# Motivation for Achievement in Mathematics: Findings, Generalizations, and Criticisms of the Research

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In this review we examine recent research in the area of motivation in mathematics education and discuss findings from research perspectives in this domain. We note consistencies across research perspectives that suggest a set of generalizable conclusions about the contextual factors, cognitive processes, and benefits of interventions that affect students' and teachers' motivational attitudes. Criticisms are leveled concerning the lack of theoretical guidance driving the conduct and interpretation of the majority of studies in the field. Few researchers have attempted to extend current theories of motivation in ways that are consistent with the current research on learning and classroom discourse. In particular, researchers interested in studying motivation in the content domain of school mathematics need to examine the relationship that exists between mathematics as a socially constructed field and students' desire to achieve.

Key Words: Achievement; Attitudes; Beliefs; Motivation; Review of research

National assessment data from the 1980s (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Dossey, Mullis, Lindquist, & Chambers, 1988) have indicated that American children tend to enjoy mathematics in the primary grades but that this level of enjoyment tends to fall dramatically when children progress into and through high school. In addition, although students feel that mathematics is important, the number of students who want to take more mathematics in school is declining steadily (Dossey et al., 1988). These statistics seem alarming when coupled with the fact that children do not possess the mathematical knowledge that they will need to function smoothly in our increasingly technological society. The problem is considered important enough for the National Council of Teachers of Mathematics (NCTM) to place the motivational domains Learning to value mathematics and Becoming confident in one's own ability as two of its foremost goals for students as an attempt to change the nature of school mathematics (NCTM, 1989).

Our purpose in this review is to describe theoretical orientations guiding research in mathematics motivation and to discuss findings in terms of how they facilitate or inhibit achievement. First, we discuss definitions of motivation and distinctions among types of motivation in education. Second, we discuss theoretical orientations and describe representative research from these orientations. Third, findings from the reviewed studies are drawn into a set of conclusions rep-

resenting what is known regarding students' motivation in mathematics, how inequities in mathematics education are reflected in students' motivational patterns, and the role of the teacher in enhancing or inhibiting students' motivation. Last, we raise criticisms regarding the role of theory in informing research and the lack of adequate conceptualization prevalent in operationally defining motivation, achievement, and mathematics as a content domain.

Although pertinent work has been done in motivation outside the domain of mathematics, in this article we focus on studies in which the participants were students in mathematics classes or mathematics teachers. We made this choice in response to criticisms that context has been largely ignored in studies of teaching and learning (Romberg & Carpenter, 1986). Moreover, there is convincing evidence that student effort and performance can be better explained by task-specific analyses of motivation in mathematics than by general measures of motivation (Seegers & Boekaerts, 1993). By focusing on studies within the mathematics education literature, we hope to draw out conclusions that are sensitive to the context of school mathematics. This analysis allows exposition and criticism regarding the limitations of our knowledge about motivation related to mathematics as a content domain.

#### JUST WHAT ARE MOTIVATIONS?

Simply stated, motivations are reasons individuals have for behaving in a given manner in a given situation. They exist as part of one's goal structures, one's beliefs about what is important, and they determine whether or not one will engage in a given pursuit (Ames, 1992). Two distinct types of academic motivation interrelate in most academic settings—intrinsic and extrinsic motivation. Academic intrinsic motivation is the drive or desire of the student to engage in learning "for its own sake." Students who are intrinsically motivated engage in academic tasks because they enjoy them. They feel that learning is important with respect to their self-images, and they seek out learning activities for the sheer joy of learning (Middleton, 1992/1993a). Their motivations tend to focus on learning goals such as understanding and mastery of mathematical concepts (Ames & Archer, 1988; Duda & Nicholls, 1992; Dweck, 1986). Students who are extrinsically motivated engage in academic tasks to obtain rewards (e.g., good grades, approval) or to avoid punishment (e.g., bad grades, disapproval). These students' motivations tend to center on such *performance goals* as obtaining favorable judgments of their competence from teachers, parents, and peers or avoiding negative judgments of their competence (Ames, 1992; Ames & Archer, 1988; Duda & Nicholls, 1992; Dweck, 1986).

When individuals engage in tasks in which they are motivated intrinsically, they tend to exhibit a number of pedagogically desirable behaviors including increased time on task, persistence in the face of failure, more elaborative processing and monitoring of comprehension, selection of more difficult tasks, greater creativity and risk taking, selection of deeper and more efficient perfor-

mance and learning strategies, and choice of an activity in the absence of an extrinsic reward (Lepper, 1988). Moreover, intrinsic motivation is related to students' perceptions of their competence in mathematics, to whether they are motivated by curiosity or by grades, and to whether their orientation toward academic achievement can be characterized as a mastery orientation. Intrinsic motivation in other subject areas seems to be only moderately correlated with these variables (Gottfried, 1985).

Researchers have found that although achievement, ability, and perceived competence each contribute to students' desire to learn mathematics, intrinsic motivation is more complex than the additive effects of these domains. When students see themselves as capable of doing well in mathematics, they tend to value mathematics more than students who do not see themselves as capable of doing well (Eccles, Wigfield, & Reuman, 1987; Midgley, Feldlaufer, & Eccles, 1989), but these expectations of success also influence short-term strategy use (Meece, Wigfield, & Eccles, 1990; Pokay & Blumenfeld, 1990), thereby inhibiting or augmenting achievement. It is likely that students must feel comfortable with mathematics, must be challenged to achieve, and must expect to succeed before the development of intrinsic motivation can begin.

The findings of these studies suggest that the decline in positive attitudes toward mathematics can be explained in part as functions of lack of teacher supportiveness and classroom environment. These findings, along with results from national assessments (Dossey et al., 1988), suggest that motivational patterns are learned and, what is particularly distressing, that students generally learn to dislike mathematics and that this dislike becomes an integral part of their mathematical self-concepts.

When one looks at the subtle ways in which motivations are formed, modified, and sustained, it becomes clear that there is no such thing as an unmotivated child. Children are motivated. Motivations help guide children's activity; they provide a structure for evaluating the outcomes of activity; and they help determine whether or not children will engage in future mathematical activity. The following discussions describe prominent approaches to investigating and applying motivational theory. We present the main theories, review research, and discuss results in terms of classroom practices that facilitate or inhibit students' developing productive motivational patterns.

#### THEORETICAL ORIENTATIONS

## Behavioral Theories of Motivation

Throughout most of the 20th century, behaviorist theories of motivation dominated the literature. In this perspective, motivations are seen as incentives for performing a given behavior (Spence, 1960). Newer reformulations of these theories (McClelland, 1965, cited in Covington, 1984) have focused on the potential conflict between an individual's perceived necessity for success and perceived necessity for avoiding failure.

Although the declining popularity of behavioral research has led to a declining number of studies in this paradigm, this theoretical orientation has provided powerful knowledge about student motivation in mathematics. First, research indicates that success in mathematics is a powerful influence on the motivation to achieve. Students perceive success as reinforcing, and they will engage in mathematics if they expect to be successful. In addition, students will not only engage more, they will also tend to enjoy tasks for which they have a moderately high probability of success more than tasks for which the probability of success is near chance (Dickinson & Butt, 1989). Although success may not be the only determinant of on-task behavior, it is clearly related to the achievement motivation of children in mathematics.

Second, and more important, an orientation toward achieving success in mathematics can be built into the mathematics classroom. When students are given incentives to achieve, the motivation and achievement of entire classes can be raised (Alschuler, 1969). When children are rewarded for choosing a high level of personal success in mathematics, they tend to enjoy mathematics more and achieve more than when they are not given incentives. Slavin (1984), for example, recommended the provision of *group incentives* to motivate students to achieve (i.e., providing a group reward for individual learning). Because the *group* score is rewarded, children are motivated to help others in the group and are pressured to learn well themselves; through this practice, individual accountability is emphasized. This practice allows students to attribute their successes to themselves and their failures to the group, thus reducing the individual's onus for failure proportionately to the number of students in the group.

Severe limitations are, however, evident in this paradigm, which depends on achievement measures that use either multiple-choice tests or well-defined problems. It is unclear how more realistic problems, ones that provide more avenues for failure, would affect the success rate of children. Also unclear is whether *success* should be defined as success with a problem as a whole or in the steps necessary to solve the problem. The operational definition of *success* inherent in behaviorist research, with a focus on discrete observable behaviors, may be too molecular in scope or too removed from children's attitudes to be a valid index of their achievement motivation. Time-on-task is often used as an index of motivation (e.g., Dickinson & Butt, 1989). Reliance on time-on-task, however, introduces a confounding variable into the research design: The difficulty level of a problem is related to the time required to solve the problem, independent of motivation. In addition, because behaviorist theories have not traditionally been concerned with individual differences, they fail to provide information on how *students* define success and failure in mathematics.

The most compelling argument against the use of incentives or coercion, however, is the "hidden costs of reward," well described by Lepper and Greene (1978). Engaging in an intrinsically motivating activity under conditions that make obvious the fact that the activity is merely a means to an end will diminish subsequent intrinsic motivation because the presence of the reward is the primary reason for the student to engage. Consequently, in the absence of the reward students become less likely to engage in similar tasks in the future. The most salient (and most misrepresented) feature of this line of research is not that rewards necessarily undermine intrinsic motivation but that the *expectation* of *tangible task-contingent* rewards tends to weaken the intrinsic desire to learn. When rewards are not expected, intrinsic interest does not seem to be affected adversely nor do noncontingent rewards seem to have any real effect on subsequent intrinsic motivation (Deci, 1972; Lepper, Greene, & Nisbett, 1973). Lepper, Keavney, and Drake (1996) even suggested that judicial application of reward contingencies can be beneficial for developing sufficient skill in a pursuit so that intrinsic motivation can develop. Although this longstanding principle has recently been contested (Cameron & Pierce, 1994, 1996), a plethora of research suggests that when rewards are used to get someone to engage in some activity, the probability of subsequent disillusionment with the activity increases significantly (Kohn, 1996; Lepper et al., 1996; Ryan & Deci, 1996).

## Attribution and Learned Helplessness Theories

Researchers in the 1960s and early 1970s, when they began to examine individuals' perceived reasons for their successes and failures, found that success is not a universal motivator. Much of an individual's intention to initiate behavior depends on the *value* that the consequences of success have for him or her (Atkinson, 1964). Researchers began to focus attention on what factors students perceive to be the causes of their successes and failures. Attribution theories deal with how the outcomes of an activity are evaluated in relation to the individual's perception of his or her own contribution (i.e., ability and effort) and the contribution of the task demands (i.e., difficulty, consistency, precedent) (Weiner, 1972).

In mathematics education, attribution theory is the most widely held of the theoretical orientations discussed in this article, perhaps because (a) attribution theories are cognitive, describing the *processes* by which motivations are acquired and changed and (b) they are applicable to a remarkable range of domains. Moreover, attribution theories provide a middle ground between competing models of motivation such that findings can be discussed in terms of reinforcers and contingencies or in terms of students' thoughts, plans, and goals.

Attributions and achievement in mathematics. Students in the lower elementary grades are generally highly motivated to learn mathematics. They believe that they are competent and that working hard will enable them to succeed. Many first and second graders do not distinguish between effort and ability as causes of success in mathematics (Kloosterman, 1993). However, there is considerable evidence that some students begin to differentiate ability for different content domains as early as kindergarten or first grade (Wigfield et al., 1992). By the middle grades, many students begin to perceive mathematics to be a special domain in which smart students succeed and other students merely "get by" or fail. They begin to believe that success and failure are attributable to ability and that effort rarely results in a significant change in their success patterns (Kloosterman & Gorman, 1990).

When students attribute their successes to ability, they tend to succeed; when they attribute their failures to lack of ability, they tend to fail. Gender studies have shown that girls tend not to attribute their successes to ability but do tend to attribute their failures to lack of ability, exactly the attributional style that leads to failure. For example, Meyer and Fennema (1985) studied the relationship between students' attributions of success in mathematics in the 8th grade and their subsequent achievement in 11th grade. This study was a departure from most attribution research, at least as it related to mathematics education, in that it assessed the relationship between attributions and future success in mathematics instead of current success. The authors found that attribution of success to ability was the most consistent correlate of Grade 11 achievement. Conversely, attribution of failure to lack of ability was the most consistent correlate of lack of achievement for both males and females. For girls in particular, when ability was controlled for, attributing failure to lack of ability was associated with lower achievement. However, attributing failure to lack of effort was also a significant predictor of lack of achievement on computation problems and high-level, conceptual mathematics tasks. Boys' attributions were not as pronounced as girls' for these variables. The authors concluded that attributions may be more important as predictors of success in mathematics for females than for males.

Kloosterman (1988) studied how seventh graders perceived the role of successes and failures in influencing their motivational attributions, their mathematical self-confidence, and their beliefs about effort as a mediator of mathematical ability and failure as an acceptable phase in learning mathematics. He found that attributional style (a combined score, scaled in the direction of internal, stable attributions) was the best predictor of mathematical self-confidence. The belief that effort is a mediator of ability and that failure is an acceptable phase in learning mathematics also contributed to students' self-confidence in mathematics. Although girls, more often than boys, felt that failure was an acceptable phase in learning mathematics, the fact that girls also thought about their failures more than boys did may have contributed to differential effects like those reported by Meyer and Fennema (1985).

These findings are significant in that when students conceive of ability as amenable to change or augmentation through effort, they tend to expend more effort in mathematics and, thus, are better achievers than students who believe that ability is fixed. Because the belief that occasional failure is acceptable in learning mathematics predicts mathematical self-confidence, the practice of allowing children to struggle with challenging problems, even in the elementary grades, is supported. When children who have not experienced difficult problems in mathematics encounter a problem that cannot be solved in a routine fashion, they may have their confidence shattered unless they believe that occasional mistakes are a part of learning mathematics.

By the time they reach college, students generally have formed stable attributions regarding their successes in mathematics. Because the attributional patterns of students in mathematics-related majors tend to focus on ability and effort as the causes for success and lack of effort for failure, females, who tend to attribute their failures to ability, may be systematically excluded from mathematics majors as a result of their prior mathematics education (Amit, 1988; Bassarear, 1986). In addition, because students with unstable attributions for the causes of failure in mathematics tend to dislike mathematics greatly (Lehmann, 1986), these students may also be filtered out of mathematics-related majors.

Amit (1988) studied the attributions of university students in five major areas and found that, overall, females tend to attribute their successes in mathematics to external and unstable causes, whereas males attribute their successes to ability, an internal and stable factor. When attributions of success were analyzed taking academic major into account, however, students tended to attribute their causes of success and failure the same way regardless of gender. Students choosing mathematics as a major tended to attribute their successes to ability and their failures to other factors. In fact, as the mathematical requirements for participation in college majors increased, so did the attribution of success to the internal factor of ability. Students who attribute their failures in mathematics to internal factors and their successes to external factors are unlikely to choose a college curriculum with substantial mathematics content.

Learned helplessness and dealing with failure. An outgrowth of attribution theory has been the specific attention of researchers to learned helplessness, a condition in which, because of lack of successes and the attribution of failure to lack of ability, individuals begin to view success as unattainable (e.g., Dweck, 1986). Unfortunately these beliefs persist as a result of educational environments that (a) place high value on ability and lower value on effort and (b) offer little opportunity for individuals with diverse learning styles to supplement their abilities with sustained effort (Covington, 1984). Because helpless individuals believe that success is out of their grasp and attribute failure to internal factors, learned helplessness often becomes perceived as a trait (i.e., stable and unchanging) (Dweck, 1986). Helpless individuals tend to show little motivation for challenging tasks, and, in fact, when facing a challenging task, they display lower achievement than can be attributed to ability.

Although the findings of most studies regarding learned helplessness are disheartening, there is some evidence that attributions can be positively influenced through classroom instruction. For example, Relich (1984) hypothesized that when students are provided attribution retraining in conjunction with skills training, their feelings of learned helplessness should be reduced and their mathematics achievement should be positively affected.

Those providing attribution training attempted to make students aware that they were achieving success on increasingly difficult problems as a result of at least average ability and high effort. Students who received the attribution training displayed superior self-efficacy gains and fewer learned-helplessness characteristics compared with students receiving no attribution training.

Relich (1984) then proposed a causal model that contrasted the direct effects of attribution training with the mediated effects on achievement and learned

helplessness. Results of a path analysis indicated that although the attribution training had a moderate direct influence on achievement, stronger paths resulted from mediation through self-efficacy. The attribution training also had a direct influence on reducing learned helplessness; reducing learned helplessness, in turn, had a direct effect on students' development of self-efficacy. Thus, it seemed reasonable to predict that the attribution training's effects on achievement were mediated through self-efficacy via reduction of learned helplessness.

Intervention and the role of the teacher. Attribution training has been found to be effective in helping students develop positive motivational patterns and increase performance in other content domains as well (Williams, 1993). However, a major difficulty in designing appropriate intervention strategies in the mathematics classroom is the tendency for teachers' attributions to parallel and reinforce those of their students. Teachers tend to initiate more concern with boys, prompt boys more, and have more social interaction with boys than with girls (Fennema & Peterson, 1984, 1985). Thus teachers may unwittingly undermine their students' achievement motivation by reinforcing failure-oriented attributions, especially for their female students.

For the most successful students, teachers tend to attribute success more to ability for boys than for girls, and teachers more often see boys as the most successful students in the class. When less successful girls fail, teachers tend to attribute their failure to lack of ability, lack of effort, and task difficulty, whereas boys' failure is more often attributed solely to lack of effort (Fennema, Peterson, Carpenter, & Lubinski, 1990). It seems then that teachers' attributions of their students' successes and failures are reflected in the ways in which they interact with boys and girls in their mathematics classes. These differences in interaction patterns, in turn, tend to contribute to differential gender-related motivation and achievement patterns.

## Goal Theories: Relating Mathematics to What Is Valued

Goal theorists delve more deeply into the cognitive bases of the reasons people do what they do. They are concerned with understanding how people think about engaging in meaningful (or meaningless) activity, and they also conduct research on people's perceptions, interpretations of academic and social information, and patterns of self-regulation (Ames & Ames, 1984). Moreover, researchers who ground their work in goal theory often incorporate the generalized findings from the attribution literature and attempt to posit how reasons for success and failure are related to what is valued (Ames & Archer, 1988; Dweck & Leggett, 1988).

Duda and Nicholls (1992) suggested that the basic dimensions of goal orientations correspond directly to distinct implicit theories (or beliefs) of how success is achieved in academic work (see also Ames, 1992; Ames & Archer, 1988; Dweck, 1986). An individual with a *mastery* (or *learning goal*) orientation values the improvement of skill or knowledge in a given domain and believes that success depends on working hard, attempting to understand the domain, and col-

laborating with others. An individual with an *ego* (or *performance goal*) orientation values establishing "superiority over others" (Duda & Nicholls, 1992, p. 290) and believes that success depends on social comparison and assertion of superior ability. A third orientation, *work avoidance*, is an especially disturbing goal pattern in which working hard is not valued. An individual with this goal orientation believes success results from, for example, "behaving nicely in class" or other behaviors superfluous to study and academic thoughtfulness. Work avoidance is often developed as a coping method for preserving feelings of adequacy by eliminating any threatening or difficult activities so that a legitimate negative evaluation of one's ability cannot be made by others (see Covington & Beery, 1976, for example).

The interplay between goal structures and intrinsic motivation. An individual's intrinsic motivation is mediated through the types of goal structures he or she has created (Meece, Blumenfeld, & Hoyle, 1988). In particular, possession of a mastery goal orientation will positively mediate intrinsic motivation such that one will become more actively involved in a cognitive task. An ego goal orientation (i.e., primarily seeking social recognition) has much less effect on one's developing active cognitive engagement patterns.

Motivational patterns have both generality and specificity. The patterns of goal orientations and beliefs about success listed above seem to be general orientations that students, at least by the time they are in high school, apply across different domains in their lives. However, feelings of personal satisfaction, relevance, and boredom seem to be created by students with respect to specific tasks (Duda & Nicholls, 1992; Seegers & Boekaerts, 1993). A child may enjoy solving story problems in arithmetic and yet feel that her ability is undervalued by her teacher or peers. In such cases, the ego goal of gaining favorable judgments of competence may begin to undermine her intrinsic enjoyment of the task. Both the saliency of goals and the strength of her intrinsic orientation toward the task are important pieces of information the child will use to determine her engagement patterns.

Because of different beliefs about the natures of different academic subjects, even mastery goals can have differential effects on learning. Students who view mathematics as a fixed body of knowledge tend to develop goals of memorization of facts and procedures. These students also tend to emphasize determining correct answers as the primary goal of mathematics learning. Students who view mathematics as a process, guided by their own search for knowledge, tend to value constructing relational understanding of concepts, and consequently they are motivated intrinsically because the knowledge they develop is their own (Underhill, 1988).

Fortunately, the ways in which teachers structure classroom inquiry can greatly influence students' views of mathematics and can lead students to develop more powerful conceptual structures in the process (Cobb et al., 1991; Cobb, Wood, Yackel, & Perlwitz, 1992). Students in inquiry-based classrooms are less likely to develop ego goals than are students in more traditional classrooms. Moreover, students in inquiry-oriented classrooms are less inclined to believe that conformity to the solutions of the teacher or others leads to success in math-

ematics, and they tend to believe more strongly that the classroom is a place where success is defined as attempts to understand mathematics and explain their thinking to others. These attitudes contribute to increased student performance on conceptual and nonroutine tasks that persists even in the face of poor instruction later on (Cobb et al., 1991; Cobb et al., 1992).

Goal orientation has been found to be a strong predictor of achievement (Henderson & Landesman, 1993). Students with mastery goals tend to perform better than those with ego goals regardless of the learning situation.

Students' goal structures also interact significantly in situations that involve extrinsic rewards. When students are provided with both coherent goals for achievement and an extrinsic reward, they tend to achieve more than students to whom stated goals are not presented (Schunk, 1984). Moreover, when an activity is not intrinsically motivating, dispensing rewards may not be productive academically unless the rewards are coupled with an appropriate goal structure. It seems likely that when goals have no intrinsic value to the students, some reward or instruction that exerts social pressure on the student must be tied to the goals to make achieving them worthwhile (Brown & Walberg, 1993).

## Theories of the Self: Personal-Construct Theories

Personal-construct theories are idiographic approaches to examining individual differences in human thought (Snow, Corno, & Jackson, 1996). They are based on the premise that individuals construct knowledge about their worlds and use this knowledge to predict outcomes of activities (Kelly, 1955). The purpose of employing personal-construct approaches in the study of motivation is to describe construct systems of individuals in order to uncover the ways they evaluate activities. Usually this description involves some sort of "mapping" of the relationships between constructs to ascertain the cognitive structure underlying the motivation. Whereas those using other approaches to the study of motivation are typically concerned with the outcomes of motivational processes (e.g., ability attributions, achievement, etc.), personal-construct psychologists are interested in the processes themselves: They assume that motivation results from rational cognitive processes, and they provide a method for understanding these processes.

Owens (1987), for example, used personal-construct theory to describe two teachers' attitudes toward mathematics and mathematics teaching. Although the teachers' conceptions of their mathematics backgrounds were remarkably similar and although they tended to rate themselves as most similar to the person they considered their "best" mathematics teacher, their concepts of what makes a good mathematics teacher differed markedly. The teacher who felt that more difficult mathematics was enjoyable also felt that inquisitiveness was a desirable trait for a mathematics teacher. The other teacher, who enjoyed mathematics that was easier, rated inquisitiveness *least* desirable as a trait for a mathematics teacher.

Owens concluded that their constructs about mathematics and mathematics education play a powerful role in determining how teachers anticipate their

teaching roles. In addition, it seems reasonable to assume that the teachers' prior mathematics education experiences, especially identification with their mathematics teachers, play a pivotal role in determining what aspects of mathematics are motivating and thus how they approach teaching mathematics.

Lucock (1987) found that children in high-ability mathematics tracks tended to find mathematics easier, tended to enjoy doing mathematics more, and tended to consider mathematics to be more useful than did children in lower ability tracks. These findings are hardly surprising. However, when children who enjoyed mathematics were asked to perform routine work (i.e., learning without understanding), they became disillusioned with mathematics and tended to give up. In addition, gender differences were found between the ways in which high-ability boys and low-ability girls internalized success in mathematics tasks. Lucock found that high-ability boys tended to fail with confidence; that is, their confidence in their abilities was fairly robust in spite of failure. Low-ability girls tended to succeed with diffidence; that is, their insecurity tended to be robust even when they were successful.

Constructing an intrinsic motivation for mathematics. Middleton, Littlefield, and Lehrer (1992) attempted to test a theory of how academic activities come to be regarded as intrinsically motivating. Their analysis revealed that children tended to organize their constructs into three general categories: arousal, or the cognitive stimulation afforded by an activity; personal control, or the degree to which the activity was considered a free choice or of appropriate difficulty; and interests (a loosely defined category), or the degree to which the students liked the activity, the importance of the activity, and their ability in performing the activity. Students, girls in particular, seemed to identify with their teachers in evaluating the motivational value of academic tasks (as was also found by Owens, 1987). In addition, children tended to rate mathematics as less fun as they progressed from elementary to junior high school. On the basis of the results of the study, Middleton et al. developed a model of academic intrinsic motivation. They asserted that when one first encounters an academic activity, she will tend to evaluate the stimulation (challenge, curiosity, fantasy) it provides and the personal control (free choice, not too difficult) the activity affords. If her arousal and control requirements are met consistently, she may choose to include the activity among her interests.

Using this model, one can gain some insight into the reasons that motivational attitudes seem to be so stable over time. If a student has classified mathematics as an interest, she will tend to engage in mathematics with enthusiasm without having to evaluate the engagement requirements of the task at hand. If she has classified mathematics as "not an interest," she will tend to avoid engagement without evaluating the task at hand. Thus, once mathematics activities have been classified with respect to interest, little further evaluation takes place. Because one must continually and consistently evaluate arousal and control to classify an activity, it seems likely that only radical and consistent change of the requirements for engagement in mathematics activities will effect change in motivational patterns.

Other research has indicated that teachers and students can be highly similar in the ways in which they define intrinsic motivation in their classrooms but that highly motivated students may tend to focus more on high arousal and less on control when engaged in mathematics activities, whereas less motivated students may tend to focus on low arousal and more on control (Middleton, 1995). In addition, teachers seem to have little background knowledge pertaining to how students view mathematics activities from a motivational perspective. The teachers' own personal constructs of what makes mathematics intrinsically motivating play a pivotal role in determining the types of activities they choose or design for their classrooms. Overall, however, teachers who are better able to predict their students' motivational constructs seem to be better able to fine-tune their instruction to meet the motivational needs of their students.

Middleton (1993b) examined the changes teachers made in their motivational constructs after a year of implementing a reform-oriented pilot curriculum that provided students with more opportunities to learn, more choices of strategies and activities, and more challenging problems than a traditional curriculum would provide; its activities were situated within real-world contexts. The data indicated that teachers' beliefs about intrinsic motivation broaden and expand before they deepen and differentiate and that carefully designed curricula, coupled with strong professional development experiences, can influence a shift in teachers' attitudes toward providing an atmosphere conducive for the development of students' intrinsic motivation. Teachers became more attuned to the conceptual complexity and challenge of the mathematics activities, placed less emphasis on task ease in defining what makes mathematics motivating, and began to perceive the importance of personalizing curricula to make the mathematics more meaningful for their students.

Results of studies in the personal-constructs paradigm have shown that motivations in mathematics education are highly individual, are related to perceived ability, and are relatively stable with regard to success and failure. Some of the individual differences in motivations can be explained in relation to students' identification with their mathematics teachers. Perhaps more important, researchers can begin to outline how academic activities can be tailored to students' individual differences such that intrinsic motivation in mathematics can be fostered by paying attention to stimulation, control, and interest factors.

Researchers in the personal-constructs paradigm, however, have provided only limited knowledge of students' motivational thought processes. The major limitation thus far has been that they have made little attempt to explicate the pertinence of *extrinsic* motivators to mathematics learning. Further research in this paradigm is critical to understanding the roles of grades and other incentives in influencing student motivation. In particular, because they deal with the processes by which students evaluate mathematics activities as worthwhile, personal-constructs methodologies seem uniquely useful for discovering why intrinsic motivation is superior to extrinsic motivation in academic areas.

A second limitation of personal-constructs studies is that they are prone to experimenter bias in the interpretation of measures of construct organization. Without well-articulated models guiding their interpretation, results of personal-constructs studies are difficult to interpret substantively.

## Descriptive Studies

The last approach discussed in this review deals with descriptive studies. Included in this category are studies that have some theoretical orientation but do not fit neatly into any of the categories mentioned previously. For reasons of clarity and cohesion, we have grouped descriptive studies according to similarities in both the variables examined and the motivational patterns discovered.

Mathematics anxiety. Individuals who perceive mathematics as difficult and their ability to do mathematics as poor generally avoid mathematics, if possible (Hilton, 1981; Otten & Kuyper, 1988). Such students are termed math anxious. Hoyles (1981), for example, examined the stories told by students about incidents (in their mathematics education histories) that they felt reflected significant influences on their learning. She was interested in discovering the perceived causes of their mathematics anxiety. Students tended to derive satisfaction from a task when they were involved in successful work, and they tended to blame their dissatisfaction on the teachers. The students seemed to appreciate teachers who provided a structured, logical progression for students' work as well as sufficient explanation, encouragement, and friendliness (see also Quilter & Harper, 1988). Although the sources of satisfaction and dissatisfaction were similar for mathematics compared to other subjects, the ways in which students internalized these experiences were markedly different. Students were much more concerned with their own roles in mathematics versus in learning other subjects. They also tended to have strong feelings about what they were capable of doing, and they tended to internalize these feelings into their self-concepts. The stories Hoyles studied showed that students' anxiety, feelings of inadequacy, and shame were common in interpreting their bad experiences in mathematics and that students generally recall more bad experiences in mathematics than in other subjects. Despite these similarities with respect to their recollections of mathematics learning, pupils differed in the ways in which they could achieve satisfaction. For some, challenge added to their satisfaction; others stressed understanding of the "whys" as well as the "hows." Some were satisfied with just being able to know what to do to solve a problem successfully, and many were quite concerned with the accuracy of their work and the grades they received.

Nakamura (1988) described motivational differences between high-achieving and underachieving mathematically gifted students. One of the primary results of her research indicated that gifted children who exhibit high achievement tend to experience *flow* (a construct that corresponds to enjoyable engagement in meaningful activity) more often and anxiety less often in schoolwork than their lower achieving counterparts. Higher achievers also tend to spend considerably more

time than low achievers in activities that afford high challenge and require well-developed skills. These activities, according to Nakamura, are those associated with the greatest amount of enjoyment for the high achievers. Lower achievers, conversely, tend to avoid challenge. Instead, they choose activities with challenge below their ability level, presumably to avoid the anxiety caused by high levels of task difficulty. In other words, higher achievers tend to enjoy academic challenge, whereas lower achievers tend to feel overwhelmed by challenge.

In short, when teachers emphasize understanding of mathematical concepts and provide facilitative classroom environments, students tend to be more receptive and less anxious with regard to mathematical activities than when teachers stress rote activities and are perceived to be authoritarian. Students who have good experiences in mathematics tend to be less math-anxious and less inhibited in pursuing mathematics-related careers than students who have bad experiences. In mathematics, perhaps because it is viewed as a difficult and important subject, students tend to internalize their experiences into their self-concept more than in other subject areas.

Motivation and underrepresented populations. Rohrkemper and Bershon (1984) examined the efficacy statements minority students used to motivate themselves to solve mathematics problems correctly. Their findings indicated that some children may begin to feel a lack of efficacy in mathematics as early as third grade. In addition, a high proportion of students reported negative inner speech (e.g., "If I don't get this right, I will maybe fail") at the *outset* of problem solving. These negative self-perceptions with regard to mathematics may undermine students' abilities and efforts to persist when faced with difficult problems.

In addition to the attribution literature, many other studies have documented gender differences in students' mathematics motivation. The consistent pattern that develops is that females are socialized into viewing mathematics as a male domain and into perceiving themselves as being less able than males to do mathematics (Fennema & Sherman, 1976). Males tend to feel more confident in learning mathematics, are more convinced of the usefulness of mathematics, and identify more with mathematics, in general, than females. Gender-role stereotyping does not solely affect females with low ability and motivation. Even girls with high ability may perceive mathematics as a male domain, or they may defer to the "dominant male role" because of other social pressures whether or not they perceive mathematics as a male domain (Jackson & Coutts, 1987).

In the middle grades, students' motivations toward mathematics tend to crystallize into their adult forms. Students who like mathematics tend to report that they started liking mathematics at about the seventh grade. Students who dislike mathematics report that they started disliking mathematics at about the seventh grade. Their reasons for liking or disliking mathematics seem to focus on the transition from elementary to middle school instructional patterns, especially the perceived supportiveness of the teacher and new rules for determining success in mathematical tasks (Eccles et al., 1987; Midgley et al., 1989). Girls in particular tend to identify with their mathematics teachers, and this identification is related to girls' interest in mathematics (Fennema & Peterson, 1985). By the time students get to high school, interest in mathematics becomes one of the best predictors of students' perceptions of the quality of their mathematical experiences, more so than ability or the desire to achieve (Schiefele & Csikszentmihalyi, 1995).

The research on gender differences in mathematics seems to paint a consistent picture. Like the research on students' attributions, other research on gender differences has indicated that mathematics is perceived by females as a male domain. Females tend to defer to males when interacting in mathematics class, even when their abilities would indicate that deference is unwarranted. Girls also tend to identify with their mathematics teachers more than boys do. In addition, inasmuch as motivational factors seem to predict academic achievement more for girls than for boys, it is reasonable to assume that girls' feelings of disinterest and even anxiety in mathematics contribute to gender-related differences in achievement. What is *not* known is what factors cause girls to be less motivated. Research regarding these causes is necessary to dispel the myth that girls are inherently less mathematically able than boys (see Secada, 1990).

Intervention studies (e.g., Croom, 1984), however, have shown that appropriate instruction, guidance, and continued support can positively influence students from underrepresented populations to continue studying high school mathematics and can foster improved attitudes toward school and toward mathematics and science in particular.

### CONCLUSIONS

The Current State of Research on Motivation in Mathematics

Although research on motivation may not be in its infancy, it has barely reached toddlerhood, and, like a toddler, it seems to be going in many directions, frequently getting into trouble. However, some consistencies are evident across studies, and these consistencies represent the current boundaries of our knowledge. Drawing together the findings from the studies reviewed in this article, we are beginning to define the body of knowledge pertaining to motivation in mathematics as it exists today.

First, findings across theoretical orientations indicate that students' perceptions of success in mathematics are highly influential in forming their motivational attitudes. Research indicates that the effort a person is willing to expend on a task is determined by the expectation that participation in the task will result in successful outcomes, mediated by how much the individual values either participation in the task itself or the extrinsic rewards associated with success in the task (Brophy, 1986). Students need a relatively high degree of success in mathematics for engagement in mathematics to be perceived as worthwhile (Alschuler, 1969), and they need to feel that success in mathematics is attributable to their ability and effort (Fennema & Peterson, 1985). In addition, students' beliefs about the nature

of mathematics and mathematics learning greatly influence their definitions of what success in mathematics is. Current practice leads students to develop attitudes that value speed of computation, following the example of the teacher, and correctness of answers over learning and understanding (Kloosterman, 1993).

Moreover, learned helplessness, lack of success, and the perception that failure is due to lack of ability seriously undermine students' motivation to learn; these factors may also affect the ability to process complex mathematical information (Dweck, 1986). Students also seem to require a healthy appreciation for the role of failure in mathematical problem solving (Kloosterman, 1988). The likelihood of failure in a task increases the task difficulty, thus increasing the value of success (e.g., Brophy, 1987). Further, learning appropriate coping strategies for failure is necessary for developing a healthy mathematical self-concept.

Motivations develop when students evaluate the demands of the mathematical task (Seegers & Boekaerts, 1993). To allow students to feel successful in mathematics without undermining either the value of success or a healthy attitude toward failure, teachers must structure tasks such that they present an appropriate level of challenge and difficulty for students (e.g., Middleton et al., 1992). Thus, mathematics activities must be difficult enough that students are not bored, yet tasks must allow for a high degree of success given appropriate effort by the student. Moreover, students should be encouraged to attribute their successes to a combination of ability and effort and their failures either to insufficient effort (so failures can be overcome through renewed diligence) or to confusion or reliance on inappropriate strategies (so failures can be overcome with additional preparation). Students must *not* be given cause to believe that their failures are due to lack of ability for fear of exacerbating their feelings of learned helplessness.

Second, motivations toward mathematics are developed early, are highly stable over time, and are influenced greatly by teacher actions and attitudes. Students seem to consolidate their motivational attitudes toward mathematics in junior high school (Eccles et al., 1987), and these attitudes in the middle grades predict the courses taken and mathematics achievement in high school and college (Amit, 1988; Meyer & Fennema, 1985). These motivations are internalized into students' self-concepts, thus affecting how they see themselves with regard to mathematics-related activities. Students with high self-concepts related to mathematics tend to be more focused on the selection and use of specific strategies for successful problem solving and are more likely to pursue further study in mathematics (Meece et al., 1990; Pokay & Blumenfeld, 1990).

The preponderance of students' recollections of bad experiences (e.g., Oldfather, 1992) explains in part why students' liking of mathematics tends to decrease when they get older and why enrollment in higher level mathematics courses has declined. These students do not see mathematics as being integral to their academic self-concepts, and they try to avoid the anxiety resulting from involvement in mathematical tasks. Because anxious or alienated students are unlikely to have or to develop the motivation to learn mathematics, the teacher

should be patient, encouraging, and supportive of students' individual learning styles. Students will feel more comfortable taking risks if they know that they will not be criticized or humiliated for making mistakes (Brophy, 1987). Students tend to attribute their feelings about mathematics to their identification with influential teachers or to their reactions to bad experiences, for which they blame teachers (Hoyles, 1981; Otten & Kuyper, 1988).

It is unclear, however, what role culture plays in the ways in which motivational strategies are implemented in the classroom. Hess and Azuma (1991), for example, found that Japanese students are expected to be more self-motivated than American students. In Japan, overt control of tasks by the teacher is minimal, effort is valued over ability, and determinations of interest and success are primarily left up to the student. In the United States, motivation is still primarily stimulus driven—that is, teachers in the United States are expected to make instruction interesting and appealing, and students are less likely to be blamed for inattention if the topic is personally unappealing. In essence, students are expected to dislike mathematics and are not provided direction or support when they fulfill this expectation.

Third, providing opportunities for students to develop intrinsic motivation in mathematics is generally superior to providing extrinsic incentives for achievement. To facilitate the development of students' intrinsic motivation, teachers must teach knowledge and skills that are worth learning. In other words, students must understand that the mathematics instruction they receive is useful, both in immediate terms and in preparing them to learn more in the fields of mathematics and in areas in which mathematics can be applied (e.g., physics, business, etc.). Use of ill-structured, real-life problem situations in which the use of mathematics facilitates uncovering important and interesting knowledge promotes this understanding. However, utility and importance are not sufficient to develop students' intrinsic motivation.

Students who come to value and enjoy mathematics increase their achievement, their persistence in the face of failure, and their confidence (Gottfried, 1985; Lehmann, 1986; Meece et al., 1990; Pokay & Blumenfeld, 1990). Tailoring activities to provide stimulation and student control and matching activities with students' interests increase intrinsic motivation (Middleton, 1993b). Providing incentives for success, however, can and does encourage students to achieve (Alschuler, 1969). Further research regarding interaction of extrinsic and intrinsic motivation in the context of the classroom is necessary because no academic task is free from the influence of either.

Fourth, inequities exist in the ways in which some groups of students in mathematics classes have been taught to view mathematics. Girls, in particular, may be influenced through gender-role stereotyping, teacher expectations, and peer pressure to view themselves negatively with respect to mathematics motivation (Fennema & Peterson, 1985; Meyer & Fennema, 1985). Girls, far more than boys, feel that their failures are due to a lack of ability in mathematics, and this attributional style may lead them to believe that success in mathematics is unat-

tainable (Benenson & Dweck, 1986). Unfortunately, teachers' thoughts and behaviors tend to reinforce learned helplessness in girls, further widening the gender gap in mathematics achievement (Fennema et al., 1990).

Last, and most important, achievement motivation in mathematics, though stable, can be affected through careful instructional design. If students realize that their successes are meaningful and result both from their abilities and from a high degree of effort, they are likely to believe that they can do mathematics if they try (Relich, 1984). Providing group incentives leads to cooperation and reciprocal instruction in mathematics problem solving so that all children are given opportunities to succeed (Slavin, 1984). Creating interesting contexts within which problems are situated stimulates students' imaginations and illustrates to them that mathematics is useful in various applications (Bransford et al., 1988). Most important, a supportive, authoritative teacher serving as a model and as a friend gives children the confidence and feelings of self-worth necessary to be comfortable in mathematics (Covington, 1984).

In addition, teachers who are more attuned to bettering their students' motivational belief systems are better able to adjust their classroom practice to motivate their students (Middleton, 1995). This finding would suggest that preservice and in-service programs could profit from detailed examination of the research findings in the field of motivation, including the studies reviewed here. Particular attention should be paid to developing strategies for assessing students' motivational beliefs in the classroom so that teachers' awareness will be linked to the instructional sequence. In such a program teachers would be able to use practical knowledge about how students' beliefs are formed and changed to tailor their instruction to better influence their students to take charge of their own learning.

Thus, it seems that there is hope after all. Motivation to achieve in mathematics is not solely a product of mathematics ability nor is it so stable that intervention programs cannot be designed to improve it. Instead, achievement motivation in mathematics is highly influenced by instructional practices, and if appropriate practices are consistent over a long period of time, children can and do learn to enjoy and value mathematics. There is a building body of evidence that indicates that the larger, more general goals of schooling can be restructured and reinvented with a fair degree of success so that the school culture becomes conducive for student learning and motivation (e.g., Maehr & Anderman, 1993). The research reviewed in this article also provides evidence that classroom practice can be positively reinvented so that the culture of the classroom can become conducive for learning and enjoying mathematics.

Little is known, however, about the socially constructed nature of motivations. What happens in the mathematics classroom when students work together and create a shared reality? Do different interpretations of mathematics support a motivating environment for some children but not others? Preliminary findings indicate that students in cooperative groups perceive the input of others in very different ways and react to the social situation in both positive and negative ways

(Mulryan, 1992). Naturalistic, observational research seems a plausible first step in examining how individual and social motivational sets become negotiated in mathematics problem-solving situations.

### A FEW CRITICISMS

Although the current research on motivation in mathematics education has provided profound insights into why students achieve and why they fail, we have some criticisms pertaining to the lack of theoretical guidance driving the conduct of, and implications drawn from, the majority of studies. The research on motivational variables in mathematics education has been primarily descriptive and inadequately conceptualized. Often motivation has been thrown "into the pot" to add a little spice to studies originally focused on other factors—such as mathematics achievement.

Particularly evident is the lack of conceptualization of how mathematics motivation develops over time. With few exceptions, researchers have neglected to examine the motivations of students while they change and develop over several years of instruction. If we as mathematics educators are interested in effecting change in students' motivational patterns, we need further research regarding the acquisition, consolidation, and maturation of students' motivations.

In addition, measurement procedures have been primarily atheoretical and poorly defined. A prime example is the operational definition of motivation as student engagement (observed affect, time on task) without the use of complementary measures. Although students' motivations should influence their engagement patterns, engagement itself is *not* motivation. Engagement can be influenced by a number of factors that distort the actual reasons behind students' levels of task involvement—fatigue, for example. At the other end of the spectrum, those conducting most motivation studies reviewed in this article have used self-report measures as indices of motivation without actually looking at and listening to children who are engaged in mathematical activity. The potential biases associated with self-report measures of attitude have been clearly delineated (Gall, Borg, & Gall, 1996; Pintrich & Schunk, 1996). When secondary measures of motivation are used, some additional measure should be administered as a validity check.

Moreover, even the theoretically driven studies are limited in their explanations of why students are motivated to achieve. Most describe personality correlates of motivation, differences or similarities in existing groups, or the correlation between motivation and achievement. Few attempts to predict and then test causal relationships between factors influencing motivation have been made. To build a more extensive body of knowledge about motivational factors in mathematics education, mathematics education researchers must attend to theoretical or model-based research that is designed to ascertain causal and interactive relationships between motivational domains and student achievement (McLeod, Reyes, Fennema, & Surber, 1984). Moreover, through these models they must begin to examine the interplay of motivational factors as they exist in the social

and cognitive worlds of the child. Researchers using causal modeling have made a first attempt at large-scale description of the web of factors influencing and affected by motivational structures. Further research along this line of inquiry holds promise for untangling the causal relationships between motivation and achievement. At the other end of the spectrum, naturalistic studies of students engaging in meaningful activity can provide powerful insight into the ways individuals and social groups define motivational constructs, modify these definitions that are based on situational variables, and abstract workable goal structures that inform future engagement.

But even with the application of appropriate methodologies, nearly all the research conducted in the area of mathematics has utilized a model of mathematics instruction that is not conceptually driven. Researchers studying a conceptual model of instruction have found that the effects of such instruction on student motivation are quite different from the effects of traditional instruction (e.g., Bransford et al., 1988; Cobb et al., 1992; Middleton, 1993b). In addition, when students who are motivated to learn mathematics concepts in a meaningful way are forced to work on routine, skills-related mathematics problems, their enjoyment of mathematics tends to plummet (Lucock, 1987). Thus, even the positive results from studies using more traditional models of mathematics teaching and achievement may be irrelevant or even misleading (Romberg & Carpenter, 1986).

One final criticism is aimed toward the use of theories in motivational research. Although studies may be situated within a theoretical framework, little attempt has been made to test the adequacy of current theories. Researchers have *used* theories to explain behavior, but they have done little to increase the accuracy, precision, and applicability of these theories. Consequently, very few new theories or reformulations of existing theories of motivation have been forthcoming. Noticeably absent are approaches that capitalize on research in the cognitive science domain. Because they are focused on individual differences, cognitive science approaches may prove to be powerful theoretical tools for the motivation researcher, especially in the area of goal theory, by providing theoretical means for examining volitional decision-making processes (e.g., Corno, 1993; Cruz, 1992). A primary goal for future researchers should be the testing and refinement of motivational theories so that their range of applicability can be delineated and exploited.

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