Rhombi

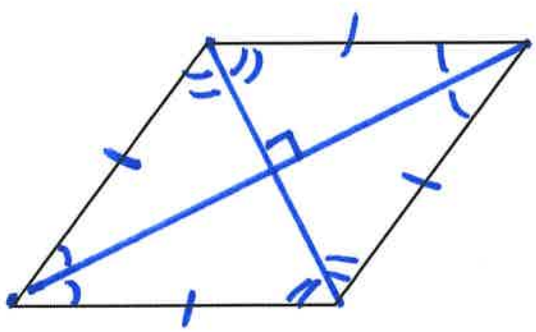
A rhombus is a ***parallelogram*** that has __2__ congruent consecutive sides and __4__ sides; the angles of a rhombus do NOT all have to be congruent or right angles.

TAKE NOTE! *Postulates & Theorems*

Theorems About the Rhombus

- All sides of a rhombus are congruent.
- The diagonals of a rhombus are perpendicular to each other.
- The diagonals of a rhombus bisect its angles.
- If a quadrilateral is equilateral, then it is a rhombus.
- If the diagonals of a parallelogram are perpendicular to each other, the parallelogram is a rhombus.

Illustrate the theorems about the rhombus in the figure below.



STUDY EDGE TIP

Recall that the rhombus method was used for the construction of parallel lines. Since the opposite sides of a rhombus are parallel, the construction of a rhombus creates the desired parallel lines.

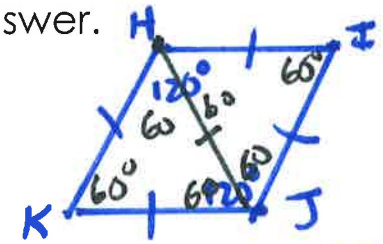
Let's Practice!

1. A diagonal of a rhombus that is on the coordinate plane can be modeled by the equation $6x + y = 13$. What is the slope of the other diagonal?

$y = -6x + 13$ $m = -6$

other diagonal $m_{\perp} = \frac{1}{6}$

2. In rhombus $HIJK$, $m\angle H$ is $120°$. Does the diagonal $\overline{HJ}$ divide the rhombus into two equilateral triangles? Justify your answer.

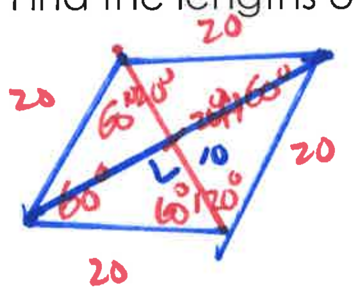


Yes.

3. The size of the acute angle of a rhombus is half the size of its obtuse angle. The side length of the rhombus is equal to 20 feet. Find the lengths of the diagonals of the rhombus.

$\frac{1}{2}x$

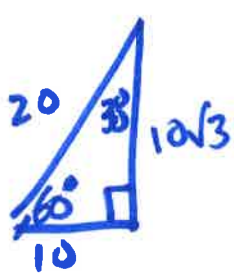
x



$x + \frac{1}{2}x = 180$

$\frac{2}{3} \cdot \frac{3}{2}x = 180 \cdot \frac{2}{3}$

$x = 120°$



$d_1 = 20$ ft.

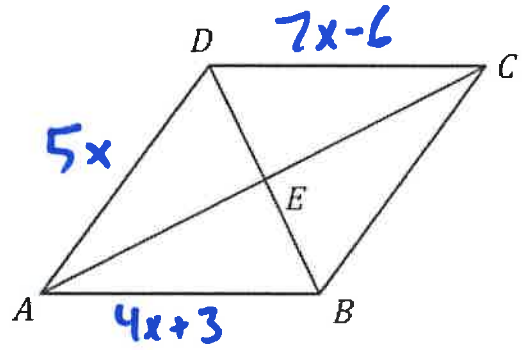
$d_2 = 20\sqrt{3}$ ft.

Try It!

4. Complete the following proof.

Given: $ABCD$ is a parallelogram with $AB = 4x + 3$, $DC = 7x - 6$, and $AD = 5x$.

Prove: $ABCD$ is a rhombus.



We are given that $ABCD$ is a parallelogram with $AB = 4x + 3$, $DC = 7x - 6$ and $AD = 5x$. Because the ___opposite___ sides of a parallelogram are ___congruent___, $7x - 6 = 4x + 3$.

Using the Addition Property of Equality, we can conclude that ___3x = 9___. Further, the Multiplication Property of Equality tells us that $x = 3$.

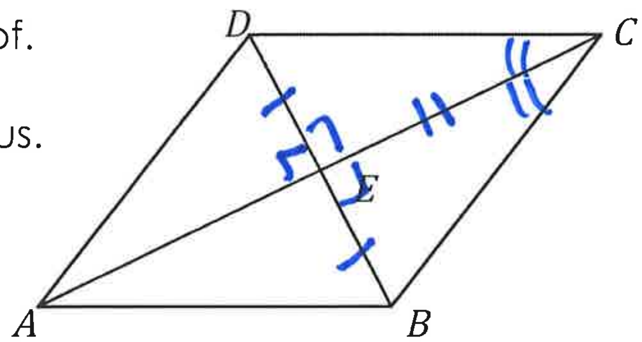
By substitution, ___5(3) = 15, 7(3) − 6 = 15, 4(3) + 3 = 15___ and $BC = 15$ since ___opposite___ sides of a parallelogram are ___congruent___.

Therefore, $ABCD$ is a rhombus, since all sides of the parallelogram are congruent.

5. Complete following proof.

Given: $ABCD$ is a rhombus.

Prove: $\overline{AC}$ bisects $\angle DCB$ of the rhombus.



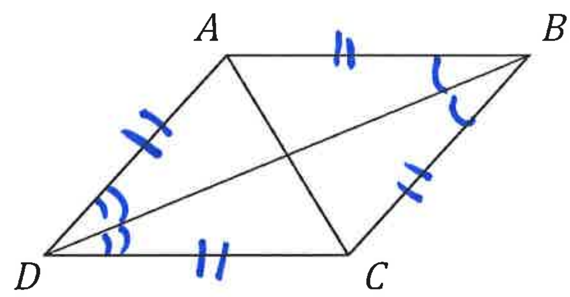
Statements	**Reasons**
1. $ABCD$ is a rhombus.	**1.** Given
2. $\overline{DB} \perp \overline{AC}$	**2.** Diagonals of a rhombus are perpendicular
3. $m\angle AEB = m\angle BEC = m\angle CED = m\angle DEA = 90^\circ$	**3.** Definition of perpendicular lines
4. $\angle DEC \cong \angle BEC$	**4.** All right angles are congruent.
5. $\overline{DE} \cong \overline{BE}$	**5.** Diagonals of a parallelogram bisect each other.
6. $\overline{EC} \cong \overline{EC}$	**6.** Reflexive Property
7. $\Delta DEC \cong \Delta BEC$	**7.** SAS
8. $\angle ECD \cong \angle ECB$	**8.** CPCTC
9. $\overline{AC}$ bisects $\angle DCB$ of the rhombus.	**9.** Definition of an angle bisector

BEAT THE TEST!

1. Complete the two-column proof below.

Given: Parallelogram $ABCD$
$\angle CBD \cong \angle ABD, \angle BDC \cong \angle BDA.$

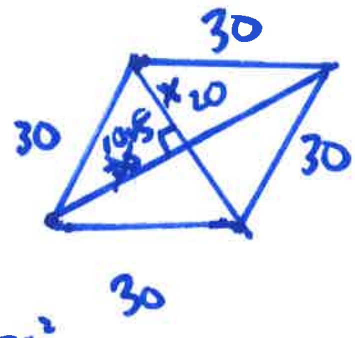
Prove: $ABCD$ is a rhombus.



Statements	Reasons
1. $ABCD$ is a parallelogram.	**1.** Given
2. $\angle CBD \cong \angle ABD, \angle BDC \cong \angle BDA$	**2.** Given
3. $\overline{BD} \cong \overline{BD}$	**3.** Reflexive Property
4. $\Delta DBA \cong \Delta DBC$	**4.** ASA congruence theorem
5. $\overline{CD} \cong \overline{AD}$	**5.** CPCTC
6. $\overline{AB} \cong \overline{BC}$	**6.** CPCTC
7. $\overline{CD} \cong \overline{AB}$, $\overline{AD} \cong \overline{BC}$	**7.** Opposite sides of a parallelogram are $\cong$
8. $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$	8. Transitive Property
9. $ABCD$ is a rhombus.	**9.** If a quadrilateral is equilateral, then it is a rhombus.

2. The perimeter of a rhombus is 120 feet and one of its diagonals has a length of 40 feet. Find the area of the rhombus.

$\frac{120}{4} = 30$ ft



The area is $\boxed{894.43}$ square feet.

$A = \frac{1}{2} d_1 d_2$

$A = \frac{1}{2}(40)(2 \cdot 10\sqrt{5}) = 400\sqrt{5}$

$20^2 + x^2 = 30^2$

$400 + x^2 = 900$

$\sqrt{x^2} = \sqrt{500}$ $x = 10\sqrt{5}$ ft

3. Consider the following statement.

All squares are rhombi.

Is the converse of the above statement also true? Justify your answer.

Converse: All rhombi are squares

False because rhombi do not have to have congruent interior angles of 90°.

Kites

A kite is a quadrilateral that has two pairs of consecutive congruent sides, but the opposite sides are not congruent.

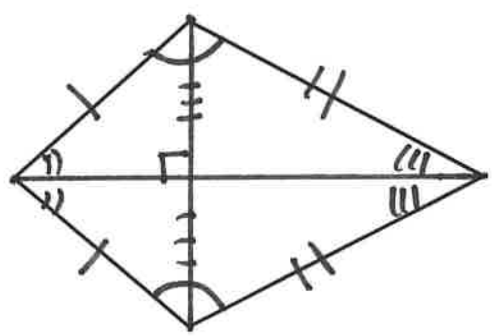
TAKE NOTE!
Postulates & Theorems

Theorems About Kites

If a quadrilateral is a kite:

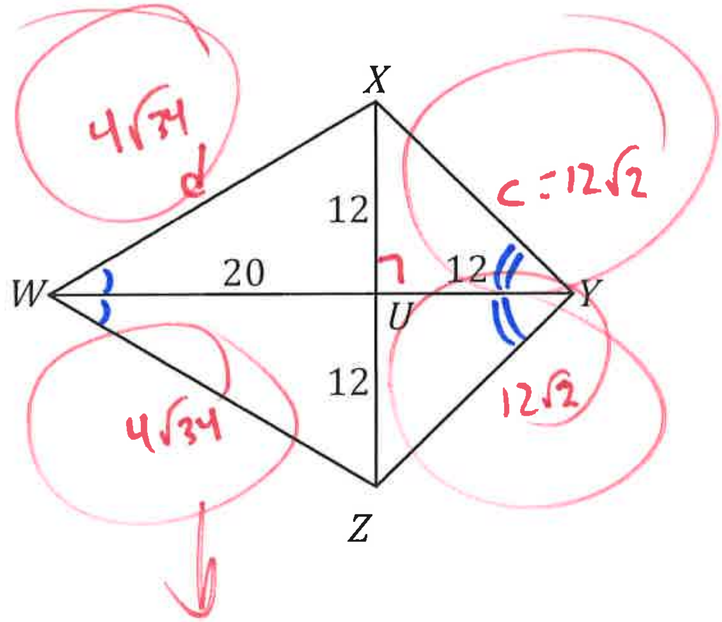
- Its diagonals are perpendicular.
- It has one pair of congruent opposite angles.
- One of its diagonals forms two isosceles triangles, and the other forms two congruent triangles.
- It has one diagonal that bisects a pair of opposite angles.
- It has one diagonal that bisects the other diagonal.
- If one of the diagonals of a quadrilateral is the perpendicular bisector of the other, the quadrilateral is a kite.

Illustrate the theorems about kites in the figure below.



Let's Practice!

1. Consider kite $WXYZ$ below.



a. Determine the lengths of each side of kite $WXYZ$.

$12^2+12^2=c^2$

$144+144=c^2$

$\sqrt{288}=\sqrt{c^2}$

$c=12\sqrt{2}$

$12^2+20^2=d^2$

$144+400=d^2$

$\sqrt{544}=\sqrt{d^2}$

$d=4\sqrt{34}$

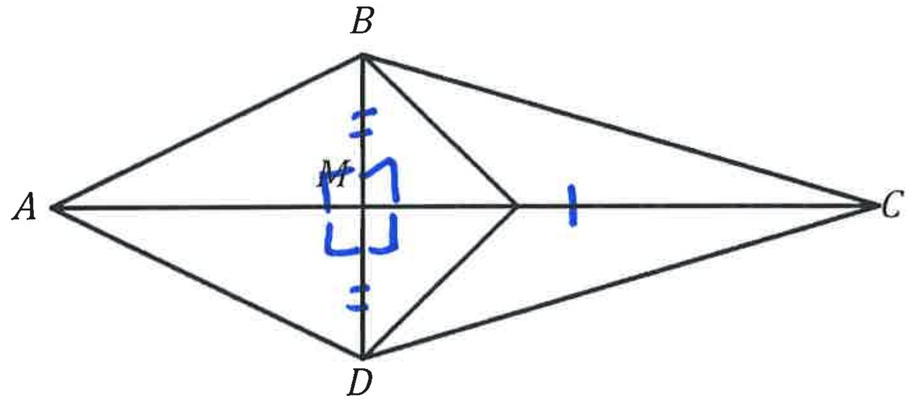
$544=16\times34$

b. Which diagonal bisects a pair of opposite angles?

$\overline{WY}$

2. Complete the following proof.

Given: $ABCD$ is a kite.
Prove: $\Delta BCM \cong \Delta DCM$.

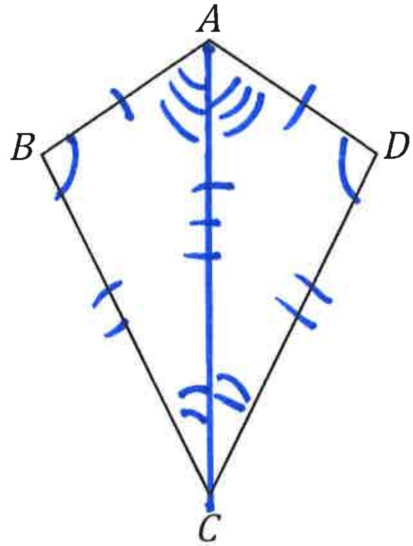


Statements	**Reasons**
1. $ABCD$ is a kite.	**1.** Given
2. $\angle BMC, \angle BMA, \angle DMA$ and $\angle DMC$ are right angles	**2.** Diagonals of a kite are perpendicular
3. $m\angle BMC = m\angle BMA = m\angle DMA = m\angle DMC = 90°$	**3.** Definition of a right angle.
4. $\angle BMC \cong \angle BMA \cong \angle DMA \cong \angle DMC$	**4.** Right angles are congruent
5. $\overline{CM} \cong \overline{CM}$	**5.** Reflexive Property
6. $\overline{BM} \cong \overline{MD}$	**6.** One diagonal bisects the other
7. $\Delta BCM \cong \Delta DCM$	**7.** SAS

Try It!

3. Complete the following proof.

Given: $ABCD$ is a kite.
Prove: $\overline{AC}$ bisects $\angle BCD$.

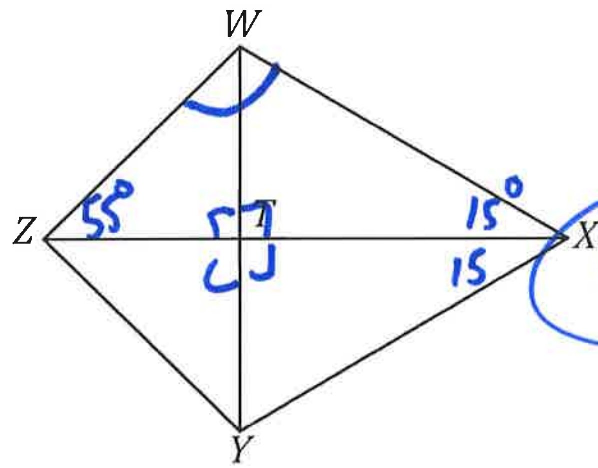


We are given that $ABCD$ is a kite. Using the properties of a kite, we can say that $\overline{AC}$ is a diagonal of the kite, $\overline{AB} \cong \overline{AD}$, and $\overline{DC} \cong \overline{BC}$.

The statement $\overline{AC} \cong \overline{AC}$ is true, because of the Reflexive Property, and using SSS, we know that $\Delta ABC \cong \Delta ADC$.

The statement $\angle ABC \cong \angle ADC$ is true because of CPCTC. Therefore, AC bisects $\angle BCD$ by the definition of angle bisector.

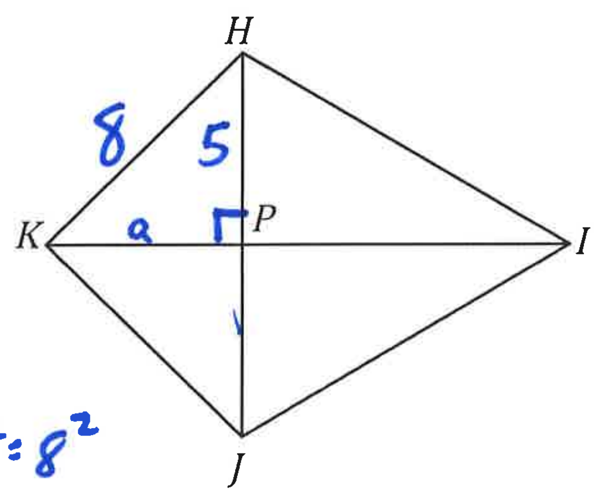
4. Consider kite $WXYZ$. If $m\angle WZT = 55°$ and $m\angle WXY = 30°$, find $m\angle ZWX$.



$180 - 55 - 15$

$m\angle ZWX = 110°$

5. Consider kite $HIJK$. If $HK = 8$ and $HP = 5$, find KP.



$a^2 + 5^2 = 8^2$

$a^2 + 25 = 64$

$\sqrt{a^2} = \sqrt{39}$

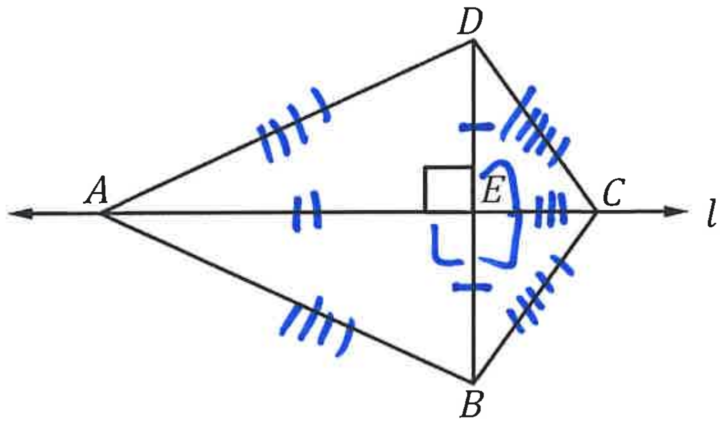
$a = \sqrt{39}$

BEAT THE TEST!

1. Complete the two-column proof below.

Given: $ABCD$ is a quadrilateral.
$\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.

Prove: $ABCD$ is a kite.



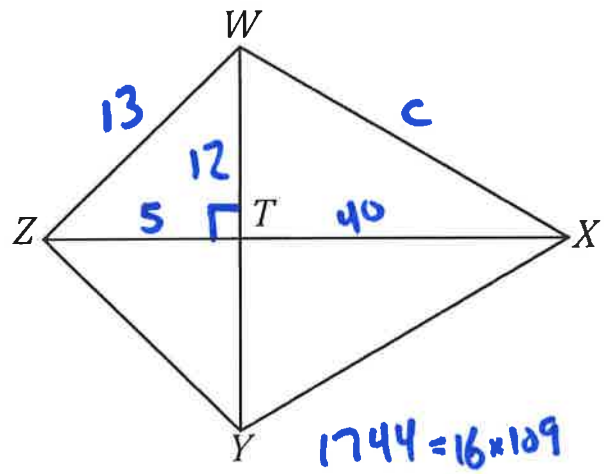
Statements	Reasons
1. $ABCD$ is a quadrilateral. $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$.	1. Given
2. $\overline{DE} \cong \overline{EB}$	2. Definition of $\perp$ bisector.
3. $m\angle DEA = m\angle BEA = m\angle CED = m\angle CEB = 90°$	3. Perpendicular lines form right angles
4. $\angle DEA \cong \angle BEA \cong \angle CED \cong \angle CEB$	4. All right angles are congruent
5. $\overline{AE} \cong \overline{AE}$; $\overline{CE} \cong \overline{CE}$	5. Reflexive Property
6. $\triangle DEA \cong \triangle BEA$, $\triangle DEC \cong \triangle BEC$	6. SAS
7. $\overline{AB} \cong \overline{AD}$, $\overline{DC} \cong \overline{BC}$	7. CPCTC
8. $ABCD$ is a kite.	8. Definition of a kite

2. Consider kite $WXYZ$.

3-4-5

5-12-13

7-24-25



$12^2 + 40^2 = d^2$

$144 + 1600 = d^2$

$\sqrt{1744} = \sqrt{d^2}$

$1744 = 16 \times 109$

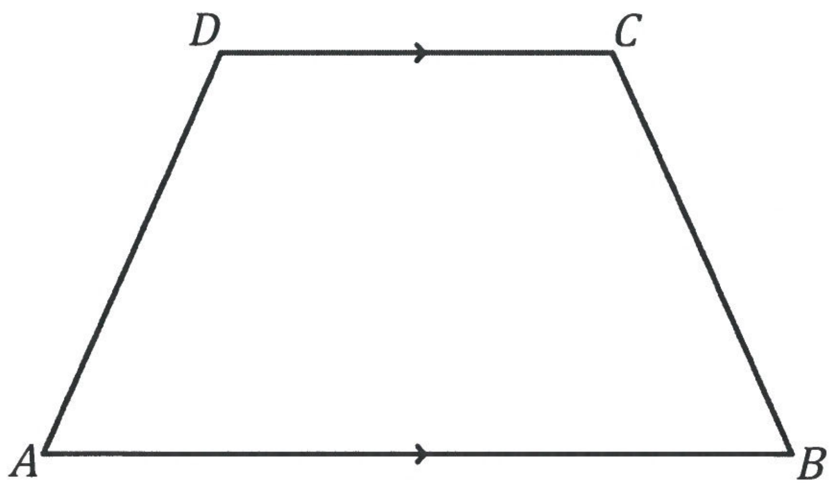
$4\sqrt{109} = d$

If $WT = 12$ yards, $TZ = 5$ yards, and $TX = 40$ yards, the perimeter of $WXYZ$ is $\boxed{109.52}$ yards.

$P = 13 + 13 + 4\sqrt{109} + 4\sqrt{109}$

Section 10 – Topic 3
Trapezoids

Consider the following trapezoid.



- A trapezoid is a quadrilateral with exactly one pair of opposites sides that are parallel.
- The parallel sides are called the bases of the trapezoid.
- The non-parallel sides are called legs.
- A trapezoid has two sets of base angles.
- A trapezoid is isosceles if it has one pair of non-consecutive sides congruent, both pair of base angles congruent, and opposite angles supplementary.

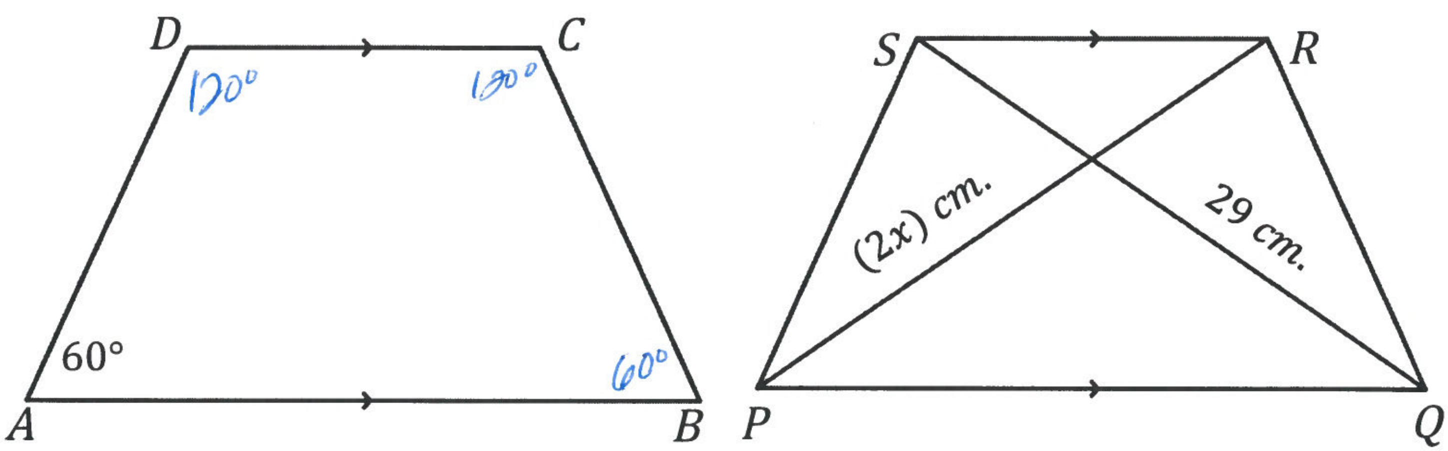
TAKE NOTE! Postulates & Theorems

Theorems About Trapezoids

- If a quadrilateral is an isosceles trapezoid, each pair of base angles is congruent.
- A trapezoid is isosceles if and only if its diagonals are congruent.

Let's Practice!

1. Consider isosceles trapezoids $ABCD$ and $PQRS$ below.



Find x, $m\angle DCB$, and PR.

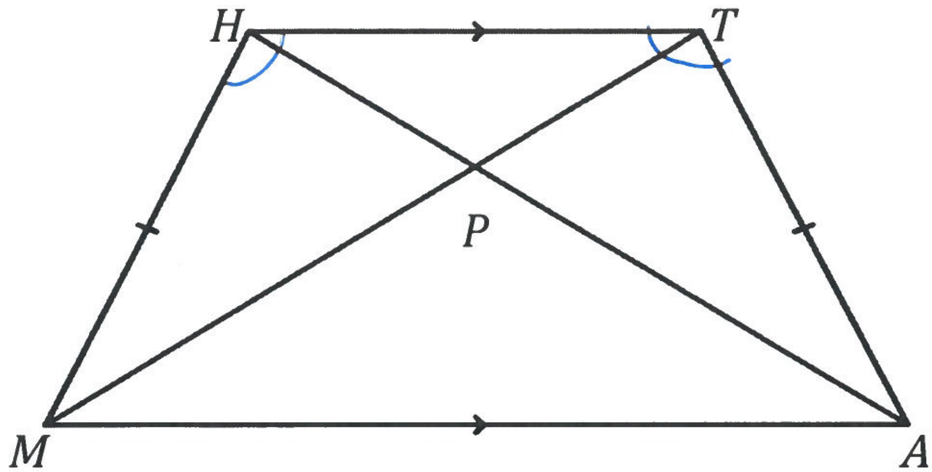
$\frac{2x}{2} = \frac{29}{2}$

$x = 14.5$

$m\angle DCB = 120^\circ$

$PR = 29$

2. Consider the figure below.



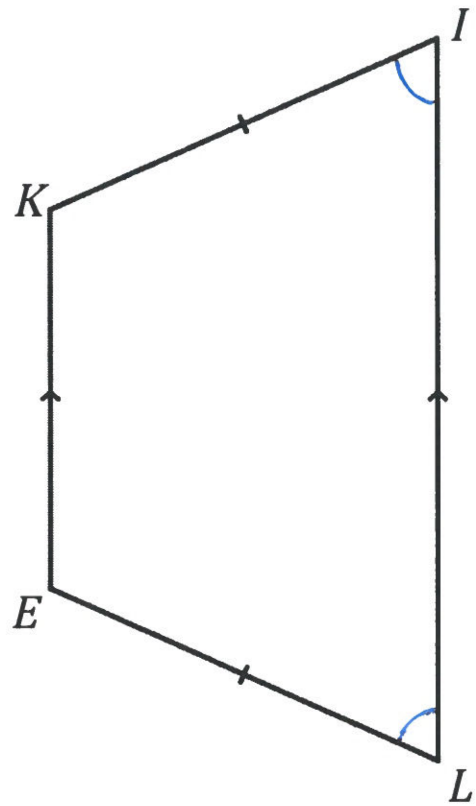
Given: $MATH$ is an isosceles trapezoid with bases $\overline{MA}$ and $\overline{TH}$.

Prove: $\overline{PH} \cong \overline{PT}$.

Statements	Reasons
1. $MATH$ is an isosceles trapezoid with bases $\overline{MA}$ and $\overline{HT}$.	**1.** Given
2. $\overline{MH} \cong \overline{AT}$	**2.** Legs are congruent in isosceles trapezoid.
3. $\angle MHT \cong \angle ATH$	**3.** Base angles are congruent.
4. $\overline{HT} \cong \overline{HT}$	**4.** Reflexive Property
5. $\Delta THM \cong \Delta HTA$	**5.** SAS
6. $\angle HMT \cong \angle TAH$	**6.** CPCTC
7. $\angle HPM \cong \angle TPA$	**7.** Vertical Angles Theorem
8. $\Delta HMP \cong \Delta TAP$	**8.** AAS
9. $\overline{PH} \cong \overline{PT}$	**9.** CPCTC

Try it!

3. Consider the figure below.



Given: $LIKE$ is a quadrilateral with $\overline{LI} \parallel \overline{EK}$ and $\overline{LE} \cong \overline{IK}$.
Prove: $\angle ELI \cong \angle KIL$.

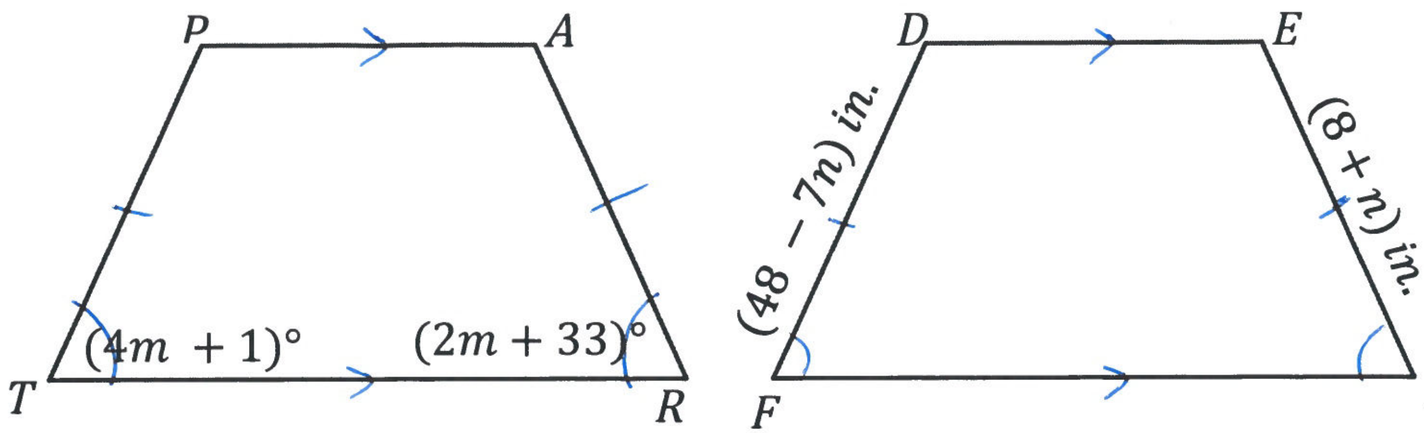
Write a paragraph proof.

LIKE is a quadrilateral with $\overline{LI} \parallel \overline{EK}$ and $\overline{LE} \cong \overline{IK}$.

LIKE is an isosceles trapezoid by definition, so $\angle ELI \cong \angle KIL$

because the base angles of an isosceles trapezoid are congruent.

4. Consider isosceles trapezoids $TRAP$ and $FRED$ below.



Find m, n, $\angle PTR$, $\angle PAR$, and $\overline{FD}$.

$4m+1 = 2m+33$

$-2m - 1 \quad -2m - 1$

$\frac{2m}{2} = \frac{32}{2}$

$\boxed{m = 16}$

$\boxed{\angle PTR} = 4(16)+1 \boxed{= 65°}$

$48 - 7n = 8 + n$

$-48 - n \quad -48 - n$

$\frac{-8n}{-8} = \frac{-40}{-8}$

$\boxed{n = 5}$

$4m+1 = 2m+33$

$4(16)+1 = 2(16)+33 = 65$

$360 - 65 - 65 = 230$

$\boxed{\angle PAR = \frac{230}{2} = 115°}$

$FD = 48 - 7n$

$= 48 - 7(5)$

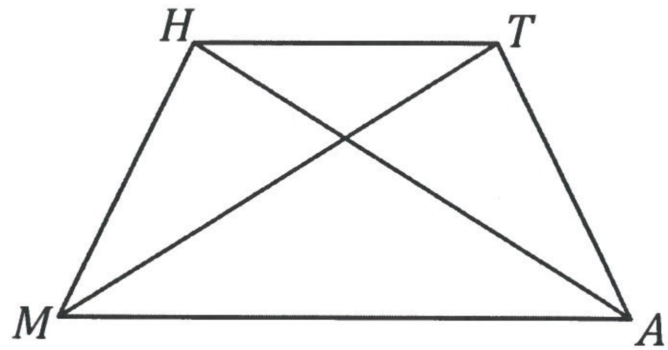
$= 48 - 35$

$\boxed{FD = 13\text{in}}$

BEAT THE TEST!

1. Complete the following proof.

Given: $MATH$ is an isosceles trapezoid.
Prove: $\angle MHA \cong \angle ATM$



Complete the paragraph proof using the bank of terms below.

Is is given that $MATH$ is an isosceles trapezoid. We can prove that $\overline{MH} \cong \overline{AT}$ Definition of isosceles trapezoid. Then, $\overline{MA} \cong \overline{MA}$ by the Reflexive Property. We can state that $\overline{AH} \cong \overline{MT}$ by the use of Trapezoid Diagonals Theorem. Now, we have triangles $\Delta HMA \cong \Delta TAM$ by SSS. Finally, using CPCTC we can prove that $\angle MHA \cong \angle ATM$.

~~Reflexive Property~~	Midsegment Theorem for Trapezoids
Trapezoid Base Angles Theorem	~~Definition of Isosceles Trapezoid~~
ASA	~~SSS~~
~~Trapezoid Diagonals Theorem~~	Transitive Property
Alternate Interior Angles	~~CPCTC~~

Section 10 – Topic 4
Midsegments of Trapezoids

Consider the mid-segment of a trapezoid and its characteristics.

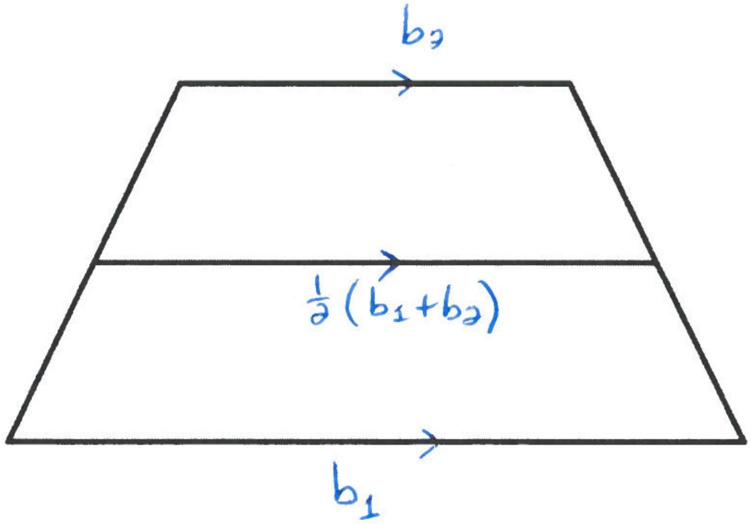
- The midsegment of a trapezoid is the segment that connects the two midpoints of the opposite sides.
- A median of a trapezoid is another term for the midsegment.

TAKE NOTE! *Postulates & Theorems*

Midsegment Theorem for Trapezoids

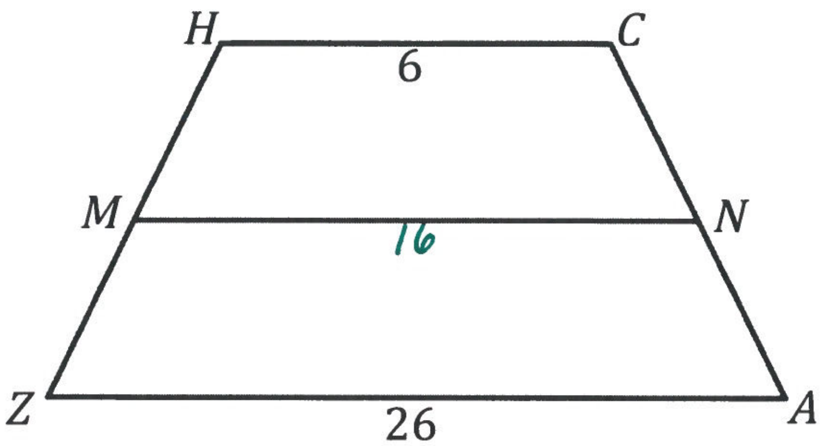
The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

Illustrate the theorems and characteristics of the mid-segment of a trapezoid in the figure below.



Let's Practice!

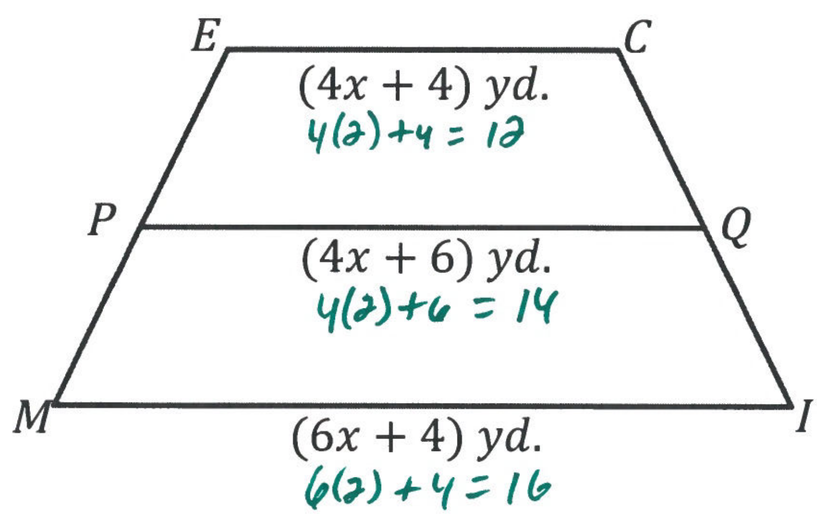
1. $ZACH$ is an isosceles trapezoid with midsegment $\overline{MN}$.



Determine the length of $\overline{MN}$.

$\overline{MN} = \frac{1}{2}(26+6) = \frac{1}{2}(32) = 16$

2. $MICE$ is an isosceles trapezoid with midsegment $\overline{PQ}$.



Determine the lengths of $\overline{MI}$, $\overline{PQ}$, and $\overline{EC}$.

$4x+6 = \frac{1}{2}(4x+4+6x+4) = \frac{1}{2}(10x+8) = 5x+4$

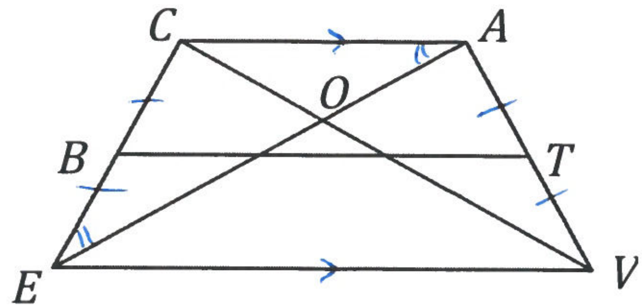
$4x+6 = 5x+4$

$x = 2$

$\boxed{\overline{MI} = 16 \quad \overline{EC} = 12 \quad \overline{PQ} = 14}$

Try It!

3. Complete the following proof.

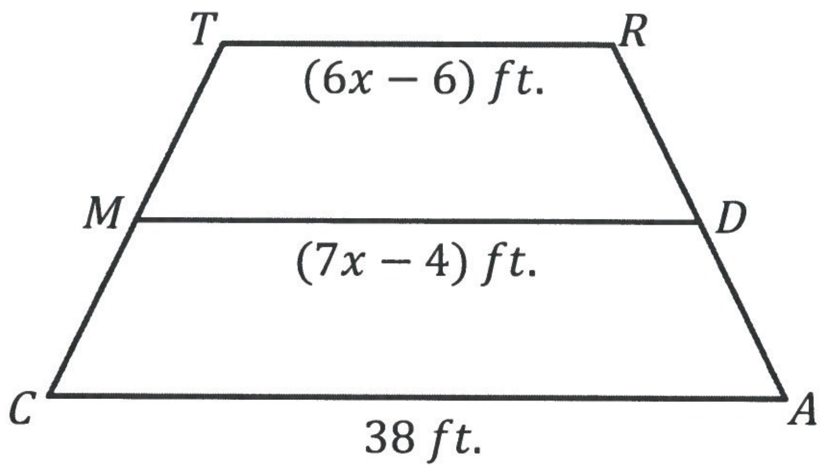


Given: $CAVE$ is a quadrilateral. $\Delta COE \cong \Delta AOV$, $\Delta CAO \sim \Delta EOV$, $\overline{AT} \cong \overline{BE}$ where T is the midpoint of $\overline{AV}$.

Prove: $CAVE$ is an isosceles trapezoid with midsegment $\overline{BT}$.

Statements	**Reasons**
1. $CAVE$ is a quadrilateral. $\Delta COE \cong \Delta AOV$, $\Delta CAO \sim \Delta EOV$, $\overline{AT} \cong \overline{BE}$ where T is the midpoint of $\overline{AV}$.	**1.** Given
2. $\overline{CE} \cong \overline{AV}$	**2.** CPCTC
3. $CB + BE = CE$; $AT + TV = AV$	**3.** Segment Addition Postulate
4. $CB + BE = AT + TV$	**4.** Substitution
5. $CB + BE = BE + TV$	**5.** Substitution
6. $CB = TV$	**6.** Subtraction Property of Equality
7. $\overline{CB} \cong \overline{TV}$	**7.** Definition of Congruence
8. B is the midpoint of $\overline{CE}$	**8.** Definition of Midpoint
9. $\overline{BE} \cong \overline{CB} \cong \overline{AT} \cong \overline{TV}$	**9.** Substitution
10. $\angle CAE \cong \angle AEV$	**10.** $\Delta CAO \sim \Delta EOV$
11. $\overline{CA} \parallel \overline{EV}$	**11.** Converse of Alternate Interior Angles Theorem
12. $CAVE$ is an isosceles trapezoid with midsegment $\overline{BT}$.	**12.** Definition of Isosceles Trapezoid and Midsegment

4. *CART* is an isosceles trapezoid with midsegment $\overline{MD}$.



Determine the length of $\overline{TR}$ and $\overline{MD}$.

$7x-4=\frac{1}{2}(6x-6+38)$

$7x-4=\frac{1}{2}(6x+32)$

$7x-4=3x+16$

$-3x+4 \quad -3x \quad +4$

$\frac{4x}{4}=\frac{20}{4}$

$x=5$

$\overline{TR}=6x-6$

$\overline{TR}=6(5)-6$

$\overline{TR}=30-6$

$\boxed{\overline{TR}=24}$

$\overline{MD}=7x-4$

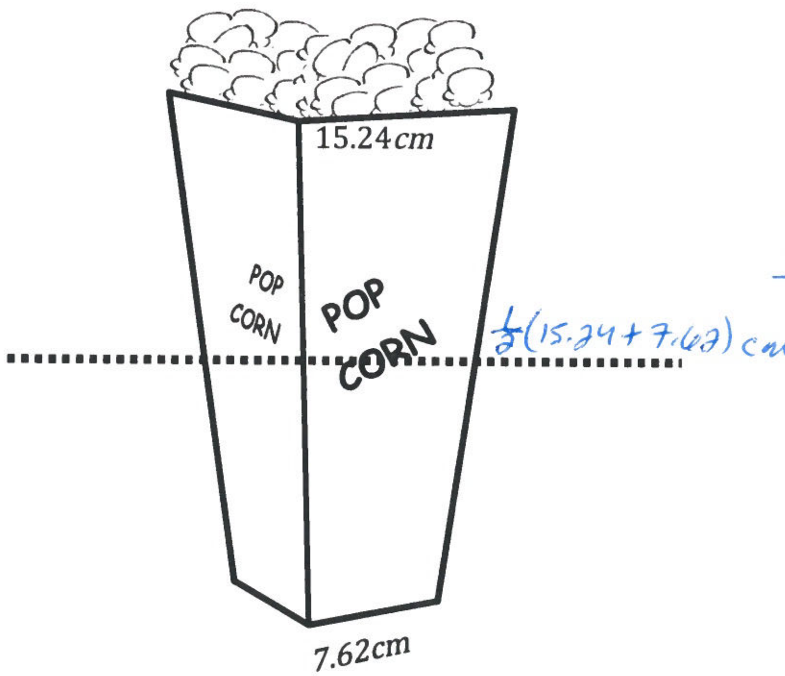
$\overline{MD}=7(5)-4$

$\overline{MD}=35-4$

$\boxed{\overline{MD}=31}$

BEAT THE TEST!

1. Julia is designing a popcorn box. She wants the end of the box to be a trapezoid with the dimensions shown. If she wants to cut the box through the middle to make the box smaller for her little sister, about how wide would the top base of the smaller box be?



$$\frac{15.24+7.62}{2}$$

$\boxed{11.43}$ centimeters.

Section 10 – Topic 5
Quadrilaterals in Coordinate Geometry - Part 1

Let's discuss writing proofs using Coordinate Geometry.

- Coordinate geometry involves placing geometric figures in a coordinate plane.
- Coordinate geometry proofs use several kinds of formulas.

Distance Formula	Slope Formula	Midpoint Formula
$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	$m=\dfrac{y_2-y_1}{x_2-x_1}$	$M_{x,y}=\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$

When developing a coordinate geometry proof, we should complete the following steps:

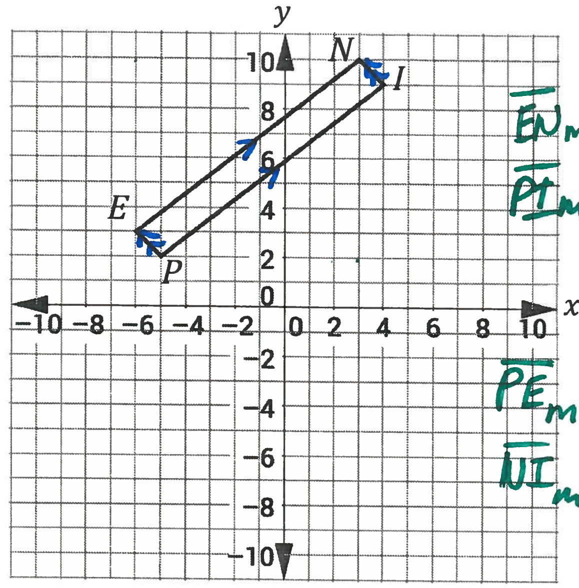
- Draw and label the graph (or identify a graph in a plane).
- Decide which formula(s) are needed to prove the type of quadrilateral.
- Develop a two-column, paragraph, or flow map proof.

Let's Practice!

1. Consider the information and figure below.

Given:
$PINE$ is a quadrilateral with vertices at $P(-5,2), I(4,9)$, $N(3,10)$, and $E(-6,3)$.

Prove:
$PINE$ is a parallelogram.



$\overline{EN}_m = \frac{7}{9}$

$\overline{PI}_m = \frac{7}{9}$

$\overline{PE}_m = -1$

$\overline{NI}_m = -1$

Write a paragraph proof based on the above information and diagram.

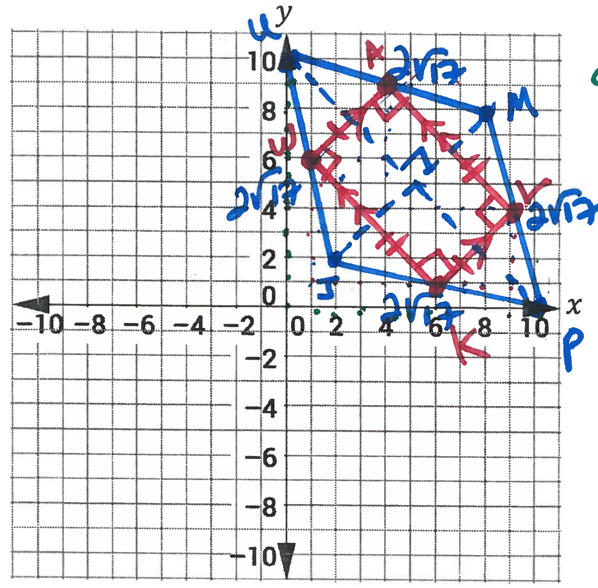
Given that PINE is a quadrilateral with vertices at P(-5,2), I(4,9), N(3,10), and E(-6,3), the slopes of opposite sides $\overline{EN}$ and $\overline{PI}$ must be equal, and the slopes of opposite sides $\overline{PE}$ and $\overline{NI}$ must also be equal. The slopes of $\overline{EN}$ and $\overline{PI}$ are $\frac{7}{9}$ for both. The slopes of $\overline{PE}$ and $\overline{NI}$ are -1 for both. Opposite sides of PINE are parallel. Using properties of parallelograms, we can conclude that PINE is a parallelogram.

2. Consider the information and figure below.

Given: $J(2,2), U(0,10), M(8,8), P(10,0)$.
Prove: $JUMP$ is a rhombus

$m_{UP} = -1$

$m_{JM} = 1$



$d_{JU} = \sqrt{4+64}$
$= \sqrt{68}$
$= \sqrt{4} \cdot \sqrt{17}$
$= 2\sqrt{17}$

Write a paragraph proof.

Given the coordinates of JUMP, $JU = UM = MP = PJ = 2\sqrt{17}$ units. Therefore, $\overline{JU} \cong \overline{UM} \cong \overline{MP} \cong \overline{PJ}$ by definition of congruence. $\overline{JM}$ and $\overline{UP}$ are diagonals of JUMP. The slope of $\overline{JM}$ (1) is the opposite reciprocal of the slope of $\overline{UP}$ (-1). Therefore, $\overline{JM} \perp \overline{UP}$. Besides, $\overline{JM}$ is shorter than $\overline{UP}$. Thus, JUMP is a rhombus by definition.

3. Consider the quadrilateral $JUMP$ from the previous exercise. The midpoint of $\overline{JU}$ is $(1, 6)$, called W. The midpoint of $\overline{UM}$ is $(4, 9)$, called A. The midpoint of $\overline{MP}$ is $(9, 4)$, called L. The midpoint of $\overline{PJ}$ is $(6, 1)$, called K.

Use the information above to prove that the line segments joining the midpoints of the consecutive sides of a rhombus form a rectangle. Complete the following two-column proof.

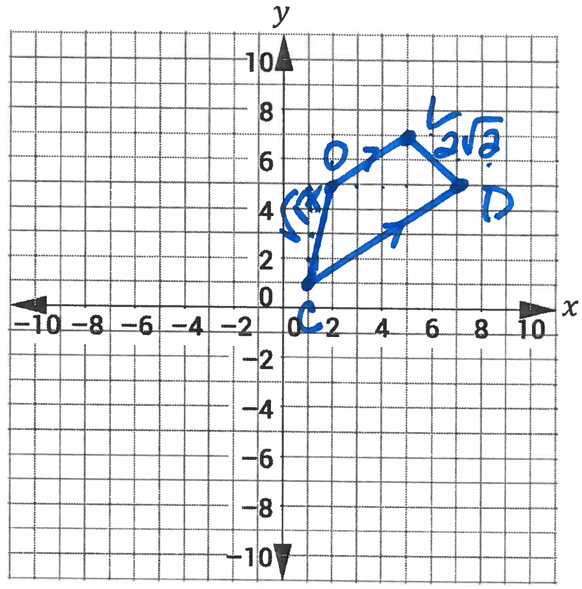
Statements	Reasons
1. $JUMP$ is a rhombus where the midpoint of $\overline{JU}$ is $(1, 6)$, the midpoint of $\overline{UM}$ is $(4, 9)$, the midpoint of $\overline{MP}$ is $(9, 4)$, and the midpoint of $\overline{PJ}$ is $(6, 1)$.	**1.** Given
2. $\overline{WA} \parallel \overline{LK}$ $\overline{AL} \parallel \overline{KW}$	**2.** Slope Formula, because opposite sides of a rectangle are parallel
3. $\overline{WA} \perp \overline{AL}$ $\overline{AL} \perp \overline{LK}$ $\overline{LK} \perp \overline{KW}$ $\overline{KW} \perp \overline{WA}$	**3.** Perpendicular lines have slopes that are opposite reciprocals.
4. $\overline{WA} \cong \overline{LK}$ $\overline{AL} \cong \overline{KW}$	**4.** Distance Formula, because opposite sides of a rectangle are congruent
5. WALK forms a rectangle.	**5.** Definition of rectangle

Try It!

4. Graph $COLD$ and complete the following paragraph proof.

Given: $COLD$ is a quadrilateral with vertices at $C(1,1), O(2,5), L(5,7)$, and $D(7,5)$.

Prove: $COLD$ is a trapezoid, but is *NOT* an isosceles trapezoid.



Given that $COLD$ is a quadrilateral with vertices at $C(1,1), O(2,5), L(5,7)$, and $D(7,5)$, the slope of $\overline{OL}$ is __2/3__ and the slope of $\overline{CD}$ is __2/3__, so __$\overline{OL} \parallel \overline{CD}$__ because their slopes are equal. However, the slopes of $\overline{CO}$ and $\overline{LD}$ are not __equal__, so these lines are not parallel. Therefore, $COLD$ is a trapezoid by definition.

$\overline{CO}$ is __$\sqrt{17}$__ units long and $\overline{LD}$ is __$2\sqrt{2}$__ units long, by the distance formula. Since, __$CO \neq LD$__ and __$\overline{CO} \ncong \overline{LD}$__, trapezoid $COLD$ is not isosceles. Their __legs__ are not congruent.

Section 10 – Topic 6
Quadrilaterals in Coordinate Geometry – Part 2

Let's Practice!

1. Consider quadrilateral $JKLM$ with the following coordinates: $J(1,-2), K(-1,-4), L(-3,-2), M(-1,10)$. What kind of quadrilateral is $JKLM$? Remember to justify your answer!

$JK = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$KL = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$LM = \sqrt{4+144} = \sqrt{148} = 2\sqrt{37}$

$MJ = \sqrt{4+144} = \sqrt{148} = 2\sqrt{37}$

Slope $\overline{JL} = 0$

Slope $\overline{KM}$ = undefined

$\overline{JL} \perp \overline{KM}$

JKLM is a kite : 2 pairs of consecutive sides congruent and diagonals perpendicular to each other.

2. Which of the following conclusions cannot always be drawn using coordinate geometry?

- Ⓐ Two sides are congruent. ✓
- Ⓑ A segment bisects another segment. ✓
- ● Two angles are congruent. ✗
- Ⓓ A quadrilateral is a trapezoid with a midsegment. ✓

3. Quinn graphs parallelogram $GRIT$ with the coordinates $G(10,8),\ R(10,20), I(18,20), T(18,8)$.

The diagonals meet at point $\boxed{(14,14)}$.

Midpoint $\overline{RT} = \left(\frac{18+10}{2}, \frac{8+20}{2}\right)$

$= (14, 14)$ ✓

Midpoint $\overline{GI} = \left(\frac{10+18}{2}, \frac{8+20}{2}\right)$

$= (14, 14)$ ✓

Try It!

4. Quadrilateral $ABCD$ has the following coordinates: $A(0,0), B(0,3), C(5,5), D(5,2)$. What kind of quadrilateral is $ABCD$? Prove your answer.

$AB = \sqrt{9} = 3$

$BC = \sqrt{4+25} = \sqrt{29}$

$CD = \sqrt{9} = 3$

$DA = \sqrt{25+4} = \sqrt{29}$

$\text{Slope}_{AB} = \frac{3}{0} = \text{undefined}$

$\text{Slope}_{BC} = \frac{2}{5}$

$\text{Slope}_{CD} = \frac{-3}{0} = \text{undefined}$

$\text{Slope}_{DA} = \frac{2}{5}$

Opposite sides are congruent and parallel. ∴ ABCD is a parallelogram.

5. Quadrilateral $GRIT$ has coordinates $G(10,8), R(10,20), I(18,20)$, and $T(18,8)$.

Part A: Circle the correct answer that completes the statement below.

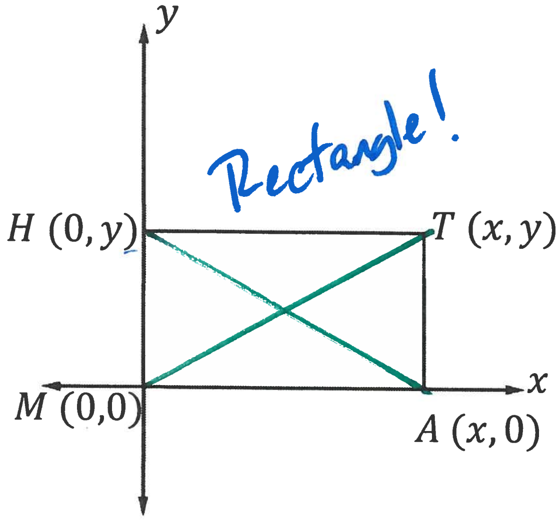
$GRIT$ is a rectangle | rhombus | isosceles trapezoid.

Part B: Which of the following statements is enough to justify your answer?

- (A) $GRIT$ has four right angles and two pairs of congruent sides. ✓
- (B) $GRIT$ has opposite angles that are congruent but not right angles.
- (C) $GRIT$ has diagonals that intersect at $90°$.
- (D) $GRIT$ has one pair of parallel opposite sides and one pair of non-parallel but congruent sides.

BEAT THE TEST!

1. Consider quadrilateral $MATH$ below.



We can prove that $MATH$ is a rectangle by calculating the length of each diagonal.

Write the algebraic expression for the length of each diagonal.

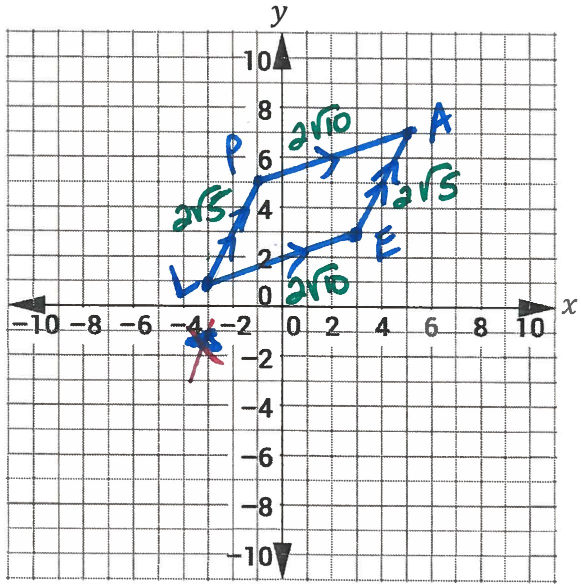
$MT = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(x-0)^2 + (y-0)^2}$

$MT = \sqrt{x^2+y^2}$ ✓

$HA = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(x-0)^2 + (0-y)^2}$

$HA = \sqrt{x^2 + (-y)^2} = \sqrt{x^2+y^2}$ ✓

2. Prove that quadrilateral $LEAP$ with vertices $L(-3,1)$, $E(3,3)$, $A(5,7)$, and $P(-1,5)$ is a parallelogram.



Slopes:

$PA = \frac{2}{6} = \frac{1}{3}$

$AE = \frac{4}{2} = 2$

$LE = \frac{2}{6} = \frac{1}{3}$

$LP = \frac{4}{2} = 2$

Which of the following statements help to prove that $LEAP$ is a parallelogram? Select all that apply.

- ☑ $LE = 2\sqrt{10}$, $EA = 2\sqrt{5}$, $AP = 2\sqrt{10}$, $LP = 2\sqrt{5}$, so $\overline{LE} \cong \overline{AP}$ and $\overline{EA} \cong \overline{LP}$. Opposite sides of a parallelogram are congruent.
- ☐ ~~$LE = EA = AP = LP = 2\sqrt{10}$, so $\overline{LE} \cong \overline{EA} \cong \overline{AP} \cong \overline{LP}$. All sides are congruent, depicting a square, which is a type of parallelogram.~~
- ☑ The slope of $\overline{LE}$ and $\overline{AP}$ is $\frac{1}{3}$. The slope of $\overline{EA}$ and $\overline{LP}$ is 2. Since $\overline{LE} \parallel \overline{AP}$ and $\overline{EA} \parallel \overline{LP}$, opposite sides of a parallelogram are parallel.
- ☐ ~~The slope of $\overline{LE}$ and $\overline{AP}$ is $\frac{1}{3}$. Since $\overline{LE} \parallel \overline{AP}$, parallelograms have one pair of parallel sides.~~
- ☐ ~~The slope of $\overline{LE}$ and $\overline{AP}$ is $\frac{1}{3}$, while the slope of $\overline{EA}$ and $\overline{LP}$ is -3. These slopes are opposite reciprocals of each other, so $LEAP$ is a rectangle, which is a type of parallelogram.~~

Section 10 – Topic 7
Constructions of Quadrilaterals

How can we construct quadrilaterals?

- Different quadrilaterals can be identified based on the properties of __angles__, __sides__, and __diagonals__.

If we know the properties of the quadrilateral that we want to construct, then we can apply the skills we learned in previous sections to construct line segments, angles and triangles.

What questions should we ask ourselves before starting the construction of a square?

- What are the properties of a square?
  - All sides congruent
  - 2 pair of opposite sides parallel
  - diagonals are congruent, perpendicular and bisect each other
  - Angles are 90° in the interior.
- What basic construction skills do we need to make a square with the correct properties?
  - Perpendicular lines
  - Copy segment and parallel

Let's Practice!

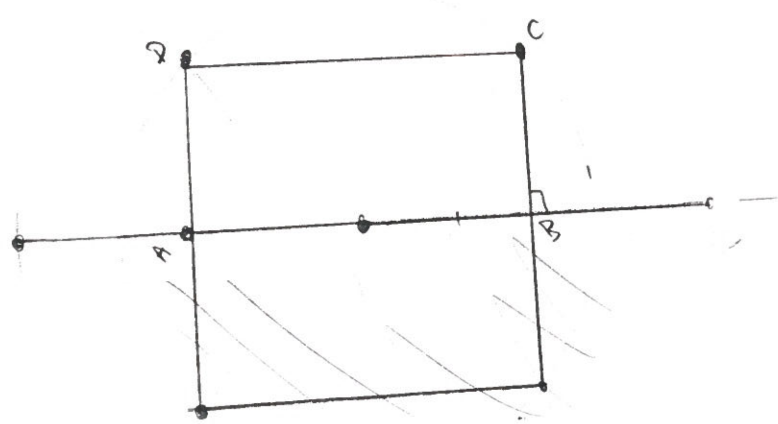
1. Construct rectangle $ABCD$.

   Step 1. Draw line segment $\overline{AB}$, which will be the base of the rectangle.

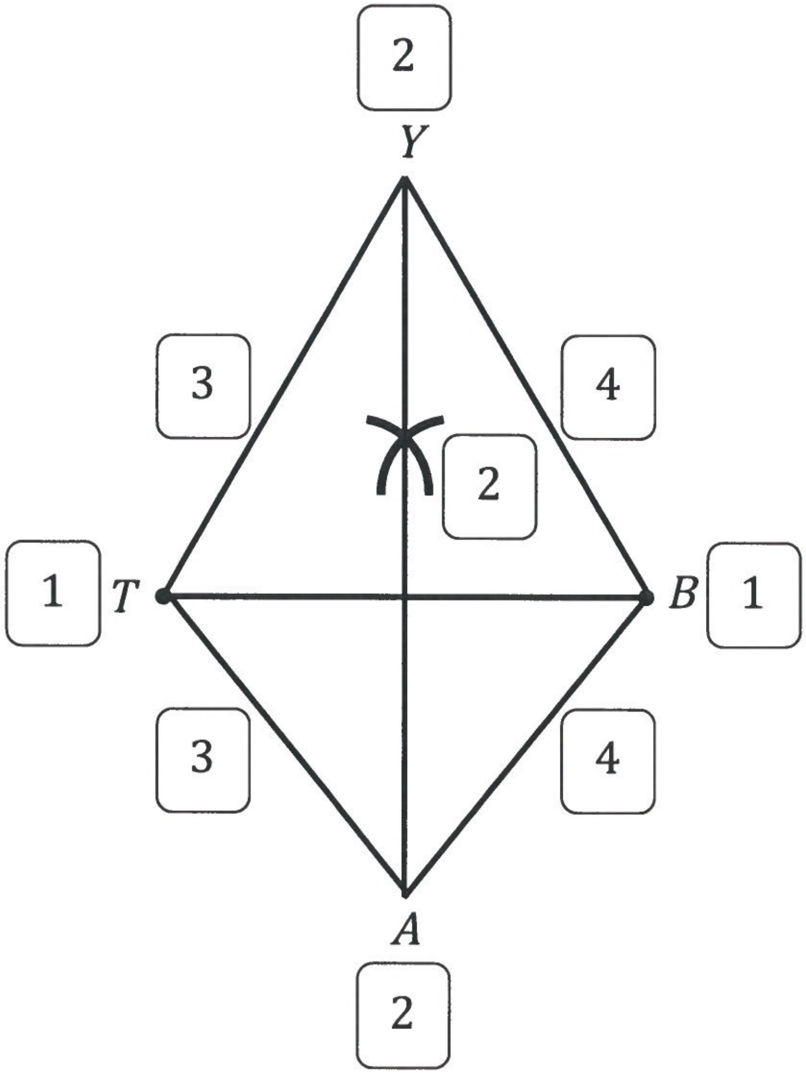
   Step 2. Construct two congruent perpendicular line segments to $\overline{AB}$, one passing through point A and one passing through point B. Label the line segments $\overline{AD}$ and $\overline{BC}$.

   Step 3. Draw line segment $\overline{CD}$ parallel to $\overline{AB}$.

   Step 4. Check your work by placing the compass on any vertex. Open the compass to the opposite vertex. The opening of the compass must match the measurement of the two other opposite vertices.



2. List the steps in the construction of kite $TABY$ shown below.



1. Draw segment $\overline{AB}$.

2. Create $\overline{YA}$ perpendicular bisector of $\overline{TB}$.

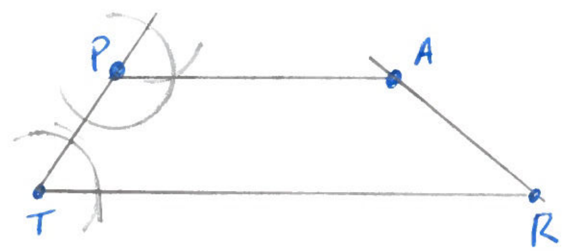
3. Create $\overline{TY}$ and $\overline{TA}$

4. Create and finish kite with $\overline{BA}$ and $\overline{BY}$.

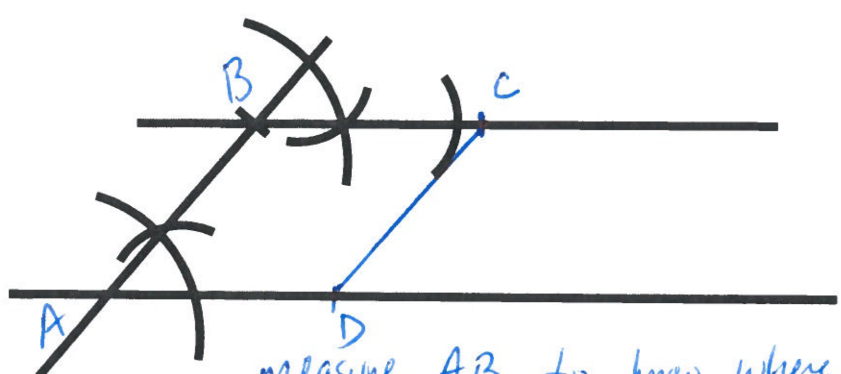
Try It!

3. Construct trapezoid $TRAP$.

   Step 1. Draw line segment $\overline{TR}$.
   Step 2. Construct a segment from point T using any angle. Name the segment $\overline{PT}$.
   Step 3. Construct a line parallel to $\overline{TR}$ and passing through P. Label the line segment $\overline{PA}$.
   Step 4. Draw line segment $\overline{RA}$.



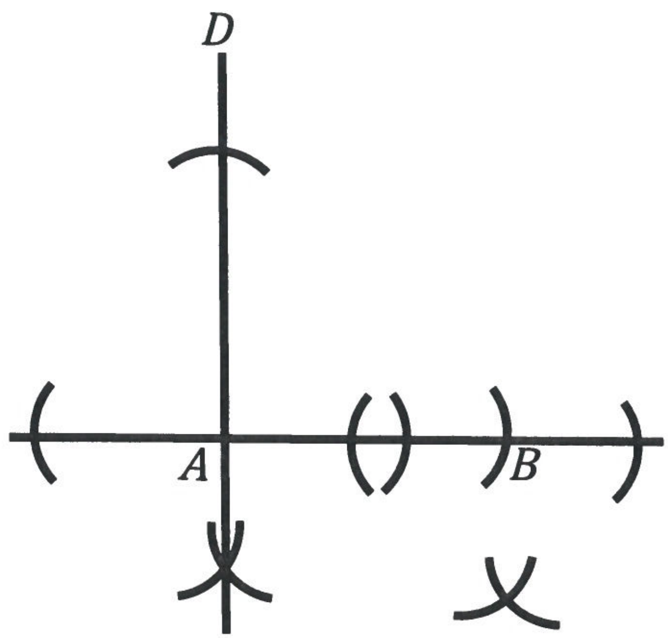
4. Abby is constructing the rhombus below. What would be the next step in her construction?



measure AB to know where to put point C and D.

BEAT THE TEST!

1. Consider the quadrilateral "under construction" below.



Part A: Which statement represents the next logical step to complete the construction.

Ⓐ The next step is drawing a line from B to D. This will complete the construction.

Ⓑ The next step is to swing the compass around and mark off the same length as AB on the opposite angle of A.

Ⓒ The next step is to draw a line parallel to AB passing through D.

Ⓓ The next step is to draw a line through the intersection under B that connects it with B.

Part B: Use the table to categorize which of the following quadrilaterals can be the outcome of the construction described above. Write each of the answer choices below in the appropriate column of the following table.

- *Concave quadrilateral*
- *Kite*
- *Isosceles Trapezoid*
- *Parallelogram*
- *Rectangle*
- *Rhombus*
- *Square*
- *Trapezoid*

Can be the outcome	Cannot be the outcome
Parallelogram rectangle rhombus square trapezoid	Concave quadrilateral kite Isosceles Trapezoid