

8-6

Skills Practice

Trapezoids

COORDINATE GEOMETRY $ABCD$ is a quadrilateral with vertices $A(-4, -3)$, $B(3, -3)$, $C(6, 4)$, $D(-7, 4)$.

1. Verify that $ABCD$ is a trapezoid.

$$m_{AB} = \frac{-3-3}{3-(-4)} = \frac{-6}{7} = -\frac{6}{7}$$

$$m_{BC} = \frac{4-(-3)}{6-3} = \frac{7}{3}$$

$$m_{CD} = \frac{4-4}{-7-6} = \frac{0}{-13} = 0$$

$$m_{AD} = \frac{-3-4}{-4-(-7)} = \frac{-7}{3}$$

$AB \parallel CD$, $BC \nparallel AD$
Yes

2. Determine whether $ABCD$ is an isosceles trapezoid. Explain.

$$BC = \sqrt{(3-6)^2 + (-3-4)^2} = \sqrt{9+49} = \sqrt{58}$$

$$AD = \sqrt{(-4-(-7))^2 + (-3-4)^2} = \sqrt{9+49} = \sqrt{58}$$

Yes, legs $\cong$

COORDINATE GEOMETRY $EFGH$ is a quadrilateral with vertices $E(1, 3)$, $F(5, 0)$, $G(8, -5)$, $H(-4, 4)$.

3. Verify that $EFGH$ is a trapezoid.

$$m_{EF} = \frac{0-3}{5-1} = -\frac{3}{4}$$

$$m_{FG} = \frac{-5-0}{8-5} = -\frac{5}{3}$$

$$m_{GH} = \frac{4-(-5)}{-4-8} = \frac{9}{-12} = -\frac{3}{4}$$

$$m_{EH} = \frac{4-3}{-4-1} = \frac{1}{-5} = -\frac{1}{5}$$

$EF \parallel GH$, $FG \nparallel EH$
Yes

4. Determine whether $EFGH$ is an isosceles trapezoid. Explain.

$$FG = \sqrt{(5-8)^2 + (0-(-5))^2} = \sqrt{9+25} = \sqrt{34}$$

$$EH = \sqrt{(-4-1)^2 + (4-3)^2} = \sqrt{25+1} = \sqrt{26}$$

No, legs $\neq$

COORDINATE GEOMETRY $LMNP$ is a quadrilateral with vertices $L(-1, 3)$, $M(-4, 1)$, $N(-6, 3)$, $P(0, 7)$.

$$m_{LM} = \frac{1-3}{-4-(-1)} = \frac{-2}{-3} = \frac{2}{3}$$

$$m_{NP} = \frac{7-3}{0-(-6)} = \frac{4}{6} = \frac{2}{3}$$

$LM \parallel NP$,
 $MN \nparallel LP$ Yes

5. Verify that $LMNP$ is a trapezoid. $m_{MN} = \frac{3-1}{-6-(-4)} = \frac{2}{-2} = -1$ $m_{LP} = \frac{7-3}{0-(-1)} = \frac{4}{-1} = -4$

6. Determine whether $LMNP$ is an isosceles trapezoid. Explain.

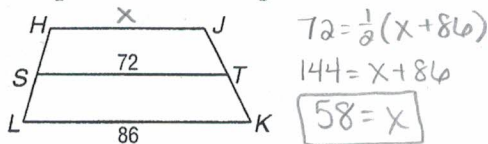
$$MN = \sqrt{(-6-(-4))^2 + (3-1)^2} = \sqrt{4+4} = \sqrt{8}$$

$$LP = \sqrt{(-1-0)^2 + (3-7)^2} = \sqrt{1+16} = \sqrt{17}$$

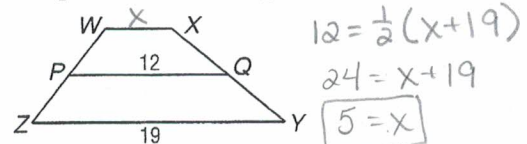
No, legs $\neq$

ALGEBRA Find the missing measure(s) for the given trapezoid.

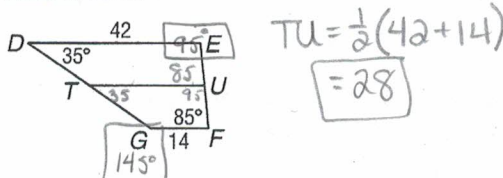
7. For trapezoid $HJKL$, S and T are midpoints of the legs. Find HJ .



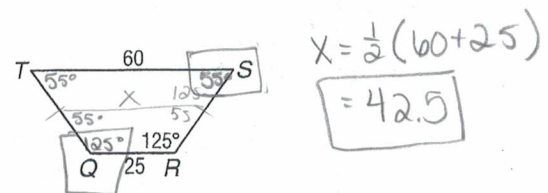
8. For trapezoid $WXYZ$, P and Q are midpoints of the legs. Find WX .



9. For trapezoid $DEFG$, T and U are midpoints of the legs. Find TU , $m\angle E$, and $m\angle G$.



10. For isosceles trapezoid $QRST$, find the length of the median, $m\angle Q$, and $m\angle S$.



8-6 Practice

Trapezoids

COORDINATE GEOMETRY $RSTU$ is a quadrilateral with vertices $R(-3, -3)$, $S(5, 1)$, $T(10, -2)$, $U(-4, -9)$.

1. Verify that $RSTU$ is a trapezoid.

$$mRS = \frac{-3-1}{-3-5} = \frac{-4}{-8} = \frac{1}{2} \quad mST = \frac{1+2}{5-10} = \frac{3}{-5} = -\frac{3}{5} \quad mTU = \frac{-2+9}{10+4} = \frac{7}{14} = \frac{1}{2} \quad mRU = \frac{-3+9}{-3+4} = \frac{6}{1} = 6$$

$RS \parallel TU$, $ST \nparallel RU$
Yes

2. Determine whether $RSTU$ is an isosceles trapezoid. Explain.

$$ST = \sqrt{(5-10)^2 + (1+2)^2} = \sqrt{25+9} = \sqrt{34} \quad RU = \sqrt{(-4+10)^2 + (-9+3)^2} = \sqrt{36+36} = \sqrt{72}$$

No, legs $\neq$

COORDINATE GEOMETRY $BGHJ$ is a quadrilateral with vertices $B(-9, 1)$, $G(2, 3)$, $H(12, -2)$, $J(-10, -6)$.

3. Verify that $BGHJ$ is a trapezoid.

$$mBG = \frac{1-3}{-9-2} = \frac{-2}{-11} = \frac{2}{11} \quad mGH = \frac{3+2}{2-12} = \frac{5}{-10} = -\frac{1}{2} \quad mHJ = \frac{-2+6}{12+10} = \frac{4}{22} = \frac{2}{11} \quad mBJ = \frac{1+6}{-9+10} = \frac{7}{1} = 7$$

$BG \parallel HJ$, $GH \nparallel BJ$
Yes

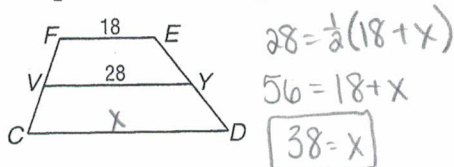
4. Determine whether $BGHJ$ is an isosceles trapezoid. Explain.

$$GH = \sqrt{(2-12)^2 + (3+2)^2} = \sqrt{100+25} = \sqrt{125} \quad BJ = \sqrt{(-9+10)^2 + (1+6)^2} = \sqrt{1+49} = \sqrt{50}$$

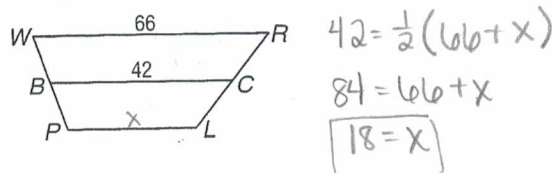
No, legs $\neq$

ALGEBRA Find the missing measure(s) for the given trapezoid.

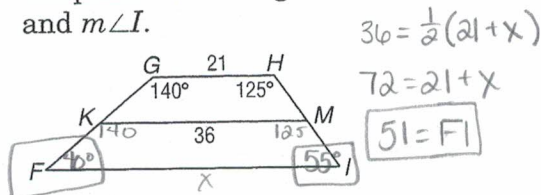
5. For trapezoid $CDEF$, V and Y are midpoints of the legs. Find CD .



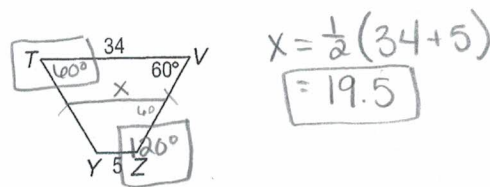
6. For trapezoid $WRLP$, B and C are midpoints of the legs. Find LP .



7. For trapezoid $FGHI$, K and M are midpoints of the legs. Find FI , $m\angle F$, and $m\angle I$.



8. For isosceles trapezoid $TVZY$, find the length of the median, $m\angle T$, and $m\angle Z$.



9. **CONSTRUCTION** A set of stairs leading to the entrance of a building is designed in the shape of an isosceles trapezoid with the longer base at the bottom of the stairs and the shorter base at the top. If the bottom of the stairs is 21 feet wide and the top is 14 feet wide, find the width of the stairs halfway to the top. $\frac{1}{2}(21+14) = 17.5 \text{ ft}$

10. **DESK TOPS** A carpenter needs to replace several trapezoid-shaped desktops in a classroom. The carpenter knows the lengths of both bases of the desktop. What other measurements, if any, does the carpenter need?

Base angle measures

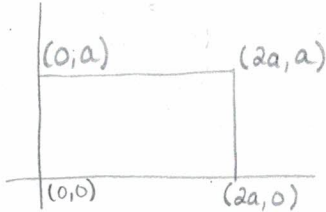
8-7

Skills Practice

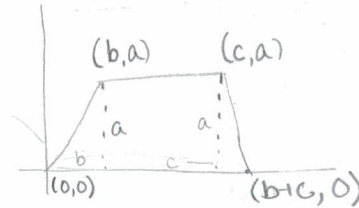
Coordinate Proof with Quadrilaterals

Position and label each quadrilateral on the coordinate plane.

1. rectangle with length $2a$ units and height a units

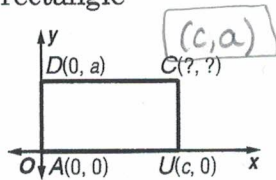


2. isosceles trapezoid with height a units, bases $c - b$ units and $b + c$ units

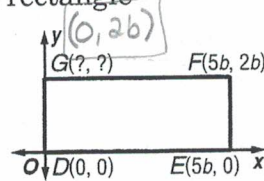


Name the missing coordinates for each quadrilateral.

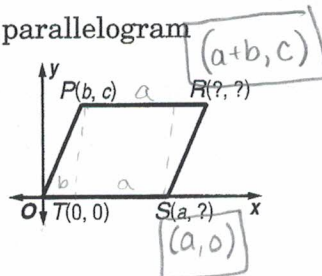
3. rectangle



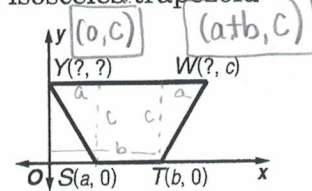
4. rectangle



5. parallelogram

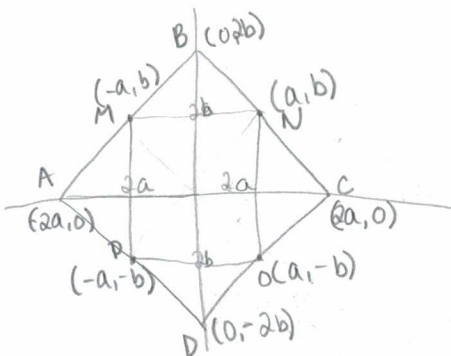


6. isosceles trapezoid



Position and label the figure on the coordinate plane. Then write a coordinate proof for the following.

7. The segments joining the midpoints of the sides of a rhombus form a rectangle.



$$MO = \frac{(-a,b) - (a,-b)}{\sqrt{4a^2 + 4b^2}} = \frac{-2a, 2b}{\sqrt{4a^2 + 4b^2}}$$

$$NP = \frac{(a,b) - (-a,-b)}{\sqrt{4a^2 + 4b^2}} = \frac{2a, 2b}{\sqrt{4a^2 + 4b^2}}$$

diagonals are $\cong$, so MNOP is a rectangle.

$$m\angle MN = \frac{b-b}{-a-a} = \frac{0}{-2a} = 0$$

$$m\angle MP = \frac{b+b}{-a-a} = \frac{2b}{0} = \text{und.}$$

cons. sides $\perp$,

so MNOP is a rectangle.

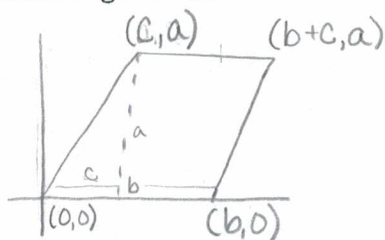
8-7

Practice

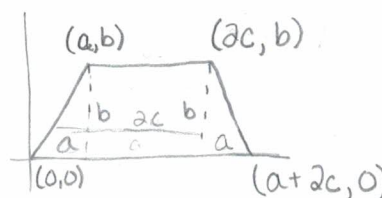
Coordinate Proof with Quadrilaterals

Position and label each quadrilateral on the coordinate plane.

1. parallelogram with side length
- b
- units and height
- a
- units

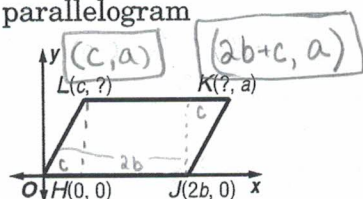


2. isosceles trapezoid with height
- b
- units, bases
- $2c - a$
- units and
- $2c + a$
- units

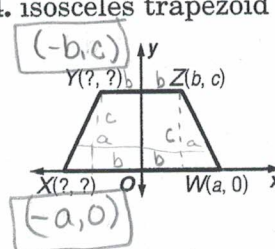


Name the missing coordinates for each quadrilateral.

3. parallelogram

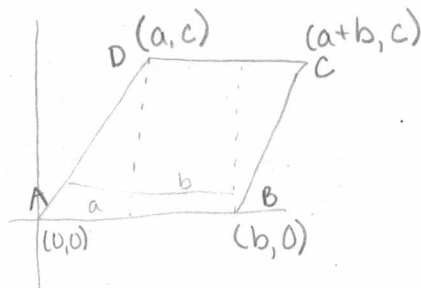


4. isosceles trapezoid



Position and label the figure on the coordinate plane. Then write a coordinate proof for the following.

5. The opposite sides of a parallelogram are congruent.



$$AD = \frac{(0,0) - (a,c)}{\sqrt{a^2 + c^2}} = \frac{-a, -c}{\sqrt{a^2 + c^2}}$$

$$BC = \frac{(b,0) - (a+b,c)}{\sqrt{a^2 + c^2}} = \frac{-a, -c}{\sqrt{a^2 + c^2}}$$

$$\overline{AD} \cong \overline{BC}$$

$$AB = \frac{(0,0) - (b,0)}{\sqrt{b^2}} = \frac{-b, 0}{b}$$

$$DC = \frac{(a,c) - (a+b,c)}{\sqrt{b^2}} = \frac{-b, 0}{b}$$

$$\overline{AB} \cong \overline{DC}$$

6. THEATER A stage is in the shape of a trapezoid. Write a coordinate proof to prove that
- $\overline{TR}$
- and
- $\overline{SF}$
- are parallel.

$$m\overline{TR} = \frac{0-0}{10-20} = \frac{0}{-10} = 0$$

$$m\overline{SF} = \frac{25-25}{0-30} = \frac{0}{-30} = 0$$

$$\overline{TR} \parallel \overline{SF}$$

