Lesson 44 – Trigonometric Identities – Double Angle Formulas

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Fast Five

- Graph the following functions and develop an alternative equation for the graphed function (i.e. Develop an identity for the given functions)
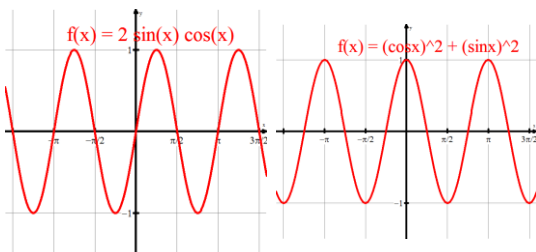
- $f(x) = 2\sin(x)\cos(x)$

- $g(x) = \cos^2(x) - \sin^2(x)$

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Fast Five

$f(x) = 2 \sin(x) \cos(x)$ $f(x) = (\cos x)^2 + (\sin x)^2$

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(A) Intro

- To continue working with trig identities, we will introduce the DOUBLE ANGLE formulas WITHOUT any attempt at a proof.

- To see a proof based upon the SUM & DIFFERENCE formulas, see:
- (a) The Math Page

- To see videos on DOUBLE ANGLE FORMULAS:
- (a) http://www.youtube.com/watch?v=VzKUIr-s5O0&feature=related
- (b) http://www.youtube.com/watch?v=FAtQrZegxfI&feature=related
- (c) http://www.youtube.com/watch?v=B8psoIRChkM&feature=related

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(B) Double Angle Formulas

- $\sin(2x) = 2\sin(x)\cos(x)$

- $\cos(2x) = \cos^2(x) - \sin^2(x)$

(B) Double Angle Formulas

- Working with $\cos(2x) = \cos^2(x) - \sin^2(x)$

- But recall the Pythagorean Identity where $\sin^2(x) + \cos^2(x) = 1$
- So $\sin^2(x) = 1 - \cos^2(x)$
- And $\cos^2(x) = 1 - \sin^2(x)$

- So $\cos(2x) = \cos^2(x) - (1 - \cos^2(x)) = 2\cos^2(x) - 1$
- And $\cos(2x) = (1 - \sin^2(x)) - \sin^2(x) = 1 - 2\sin^2(x)$

(B) Double Angle Formulas

- Working with $\cos(2x)$

- $\cos(2x) = \cos^2(x) - \sin^2(x)$
- $\cos(2x) = 2\cos^2(x) - 1$
- $\cos(2x) = 1 - 2\sin^2(x)$

(B) Double Angle Formulas

- $\cos(2x) = 2\cos^2(x) - 1$
- So we can get into "power reducing" formulas for $\cos^2 x$ ➔ $\frac{1}{2}(1 + \cos(2x))$

- $\cos(2x) = 1 - 2\sin^2(x)$
- So we can get into "power reducing" formulas for $\sin^2 x$ ➔ $\frac{1}{2}(1 - \cos(2x))$

(C) Double Angle Identities - Applications

- (A) Simplifying applications
- (B) Power reducing applications
- (C) Trig equations applications

(C) Double Angle Identities - Applications

- (a) Determine the value of sin(2θ) and cos(2θ) if
  $$\sin(\theta) = \frac{1}{4} \quad \text{and} \quad \frac{\pi}{2} \le \theta \le \pi$$
- (b) Determine the values of sin(2θ) and cos(2θ) if
  $$\cos(\theta) = -\frac{\sqrt{5}}{6} \quad \text{and} \quad \pi \le \theta \le \frac{3\pi}{2}$$
- (c) Determine the values of sin(2θ) and cos(2θ) if
  $$\tan(\theta) = \frac{2}{5} \quad \text{and} \quad \pi \le \theta \le \frac{3\pi}{2}$$

(C) Double Angle Identities - Applications

- Simplify the following expressions:

  (a) $\dfrac{\sin 2x}{\cos x}$ (b) $\cos(2x) + 1$

  (c) $\dfrac{\cos x \sin 2x}{1 + \cos 2x}$ (d) $\cos(2x) + 2\sin^2(x) + 1$

  (e) $\dfrac{\cos 2x}{\cos x + \sin x}$ (f) $\dfrac{\cos 2x}{\cos x - \sin x} - \sin x$

(C) Double Angle Identities - Applications

- Write as a single function:

  (a) $\sin x \cos x$

  (b) $2\cos^2 x - 2\sin^2 x$

  (c) $2\cos^2 x - \sin^2 x$

(C) Double Angle Identities - Applications

■ Use the double angle identities to prove that the following equations are identities:

(a) $\dfrac{\sin 2x}{1 + \cos 2x} = \tan x$

(b) $\dfrac{1 - \sin 2x}{\cos 2x} = \dfrac{\cos 2x}{1 + \sin 2x}$

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(C) Double Angle Identities - Applications

■ Solve the following equations, making use of the double angle formulas for key substitutions:

(a) $\cos 2x = \cos^2 x$ for $-\pi \le x \le \pi$

(b) $\sin 2x = \cos x$ for $-\pi \le 2x \le \pi$

(c) $\sin 2x + \sin x = 0$ for $0 \le x \le 2\pi$

(d) $\cos 2x + \cos x = 0$ for $0 \le x \le 2\pi$

(e) $\cos^2 x - 2\sin x \cos x - \sin^2 x = 0$ for $0 \le 2x \le \pi$

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(C) Double Angle Identities - Applications

■ Use the idea of "power reduction" to rewrite without powers an equivalent expression for:

(a) $y = \sin^4 x$

(b) $y = \cos^4 x$

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Homework

■ HH Text

■ EX 13J, Q1-8, page 293

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