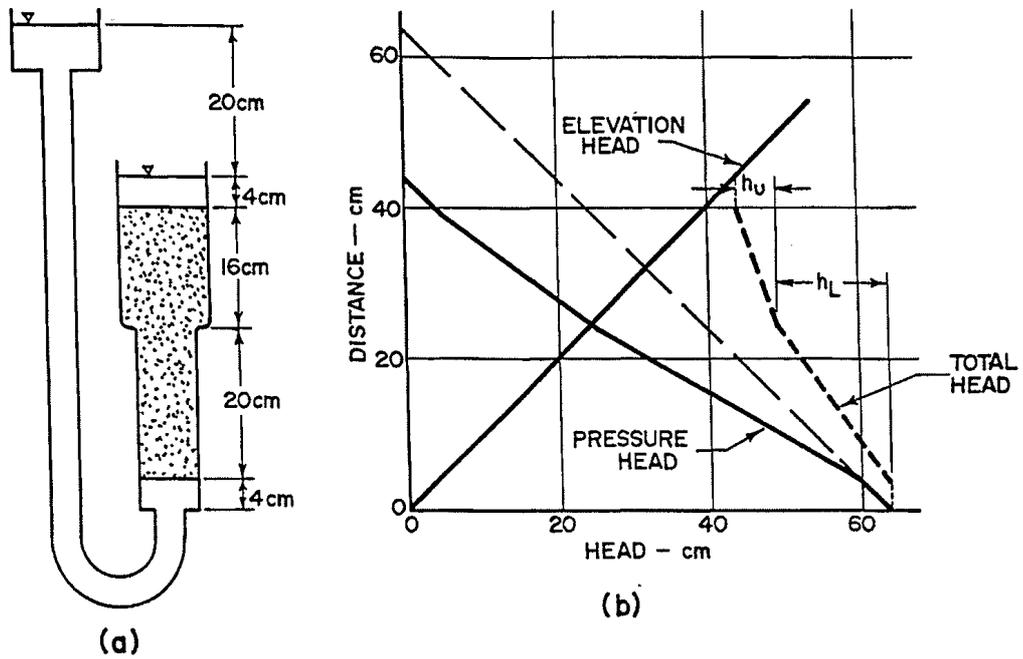


Fig. 5.7

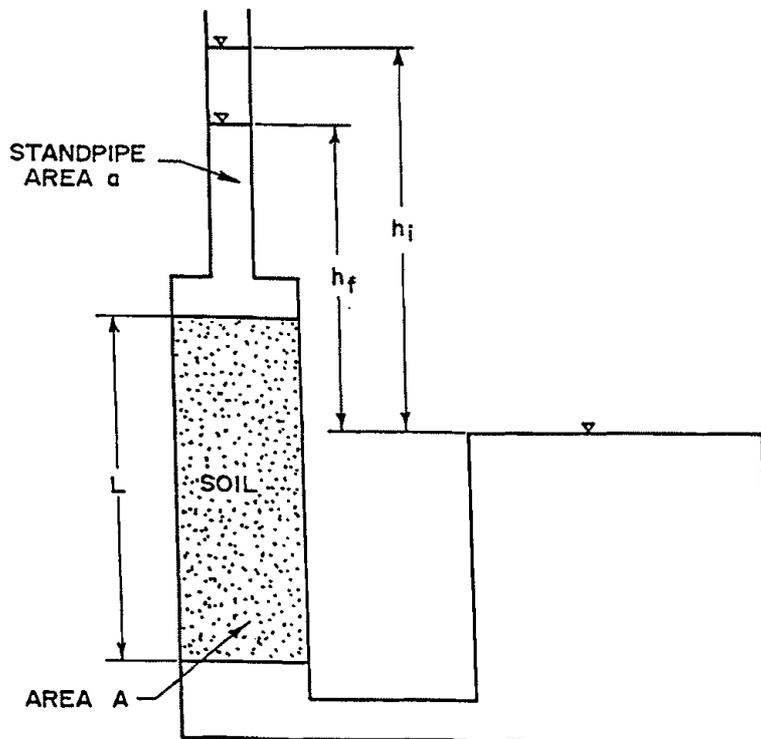


Fig. 5.8 Falling Head Permeability Test

The elevation head line can now be drawn on a plot of head against distance above the datum as illustrated in Fig. 5.7 (b).

The pressure head lines can now be drawn for the water on the upstream and downstream sides of the soil. The pressure head for the upstream (lower) side of the soil is determined by the distance below the upstream water level (left tube in Fig. 5.7 (a)). The distribution of pressure head throughout the soil cannot be drawn at this stage.

The total head lines can now be found for the water on the upstream and downstream sides of the soil by addition (represented horizontally in Fig. 5.7 (b)) of the pressure and elevation head lines. As shown in Fig. 5.7 (b) the total heads on the upstream and downstream sides of the soil are 64.0cm and 44.0cm respectively. This means that the loss in total head through the soil is 20.0cm (the same value as the elevation difference between the upstream and downstream water levels).

$$h_L - h_U = 20.0$$

where h_L and h_U are the total head losses in the lower and upper sections of soil respectively.

Since the rate of flow through the upper and lower section must be equal

$$Q = k i_L A_L = k i_U A_U$$

or
$$\frac{h_L}{L_L} \cdot A_L = \frac{h_U}{L_U} \cdot A_U$$

$$\frac{h_L}{20.0} \cdot 14.0 = \frac{h_U}{16.0} \cdot 28.0$$

\therefore
$$2h_L = 5h_U$$

The two equations involving h_L and h_U can now be solved to yield

$$h_L = 14.29\text{cm and } h_U = 5.71\text{cm}$$

and the total head line can now be completed as shown in Fig.5.7 (b).

The rate of seepage flow can now be found

$$Q = k i_L A_L$$

$$\begin{aligned}
 &= 1.2 \times 10^{-5} \times 10^2 \times \frac{14.29}{20.0} \times 14.0 \\
 &= 1.2 \times 10^{-2} \text{ cm}^3/\text{sec}
 \end{aligned}$$

The actual velocity of flow or seepage velocity, v_s in the upper section of soil is

$$\begin{aligned}
 v_s &= \frac{v}{n} & (5.10) \\
 &= \frac{k i_U}{n} \\
 &= 1.2 \times 10^{-5} \times 10^2 \times \frac{5.71}{16.0} \times \frac{1}{0.2} \\
 &= 2.14 \times 10^{-3} \text{ cm/sec}
 \end{aligned}$$

The pressure head line can be completed by subtracting the elevation head line from the total head line. The pore water pressure at any point in the soil can now be found by multiplying the pressure head at that point by $\rho_w g$.

5.6 EXPERIMENTAL DETERMINATION OF THE COEFFICIENT OF PERMEABILITY

5.6.1 Constant Head Test

The value of k could be determined by means of equation (5.4) using the experimental arrangement depicted in Fig. 5.5. The reservoir levels at A and D would be maintained constant allowing steady state seepage through the soil sample to be established. The rate of discharge Q would be observed over a convenient time interval. The permeability could then be determined from

$$k = \frac{Q}{A i} = \frac{QL}{A\Delta h} \quad (5.14)$$

The experimental details are set out in books on soil testing such as Lambe (1951) and Bowles (1970).

5.6.2 Falling Head Test

The falling head permeability test differs from the constant head permeability test in that the upstream and/or downstream total heads do not remain constant throughout the test. Fig. 4.8 depicts a situation in which the upstream water level is allowed to fall in a standpipe having a cross-sectional area a . The test commences with a head difference of h_i between the upstream and downstream water levels. At any time t after the commencement of the test the head difference will be symbolized by h . At the end of the test time t_1 , the head difference is h_f . Throughout the test the downstream water level remains constant.

At time t , the rate of discharge is

$$\begin{aligned} Q &= k i A \\ &= k \frac{h}{L} A \end{aligned}$$

and this must be equal to the rate of discharge through the standpipe

$$\begin{aligned} Q &= -a \frac{dh}{dt} \\ \therefore -a \frac{dh}{dt} &= k \frac{h}{L} A \end{aligned}$$

$$-a \int_{h_i}^{h_f} \frac{dh}{h} = \frac{kA}{L} \int_0^{t_1} dt$$

$$\therefore k = 2.3 \frac{aL}{At_1} \log_{10} \left(\frac{h_i}{h_f} \right) \quad (5.15)$$

5.6.3 Field Determination of Permeability

One of the many methods of determining k in the field is by means of a pump out test (USBR, 1974). This test can be used in a situation which is represented in Fig. 5.9. A horizontal stratum of pervious soil containing a water table overlies an impervious stratum. A well is sunk to the bottom of the pervious material and water is pumped from the well at a constant rate Q until steady state conditions are reached. Observation wells at radial distances r_1 and r_2 permit measurement of the heights h_1 and h_2 respectively.

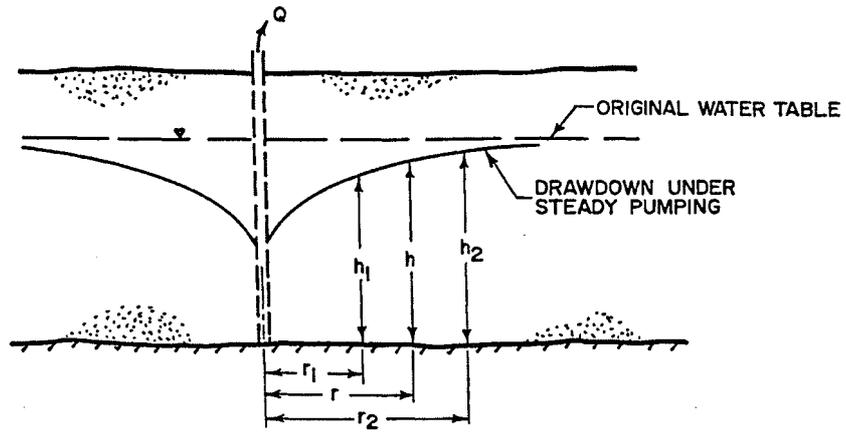


Fig. 5.9 Field Pump Out Test

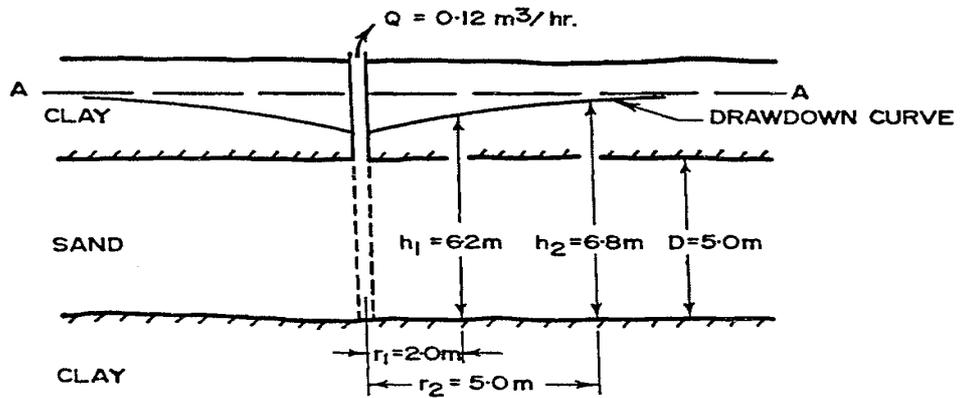


Fig. 5.10 Pump Out Test in a Confined Aquifer

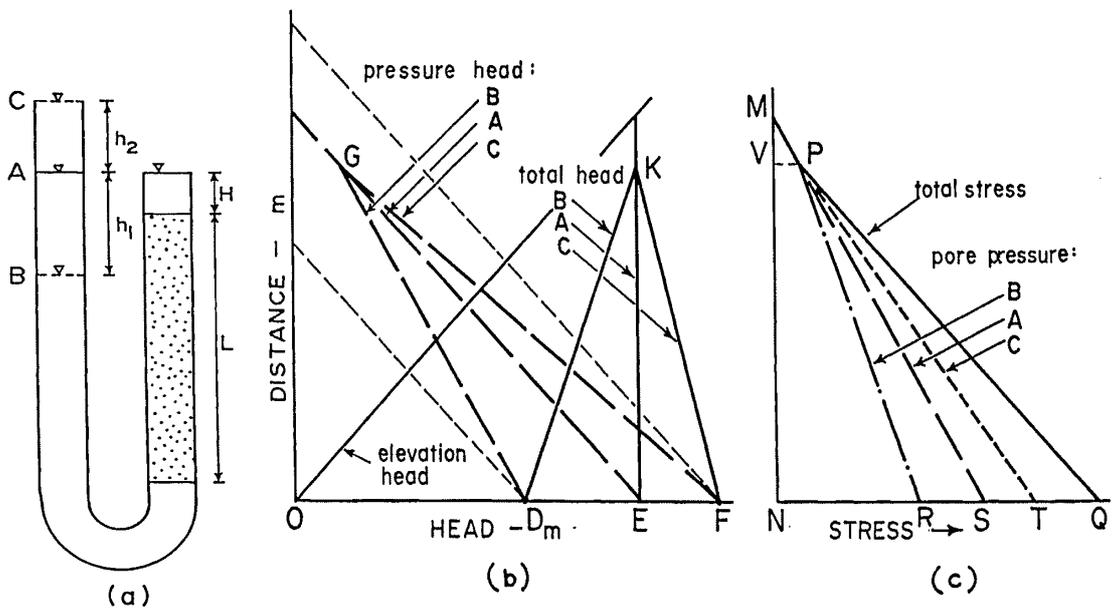


Fig. 5.11 Stress in a Soil During Seepage

At a radial distance r the cylindrically shaped area across which discharge occurs towards the well is

$$A = 2 \pi r h$$

and the hydraulic gradient at this radial distance may be approximated by

$$i = \frac{dh}{dr}$$

applying Darcy's law

$$\begin{aligned} Q &= k i A \\ &= k \frac{dh}{dr} 2\pi r h \end{aligned}$$

$$\therefore \int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k}{Q} \int_{h_1}^{h_2} h dh$$

$$\ln_e \left(\frac{r_2}{r_1} \right) = \frac{\pi k (h_2^2 - h_1^2)}{Q}$$

$$\therefore k = \frac{2.3 Q \log_{10} (r_2/r_1)}{\pi (h_2^2 - h_1^2)} \quad (4.16)$$

EXAMPLE

Fig. 5.10 represents a confined sand stratum located between two relatively impermeable clay strata. The piezometric surface (the level to which water would rise in a standpipe or piezometric tube placed in the sand stratum) is indicated by line AA. A pump out test with constant discharge is performed with a fully penetrating well. Observations of drawdown (steady state conditions) at two observation wells are shown in the figure. Determine the coefficient of permeability for the sand stratum, assuming it is homogeneous and isotropic.

Equation (5.16) cannot be used in this case since it was derived for an "unconfined" aquifer, whereas Fig. 5.10 represents a "confined" aquifer. Consequently, the appropriate equation will need to be derived.

Applying Darcy's law

$$\begin{aligned}
 Q &= k i A \\
 &= k \frac{dh}{dr} \cdot 2\pi r D \\
 \therefore \int_{r_1}^{r_2} \frac{dr}{r} &= \frac{2\pi k D}{Q} \int_{h_1}^{h_2} dh \\
 \ln_e \left(\frac{r_2}{r_1} \right) &= \frac{2\pi k D}{Q} (h_2 - h_1) \\
 \therefore k &= \frac{2.3 Q}{2\pi D} \frac{\log_{10}(r_2/r_1)}{(h_2 - h_1)} \quad (4.17)
 \end{aligned}$$

Substituting the values in Fig. 5.10 into equation (5.17)

$$\begin{aligned}
 k &= \frac{2.3 \times 0.12}{2\pi \times 5.0} \frac{\log_{10}(2.5)}{(6.8 - 6.2)} \\
 &= 5.8 \times 10^{-3} \text{ m/hr} \\
 &= 1.6 \times 10^{-6} \text{ m/sec}
 \end{aligned}$$

5.7 EFFECT OF SEEPAGE ON EFFECTIVE STRESS

In Chapter 3 the calculation of the effective vertical stress in a submerged soil stratum was demonstrated. This situation is reproduced in Fig. 5.11(a) in which a depth H of water overlies a saturated soil of depth L . The water level in the left side of the tank is at A . Under these conditions there is no flow of water.

A head diagram (similar to Fig. 5.7(b)) has been drawn in Fig. 5.11(b). The datum has been arbitrarily chosen to coincide with the underside of the soil. The pore water in the soil is under a hydrostatic pressure so the pressure head line corresponding to the water level in the left hand tube being at point A (in Fig. 5.11(a)), is given by line GE . The total head line KE shows no variation through the soil confirming that there is no flow of water.

The vertical stresses at various positions throughout the soil column are plotted horizontally in Fig. 5.11(c). The total stress line is represented MPQ and the pore water pressure

is represented by MPS. In this figure the pore pressure at the top and bottom respectively of the soil column are:

$$VP = \rho_w gH$$

$$NS = \rho_w g (H + L)$$

and the total vertical stresses are

$$VP = \rho_w g H$$

$$NQ = \rho_w gH + \rho_{\text{sat}} g L$$

$$= \rho_w g (H + L) + \rho_b g L$$

The distribution of effective vertical stress is represented by the horizontal distance between lines PQ and PS.

Now suppose that the water level in the left side of the tank is lowered by h_1 to B. The pressure head at the underside of the soil is now reduced by h_1 from OE to OD (see Fig. 4.11 (b)). The pressure head at the top of the soil remains unchanged. After steady seepage conditions (with the water level at B) have been reached, the new pressure head line is GD. The corresponding total head line (KD) confirms that downward seepage is taking place. The pore pressure in the soil may now be calculated from the pressure head line by measuring the horizontal distance between line GD and the vertical axis in Fig. 5.11(b).

Referring to Fig. 5.11(c) the total vertical stress remains unchanged at MPQ but the pore water pressure has decreased to line MPR where

$$\begin{aligned} \text{NR} &= \rho_w g (H + L - h_1) \\ \text{and RS} &= NS - NR \\ &= \rho_w g h_1 \\ &= \rho_w g i L \end{aligned}$$

where i is the gradient of total head through the soil.

The effective stress at any point in the soil is represented by the horizontal distance between lines PQ and PR, the increase in effective stress due to downward seepage being directly proportional to the total head gradient.

At the base of the soil column

$$\begin{aligned}\sigma' &= NQ - NR \\ &= \rho_b g L + \rho_w g h_1 \\ &= \rho_b g L + \rho_w g i L\end{aligned}$$

and at any depth z below the top of the soil

$$\sigma' = \rho_b g z + \rho_w g i z \quad (5.18)$$

The first term of this expression is due solely to the buoyant density of the soil and the second term is caused by the seepage force.

If steady upward seepage in the soil is to occur the water level in the left side of the tank in Fig. 4.11(a) may be raised to C. In this case the pressure head at the base of the soil column is increased by h_2 from OE to OF (see Fig. 5.11(b)). The new pressure head line becomes GF. The new total head line becomes KF confirming that upward seepage is taking place. As before, the pore water pressure in the soil may be calculated from the pressure head which is given by the horizontal distance between the vertical axis and the pressure head line GF. This increased pore pressure is indicated in Fig. 4.11(c) by the horizontal distance between the vertical axis and line PT.

$$NT = \rho_w g (H + L + h_2)$$

The effective vertical stress is represented by the horizontal distance between lines PQ and PT. The effective stress at any depth z below the top of the soil may be shown to be

$$\sigma' = \rho_b g z - \rho_w g i z \quad (5.19)$$

From the above presentation it is evident that effective stress in a soil, through which seepage is occurring, may be determined by two alternative methods

- (a) by consideration of total stresses and pore pressures, or
- (b) by consideration of buoyant density and seepage force.

The effective stress becomes zero when (from equation (5.19))

$$\rho_b = \rho_w i$$

or

$$i = \frac{\rho_b}{\rho_w} = i_c \quad (5.20)$$

This particular value of the gradient is referred to as the critical gradient and when this state is reached in a sand the phenomenon known as quicksand develops. Because the effective stress is zero the sand possesses no strength so the soil behaves as a dense fluid.

EXAMPLE

If the sand in Fig. 5.11(a) has a specific gravity (G) of 2.65 and a void ratio (e) of 0.5 determine the critical gradient at which a quicksand condition develops.

It may be shown (for example, by use of the phase diagram) that

$$\rho_b = \frac{(G-1)}{1+e} \rho_w$$

$$\therefore i_c = \frac{\rho_b}{\rho_w} = \frac{G-1}{1+e}$$

$$= \frac{2.65-1}{1+0.5}$$

$$= 1.1$$

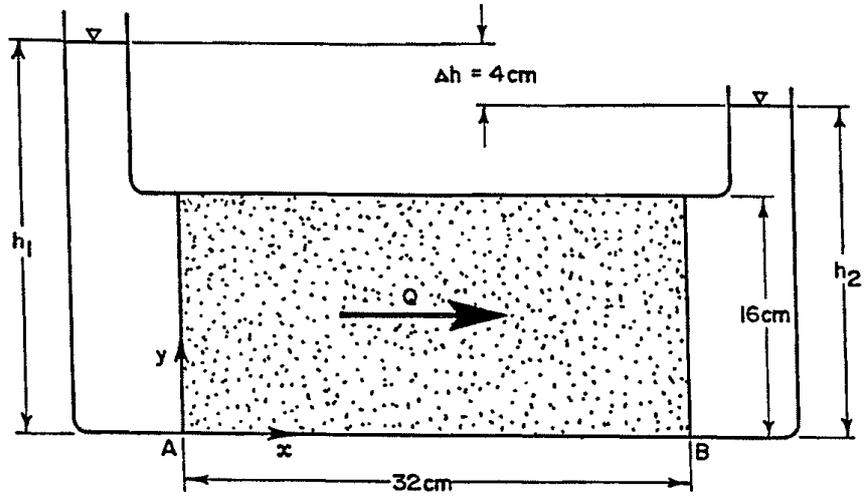
5.8 THE FLOW NET

Fig. 5.12(a) represents a one dimensional steady state seepage situation under constant head conditions. The soil is contained in a tank of rectangular section measuring 16 cm high by 8 cm wide. The loss in total head through the soil (h) is equal to 4 cm. The rate of seepage flow (Q) can be expressed in terms of the permeability, k cm/sec.

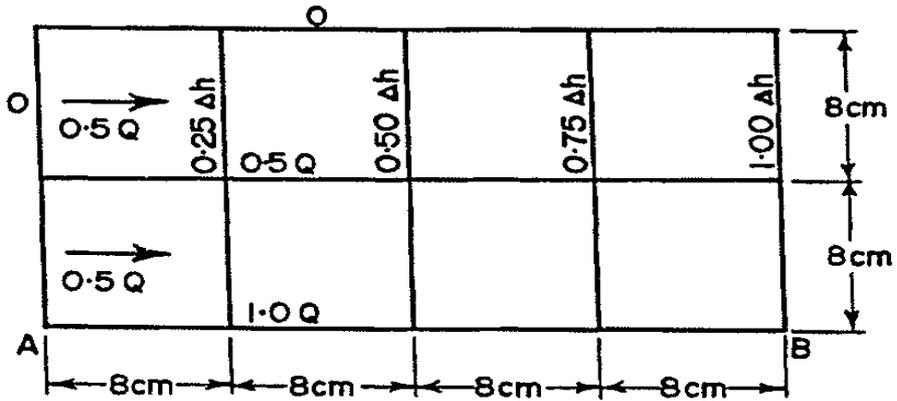
$$Q = k i A \quad (5.4)$$

$$= k \times \frac{4}{32} \times 16 \times 8$$

= 16 k cm³/sec



(a)



(b)

Fig. 5.12 Flow Net Representation of Seepage

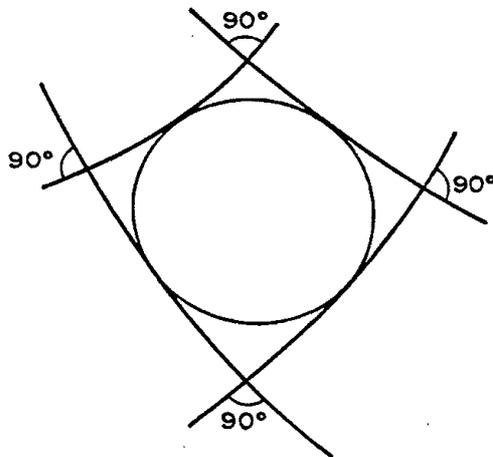


Fig. 5.13 Curvilinear Square

The soil is homogeneous and isotropic (equal values of permeability in all directions) and the total head loss is uniform throughout the 32cm length of soil. The total head loss is zero at the upstream end of the soil and equal to 4cm at the downstream end of the soil. The loss of total head throughout the length of soil is represented graphically by the vertical lines in Fig. 5.12(b).

As the flow is uniform the soil can be divided into a number of imaginary horizontal tubes through which equal amounts of seepage flow occur. For the example in Fig. 5.12 two flow tubes have been selected with one half of the total flow (Q) taking place through each tube. This subdivision of flow is represented graphically by the horizontal lines in Fig. 5.12(b). In this figure the top line has been arbitrarily selected as the original (zero flow). This leads to the bottom line as representing the total amount of seepage flow (Q).

In Fig. 5.12(b) the horizontal lines are streamlines or flowlines, which form the boundaries of the flow tubes. The vertical lines are equipotential lines, that is, lines of equal potential (total head). In this example they have been drawn to represent lines of equal total head loss. The pattern of streamlines and equipotential lines that is shown in Fig. 5.12(b) is known as a flow net. The flow net that has been drawn is only one of an infinite number of possible flow nets that could be drawn for this example. The pattern in the figure forms a mesh of squares each square measuring 8cm by 8cm. Not only could the squares have other dimensions but the basic shapes need not necessarily be squares; they could (for this particular problem) be any rectangular shape.

In the situation depicted in Fig. 5.12 the rate of seepage flow can be calculated directly from equation (5.4). In the more complicated two dimensional seepage problems the calculation cannot be carried out so simply. The calculation method which is more applicable to two dimensional problems will be demonstrated by application to the case in Fig. 5.12.

In the general case, for which a flow net consisting of a mesh of squares has been sketched, assume that the flow section has been subdivided into N_f flow tubes ($N_f = 2$ in Fig. 5.12(b)) with a rate of flow (q) passing through each tube.

$$\therefore \text{total flow rate } Q = N_f q$$

The magnitude of q may be calculated by considering the flow through one of the flow net squares. Assume that the dimensions of a flow net square are l cm x l cm ($l = 8$ cm in Fig. 5.12(b)). Let the total head loss through the soil be indicated by Δh and assume that with the sketched flow net there are N_d squares between adjacent equipotential lines (in Fig. 5.12(b), $\Delta h = 4$ cm and $N_d = 4$).

$$\therefore \text{Total head loss per square} = \frac{\Delta h}{N_d}$$

$$\text{Total head gradient through the square } i = \frac{\Delta h}{N_d l}$$

Using the Darcy expression

$$q = k i A \quad (5.4)$$

$$= k \cdot \frac{\Delta h}{N_d l} \cdot l \text{ per unit width}$$

$$= k \frac{\Delta h}{N_d} \text{ per unit width}$$

$$\begin{aligned} \text{Total flow rate } Q &= N_f q \\ &= k \Delta h \frac{N_f}{N_d} \text{ per unit width} \end{aligned} \quad (5.21)$$

For the problem in Fig. 4.12

$$\begin{aligned} q &= k \times 4 \times \frac{2}{4} \\ &= 2 k \text{ cm}^3/\text{sec. per unit width} \\ &= 2 k \times 8 \\ &= 16 k \text{ cm}^3/\text{sec.} \end{aligned}$$

in agreement with the calculation on page 5.17.

Equation (5.21) can be used in more general two dimensional flow situations in which the direct application of equation (5.4) is not appropriate.

5.9 FLOW IN TWO DIMENSIONS

In two dimensional flow problems the equipotential lines and flowlines are generally curved, but they intersect each other everywhere at right angles (see section 5.11). The flow net may be determined by one of a number of techniques; such as by mathematical calculation, by relaxation, finite difference or finite element techniques, by physical models or by manual trial and error sketching. Some of these techniques have been discussed by Halek and Svec (1979) and

by Huyakorn and Pinder (1983). If the flow net is sketched the square shapes illustrated in Fig. 5.12(b) become distorted into "curvilinear" squares. A curvilinear square as shown in Fig. 5.13 in a shape in which the four sides intersect at right angles and within which a circle touching all sides may be inscribed. In the limit, as the size becomes smaller and smaller a curvilinear square becomes an exact square.

EXAMPLE

A curved box of rectangular cross section (15 cm x 3 cm) is filled with sand as illustrated in Fig. 5.14. Steady state seepage through the sand is produced under constant head conditions. If the sand permeability is 1.8×10^{-5} m/sec determine the rate of seepage discharge.

If an attempt is made to solve this problem by means of equation (5.4) it is found that the total head gradient i cannot be evaluated with confidence since the length over which the total head loss of 4 cm occurs is not constant but depends upon the particular streamline selected. It is apparent that equation (5.21) must be used (with a square mesh flow net).

From the flow net, which has been drawn in Fig. 5.14 the numbers of flow tubes and equipotential drops may be determined for substitution into equation (5.21).

$$\begin{aligned}
 \text{Q per unit width} &= k\Delta h \frac{N_f}{N_d} && (5.21) \\
 &= 1.8 \times 10^{-5} \times 10^2 \times 4 \times \frac{2}{4} \\
 &= 3.6 \times 10^{-3} \text{ cm}^3/\text{sec/cm width} \\
 \text{Total Q} &= 3.6 \times 10^{-3} \times 3 \\
 &= 10.8 \times 10^{-3} \text{ cm}^3/\text{sec}
 \end{aligned}$$

EXAMPLE

Fig. 5.15 represents flow beneath a sheet piling wall. This flow net has been drawn by trial and error manual sketching. Determine:

- (a) the pore water pressure at point P, and
- (b) the maximum exit gradient

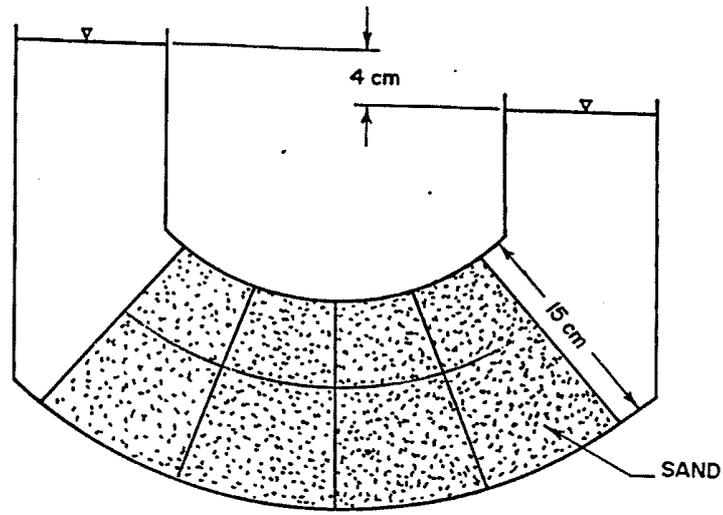


Fig. 5.14

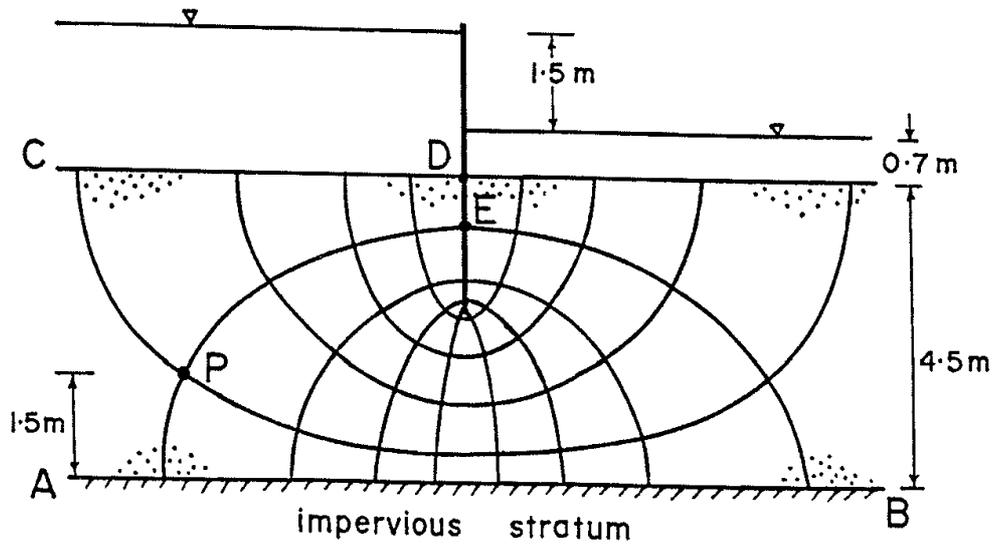


Fig. 5.15 Seepage Beneath Sheetpile Wall

(a) The total head difference of 1.5m between the upstream and downstream sides of the sheeting piling is lost over nine equipotential drops for the flow net that is drawn. If the impervious boundary AB is arbitrarily chosen as datum then the total head for the equipotential line CD along the upstream boundary of the flow region may be calculated.

$$\begin{aligned} \text{total head for CD} &= \text{elevation head} + \text{pressure head} \\ &= 4.5 + 2.2 \\ &= 6.7\text{m} \end{aligned}$$

The total head for the equipotential line passing through point P is less than this 6.7m by the amount of total head lost in one equipotential drop.

$$\begin{aligned} \text{total head for equipotential line through P} \\ &= 6.7 - \frac{1.5}{9} \\ &= 6.53\text{m} \end{aligned}$$

This means that the total head for every point on the equipotential line passing through point P is 6.53m. For point P in particular the elevation head is 1.5m.

$$\begin{aligned} \therefore \text{pressure head at point P} &= \text{total head} - \text{elevation head} \\ &= 6.53 - 1.5 \\ &= 5.03\text{m} \\ \therefore \text{pore pressure} &= 5.03 \times 9.81 \times 1000 \text{ N/m}^2 \\ &= 49.4 \text{ kN/m}^2 \end{aligned}$$

(b) The exit gradient will be calculated for the last equipotential drop in the flow net in Fig. 5.15.

$$\text{total head loss} = \frac{1.5}{9} = 0.17\text{m.}$$

The length over which this head is lost varies with the flow line considered and increases with increasing distance from the sheet pile. The minimum length is immediately adjacent to the sheet pile, that is, distance DE which measures 0.75m from the flow net.

$$\begin{aligned}\therefore \text{maximum gradient} &= \frac{0.17}{0.75} \\ &= 0.22\end{aligned}$$

EXAMPLE

Suppose that a weir 5m long with a cutoff wall as illustrated in Fig. 4.16 is to be considered as an alternative to the sheet pile wall in Fig. 4.15. Sketch the flow net for the new geometry and compare the rate of seepage discharge for this case with that depicted in Fig. 4.15. The permeability of the sand may be taken as 5μ m/sec.

In sketching the flow net in Fig. 4.16 the following points need to be remembered:

- (a) All flow lines commence on the upstream equipotential line CD and terminate on the downstream equipotential line HJ with right angle intersections.
- (b) DFGH and AB are flowlines.
- (c) All equipotential lines must intersect the two flowlines identified in (b) as well as every other flowline at right angles.
- (d) No two flowlines should touch each other.
- (e) No two equipotential lines should touch each other.

Suggestions for the beginner in learning graphical sketching of flownets have been given by Casagrande (1937) and Cedergren (1977). Using equation (4.21) for the flownet sketched in Fig. 5.16:

$$\begin{aligned}Q &= k\Delta h \frac{N_f}{N_d} \\ &= 5 \times 10^{-6} \times 1.5 \times \frac{4}{11} \\ &= 2.73 \times 10^{-6} \text{ m}^3/\text{sec for unit width.}\end{aligned}$$

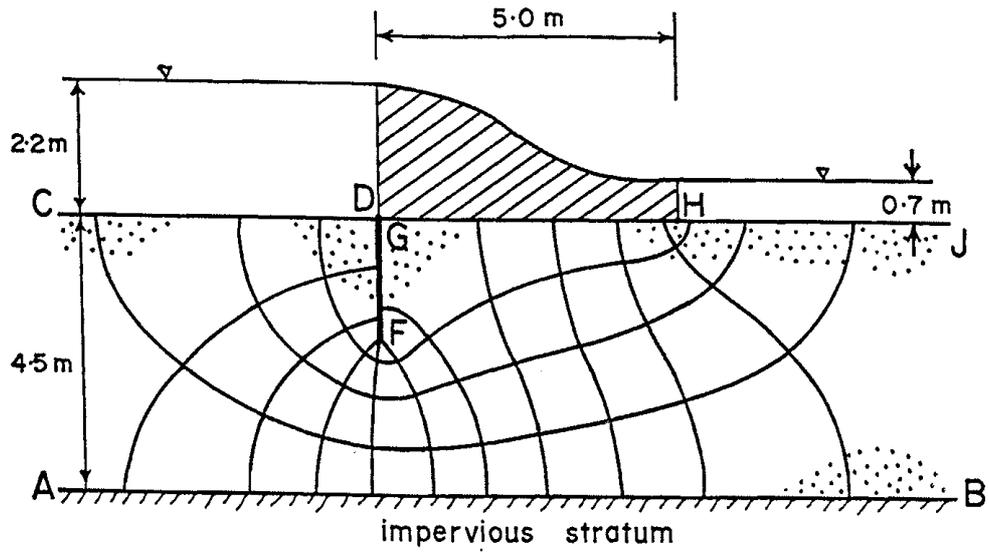


Fig. 5.16 Seepage Beneath Weir

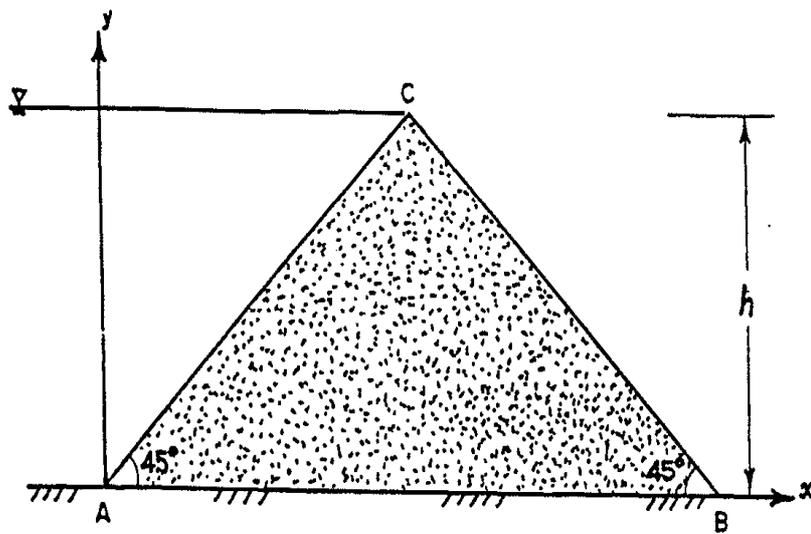


Fig. 5.17 Flow Through a Triangular Dam

By comparison, for the sheet pile wall in Fig. 5.15

$$\begin{aligned}
 Q &= 5 \times 10^{-6} \times 1.5 \times \frac{4.5}{9} \\
 &= 3.75 \times 10^{-6} \text{ m}^3/\text{sec per unit width}
 \end{aligned}$$

indicating that a slightly greater rate of underseepage occurs with the sheet pile wall.

REFERENCES

Bowles, J.E., (1970), "Engineering Properties of Soils and Their Measurement", McGraw Hill Book Co., New York.

Casagrande, A., "Seepage through Dams" Jnl. New England Water Works Assn., Vol. 51, No. 2, June 1937, also published in "Contributions to Soil Mechanics, 1925-1940", Boston Society of Civil Engineers, pp 295-336, 1940.

Cedergren, H.R., (1977) "Seepage, Drainage and Flow Nets", John Wiley and Sons, New York, 534p.

Halek, V. and Svec, J., (1979), "Groundwater Hydraulics" Elsevier, Amsterdam, 620p.

Harr, M.E., (1962) "Groundwater and Seepage", McGraw-Hill, New York, 315p.

Huyakorn, P.S. and Pinder, G.F., (1983), "Computational Methods in Subsurface Flow", Academic Press, New York, 473p.

Lambe, T.W., and Whitman, R.V., (1979), "Soil Mechanics SI Version", John Wiley and Sons, New York, 553p.

Lambe, T.W., (1951), "Soil Testing for Engineers", John Wiley and Sons, New York, 165p.

Mitchell, J.K. (1976), "Fundamental of Soil Behaviour", John Wiley and Sons, New York.

Olsen, H.W., (1962) "Hydraulic Flow through Saturated Clays", Proc. Ninth National Conference on Clays and Clay Minerals, pp 131 - 161.

United States Bureau of Reclamation, (1974) "Earth Manual", Dept. of the Interior, 810p.